Documentos de trabajo Número II – 2025

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El Colegio de México

febrero de 2025

http://cee.colmex.mx/documentos-de-trabajo

Bounded Rationality and Macroeconomic (In)Stability

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February 2025

Abstract

We analyze how bounded rationality affects the equilibrium determinacy properties of forecast-based interest-rate rules in a behavioural New Keynesian model with limited asset market participation (LAMP). We show that the key policy prescriptions of rational expectation models do not carry over to behavioural frameworks with myopic agents. In high participation economies, the Taylor principle is more likely to induce indeterminacy when bounded rationality is introduced following the cognitive discounting approach of Gabaix (2020). Indeterminacy arises from a discounting channel and the problem is exacerbated under flexible prices and nominal illusion. In contrast, cognitive discounting plays a stabilizing role in LAMP economies, where passive policy is no longer required to prevent indeterminacy, and determinacy can potentially be restored under the Taylor principle. We investigate how our results depend on the timing of the interest-rate rule, alternative forms of bounded rationality, and the presence of a cost-channel of monetary policy.

JEL Classification: E31; E32; E44; E52; E71

Keywords: Bounded rationality; Cognitive discounting; Equilibrium determinacy; Limited asset markets participation; Taylor principle; Monetary policy.

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1 Introduction

Modern general equilibrium models that analyze monetary policy are based on the assumption that agents form expectations rationally about future economic outcomes. However, recent empirical evidence suggests that real-world decision-making is subject to cognitive limitations and inattention, which is not consistent with fully rational behavior (e.g., Coibion and Gorodnichenko, 2015; Coibion, et al., 2018; Kohlhas and Walther, 2021). This raises important questions for Inflation Forecast Targeting (IFT) central banks in terms of how to conduct monetary policy in the presence of myopic agents. Of particular importance concerns the suitability of the traditional Taylor principle - where the nominal interest rate should respond more than inflation - to induce macroeconomic stability by preventing the emergence of indeterminacy and sunspot equilibria. While there is a large literature analyzing the stability properties of interest-rate policies under rational expectations,[1] to date there is little work exploring the implications of bounded rationality for equilibrium determinacy. The goal of this paper is to fill this gap.

The model economy we consider is a tractable Two-Agent New Keynesian (TANK) model featuring both financially constrained and unconstrained households. The former are non-savers and live hand-to-mouth by consuming all their current wage income, whereas the latter are asset-holders who can borrow and lend from financial markets. We follow Bilbiie (2008) and include Limited Asset Market Participation (LAMP) since the empirical evidence suggests it is an important feature of developed and emerging market economies.[2] Differently from Bilbiie (2008), we assume that agents exhibit bounded rationality. There are many different ways bounded rationality can be introduced into New Keynesian-type models.[3] We initially adopt the popular cognitive discounting approach of Gabaix (2020) by assuming that agents discount future deviations from the deterministic steady state.[4] This captures the idea that behavioural agents are less attentive than rational agents to events that will take place in the distant future. However, we test the robustness of our results by introducing myopia following Angeletos and Lian (2018), where the

[1] See, e.g., Bullard and Mitra (2002), Woodford (2003), Llosa and Tuesta (2009), Kurozumi and Van Zandweghe (2010), Airaudo and Zanna (2012), Buffie (2013), McKnight and Mihailov (2015), Levine et al. (2025).

[2] Aguiar et al. (2020) estimate that approximately 40% of US households are non-savers. The share of non-saving households is estimated to be potentially much higher in smaller developed countries (e.g., Canada and Japan) and emerging market economies. For further details, see Buffie (2013), Buffie and Zanna (2017) and Levine et al. (2025).

[3] E.g., cognitive discounting is introduced in Angeletos and Lian (2018) by allowing for imperfect common knowledge, whereas in Gabaix (2020) it arises from macroeconomic inattention. Farhi and Werning (2019) and García-Schmidt and Woodford (2019) introduce myopia via *level-k reasoning*, whereas Woodford (2019) and Woodford and Xie (2022) impose finite-horizon planning.

[4] Due to its tractability, the Gabaix (2020) approach is commonly used to introduce bounded rationality in the optimal monetary policy literature. See, e.g., Benchimol and Bounader (2023), Wagner et al. (2023), Bonciani and Oh (2025).

additional discounting of behavioural agents arises due to deviations from common knowledge. Consistent with the empirical evidence, the central bank adjusts the policy rate in response to changes in expected future inflation.[5]

The structural equations of our TANK model can be reduced to a two-dimensional dynamic system comprised of behavioural versions of the New Keynesian Phillips Curve (NKPC), representing the supply side of the economy, and an intertemporal Euler (or IS) equation which characterizes the demand side. If the degree of LAMP exceeds a threshold level, the slope of the IS equation changes sign, and monetary policy is transmitted according to an 'Inverted Aggregate Demand Logic' (IADL), where aggregate demand responds *positively* to increases in the real interest rate.[6] As shown by Bilbiie (2008), in this case an *inverted* Taylor principle is needed to induce determinacy of the Rational Expectations Equilibrium (REE). We show that bounded rationality introduces discounting into both the IS equation and the NKPC, as both aggregate demand and inflation become less forward-looking. Myopic firms affect the slope of the long-run NKPC, where discounting increases the long-run inflation elasticity of output. Myopic households affect how future changes in the real interest rate are discounted, reducing their effect on current output from the IS equation. The central message of our paper is that this *discounting channel* significantly alters the conditions for equilibrium determinacy, and key monetary policy prescriptions obtained under rational expectations do not carry over to frameworks with myopic agents.

Our main findings are summarized as follows. For economies with high asset market participation rates, the traditional Taylor principle, where the central bank adopts an 'active' policy stance, is no longer a necessary condition for determinacy under bounded rationality. By modifying the Taylor principle, cognitive discounting exerts a stabilizing effect on the economy, such that the long-run nominal interest rate no longer has to rise by proportionally more than permanent increases in inflation. Consequently, under the *behavioural Taylor principle*, a 'passive' policy can also be consistent with equilibrium determinacy. The stabilizing benefits are shown to larger as the degree of agent inattention and asset market participation rate increase, and as prices become more sticky in the economy. However, the scope for active policy is restricted by an upper bound on the forward-looking Taylor coefficient, which is reduced as the degree of agent myopia increases. The indeterminacy problem of active policy is found to be particularly serious as prices in the economy

[5] For further discussion, see McKnight and Mihailov (2015) and the references therein.

[6] Intuitively, the IADL arises in LAMP economies when the increase in firm profits from higher real interest rates dominates the lower real wages induced by the intertemporal effects of delayed consumption, resulting in higher aggregate demand as income is redistributed towards unconstrained households.

become more flexible, and under nominal illusion, where the real interest rate is also incorrectly perceived by households.[7]

In LAMP economies, the inversion of the Taylor principle is no longer a necessary condition for equilibrium determinacy when agents are not fully rational. We show that the discounting channel exerts a strong stabilizing effect in the presence of the IADL, such that determinacy can now also be induced under an active policy. Indeed, the determinate policy space under the traditional Taylor principle is potentially large with high degrees of LAMP and sticky prices, typical features of many less developed countries (e.g., Buffie and Zanna, 2017).

We examine the robustness of our findings for a variety of popular IFT feedback rules that also include a policy response to output. For both forward-looking and hybrid Taylor-type rules, the ability of the Taylor principle to induce determinacy in high-participation economies is once again reduced relative to the REE benchmark, whereas myopia significantly expands the determinacy regions under active and passive policy in IADL economies. We also investigate the determinacy implications of a contemporaneous Taylor rule which responds to *current* inflation. In contrast to forecast-based rules, due to the absence of an upper bound on the Taylor coefficient, cognitive discounting now exerts a stabilizing influence in both LAMP and non-LAMP economies.

Finally, we consider whether our results generalize by extending the baseline TANK model in two important directions. First, we consider an alternative model of bounded rationality, where cognitive discounting emerges due to deviations from common knowledge. In stark contrast to the macroeconomic inattention model, the upper bound on the forward-looking Taylor coefficient shifts outwards under imperfect common knowledge, *increasing* the region of determinacy under the Taylor principle for high-participation economies. However, the stabilizing benefits of cognitive discounting for IADL economies is now significantly reduced. Second, we introduce a supply-side effect of monetary policy (the so-called 'cost channel') by allowing nominal interest rate changes to directly affect the real marginal cost of firms. Regardless of the degree of asset market participation, cognitive discounting is shown to play a key stabilizing role in ameliorating the indeterminacy problem that emerges under REE from the cost channel.

This paper contributes to an important literature that considers the local stability of forecast-based interest rate policy. Using the standard (full-participation) NK model, Bullard and Mitra (2002) show that the Taylor principle is constrained under IFT rules, as indeterminacy of REE

[7] Nominal illusion arises when (unconstrained) households only have access to a nominal savings market. See Gabaix (2020), for further discussion.

can arise if central banks react too aggressively to expected future inflation. The presence of a cost-channel of monetary policy arising from working capital loans (e.g., Llosa and Tuesta, 2009), labor market search and matching frictions (e.g., Kurozumi and Van Zandweghe, 2010), or real balance effects (e.g., McKnight and Mihailov, 2015), increases the severity of the indeterminacy problem. We show that bounded rationality can worsen the indeterminacy problem of forecast-based monetary policy in economies with high asset market participation rates, whereas it helps mitigate the destabilizing effects that arise under the cost channel. For LAMP economies, Bilbiie (2008) finds that determinacy becomes impossible under the Taylor principle and IFT central banks are required to adopt the inverted Taylor principle. Buffie (2013), however, shows that the traditional Taylor principle can be restored when real wages adjust gradually to clear the labor market. We find that bounded rationality exerts a large stabilizing effect on LAMP economies, such that determinacy can also be restored under the Taylor principle without resorting to real wage rigidities.

This paper also closely relates to a recent literature that explores the implications of bounded rationality for the design of monetary policy. It has been established by Angeletos and Lian (2018), Gabaix (2020), and Woodford and Xie (2022), among others, that introducing bounded rationality into NK models can help resolve the 'forward guidance puzzle' of Del Negro et al. (2023). Focusing on contemporaneous interest-rate rules, Gabaix (2020) also shows that cognitive discounting exerts a stabilizing effect in a full-participation model by reducing the likelihood of indeterminacy. In contrast, we find that determinacy can actually be undermined when agents are not fully rational, under empirically-appealing forecast-based rules that are of most interest to policymakers. Moreover, we further show that different models of bounded rationality, by altering the behavioural NKPC and IS equations, can generate contrasting policy prescriptions for avoiding indeterminacy.

Finally, this paper is also related to a literature that investigates the conditions under which forecast-based policy can induce learning (or E-stablility) of the REE. Bullard and Mitra (2002) show that the Taylor principle yields a unique E-stable REE, whereas both Llosa and Tuesta (2009) and Kurozumi and Van Zandweghe (2012) find that an indeterminate E-unstable REE can arise in the presence of a cost channel. However, in our framework behavioural agents do not learn, and consequently, convergence to the REE is never possible under cognitive discounting.

The rest of the paper is organized as follows. Section 2 outlines the behavioural model under LAMP and the main results from the determinacy analysis are given in Section 3. Section 4 discusses the determinacy implications of cognitive discounting under different timing specifications for the interest-rate rule. Section 5 examines the robustness of our results to changes in the modelling environment by considering an alternative model of bounded rationality, the role of nominal illusion, and the inclusion of a cost-channel of monetary policy. Finally, Section 6 concludes. Proofs and additional results are given in an online appendix.

2 The LAMP model under bounded rationality

We introduce cognitive discounting, or myopia, into the influential LAMP model of Bilbiie (2008). Following Gabaix (2020), behavioural agents are not able to perfectly measure expected events in the future: the further into the future an event is, the less accurate agents are in measuring the event. In what follows, let E_t denote the rational expectation operator and E_t^{BR} denote the behavioural expectation operator. With behavioural agents, the state vector $\boldsymbol{X}_t$ with mean $\boldsymbol{X}^*$ is assumed to evolve according to the following log-linearized law of motion:

$$\boldsymbol{X}_{t+1} = (1-m)\boldsymbol{X}^* + m\left(\mathit{\Gamma}\boldsymbol{X}_t + \epsilon_{t+1}\right), \tag{1}$$

for some matrix $\mathit{\Gamma}$ and innovations ϵ_{t+1}. The parameter $m \in [0,1]$ is the cognitive discounting parameter. Under rational expectations, $m = 1$, and the rational law of motion collapses to $\boldsymbol{X}_{t+1} = \mathit{\Gamma}\boldsymbol{X}_t + \epsilon_{t+1}$. It follows from Lemma 1 of Gabaix (2020) that for any variable $Z(\boldsymbol{X}_t)$, the beliefs of the behavioral agent for $k \geq 0$ satisfy:

$$\mathrm{E}_t^{BR}[Z(\boldsymbol{X}_{t+k}) - \boldsymbol{X}^*] = m^k \mathrm{E}_t[Z(\boldsymbol{X}_{t+k}) - \boldsymbol{X}^*], \tag{2}$$

where E_t^{BR} uses the misperceived law of motion (1) and E_t uses the rational law of motion. Letting a variable with a hat $\widehat{Z}_t$ denote the log-deviation from its steady state Z^{ss} $\left(\text{i.e., } \widehat{Z}_t = \ln Z_t - \ln Z^{ss}\right)$, it follows from (2) that:

$$\mathrm{E}_t^{BR}\widehat{Z}_{t+k} = m^k \mathrm{E}_t \widehat{Z}_{t+k}. \tag{3}$$

Behavioural agents discount deviations from the steady state by the cognitive discount factor m^k. For events that are expected to occur further in the future at horizon k, this results in smaller deviations from the steady state than under rational expectations.

2.1 Households

There are two types of households. There exists a proportion of *constrained* households $\lambda \in (0,1)$, denoted by the subscript H, and a proportion of *unconstrained* behavioral households $1-\lambda$, denoted by the subscript S. Constrained households hold no assets and have no access to financial markets. Consequently, they consume $C_{H,t}$ only from income obtained from supplying labor $N_{H,t}$:

$$C_{H,t} = w_t N_{H,t}, \tag{4}$$

where w_t denotes the real wage. In contrast, unconstrained households can purchase one-period nominal risk-free bonds B_t, which pay the (gross) interest rate R_t, and receive an equal share $1/(1-\lambda)$ of real profit income D_t from firm ownership. Letting P_t denote the aggregate price level, the period budget constraint of unconstrained households is given by:

$$B_t + P_t C_{S,t} = R_{t-1} B_{t-1} + P_t w_t N_{S,t} + \frac{P_t D_t}{1-\lambda}. \tag{5}$$

For both household types $j = \{H, S\}$, preferences are assumed to be separable between consumption $C_{j,t}$ and labor supply $N_{j,t}$:

$$u(C_{j,t}, N_{j,t}) = \frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{N_{j,t}^{1+\varphi}}{1+\varphi}, \tag{6}$$

where $\sigma > 0$ measures the inverse of the elasticity of intertemporal substitution and $\varphi > 0$ is the inverse of the labor supply elasticity.

The first-order condition (expressed in log deviations from the steady state) for constrained households is:

$$\varphi \widehat{N}_{H,t} = \widehat{w}_t - \sigma \widehat{C}_{H,t}. \tag{7}$$

Combining the log-linearized version of (4) with (7) yields the following solution to the constrained households problem:

$$\widehat{N}_{H,t} = \eta \widehat{w}_t, \qquad \widehat{C}_{H,t} = (1+\eta)\widehat{w}_t, \qquad \text{where} \qquad \eta \equiv \frac{1-\sigma}{\sigma+\varphi}. \tag{8}$$

For a representative unconstrained behavioural household, maximizing (6) with respect to (5) and log-linearizing around the steady state yields the following optimality conditions:

$$\varphi \widehat{N}_{S,t} = \widehat{w}_t - \sigma \widehat{C}_{S,t}, \tag{9}$$

$$\mathrm{E}_t^{BR} \widehat{C}_{S,t+1} = \widehat{C}_{S,t} + \frac{1}{\sigma} \left(\widehat{R}_t - \mathrm{E}_t \widehat{\pi}_{t+1} \right), \tag{10}$$

where $\widehat{\pi}_{t+1} \equiv \widehat{P}_{t+1} - \widehat{P}_t$ denotes the (gross) inflation rate assuming a zero inflation steady state (i.e., $\pi^{ss} = 1$). We initially follow Gabaix (2020) and assume that unconstrained households make saving decisions from (10) at a guaranteed real interest rate, $\widehat{R}_t - \mathrm{E}_t \widehat{\pi}_{t+1}$. In section 4.1, we relax this assumption and allow for nominal illusion, where future inflation is not correctly perceived by behavioural households (i.e., $\widehat{R}_t - \mathrm{E}_t^{BR} \widehat{\pi}_{t+1}$).

2.2 Firms

The economy is comprised of a continuum of monopolistically competitive firms each indexed by $i \in [0, 1]$ who produce differentiated products $Y_t(i)$ using the technology:

$$Y_t(i) = N_t(i) - F, \tag{11}$$

where F is a fixed cost assumed to be equal across all firms. Given competitive wages, cost minimization implies that real marginal cost $\widehat{mc}_t$ equals the real wage:

$$\widehat{mc}_t = \widehat{w}_t. \tag{12}$$

Following Calvo (1983), in each period there is a constant probability $1 - \theta$ that a firm will be randomly selected to adjust its price. Otherwise, the firm will keep the price fixed. The objective of a behavioural firm i faced with resetting its price at time t is to choose P_t^* to maximize discounted nominal profits:

$$\sum_{j=0}^{\infty} \theta^j \mathrm{E}_t^{BR} \left[\Lambda_{t,t+j} \left(P_t^* Y_{t+j}(i) - P_{t+j} mc_{t+j} Y_{t+j}(i) \right) \right],$$

subject to the demand constraint:

$$Y_{t+j}(i) = \left(\frac{P_t^*}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j},$$

where $\varepsilon > 1$ is the elasticity of substitution across goods and $\Lambda_{t,t+j}$ denotes the stochastic discount factor, which in the steady state equals the subjective discount factor $\Lambda^{ss} = \beta^j$. The optimal price, expressed in log deviations from a zero inflation steady state, is given by:

$$\widehat{P}_t^* = (1-\theta\beta) \sum_{j=0}^{\infty} (\theta\beta)^j \, \mathrm{E}_t^{BR} \left[\widehat{mc}_{t+j} + \widehat{P}_{t+j} \right], \tag{13}$$

and the aggregate price level is:

$$\widehat{P}_t = \theta \widehat{P}_{t-1} + (1-\theta) \widehat{P}_t^*, \tag{14}$$

$$\Rightarrow \widehat{\pi}_t = (1-\theta) \left(\widehat{P}_t^* - \widehat{P}_{t-1} \right). \tag{15}$$

2.3 Monetary policy

Monetary policy is specified as an interest-rate feedback rule in which the nominal interest rate is a function of expected future inflation and contemporaneous output:

$$\widehat{R}_t = \mu_\pi \mathrm{E}_t \widehat{\pi}_{t+1} + \mu_y \widehat{Y}_t, \tag{16}$$

where $\mu_\pi, \mu_y \geq 0$ are the inflation and output response coefficients, respectively. Our motivation for considering a hybrid Taylor rule of the form (16) is twofold. First, due to the widely observed time lags in the transmission of monetary policy (see, e.g., Batini and Haldane, 1999), many central banks set the nominal interest rate in response to expectations of future inflation. Second, as discussed by McKnight and Mihailov (2015), there is sizable empirical evidence to suggest that real economic activity is used to help forecast inflation. We assume throughout that the central bank does not suffer from myopia.

2.4 Market clearing, aggregation and equilibrium

Defining aggregate output, $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$, and aggregate employment, $N_t = \int_0^1 N_t(i) di$, aggregating the production function (11) yields:

$$\widehat{Y}_t = (1+\mu) \widehat{N}_t, \tag{17}$$

where we follow Bilbiie (2008) and set F^{ss}/Y^{ss} equal to the steady state net markup $\mu = (\varepsilon - 1)^{-1}$, implying that profits are zero in steady state. Consequently, in the steady state the household budget constraints (4) and (5) are the same, and steady-state consumption and labor supply will be equal for both household types (an *equitable* steady state).

Labor market clearing requires:

$$\widehat{N}_t = \lambda \widehat{N}_{H,t} + (1-\lambda)\widehat{N}_{S,t}, \tag{18}$$

and aggregate consumption C_t can be expressed as:

$$\widehat{C}_t = \lambda \widehat{C}_{H,t} + (1-\lambda)\widehat{C}_{S,t}. \tag{19}$$

Aggregate real firm profits D_t is given by:

$$D_t = \int_0^1 D_t(i)di = Y_t - w_t N_t.$$

Eliminating D_t from (5), imposing the bond market clearing condition $B_t = 0$, and using (18) and (19), yields the goods market clearing condition:

$$\widehat{Y}_t = \widehat{C}_t. \tag{20}$$

Equilibrium For the variables $\{\widehat{C}_t,\ \widehat{C}_{H,t},\ \widehat{C}_{S,t},\ \widehat{N}_t,\ \widehat{N}_{H,t},\ \widehat{N}_{S,t},\ \widehat{Y}_t,\ \widehat{R}_t,\ \widehat{w}_t,\ \widehat{mc}_t,\ \widehat{\pi}_t,\ \widehat{P}_t,\ \widehat{P}_t^*\}$, the system of equilibrium conditions is summarized by the log-linearized equations given in (8)–(10) and (12)–(20).

2.5 Local equilibrium dynamics

The system of log-linearized equilibrium conditions can be reduced to a two-dimensional system of difference equations comprising of a behavioural IS equation and a behavioural New Keynesian Phillips Curve (NKPC). To derive the behavioural IS equation first combine conditions (17) and (18) with (8) and (9) to obtain:

$$\widehat{N}_{S,t} = \frac{1}{[1-\lambda(1-\eta\varphi)]}\frac{\widehat{Y}_t}{(1+\mu)} - \frac{\eta\lambda\sigma}{1-\lambda(1-\eta\varphi)}\widehat{C}_{S,t}. \tag{21}$$

Next, combining (19) and (20), and using (8), (9), and (21) yields:

$$\widehat{C}_{S,t} = \delta \widehat{Y}_t, \qquad \text{where} \qquad \delta \equiv 1 - \left(\frac{\lambda}{1-\lambda}\right) \frac{\varphi(1-\eta\mu)}{1+\mu} \gtrless 0. \tag{22}$$

Using (22) to eliminate $\widehat{C}_{S,t}$ from the Euler equation (10) and noting that equation (3) implies that $\mathrm{E}_t^{BR}\widehat{C}_{S,t+1} = m^s \mathrm{E}_t \widehat{C}_{S,t+1}$, where $m^s \in [0,1]$ is the degree of myopia of unconstrained households, the following behavioural IS equation is obtained:

$$\widehat{Y}_t = m^s \mathrm{E}_t \widehat{Y}_{t+1} - \frac{1}{\sigma\delta}\left(\widehat{R}_t - \mathrm{E}_t \widehat{\pi}_{t+1}\right). \tag{23}$$

Discounting arises in the IS equation with the presence of myopic households. Iterating equation (23) forward yields:

$$\widehat{Y}_t = -\frac{1}{\sigma\delta}\mathrm{E}_t \sum_{j=0}^{\infty} (m^s)^j \left(\widehat{R}_{t+j} - \mathrm{E}_{t+j}\widehat{\pi}_{t+j+1}\right). \tag{24}$$

Future changes in the real interest rate are discounted by a factor $(m^s)^j$ and thus have smaller effects on current output than changes in the real interest rate today.[8]

While the degree of constrained households λ is not important for the emergence of discounting in (23), it does affect the slope of the behavioural IS equation, where the threshold is given by:

$$\lambda^* = \frac{1+\mu}{1+\mu+\varphi(1-\eta\mu)}. \tag{25}$$

For $\lambda < \lambda^*$, $\delta > 0$, and the IS equation is negatively sloped. This generates the standard aggregate demand logic (SADL), where both output and consumption respond negatively to an increase in the real interest rate. However, if $\lambda > \lambda^*$, $\delta < 0$, and the IS equation is now positively sloped. In this case we have an inverted aggregate demand logic (IADL), where output responds positively to an increase in the real interest rate. With sufficiently low participation in the asset market, the increase in firm profits from a higher real interest rate dominates the fall in real wages induced by lower aggregate demand from the intertemporal effects of delayed consumption. This redistribution from constrained to unconstrained households, results in an expansionary effect on aggregate output.

[8] Under rational expectations ($m^s = 1$), there is no discounting in (23). Therefore, future real interest rate changes have the same effect on current output as a similar change in the interest rate today.

To derive the behavioural NKPC, first rearrange the optimal price-setting condition (13):

$$\widehat{P}_t^* - \widehat{P}_t = (1-\theta\beta)\widehat{mc}_t + \theta\beta \mathrm{E}_t^{BR}\left[\widehat{P}_{t+1}^* - \widehat{P}_t\right],$$

and using (14) and (15) yields:

$$\frac{\theta}{1-\theta}\widehat{\pi}_t = (1-\theta\beta)\widehat{mc}_t + \frac{\theta\beta}{1-\theta}\mathrm{E}_t^{BR}\widehat{\pi}_{t+1}. \tag{26}$$

Next, it follows from equations (9) and (12) that:

$$\widehat{mc}_t = \varphi\widehat{N}_{S,t} + \sigma\widehat{C}_{S,t} = \chi\widehat{Y}_t, \tag{27}$$

where $\chi \equiv \dfrac{\varphi}{[1-\lambda(1-\eta\varphi)](1+\mu)} + \sigma\delta\left[1 - \dfrac{\eta\lambda\varphi}{1-\lambda(1-\eta\varphi)}\right] = \sigma + \dfrac{\varphi}{1+\mu} > 0,$

after using (21) and (22) to eliminate $\widehat{N}_{S,t}$ and $\widehat{C}_{S,t}$. Noting that (3) implies that $\mathrm{E}_t^{BR}\widehat{\pi}_{t+1} = m^f\mathrm{E}_t\widehat{\pi}_{t+1}$, where $m^f \in [0,1]$ is the degree of myopia of firms, it follows from (26) and (27):

$$\widehat{\pi}_t = \kappa\chi\widehat{Y}_t + \beta m^f\mathrm{E}_t\widehat{\pi}_{t+1}, \qquad \text{where} \qquad \kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta} > 0. \tag{28}$$

Equation (28) is the behavioural NKPC, where the coefficient κ denotes the real marginal cost elasticity of inflation, which is the same as under rational expectations.[9] The *long-run* version of (28) directly follows after eliminating the time subscripts and the expectation operator:

$$\widehat{Y}^* = \alpha_{\pi,y}^m\widehat{\pi}^*, \qquad \text{where} \qquad \alpha_{\pi,y}^m \equiv \frac{1-\beta m^f}{\kappa\chi}. \tag{29}$$

The long-run inflation elasticity of output, $\alpha_{\pi,y}^m$, determines the slope of the long-run NKPC, $\partial\widehat{Y}^*/\partial\widehat{\pi}^* = \alpha_{\pi,y}^m$, which is steeper under cognitive discounting compared to rational expectations.

2.6 Parameters used in the numerical examples

In the following sections, we derive analytical results investigating the implications of myopic agents for equilibrium determinacy. It will also be insightful to illustrate our results numerically. Table 1 summarizes the parameter values used in the numerical examples. As is standard in literature, we

[9] Our derivation of the behavioural NKPC is in agreement with both Benchimol and Bounader (2023) and Meggiorini (2023), and differs from Gabaix (2020) due to a methodological error. For further discussion, see Meggiorini (2023).

Table 1: Baseline parameter values

β	Subjective discount factor	0.99
ε	Elasticity of substitution of differentiated goods	11
σ^{-1}	Intertemporal elasticity of substitution in consumption	2
φ	Inverse of the labor supply elasticity	2
θ	Degree of price stickiness	0.75 or 0.5
κ	Real marginal cost elasticity of inflation	0.086 or 0.505
λ	Share of constrained households in IADL economy	0.5 or 0.7
m^s	Cognitive discounting by unconstrained households	0.85 or 0.5
m^f	Cognitive discounting by firms	0.8 or 0.5
μ_π	Inflation response coefficient	$\mu_\pi \in [0, 5]$
μ_y	Output response coefficient	$\mu_y \in [0, 1]$

set the (quarterly) discount factor $\beta = 0.99$, consistent with an annualized real interest rate of 4%, and $\varepsilon = 11$ implying a markup of 10%. Following Bilbiie (2008), we set $\varphi = 2$ implying a Frisch labor supply elasticity of 0.5. For the intertemporal elasticity of substitution in consumption σ^{-1}, values found in the literature typically range from 0.5 to 6.4. We follow McKnight (2018) and choose a midway value of $\sigma^{-1} = 2$. For the degree of price stickiness, empirical estimates of θ vary considerably. To adequately cover the range of empirical estimates, we consider two values of $\theta = 0.5, 0.75$. Setting $\theta = 0.75$ constitutes an average price duration of one year, which is roughly consistent with the estimates of Nakamura and Steinsson (2008) and implies a value for the real marginal cost elasticity of inflation $\kappa = 0.086$. A lower value of $\theta = 0.5$, consistent with the estimates of Bils and Klenow (2004), implies prices are fixed on average for two quarters and suggests a value of $\kappa = 0.505$. For the IADL economy, we follow Buffie (2013) and Buffie and Zanna (2018) and consider two values for LAMP, $\lambda = \{0.5, 0.7\}$, consistent with the empirical estimates for a number of developed and less developed countries.

For the myopic parameters m^s and m^f, as a benchmark we follow Gabaix (2020) and set $m^f = 0.8$ and $m^s = 0.85$, the latter implies that the half-life of (unconstrained) household attention is one year. However, several recent Bayesian studies suggest that the degree of cognitive discounting is likely to be much lower. Ilabaca et al. (2020) estimate $m^f = 0.41$ and $m^s = 0.71$, whereas Kolasa et al. (2022) sets $m = m^s = m^f$ and estimates a value of $m = 0.53$. Estimates by Meggiorini (2023) suggest that m lies in the interval $m \in [0.43, 0.62]$. Consequently, we also consider a value of 0.5 for both m^s and m^f. Finally, for the monetary policy coefficients μ_π and μ_y, we search in the range $\mu_\pi \in [0, 5]$ and $\mu_y \in [0, 1]$ covering the vast majority of empirical estimates.

3 Determinacy analysis

This section considers the issue of local equilibrium determinacy with behavioural agents. We start by assuming that monetary policy takes the form of a strict inflation-targeting rule, where the central bank responds only to expected future inflation, before considering a policy response to output.

3.1 Strict inflation-targeting rule

Given the interest-rate rule $\widehat{R}_t = \mu_\pi \mathrm{E}_t \widehat{\pi}_{t+1}$, the behavioural IS equation (23), and the behavioural NKPC (28), the minimal state-space representation of the model can be expressed as:

$$\mathrm{E}_t \mathbf{z}_{t+1} = \mathbf{A_1} \mathbf{z}_t, \qquad \mathbf{z}_t = \begin{bmatrix} \widehat{Y}_t \ \widehat{\pi}_t \end{bmatrix}', \qquad \mathbf{A_1} \equiv \begin{bmatrix} \frac{1}{m^s} - \frac{\kappa\chi(\mu_\pi - 1)}{\beta\delta\sigma m^s m^f} & \frac{\mu_\pi - 1}{\beta\delta\sigma m^s m^f} \\ -\frac{\kappa\chi}{\beta m^f} & \frac{1}{\beta m^f} \end{bmatrix}.$$

Proposition 1 *If the interest-rate rule reacts to forward-looking inflation, the necessary and sufficient conditions for equilibrium determinacy with behavioural agents are:*

$$\textit{Case I:} \qquad \delta > 0, \quad \textit{and} \quad 1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi} < \mu_\pi < 1 + \frac{\delta\sigma(1+m^s)(1+\beta m^f)}{\kappa\chi}; \tag{30}$$

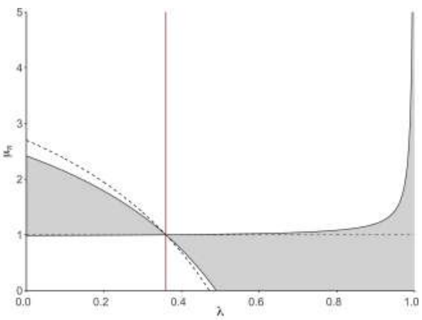
$$\textit{Case II:} \qquad \delta < 0, \quad \textit{and} \quad 1 + \frac{\delta\sigma(1+m^s)(1+\beta m^f)}{\kappa\chi} < \mu_\pi < 1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}. \tag{31}$$

Proof. The determinant (det) and trace (tr) of the coefficient matrix $\mathbf{A_1}$ are:

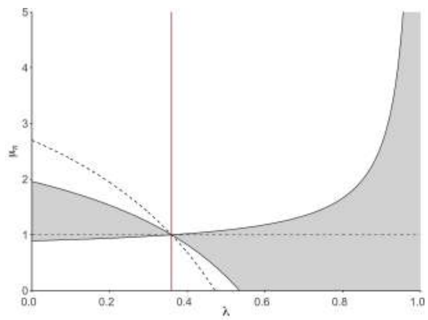
$$\det \mathbf{A_1} = \frac{1}{\beta m^s m^f} > 1 \quad \text{and} \quad \mathrm{tr}\mathbf{A_1} = \frac{1}{m^s} + \frac{1}{\beta m^f} - \frac{(\mu_\pi - 1)\kappa\chi}{\beta\delta\sigma m^s m^f}.$$

Since there are no predetermined variables, determinacy requires that both eigenvalues are outside the unit circle. According to the Schur-Cohn criterion (see, e.g., LaSalle, 1986), this requires that $|\mathrm{tr}\mathbf{A_1}| < 1 + \det \mathbf{A_1}$, which implies $\mathit{\Gamma}_1 < \mu_\pi < \mathit{\Gamma}_2$ if $\delta > 0$, and $\mathit{\Gamma}_2 < \mu_\pi < \mathit{\Gamma}_1$ if $\delta < 0$, where $\mathit{\Gamma}_1,\ \mathit{\Gamma}_2$ are given in (30) and (31). This completes the proof. ■

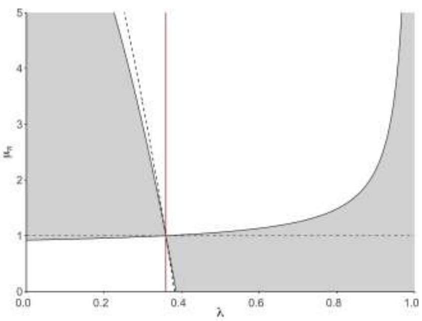
Using the baseline parameter values, Figure 1 plots the results of Proposition 1 for combinations of the degree of LAMP (λ) and the monetary policy parameter μ_π. The dashed lines illustrate the determinacy regions under rational expectations ($m^s = m^f = 1$) and the red vertical line depicts the threshold value of λ^* above which the IADL holds. For standard SADL economies, a necessary condition for determinacy under rational expectations is the so-called *Taylor Principle* ($\mu_\pi > 1$),



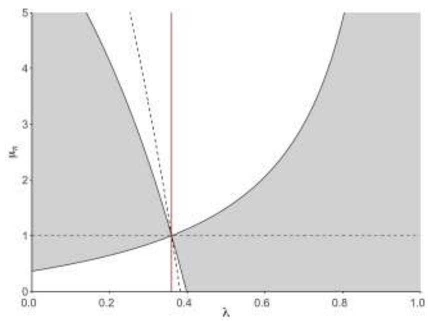
(a) $\theta = 0.5$; $m^s = 0.85$, $m^f = 0.8$



(b) $\theta = 0.5$; $m^s = m^f = 0.5$



(c) $\theta = 0.75$; $m^s = 0.85$, $m^f = 0.8$



(d) $\theta = 0.75$; $m^s = m^f = 0.5$

Figure 1: Determinacy regions (grey areas) under a strict inflation-targeting rule. Parameter values are $\beta = 0.99$, $\sigma = 0.5$, $\varphi = 2$, $\varepsilon = 11$. The dashed lines illustrate the regions of determinacy under rational expectations. The red vertical line depicts λ^* above which IADL holds.

where the central bank should adopt an 'active' policy stance. However, the central bank should not react too aggressively to expected future inflation when prices are sufficiently flexible ($\theta = 0.5$), since the upper-bound on μ_π is potentially binding. In stark contrast, the *inverted Taylor principle* ($\mu_\pi < 1$) is required for IADL economies under rational expectations, where the central bank is required to adopt a 'passive' policy stance to prevent indeterminacy.

Under cognitive discounting, some interesting findings emerge. First, determinacy can arise in SADL economies even if the Taylor principle is violated, where the lower bound on μ_π given in (30) is increasing in m^s and m^f and decreasing in the degree of price stickiness θ. This region of determinacy is largest in panel (d) of Figure 1 for the parameter combinations $m^s = m^f = 0.5$ and $\theta = 0.75$. Second, since the upper bound given in (30) is increasing in m^s and m^f and decreasing in θ, indeterminacy is more likely to arise under the Taylor principle in SADL economies with cognitive discounting and relatively flexible prices (see, e.g., panel (b) of Figure 1). Third, myopia exerts a stabilizing effect in IADL economies, as determinacy can now also arise under the Taylor principle. As illustrated by panel (d) of Figure 1, this region of determinacy is potentially large under sticky prices for $\lambda > \lambda^*$. In sum, cognitive discounting reduces the ability of the Taylor principle to achieve determinacy in SADL economies but increases its ability in IADL economies.

For some intuition, consider a sunspot-induced increase in inflationary expectations. Under the Taylor principle, the nominal interest rate $\widehat{R}_t$ rises by more than the increase in expected inflation resulting in an increase in the real interest rate. In SADL economies, provided the central bank is not overly aggressive in raising $\widehat{R}_t$, real marginal cost and output fall under the Taylor principle, which from the NKPC, exerts downward pressure on inflation contradicting the initial inflationary belief. In LAMP economies, the conventional aggregate demand channel of monetary policy is inverted, and real marginal cost and output increase under the Taylor principle. Consequently, in response to higher expected inflation, the real interest rate falls under a passive policy (the 'inverted Taylor principle') resulting in lower output and inflation. With myopic households ($m^s < 1$), however, discounting is now present in the IS equation as aggregate demand becomes less forward-looking, and this *discounting channel* weakens the transmission mechanism of future monetary policy on current outcomes. Using the interest-rate rule to eliminate $\widehat{R}_t$ from (23) yields:

$$\widehat{Y}_t - m^s \mathrm{E}_t \widehat{Y}_{t+1} = -\left(\frac{\mu_\pi - 1}{\sigma\delta}\right) \mathrm{E}_t \widehat{\pi}_{t+1}. \tag{32}$$

Consider a permanent increase in inflation $\widehat{\pi}^*$. From (29), this results in a permanent increase in output $\widehat{Y}^*$ of $\alpha^m_{\pi,y} \equiv \frac{1-\beta m^f}{\kappa\chi}$ percentage units and it follows from the long-run version of (32):[10]

$$\begin{aligned} (1 - m^s)\widehat{Y}^* &= -\left(\frac{\mu_\pi - 1}{\sigma\delta}\right)\widehat{\pi}^*, \\ \Rightarrow 1 - \sigma\delta(1 - m^s)\alpha^m_{\pi,y} - \mu_\pi &= 0, \end{aligned} \tag{33}$$

which is just the lower bound given in (30) with $\delta > 0$ and the upper bound given in (31) with $\delta < 0$. In the event of a permanent rise in inflation, it is not necessary in SADL (IADL) economies that the real interest rate increase (decrease) provided $m^s < 1$, where the parameters m^f and θ both affect the long-run inflation elasticity of output $\alpha^m_{\pi,y}$. This is in stark contrast to the rational expectations benchmark where the LHS of (32) is zero in the long run, and it directly follows that a necessary condition for determinacy is $\mu_\pi > 1$ with $\delta > 0$ and $\mu_\pi < 1$ with $\delta < 0$.

In SADL economies, indeterminacy also arises under the Taylor principle when policy is 'too active', whereas in IADL economies indeterminacy can arise by violating the lower bound given

[10] In the long run, $\widehat{Y}_t = \mathrm{E}_t\widehat{Y}_{t+1} = \widehat{Y}^*$.

in condition (31) when policy becomes 'too passive'. Bounded rationality makes indeterminacy more likely to happen in both these cases. For some intuition, we numerically solved the model under indeterminacy using the Farmer-Khramov-Nicolo (2015) solution method. The impulse response functions suggest that when monetary policy is overly aggressive in SADL economies, sunspot shocks generate oscillatory behaviour for inflation $\widehat{\pi}_t > 0 > \mathrm{E}_t\widehat{\pi}_{t+1}$ and output $\widehat{Y}_t > 0 > \mathrm{E}_t\widehat{Y}_{t+1}$.[11] Since cognitive discounting reduces the attentiveness of behavioural consumers and firms, smaller values of m^s and m^f reduce the weights of $\mathrm{E}_t\widehat{Y}_{t+1}$ and $\mathrm{E}_t\widehat{\pi}_{t+1}$ in equations (23) and (28), respectively. Consequently, $\widehat{\pi}_t > 0$ and $\widehat{Y}_t > 0$ can be more easily supported under deflationary beliefs $\mathrm{E}_t\widehat{\pi}_{t+1} < 0$, increasing the possibility of sunspot equilibria.

3.2 Reacting to output

We now consider the determinacy implications of a policy response to contemporaneous output.

Proposition 2 *If the interest-rate rule reacts to expected future inflation and contemporaneous output* (16)*, the necessary and sufficient conditions for determinacy with behavioural agents are:*

$$\textit{Case I:} \quad \delta > 0, \quad \mu_\pi + \frac{1-\beta m^f}{\kappa\chi}\mu_y > 1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}, \tag{34}$$

$$\textit{and} \quad \mu_\pi < 1 + \frac{1+\beta m^f}{\kappa\chi}\mu_y + \frac{\delta\sigma(1+m^s)(1+\beta m^f)}{\kappa\chi}; \tag{35}$$

$$\textit{Case IIA:} \quad \delta < 0, \quad 0 < \mu_y < -\delta\sigma(1-\beta m^s m^f), \tag{36}$$

$$\textit{and} \quad \mu_\pi + \frac{1-\beta m^f}{\kappa\chi}\mu_y < 1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}, \tag{37}$$

$$\textit{and} \quad \mu_\pi > 1 + \frac{1+\beta m^f}{\kappa\chi}\mu_y + \frac{\delta\sigma(1+m^s)(1+\beta m^f)}{\kappa\chi}; \tag{38}$$

$$\textit{Case IIB:} \quad \delta < 0, \quad \mu_y > -\delta\sigma(1+\beta m^s m^f), \tag{39}$$

and conditions (34) *and* (35) *are satisfied.*

Proof. The model (16), (23), and (28) can be reduced to the following two-dimensional system:

$$\mathrm{E}_t\mathbf{z}_{t+1} = \mathbf{A_2}\mathbf{z}_t, \qquad \mathbf{z}_t = \begin{bmatrix}\widehat{Y}_t & \widehat{\pi}_t\end{bmatrix}', \qquad \mathbf{A_2} \equiv \begin{bmatrix} \frac{1}{m^s} + \frac{\mu_y}{\delta\sigma m^s} - \frac{\kappa\chi(\mu_\pi - 1)}{\beta\delta\sigma m^s m^f} & \frac{\mu_\pi - 1}{\beta\delta\sigma m^s m^f} \\ -\frac{\kappa\chi}{\beta m^f} & \frac{1}{\beta m^f} \end{bmatrix}.$$

[11] Similar oscillatory sunspot equilibria arise in IADL economies under a policy stance that is too passive.

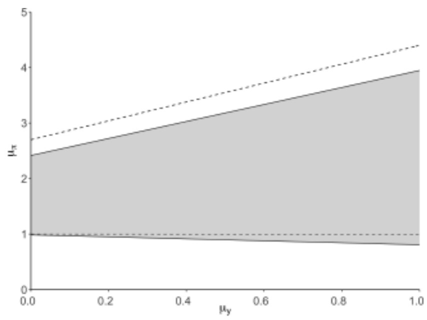
The determinate (det) and trace (tr) of the coefficient matrix $\mathbf{A_2}$ are:

$$\det \mathbf{A_2} = \frac{1}{\beta m^s m^f} + \frac{\mu_y}{\beta\delta\sigma m^s m^f} \quad \text{and} \quad \mathrm{tr}\mathbf{A_2} = \frac{1}{m^s} + \frac{1}{\beta m^f} + \frac{\mu_y}{\delta\sigma m^s} - \frac{(\mu_\pi - 1)\kappa\chi}{\beta\delta\sigma m^s m^f}.$$

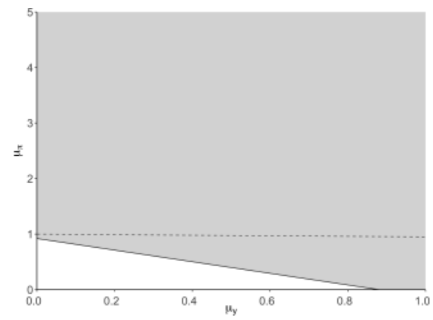
Since there are no predetermined variables, determinacy requires that both eigenvalues are outside the unit circle. According to the Schur-Cohn criterion, this requires that (i) $|\det \mathbf{A_2}| > 1$ and (ii) $|\mathrm{tr}\mathbf{A_2}| < 1 + \det \mathbf{A_2}$. If $\delta > 0$, $\det \mathbf{A_2} > 1$ is always satisfied, and $1 + \det \mathbf{A_2} - \mathrm{tr}\mathbf{A_2} > 0$ implies (34), while $1 + \det \mathbf{A_2} + \mathrm{tr}\mathbf{A_2} > 0$ implies (35). If $\delta < 0$, $\det \mathbf{A_2} > 1$ requires $0 < \mu_y < -\delta\sigma(1 - \beta m^s m^f)$ and condition (ii) yields (37) and (38). Next note that $\det \mathbf{A_2} < -1$ requires $\mu_y > -\delta\sigma(1 + \beta m^s m^f)$ and $\delta < 0$. Then $|\mathrm{tr}\mathbf{A_2}| < -1 - \det \mathbf{A_2}$ gives (34) and (35). This completes the proof. ■

In what follows, we refer to condition (34) as the *behavioral Taylor principle* for SADL economies. This differs from the *generalized (or long-run) Taylor principle* derived by Bullard and Mitra (2002) and Woodford (2003) under rational expectations, $\mu_\pi + \alpha_{\pi,y}\mu_y > 1$, obtained by setting $m^s = m^f = 1$ in (34), where $\alpha_{\pi,y} \equiv \frac{1-\beta}{\kappa\chi} > 0$. The generalized Taylor principle implies that each percentage point of permanently higher inflation results in a permanent increase in output of $\alpha_{\pi,y}$ percentage points. Consequently, determinacy requires that the long-run nominal interest rate should rise by proportionally more than the increase in inflation. Since the subjective discount factor β is calibrated to be very close to one, the trade-off between the policy parameters μ_π and μ_y is very weak under rational expectations ($\alpha_{\pi,y} \approx 0$). In contrast, the presence of myopic firms increases the long-run inflation elasticity of output, $\alpha^m_{\pi,y} \equiv (1 - \beta m^f)/\kappa\chi$, implying that the slope of the long-run behavioral NKPC (29) is relatively steeper than its rational expectations counterpart. In the presence of discounting in the behavioural IS equation ($m^s < 1$), it is no longer necessary for the long-run nominal interest rate to rise by proportionally more than a permanent increase in inflation, and determinacy can therefore be achieved under a passive policy provided (34) is satisfied. As illustrated in Figure 2, the myopic parameters and the degree of price stickiness have two key implications for the determinacy boundary determined by the behavioral Taylor principle (34). The lower the values of m^s, m^f and the higher the value of θ: (i) reduce the vertical intercept term $1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}$ shifting the determinacy boundary downwards; and (ii) increase the slope of the determinacy boundary, given by $-\alpha^m_{\pi,y}$, on the plane (μ_y, μ_π). Similar to the previous section, this mechanism helps to weaken the impact of future aggregate demand changes on current outcomes, resulting in an increase in the determinacy region.

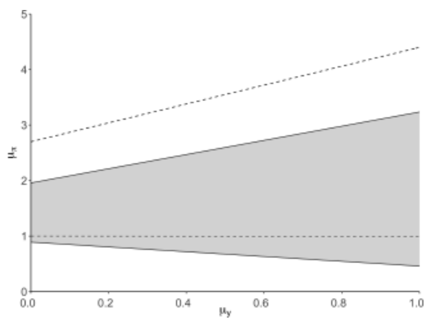
In contrast, it follows from condition (35) that the upper bound on the inflation response



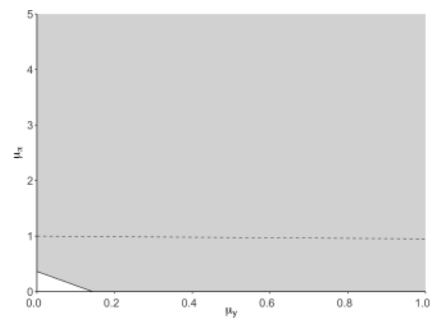
(a) $\theta = 0.5$; $m^s = 0.85$, $m^f = 0.8$



(b) $\theta = 0.75$; $m^s = 0.85$, $m^f = 0.8$



(c) $\theta = 0.5$; $m^s = m^f = 0.5$

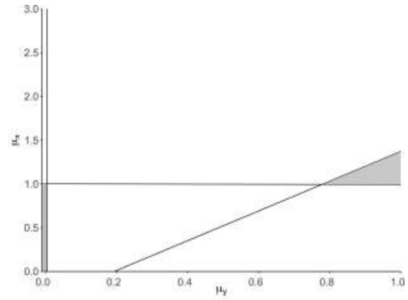


(d) $\theta = 0.75$; $m^s = m^f = 0.5$

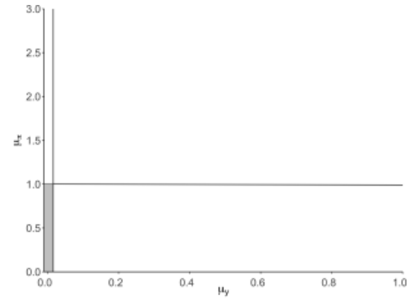
Figure 2: Determinacy regions (grey areas) with a policy response to output: SADL economy. Parameter values are $\beta = 0.99$, $\sigma = 0.5$, $\varphi = 2$, $\varepsilon = 11$, $\lambda = 0$. Dashed lines illustrate the determinacy regions under rational expectations.

coefficient is increasing in μ_y. Similar to the strict inflation-targeting policy of the previous section, this upper bound is reduced under behavioural agents, increasing the indeterminacy region that arises under the Taylor principle in SADL economies. By inspection of Figure 2, the net effect of cognitive discounting for determinacy depends on the degree of price stickiness. For empirically plausible combinations of the policy parameters (μ_y, μ_π), the determinate policy space expands under a high degree of price stickiness ($\theta = 0.75$), whereas the net effect of myopic agents is to shrink the determinate policy space when prices are more flexible ($\theta = 0.5$).

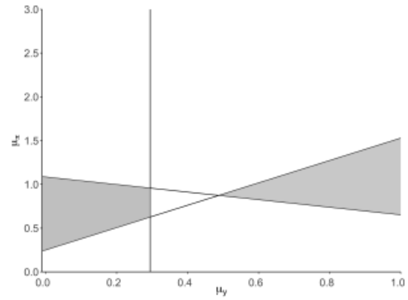
For the IADL case, first note it follows from (36) and (39) of Case II of Proposition 2 that there is an interval of μ_y where indeterminacy always exists: $-\delta\sigma(1-\beta m^s m^f) < \mu_y < -\delta\sigma(1+\beta m^s m^f)$, regardless of the value of μ_π. While the behavioral Taylor principle (34) is required in Case IIB, the *inverted* behavioral Taylor principle (37) is necessary for Case IIA. Using the baseline parameter values, Figures 3 and 4 illustrate the regions of determinacy and indeterminacy for two values of LAMP, $\lambda = 0.5, 0.7$. Panels (a) and (b) of these figures graph Case II of Proposition 2 under rational expectations ($m^s = m^f = 1$), while panels (c) and (d) illustrate the results under cognitive



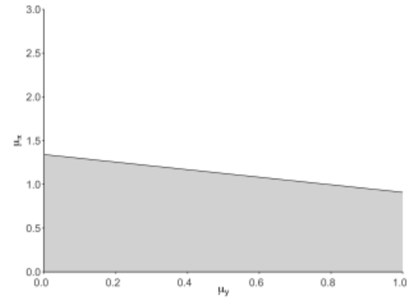
(a) $\lambda = 0.5$; $m^s = m^f = 1$



(b) $\lambda = 0.7$; $m^s = m^f = 1$

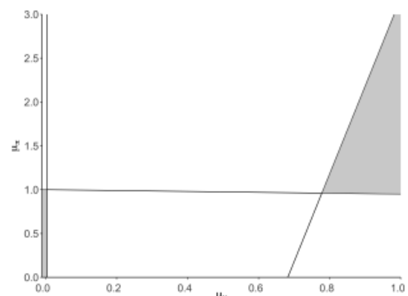


(c) $\lambda = 0.5$; $m^s = m^f = 0.5$

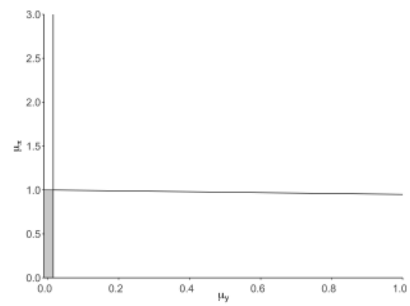


(d) $\lambda = 0.7$; $m^s = m^f = 0.5$

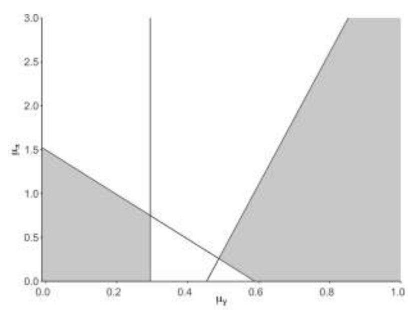
Figure 3: Determinacy regions (grey areas) with a policy response to output: IADL economy ($\theta = 0.5$).



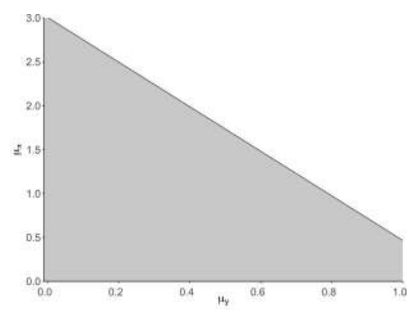
(a) $\lambda = 0.5$; $m^s = m^f = 1$



(b) $\lambda = 0.7$; $m^s = m^f = 1$



(c) $\lambda = 0.5$; $m^s = m^f = 0.5$



(d) $\lambda = 0.7$; $m^s = m^f = 0.5$

Figure 4: Determinacy regions (grey areas) with a policy response to output: IADL economy ($\theta = 0.75$).

discounting setting $m^s = m^f = 0.5$. To satisfy Case IIA under rational expectations, the inverted generalized Taylor principle, given by $\mu_\pi + \alpha_{\pi,y}\mu_y < 1$, is a necessary condition for determinacy. However, by inspection of panels (a) and (b), condition (36) can only be satisfied in this case for very small values of μ_y. To satisfy Case IIB under rational expectations, central banks of IADL economies should follow the generalized Taylor principle and be sufficiently aggressive in their policy response to output in order to satisfy (39). However, by inspection of the figures, the determinate policy space is small for $\lambda = 0.5$, and non-existent for $\lambda = 0.7$, regardless of the degree of price stickiness. The presence of myopic agents significantly increase the regions of determinacy for both Cases IIA and IIB. This difference is particularly stark under $\lambda = 0.7$ under both values of price stickiness.[12] These conclusions mirror the strict inflation-targeting policy findings of the previous section; namely that cognitive discounting exerts a strong stabilizing effect on IADL economies.

4 The timing of the interest-rate rule

In this section, we examine the robustness of our previous findings under alternative timing specifications for the Taylor rule. Specifically, we consider interest-rate feedback rules of the form:

$$\widehat{R}_t = \mu_\pi \mathrm{E}_t \widehat{\pi}_{t+j} + \mu_y \mathrm{E}_t \widehat{Y}_{t+j}. \tag{40}$$

If $j = 1$, the Taylor rule is forward looking with respect to both inflation and output, whereas, if $j = 0$, the policy rule responds to contemporaneous inflation and output. Both of these specifications for the interest-rate rule are often adopted in the determinacy literature.[13]

4.1 Forward-looking Taylor rule

Proposition 3 *Under a forward-looking Taylor rule (setting $j = 1$ in eq. (40)), the necessary and sufficient conditions for equilibrium determinacy with behavioural agents are:*

[12] Since δ tends to minus infinity as $\lambda \to 1$, it follows that conditions (36) and (38) of Case IIA are always satisfied in our numerical example under cognitive discounting after setting $\lambda = 0.7$. Consequently, determinacy requires that the inverted generalized Taylor principle (37) holds.

[13] See, e.g., Bullard and Mitra (2002), Woodford (2003), Airaudo and Zanna (2012), McKnight and Mihailov (2015).

$$\textit{Case I:} \quad \delta > 0, \quad 0 < \mu_y < \delta\sigma\left(m^s + \frac{1}{\beta m^f}\right), \tag{41}$$

$$\textit{and} \quad \mu_\pi + \frac{1-\beta m^f}{\kappa\chi}\mu_y > 1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}, \tag{42}$$

$$\textit{and} \quad \mu_\pi < 1 - \frac{1+\beta m^f}{\kappa\chi}\mu_y + \frac{\delta\sigma(1+m^s)(1+\beta m^f)}{\kappa\chi}; \tag{43}$$

$$\textit{Case II:} \quad \delta < 0, \quad 0 < \mu_y < -\delta\sigma\left(\frac{1}{\beta m^f} - m^s\right), \tag{44}$$

$$\textit{and} \quad \mu_\pi + \frac{1-\beta m^f}{\kappa\chi}\mu_y < 1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}, \tag{45}$$

$$\textit{and} \quad \mu_\pi > 1 - \frac{1+\beta m^f}{\kappa\chi}\mu_y + \frac{\delta\sigma(1+m^s)(1+\beta m^f)}{\kappa\chi}. \tag{46}$$

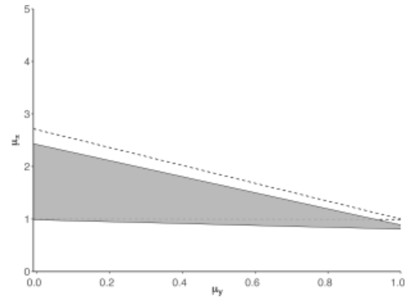
Proof. See online appendix A. ■

Both the standard and inverted versions of the behavioural Taylor principle remain unchanged after replacing the explicit policy response to contemporaneous output with expected future output; that is, equations (42) and (45) are identical to (34) and (37). Therefore, exactly as in section 3.2 a discounting channel arises under myopia, which shifts the determinacy boundary inwards (outwards) in SADL (IADL) economies, expanding the region of determinacy. For the SADL case, however, the upper bound on the inflation response coefficient given by condition (43) is now *decreasing* in μ_y, such that determinacy becomes impossible under the behavioural Taylor principle once the output response coefficient becomes sufficiently large: $\mu_y > \delta\sigma(1+m^s)+\kappa\chi/(1+\beta m^f)$.[14] As illustrated by Figure 5, indeterminacy is significantly more likely under a forward-looking Taylor rule, and in contrast to the baseline hybrid rule (16), the determinacy regions shrink under myopic agents even with a high degree of price stickiness ($\theta = 0.75$).

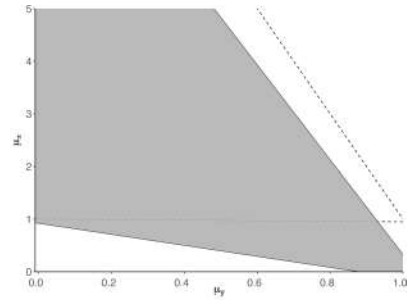
As shown by Case II of Proposition 3, determinacy can no longer be supported in IADL economies under the behavioural Taylor principle with a policy response to expected future output. Figure 6 gives a graphical representation of the IADL case for two values of LAMP, $\lambda = 0.5, 0.7$, setting the price stickiness parameter equal to 0.75.[15] Under rational expectations, the determinacy region associated with the inverted generalized Taylor principle is barely visible from panels (a) and (b). Setting the values of the myopic parameters to be $m^s = m^f = 0.5$, panels (c) and (d) show that with behavioral agents the determinacy region is now significantly increased. Therefore,

[14] While condition (41) also imposes an upper-bound on μ_y, this is non-binding within the interval $\mu_y \in [0, 1]$ using our baseline parameter values under $\lambda = 0$.

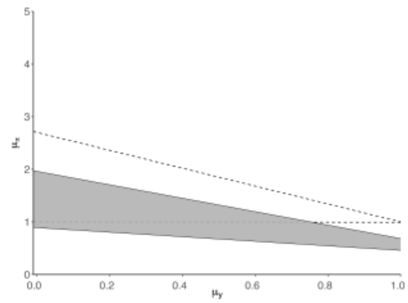
[15] Note that condition (46) is always satisfied in this numerical example.



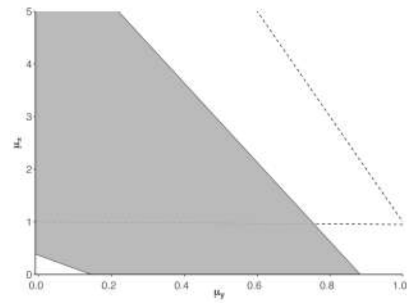
(a) $\theta = 0.5$; $m^s = 0.85$, $m^f = 0.8$



(b) $\theta = 0.75$; $m^s = 0.85$, $m^f = 0.8$

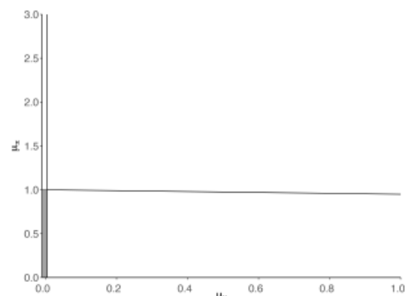


(c) $\theta = 0.5$; $m^s = m^f = 0.5$

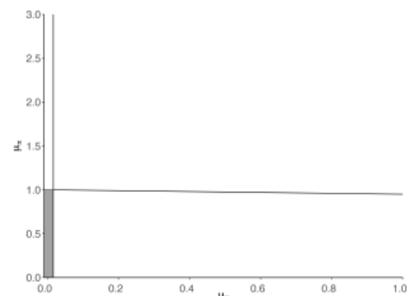


(d) $\theta = 0.75$; $m^s = m^f = 0.5$

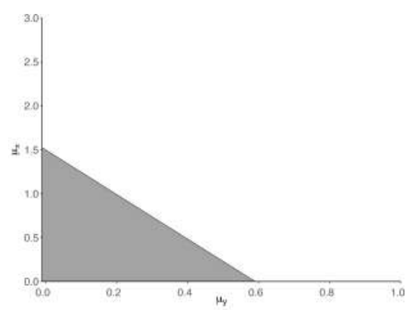
Figure 5: Determinacy regions (grey areas) under a forward-looking Taylor rule: SADL economy ($\lambda = 0$). Dashed lines illustrate the determinacy regions under rational expectations.



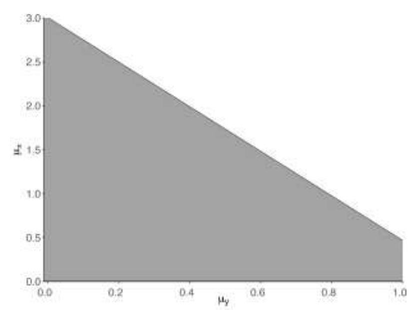
(a) $\lambda = 0.5$; $m^s = m^f = 1$



(b) $\lambda = 0.7$; $m^s = m^f = 1$



(c) $\lambda = 0.5$; $m^s = m^f = 0.5$



(d) $\lambda = 0.7$; $m^s = m^f = 0.5$

Figure 6: Determinacy (grey areas) under a forward-looking Taylor rule: IADL economy ($\theta = 0.75$).

while the determinate policy space is partially reduced under a forward-looking Taylor rule, cognitive discounting continues to exert a strong stabilizing effect on IADL economies.

4.2 Contemporaneous-looking Taylor rule

Proposition 4 *Under a current-looking Taylor rule (setting $j = 0$ in eq. (40)), the necessary and sufficient conditions for equilibrium determinacy with behavioural agents are:*

$$\textit{Case I:} \qquad \delta > 0, \qquad \mu_\pi + \frac{1-\beta m^f}{\kappa\chi}\mu_y > 1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}, \tag{47}$$

$$\textit{Case IIA:} \qquad \delta < 0, \qquad \kappa\chi\mu_\pi + \mu_y < -\delta\sigma(1-\beta m^s m^f), \tag{48}$$

$$\textit{and} \qquad \kappa\chi(\mu_\pi + 1) + \mu_y(1+\beta m^f) < -\delta\sigma(1+m^s)(1+\beta m^f), \tag{49}$$

$$\textit{and} \qquad \mu_\pi + \frac{1-\beta m^f}{\kappa\chi}\mu_y < 1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}; \tag{50}$$

$$\textit{Case IIB:} \qquad \delta < 0, \qquad \kappa\chi\mu_\pi + \mu_y > -\delta\sigma(1+\beta m^s m^f), \tag{51}$$

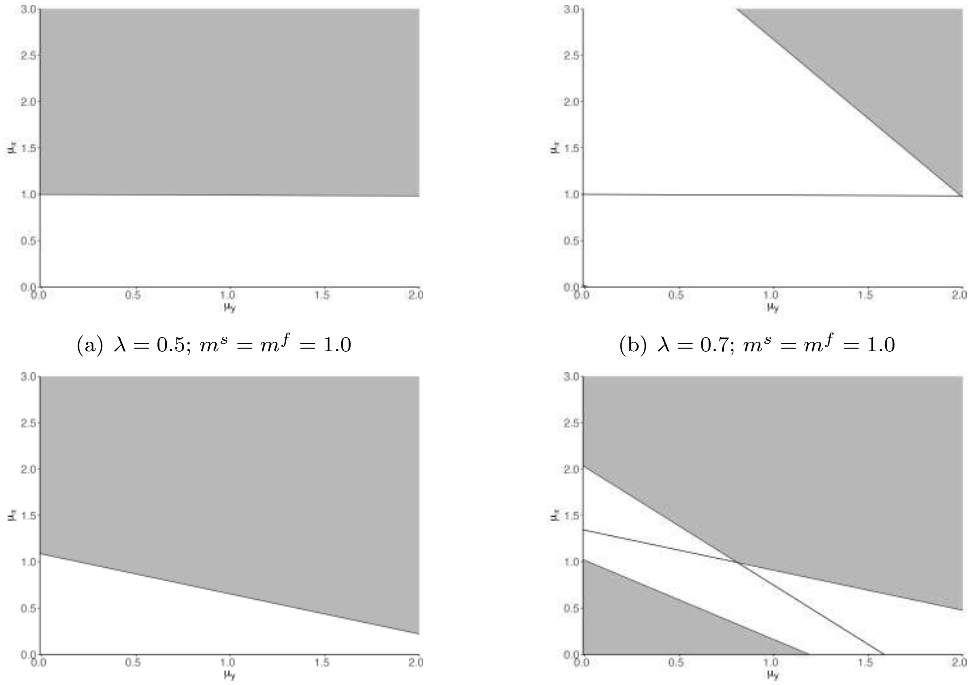
$$\textit{and} \qquad \kappa\chi(\mu_\pi + 1) + \mu_y(1+\beta m^f) > -\delta\sigma(1+m^s)(1+\beta m^f), \tag{52}$$

$$\textit{and} \qquad \mu_\pi + \frac{1-\beta m^f}{\kappa\chi}\mu_y > 1 - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}. \tag{53}$$

Proof. See online appendix B. ■

In stark contrast to forward-looking policy, cognitive discounting unambiguously exerts a stabilizing effect on both SADL and IADL economies under a current-looking Taylor rule. While the behavioural Taylor principle (47) remains exactly the same as before, in the absence of an upper-bound on μ_π, the determinacy region always expands with lower values of m^s and m^f in the SADL case. This is consistent with the findings of Gabaix (2020).

For LAMP economies, Figure 7 illustrates Proposition 4 using the baseline parameter values for two values of LAMP, $\lambda = 0.5, 0.7$, setting the price stickiness parameter equal to 0.5. In our numerical example, Case IIA never applies under rational expectations, and a necessary condition for determinacy is the generalized Taylor principle. While the Taylor principle achieves determinacy with $\lambda = 0.5$ under rational expectations (panel (a)), for a higher value of LAMP, condition (52) can only be satisfied in the interval $\mu_\pi \in [1, 3]$ for values of μ_y greater than one (panel (b)). Cognitive discounting has three important effects in expanding the regions of determinacy. First, the slope of the determinacy boundary associated with the behavioral Taylor principle becomes significantly steeper (panel (c)). Second, cognitive discounting enables condition (52) to hold under smaller



(a) $\lambda = 0.5$; $m^s = m^f = 1.0$ (b) $\lambda = 0.7$; $m^s = m^f = 1.0$

(c) $\lambda = 0.5$; $m^s = m^f = 0.5$ (d) $\lambda = 0.7$; $m^s = m^f = 0.5$

Figure 7: Determinacy (grey areas) under a current-looking Taylor rule: IADL economy ($\theta = 0.5$).

values of μ_y (panel (d)). Third, determinacy can also now arise under the inverted behavioral Taylor principle (50) (panel (d)). Therefore, regardless if the Taylor rule is forward looking or contemporaneous looking, cognitive discounting has a strong stabilizing effect on IADL economies.

5 Model extensions

In this section, we investigate the robustness of our results by relaxing some of the model assumptions. Specifically, we consider the determinacy implications of (i) the introduction of nominal illusion; (ii) generating cognitive discounting using the imperfect common knowledge approach of Angeletos and Lian (2018); and (iii) the inclusion of a cost-channel of monetary policy.

5.1 The role of nominal illusion

In the baseline model, the real interest rate in the Euler equation (10) is correctly perceived by behavioural households. Following Gabaix (2020), we now assume that unconstrained households only have access to a nominal savings market, where future inflation is perceived as $\mathrm{E}_t^{BR}\widehat{\pi}_{t+1} =$

$m^s_\pi \mathrm{E}_t \widehat{\pi}_{t+1}$. Consequently, the behavioural IS equation (23) becomes:

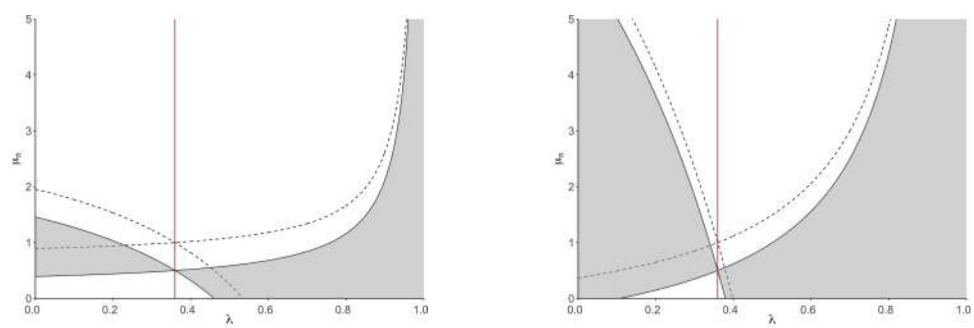
$$\widehat{Y}_t = m^s \mathrm{E}_t \widehat{Y}_{t+1} - \frac{1}{\sigma\delta}\left(\widehat{R}_t - m^s_\pi \mathrm{E}_t \widehat{\pi}_{t+1}\right). \tag{54}$$

Under nominal illusion, the discounting channel is strengthened as expected future inflation is also now discounted at rate $m^s_\pi < 1$. The behavioral NKPC (28) remains unchanged.

The conditions for determinacy under a hybrid Taylor rule (16) are given in Proposition 7 of the online appendix C. However, the determinacy effects of nominal illusion are most evident in the absence of a policy response to output (i.e., setting $\mu_y = 0$), where the local stability properties of the model collapse to:

$$\begin{aligned}
&\text{Case I:} \quad \delta > 0, \quad \textit{and} \quad m^s_\pi - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi} < \mu_\pi < m^s_\pi + \frac{\delta\sigma(1+m^s)(1+\beta m^f)}{\kappa\chi};\\
&\text{Case II:} \quad \delta < 0, \quad \textit{and} \quad m^s_\pi + \frac{\delta\sigma(1+m^s)(1+\beta m^f)}{\kappa\chi} < \mu_\pi < m^s_\pi - \frac{\delta\sigma(1-m^s)(1-\beta m^f)}{\kappa\chi}.
\end{aligned}$$

Relative to the $m^s_\pi = 1$ baseline, nominal illusion results in a larger reduction in both the upper and lower bounds on μ_π given in Cases I and II above. For example, while the lower bound is less likely to bind in SADL economies, expanding the determinacy region under a passive monetary policy, the upper bound is more likely to bind reducing the determinacy region under the Taylor principle. Figure 8 depicts the regions of (in)determinacy for combinations of λ and μ_π setting $m^s = m^f = m^s_\pi = 0.5$. The dashed lines depict the determinacy regions in the absence of nominal illusion ($m^s_\pi = 1$). While the net effect on the determinate policy space is ambiguous



(a) $\theta = 0.5$ (b) $\theta = 0.75$

Figure 8: Determinacy regions (grey areas) under a strict inflation-targeting rule with nominal illusion. Parameter values are $\beta = 0.99$, $\sigma = 0.5$, $\varphi = 2$, $\varepsilon = 11$, $m^s = m^f = m^s_\pi = 0.5$. The dashed lines illustrate the determinacy regions without nominal illusion ($m^s_\pi = 1$). The red vertical line depicts λ^* above which IADL holds.

for SADL economies, by reducing the determinacy region that arises under the Taylor principle, it is unambiguously smaller for IADL economies. Therefore, the stabilizing effects of bounded rationality on LAMP economies is reduced under nominal illusion.[16]

5.2 A behavioral LAMP model under imperfect common knowledge

Does the form of myopia that is introduced into the behavioral model matter for our results? To answer this question, we replace the macroeconomic inattention approach of Gabaix (2020) and instead introduce cognitive discounting due to deviations from common knowledge as in Angeletos and Lian (2018). Letting $\alpha \in (0,1]$ denote the degree of common knowledge, the behavioral versions of the IS equation and NKPC curve under imperfect common knowledge are given by:[17]

$$\widehat{Y}_t = m^s_\alpha E_t \widehat{Y}_{t+1} - \frac{1}{\sigma\delta}\left(\widehat{R}_t - \alpha \mathrm{E}_t \widehat{\pi}_{t+1}\right), \qquad m^s_\alpha \equiv \beta + (1-\beta)\alpha \in (\beta, 1], \tag{55}$$

$$\widehat{\pi}_t = \kappa\chi\alpha\widehat{Y}_t + \beta m^f_\alpha \mathrm{E}_t \widehat{\pi}_{t+1}, \qquad m^f_\alpha \equiv \theta + (1-\theta)\alpha \in (\theta, 1]. \tag{56}$$

In the behavioural IS equation (55), expected future output is discounted by $m^s_\alpha \approx 1$ with $\beta = 0.99$, but similar to the case of nominal illusion discussed above, expected future inflation is also discounted by a factor α. In (56), the slope of the behavioural NKPC is decreasing in α, and both θ and α affect the effective discount factor of firms m^f_α. The model collapses to the rational expectations benchmark by setting $\alpha = 1$.

Proposition 5 *Under a hybrid Taylor rule* (16)*, the necessary and sufficient conditions for equilibrium determinacy with imperfect common knowledge are:*

$$\textit{Case I:} \qquad \delta > 0, \qquad \mu_\pi + \frac{1-\beta m^f_\alpha}{\kappa\chi\alpha}\mu_y > \alpha - \frac{\delta\sigma(1-m^s_\alpha)(1-\beta m^f_\alpha)}{\kappa\chi\alpha}, \tag{57}$$

$$\textit{and} \qquad \mu_\pi < \alpha + \frac{1+\beta m^f_\alpha}{\kappa\chi\alpha}\mu_y + \frac{\delta\sigma(1+m^s_\alpha)(1+\beta m^f_\alpha)}{\kappa\chi\alpha}; \tag{58}$$

[16] As shown in the online appendix C, similar conclusions are obtained if the policy rule also responds to output.

[17] In the absence of LAMP, equations (55) and (56) are in agreement with the structural equation system given in Proposition 10 of the online appendix of Angeletos and Lian (2018).

$$\textit{Case IIA:} \quad \delta < 0, \quad 0 < \mu_y < -\delta\sigma(1 - \beta m_\alpha^s m_\alpha^f), \tag{59}$$

$$\textit{and} \quad \mu_\pi + \frac{1 - \beta m_\alpha^f}{\kappa\chi\alpha}\mu_y < \alpha - \frac{\delta\sigma(1 - m_\alpha^s)(1 - \beta m_\alpha^f)}{\kappa\chi\alpha}, \tag{60}$$

$$\textit{and} \quad \mu_\pi > \alpha + \frac{1 + \beta m_\alpha^f}{\kappa\chi\alpha}\mu_y + \frac{\delta\sigma(1 + m_\alpha^s)(1 + \beta m_\alpha^f)}{\kappa\chi\alpha}; \tag{61}$$

$$\textit{Case IIB:} \quad \delta < 0, \quad \mu_y > -\delta\sigma(1 + \beta m_\alpha^s m_\alpha^f), \tag{62}$$

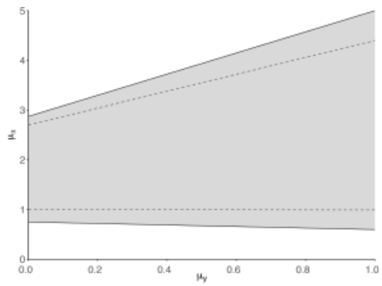
and *conditions* (57) *and* (58) *are satisfied.*

Proof. See online appendix D. ∎

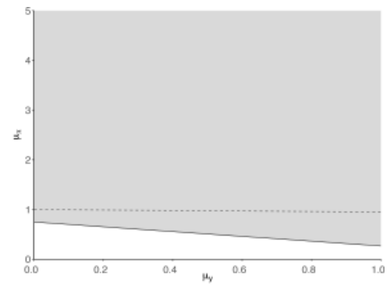
First consider the SADL case. Condition (57) is the imperfect common knowledge version of the behavioral Taylor principle. Similar to the macroeconomic inattention model, determinacy arises under a passive monetary policy, where the long-run inflation elasticity of output (or slope of the long-run NKPC) is now equal to $\frac{1-\beta m_\alpha^f}{\kappa\chi\alpha}$. Using the baseline parameter values, Figure 9 plots the results of Case I of Proposition 5 for combinations of the policy parameters μ_π and μ_y. We consider two values of $\alpha = 0.5, 0.75$, as justified by Angeletos and Lian (2018). The determinacy boundary given by the behavioral Taylor principle (57) behaves very similar to its macroeconomic inattention counterpart. The lower the value of α and the higher the value of θ, the larger the determinate policy space that arises under a passive policy. The crucial difference between the two behavioural models relates to the upper bound on μ_π given by (58), which shifts outwards (relative to the rational expectations baseline) with imperfect common knowledge, but shifts inwards under macroeconomic inattention. This difference arises from the fact that α directly affects the slope of the behavioral NKPC (56). As previously discussed, for sunspot equilibria to arise under an active monetary policy requires $\widehat{\pi}_t > 0 > \mathrm{E}_t\widehat{\pi}_{t+1}$. While a lower α reduces the weight of $\mathrm{E}_t\widehat{\pi}_{t+1}$ in (56), it also dampens its slope reducing the weight of $\widehat{Y}_t > 0$. Consequently, $\widehat{\pi}_t > 0$ is less likely to be supported under deflationary beliefs $\mathrm{E}_t\widehat{\pi}_{t+1} < 0$, reducing the possibility of sunspot equilibria. Therefore, in stark contrast to the baseline results of section 3, forward-looking policy in the absence of common knowledge unambiguously exerts a stabilizing effect on SADL economies.

For IADL economies, however, the stabilizing benefits of discounting are significantly reduced under imperfect common knowledge. Figure 10 plots the determinacy regions for the two values of $\alpha = 0.5, 0.75$ and LAMP, $\lambda = 0.5, 0.7$, setting $\theta = 0.75$.[18] First, the determinate policy space associated with the inverted behavioral Taylor principle (60) shrinks dramatically relative to the

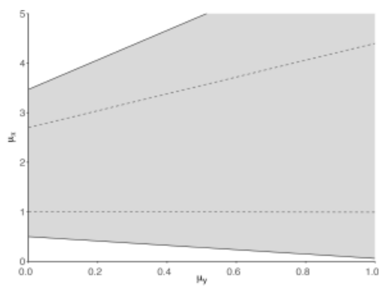
[18]As shown in the online appendix D, very similar findings are obtained with a lower degree of price stickiness.



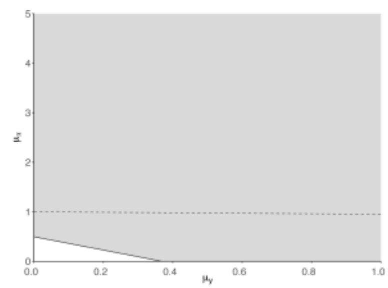
(a) $\theta = 0.5$; $\alpha = 0.75$



(b) $\theta = 0.75$; $\alpha = 0.75$

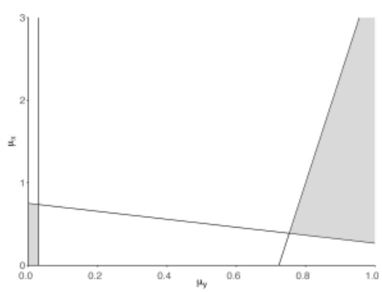


(c) $\theta = 0.5$; $\alpha = 0.5$

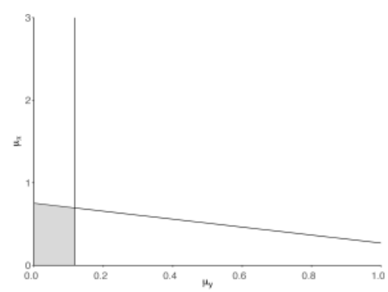


(d) $\theta = 0.75$; $\alpha = 0.5$

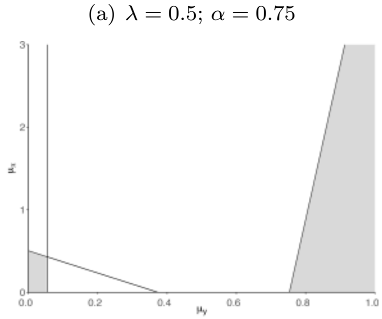
Figure 9: Determinacy regions (grey areas) under a hybrid Taylor rule and imperfect common knowledge: SADL economy. Parameter values are $\beta = 0.99$, $\sigma = 0.5$, $\varphi = 2$, $\varepsilon = 11$, $\lambda = 0$. Dashed lines illustrate the determinacy regions under rational expectations.



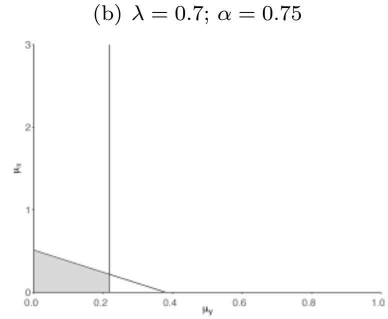
(a) $\lambda = 0.5$; $\alpha = 0.75$



(b) $\lambda = 0.7$; $\alpha = 0.75$



(c) $\lambda = 0.5$; $\alpha = 0.5$



(d) $\lambda = 0.7$; $\alpha = 0.5$

Figure 10: Determinacy regions (grey areas) under a hybrid Taylor rule and imperfect common knowledge: IADL economy ($\theta = 0.75$).

macroeconomic inattention model (see panels (c) and (d) of Figure 4). This arises because condition (59) only holds for small values of μ_y. Second, for larger values of μ_y, determinacy requires satisfying Case IIB of Proposition 5. Since the upper threshold (58) on μ_π is relatively smaller than its cognitive discounting counterpart, the determinate policy space is also reduced in this case. Indeed, as shown in panels (b) and (d) of Figure 10, the determinacy region associated with the behavioral Taylor principle (57) becomes non-existent when $\lambda = 0.7$, since to satisfy condition (62) in our numerical example, requires an unrealistically large policy coefficient of $\mu_y > 2.94$.

5.3 Bounded rationality and the cost channel of monetary policy

In the baseline model, monetary policy is transmitted by affecting aggregate demand. As a final robustness check, we now introduce a second transmission channel of monetary policy, where changes in the interest rate affect real marginal cost, and thus, the price-setting behaviour of firms.[19] Following Ravenna and Walsh (2006) and Llosa and Tuesta (2009), suppose that to pay their wage bill all firms must borrow from competitive intermediaries at the nominal interest rate R_t. Unconstrained households are assumed to receive profit income from both firms and intermediaries. In terms of log deviations from the steady state, aggregate real marginal cost is given by $\widehat{mc}_t = \chi \widehat{Y}_t + \widehat{R}_t$. Consequently, in the presence of a cost channel, the behavioural NKPC becomes:

$$\widehat{\pi}_t = \kappa\chi\widehat{Y}_t + \kappa\widehat{R}_t + \beta m^f \mathrm{E}_t\widehat{\pi}_{t+1}. \tag{63}$$

The behavioural IS equation (23) remains unchanged.

Proposition 6 *Under a hybrid Taylor rule* (16)*, the necessary and sufficient conditions for equilibrium determinacy with a cost channel of monetary policy are:*

$$\textit{Case IA:} \quad 0 < \frac{\chi}{\sigma(1+m^s)} < \delta, \qquad 0 < \mu_\pi < \frac{1-\beta m^s m^f}{\kappa m^s} + \frac{1}{\delta\sigma\kappa m^s}\mu_y \equiv \mathit{\Gamma}_1, \tag{64}$$

$$\textit{and} \quad \mu_\pi + \frac{1}{\kappa}\frac{1-\beta m^f-\kappa}{\chi-\delta\sigma(1-m^s)}\mu_y > 1 - \frac{1}{\kappa}\frac{\delta\sigma(1-m^s)(1-\beta m^f-\kappa)}{\chi-\delta\sigma(1-m^s)}; \tag{65}$$

$$\textit{Case IB:} \quad 0 < \delta < \frac{\chi}{\sigma(1+m^s)}, \quad \textit{conditions (64) and (65) are satisfied,}$$

$$\textit{and} \quad \mu_\pi < 1 + \frac{1}{\kappa}\frac{1+\beta m^f+\kappa}{\chi-\delta\sigma(1+m^s)}\mu_y + \frac{1}{\kappa}\frac{\delta\sigma(1+m^s)(1+\beta m^f+\kappa)}{\chi-\delta\sigma(1+m^s)} \equiv \mathit{\Gamma}_2; \tag{66}$$

[19] This cost channel of monetary policy is strongly supported by the empirical literature. See, e.g., Chowdhury et al. (2006), Ravenna and Walsh (2006), and Tillmann (2008), and the references therein.

Case IIA: $\quad \delta < 0, \quad$ *condition* (64) *is satisfied,*

$$\text{and} \quad \mu_\pi + \frac{1}{\kappa}\frac{1-\beta m^f-\kappa}{\chi-\delta\sigma(1-m^s)}\mu_y < 1 - \frac{1}{\kappa}\frac{\delta\sigma(1-m^s)(1-\beta m^f-\kappa)}{\chi-\delta\sigma(1-m^s)}, \tag{67}$$

$$\text{and} \quad \mu_\pi > 1 + \frac{1}{\kappa}\frac{1+\beta m^f+\kappa}{\chi-\delta\sigma(1+m^s)}\mu_y + \frac{1}{\kappa}\frac{\delta\sigma(1+m^s)(1+\beta m^f+\kappa)}{\chi-\delta\sigma(1+m^s)}; \tag{68}$$

Case IIB: $$\delta < 0, \quad 0 < \mu_\pi < -\frac{1+\beta m^s m^f}{\kappa m^s} - \frac{1}{\delta\sigma\kappa m^s}\mu_y \equiv \varGamma_3, \tag{69}$$

and conditions (65) *and* (66) *are satisfied.*

Proof. See online appendix E. ■

First note that for the parameter values used in the numerical analysis, Case IA of Proposition 6 never applies. Under rational expectations, the cost channel has two important implications for the determinacy of SADL economies. First, condition (65) implies that the long-run inflation elasticity of output $\frac{1-\beta-\kappa}{\kappa\chi}$, and hence the slope of the long-run NKPC, is negative for empirically realistic values of κ.[20] Consequently, a permanent increase in inflation results in a permanent *fall* in output and the central bank has to be more aggressive in reacting to inflation to prevent indeterminacy: $\mu_\pi > 1 + \frac{\kappa+\beta-1}{\kappa\chi}\mu_y$. Second, the upper-bound $\varGamma_1$ on μ_π given by condition (64) is likely to bind especially for low values of μ_y.[21] Similar to Llosa and Tuesta (2009), determinacy becomes impossible in the presence of the cost channel after setting $\mu_y = 0$: while condition (64) requires a passive monetary policy, this contradicts the Taylor principle implied by (65). Cognitive discounting dampens significantly the destabilizing effects of the cost channel. First, as illustrated in panels (b)–(d) of Figure 11, the long-run inflation elasticity of output switches sign and becomes positive once again. Second, the upper bounds $\varGamma_1$ and $\varGamma_2$ on μ_π both increase significantly, expanding the determinate policy space.[22]

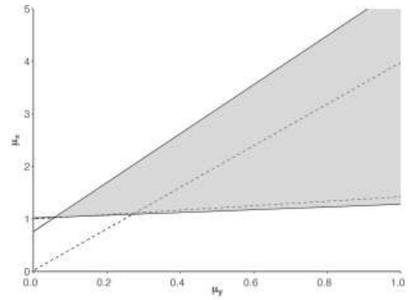
Similar conclusions arise for IADL economies. Using the baseline parameter values, Figure 12 illustrates Cases IIA and IIB of Proposition 6 setting $\theta = 0.75$. By inspection of panels (a) and (b), the cost channel drastically shrinks the determinate policy space under rational expectations. While determinacy can arise under the generalized Taylor principle (65) with $\lambda = 0.5$, in order to satisfy the upper-bound $\varGamma_3$ given by condition (69), a sufficiently large value of μ_y is required.[23]

[20] For example, setting $\beta = 0.99$ implies that $1 - \beta - \kappa < 0$ for all $\theta \leq 0.9$.

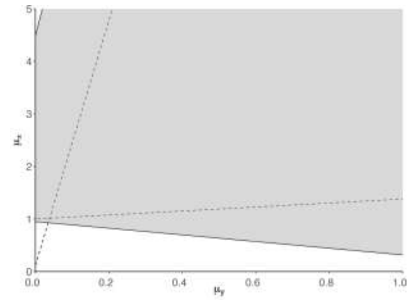
[21] Note that the cost channel results in indeterminacy of dimension two provided $\varGamma_1 < \mu_\pi < \varGamma_2$.

[22] Similar to the rational expectations benchmark, under our baseline parameter values the upper bounds $\varGamma_1 < \varGamma_2$ if $m^s = 0.85$ and $m^f = 0.8$. However, the reverse holds and $\varGamma_2 < \varGamma_1$ with $m^s = 0.5$ and $m^f = 0.5$ under both values of $\theta = 0.5, 0.75$.

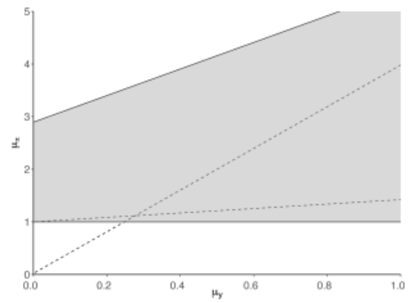
[23] Note that $\varGamma_1 < 0$ for all $\mu_y > 0$ with $\lambda = 0.5$ and thus Case IIA of Proposition 6 can only be satisfied in the absence of a policy response to output, $\mu_y = 0$.



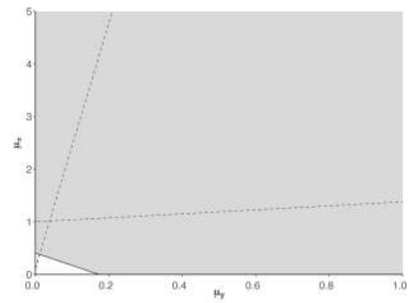
(a) $\theta = 0.5$; $m^s = 0.85$, $m^f = 0.8$



(b) $\theta = 0.75$; $m^s = 0.85$, $m^f = 0.8$

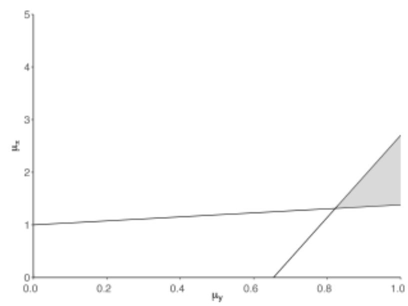


(c) $\theta = 0.5$; $m^s = m^f = 0.5$

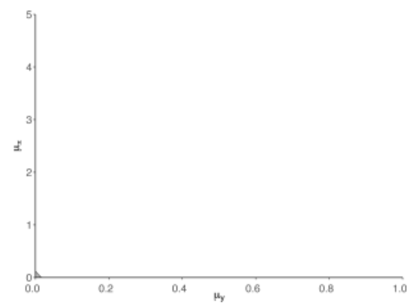


(d) $\theta = 0.75$; $m^s = m^f = 0.5$

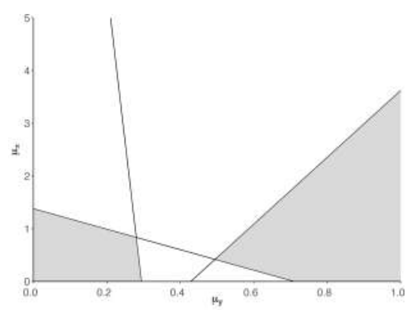
Figure 11: Determinacy (grey areas) under a hybrid Taylor rule and cost channel: SADL economy ($\lambda = 0$). Dashed lines illustrate the determinacy regions under rational expectations.



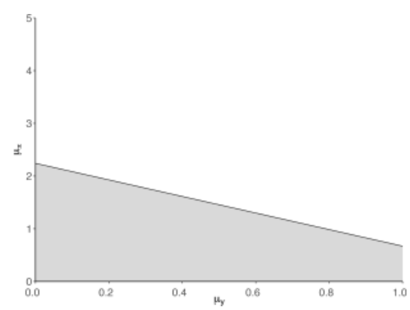
(a) $\lambda = 0.5$; $m^s = m^f = 1.0$



(b) $\lambda = 0.7$; $m^s = m^f = 1.0$



(c) $\lambda = 0.5$; $m^s = m^f = 0.5$



(d) $\lambda = 0.7$; $m^s = m^f = 0.5$

Figure 12: Determinacy (grey areas) under a hybrid rule & cost channel: IADL economy ($\theta = 0.75$).

Under $\lambda = 0.7$, condition (68) is always satisfied within the parameter space (μ_π, μ_y), and determinacy requires satisfying the inverted generalized Taylor principle (67). However, the upper-bound $\mathit{\Gamma}_1$ given by (64) is very close to zero, rendering the equilibrium indeterminate for nearly all possible parameter combinations (μ_π, μ_y). As highlighted by panels (c) and (d) of Figure 12, cognitive discounting helps ameliorate the indeterminacy problem for IADL economies. Myopia expands the upper-bound $\mathit{\Gamma}_1$ increasing the determinacy region under the inverted generalized Taylor principle for both values of λ. Moreover, by increasing the upper-bound $\mathit{\Gamma}_3$, the determinate policy space also expands significantly under the generalized Taylor principle for $\lambda = 0.5$ (panel (c)).

6 Conclusions

This paper has examined the implications of bounded rationality for equilibrium determinacy. Our modelling framework includes limited asset market participation, which is an important feature of many real-world economies, and focuses on forecast-based interest rate policy, which is empirically-relevant for inflation targeting central banks.

Our main findings reveal that bounded rationality plays a crucial role in the ability of active and passive policies to achieve determinacy in both SADL and IADL economies. Cognitive discounting exerts a strong stabilizing effect in IADL economies, such that passive policy is no longer a necessary condition for preventing indeterminacy, and determinacy can be restored under the traditional Taylor principle. In SADL economies, determinacy is undermined with myopia under the Taylor principle, whereas the determinacy region expands under a passive policy. Our results are shown to be robust for a variety of strict and flexible inflation targeting rules that also respond to output.

Our baseline results suggest that the commonly-advocated use of the Taylor principle in SADL economies, and the inverted Taylor principle in IADL economies, may be ill-advised. However, it is important to stress that these policy prescriptions are a feature of adopting the popular cognitive discounting approach of Gabaix (2020) and do not generalize to other bounded rationality frameworks, such as the imperfect common knowledge model of Angeletos and Lian (2018). We leave it to the empirical literature to determine which behavioral model the data prefers. One practical conclusion that does appear to be robust and should be of interest to IFT central banks relates to the indeterminacy problem that arises under a cost-channel of monetary policy. Bounded rationality is shown to exert an important stabilizing role in both SADL and IADL economies by helping to ameliorate the indeterminacy that arises from the cost-channel effect.

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Appendix: Proofs and Additional Results
(for online publication)

A Proof of proposition 3

The model equations (23), (28), along with the Taylor rule (40) with $j = 1$, can be reduced to the following two-dimensional system:

$$\mathrm{E}_t\mathbf{z}_{t+1} = \mathbf{A_3}\mathbf{z}_t, \qquad \mathbf{z}_t = \left[\widehat{Y}_t\ \widehat{\pi}_t\right]', \qquad \mathbf{A_3} \equiv \begin{bmatrix} \frac{\delta\sigma}{\delta\sigma m^s-\mu_y} - \frac{\kappa\chi(\mu_\pi-1)}{\beta m^f(\delta\sigma m^s-\mu_y)} & \frac{\mu_\pi-1}{\beta m^f(\delta\sigma m^s-\mu_y)} \\ -\frac{\kappa\chi}{\beta m^f} & \frac{1}{\beta m^f} \end{bmatrix}.$$

The determinate and trace of the coefficient matrix $\mathbf{A_3}$ are:

$$\det \mathbf{A_3} = \frac{\delta\sigma}{\beta m^f(\delta\sigma m^s - \mu_y)} \quad \text{and} \quad \mathrm{tr}\mathbf{A_3} = \frac{\delta\sigma}{\delta\sigma m^s - \mu_y} + \frac{1}{\beta m^f} - \frac{(\mu_\pi - 1)\kappa\chi}{\beta m^f(\delta\sigma m^s - \mu_y)}.$$

Since there are no predetermined variables, determinacy requires that both eigenvalues are outside the unit circle. According to the Schur-Cohn criterion, this requires that (i) $|\det \mathbf{A_3}| > 1$ and (ii) $|\mathrm{tr}\mathbf{A_3}| < 1 + \det \mathbf{A_3}$. If $\delta > 0$, $\det \mathbf{A_3} > 1$ requires $0 < \mu_y < \delta\sigma m^s$, and $1 + \det \mathbf{A_3} - \mathrm{tr}\mathbf{A_3} > 0$ implies (42), while $1 + \det \mathbf{A_3} + \mathrm{tr}\mathbf{A_3} > 0$ implies (43). Next note that $\det \mathbf{A_3} < -1$ requires $\delta\sigma m^s < \mu_y < \delta\sigma(m^s + 1/\beta m^f)$ and $|\mathrm{tr}\mathbf{A_3}| < -1 - \det \mathbf{A_3}$ implies (42) and (43). If $\delta < 0$, $\det \mathbf{A_3} > 1$ requires $0 < \mu_y < -(\delta\sigma/\beta m^f)(1 - \beta m^s m^f)$ and condition (ii) implies (45) and (46). Finally note that $\det \mathbf{A_3} < -1$ can never be satisfied under $\delta < 0$. This completes the proof. ■

B Proof of proposition 4

The model equations (23), (28), along with the Taylor rule (40) with $j = 0$, can be reduced to the following two-dimensional system:

$$\mathrm{E}_t\mathbf{z}_{t+1} = \mathbf{A_4}\mathbf{z}_t, \qquad \mathbf{z}_t = \left[\widehat{Y}_t\ \widehat{\pi}_t\right]', \qquad \mathbf{A_4} \equiv \begin{bmatrix} \frac{1}{m^s} + \frac{\mu_y}{\delta\sigma m^s} + \frac{\kappa\chi}{\beta\delta\sigma m^s m^f} & \frac{\mu_\pi}{\delta\sigma m^s} - \frac{1}{\beta\delta\sigma m^s m^f} \\ -\frac{\kappa\chi}{\beta m^f} & \frac{1}{\beta m^f} \end{bmatrix}.$$

The determinate and trace of the coefficient matrix $\mathbf{A_4}$ are:

$$\det \mathbf{A_4} = \frac{1}{\beta m^s m^f} + \frac{\kappa\chi\mu_\pi + \mu_y}{\beta\delta\sigma m^s m^f} \quad \text{and} \quad \mathrm{tr}\mathbf{A_4} = \frac{1}{m^s} + \frac{1}{\beta m^f} + \frac{\mu_y}{\delta\sigma m^s} + \frac{\kappa\chi}{\beta\delta\sigma m^s m^f}.$$

With no predetermined variables, determinacy requires that both eigenvalues are outside the unit circle. According to the Schur-Cohn criterion, this requires that (i) $|\det \mathbf{A_4}| > 1$ and (ii) $|\mathrm{tr}\mathbf{A_4}| < 1 + \det \mathbf{A_4}$. If $\delta > 0$, $\det \mathbf{A_4} > 1$ and $1 + \det \mathbf{A_4} + \mathrm{tr}\mathbf{A_4} > 0$ are always satisfied, and $1 + \det \mathbf{A_4} - \mathrm{tr}\mathbf{A_4} > 0$ implies (47). If $\delta < 0$, $\det \mathbf{A_4} > 1$ requires $\kappa\chi\mu_\pi + \mu_y < -\delta\sigma(1 - \beta m^s m^f)$ and condition (ii) implies (49) and (50). Next note that $\det \mathbf{A_4} < -1$ requires $\kappa\chi\mu_\pi + \mu_y > -\delta\sigma(1 + \beta m^s m^f)$ and $\delta < 0$. Then $|\mathrm{tr}\mathbf{A_4}| < -1 - \det \mathbf{A_4}$ implies (52) and (53). This completes the proof. ■

C Nominal illusion

Proposition 7 *Under a hybrid Taylor rule* (16)*, the necessary and sufficient conditions for equilibrium determinacy with nominal illusion are:*

$$\textit{Case I:} \quad \delta > 0, \quad \mu_\pi + \frac{1 - \beta m^f}{\kappa\chi}\mu_y > m_\pi^s - \frac{\delta\sigma(1 - m^s)(1 - \beta m^f)}{\kappa\chi}, \tag{70}$$

$$\textit{and} \quad \mu_\pi < m_\pi^s + \frac{1 + \beta m^f}{\kappa\chi}\mu_y + \frac{\delta\sigma(1 + m^s)(1 + \beta m^f)}{\kappa\chi}; \tag{71}$$

$$\textit{Case IIA:} \quad \delta < 0, \quad 0 < \mu_y < -\delta\sigma(1 - \beta m^s m^f), \tag{72}$$

$$\textit{and} \quad \mu_\pi + \frac{1 - \beta m^f}{\kappa\chi}\mu_y < m_\pi^s - \frac{\delta\sigma(1 - m^s)(1 - \beta m^f)}{\kappa\chi}, \tag{73}$$

$$\textit{and} \quad \mu_\pi > m_\pi^s + \frac{1 + \beta m^f}{\kappa\chi}\mu_y + \frac{\delta\sigma(1 + m^s)(1 + \beta m^f)}{\kappa\chi}; \tag{74}$$

$$\textit{Case IIB:} \quad \delta < 0, \quad \mu_y > -\delta\sigma(1 + \beta m^s m^f), \tag{75}$$

and conditions (70) *and* (71) *are satisfied.*

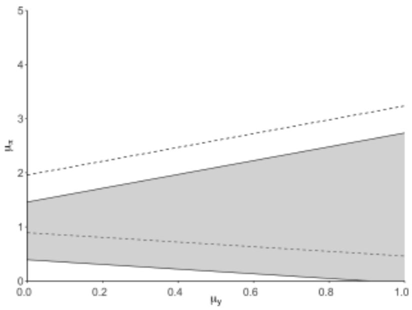
Proof. The model equations (16), (28), and (54) can be reduced to:

$$\mathrm{E}_t \mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t, \qquad \mathbf{z}_t = \left[\widehat{Y}_t \ \widehat{\pi}_t\right]', \qquad \mathbf{A} \equiv \begin{bmatrix} \frac{1}{m^s} + \frac{\mu_y}{\sigma\delta m^s} - \frac{\kappa\chi(\mu_\pi - m_\pi^s)}{\beta\delta\sigma m^s m^f} & \frac{\mu_\pi - m_\pi^s}{\beta\delta\sigma m^s m^f} \\ -\frac{\kappa\chi}{\beta m^f} & \frac{1}{\beta m^f} \end{bmatrix}.$$

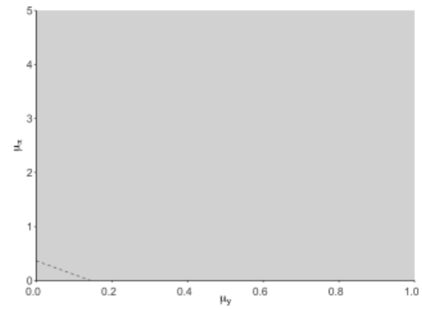
The determinate and trace of the coefficient matrix $\mathbf{A}$ are:

$$\det \mathbf{A} = \frac{1}{\beta m^s m^f} + \frac{\mu_y}{\beta\delta\sigma m^s m^f} \quad \text{and} \quad \mathrm{tr}\mathbf{A} = \frac{1}{m^s} + \frac{1}{\beta m^f} + \frac{\mu_y}{\delta\sigma m^s} - \frac{(\mu_\pi - m_\pi^s)\kappa\chi}{\beta\delta\sigma m^s m^f}.$$

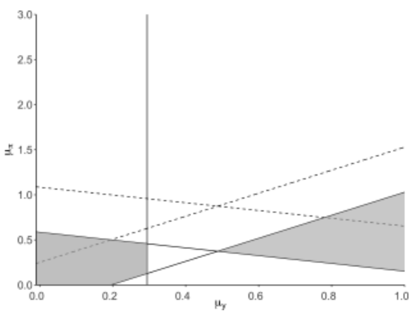
With no predetermined variables, determinacy requires that both eigenvalues are outside the unit circle, which requires that (i) $|\det \mathbf{A}| > 1$ and (ii) $|\mathrm{tr}\mathbf{A}| < 1 + \det \mathbf{A}$. If $\delta > 0$, $\det \mathbf{A} > 1$ is always satisfied, and $1 + \det \mathbf{A} - \mathrm{tr}\mathbf{A} > 0$ implies (70), while $1 + \det \mathbf{A} + \mathrm{tr}\mathbf{A} > 0$ implies (71). If $\delta < 0$, $\det \mathbf{A} > 1$ requires $0 < \mu_y < -\delta\sigma(1 - \beta m^s m^f)$ and condition (ii) implies (73) and (74). Next note that $\det \mathbf{A} < -1$ requires $\mu_y > -\delta\sigma(1 + \beta m^s m^f)$ and $\delta < 0$. Then $|\mathrm{tr}\mathbf{A}| < -1 - \det \mathbf{A}$ implies (70) and (71). This completes the proof. ∎



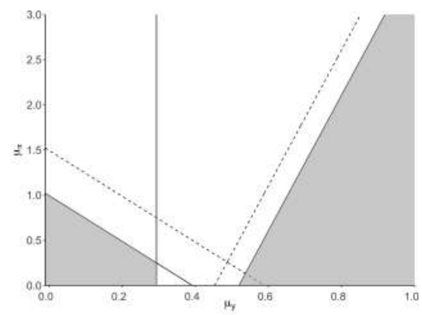
(a) $\theta = 0.5$; $\lambda = 0$



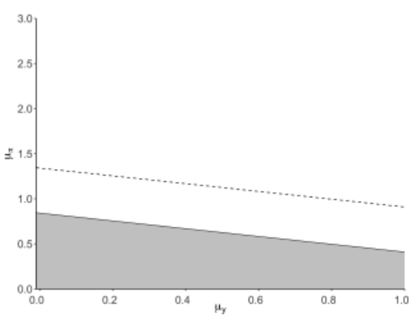
(b) $\theta = 0.75$; $\lambda = 0$



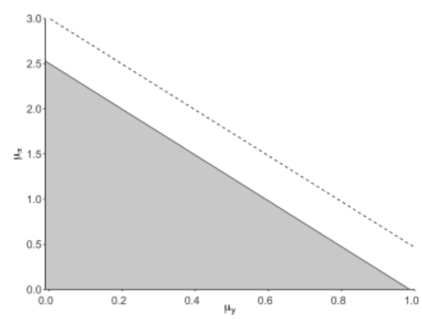
(c) $\theta = 0.5$; $\lambda = 0.5$



(d) $\theta = 0.75$; $\lambda = 0.5$



(e) $\theta = 0.5$; $\lambda = 0.7$



(f) $\theta = 0.75$; $\lambda = 0.7$

Figure A.1: Determinacy regions (grey areas) under a hybrid Taylor rule with nominal illusion. Parameter values are $\beta = 0.99$, $\sigma = 0.5$, $\varphi = 2$, $\varepsilon = 11$, $m^s = m^f = m^s_\pi = 0.5$. The dashed lines illustrate the regions of determinacy in the absence of nominal illusion ($m^s_\pi = 1$).

D The imperfect common knowledge LAMP model

D.1 Proof of proposition 5

The model equations (16), (55), and (56) can be reduced to:

$$\mathrm{E}_t\mathbf{z}_{t+1} = \mathbf{A_5}\mathbf{z}_t, \qquad \mathbf{z}_t = \left[\widehat{Y}_t \ \widehat{\pi}_t\right]', \qquad \mathbf{A_5} \equiv \begin{bmatrix} \frac{1}{m_\alpha^s} + \frac{\mu_y}{\delta\sigma m_\alpha^s} - \frac{\kappa\chi\alpha(\mu_\pi-\alpha)}{\beta\delta\sigma m_\alpha^s m_\alpha^f} & \frac{\mu_\pi-\alpha}{\beta\delta\sigma m_\alpha^s m_\alpha^f} \\ -\frac{\kappa\chi\alpha}{\beta m_\alpha^f} & \frac{1}{\beta m_\alpha^f} \end{bmatrix}.$$

The determinate and trace of the coefficient matrix $\mathbf{A_5}$ are:

$$\det \mathbf{A_5} = \frac{1}{\beta m_\alpha^s m_\alpha^f} + \frac{\mu_y}{\beta\delta\sigma m_\alpha^s m_\alpha^f} \quad \text{and} \quad \mathrm{tr}\mathbf{A_5} = \frac{1}{m_\alpha^s} + \frac{1}{\beta m_\alpha^f} + \frac{\mu_y}{\delta\sigma m_\alpha^s} - \frac{(\mu_\pi-\alpha)\kappa\chi\alpha}{\beta\delta\sigma m_\alpha^s m_\alpha^f}.$$

With no predetermined variables, determinacy requires that both eigenvalues are outside the unit circle, which requires that (i) $|\det \mathbf{A_5}| > 1$ and (ii) $|\mathrm{tr}\mathbf{A_5}| < 1 + \det \mathbf{A_5}$. If $\delta > 0$, $\det \mathbf{A_5} > 1$ is always satisfied, and $1+\det \mathbf{A_5} - \mathrm{tr}\mathbf{A_5} > 0$ implies (57), while $1+\det \mathbf{A_5} + \mathrm{tr}\mathbf{A_5} > 0$ implies (58). If $\delta < 0$, $\det \mathbf{A_5} > 1$ requires $0 < \mu_y < -\delta\sigma(1-\beta m_\alpha^s m_\alpha^f)$ and condition (ii) yields (60) and (61). Next note that $\det \mathbf{A_5} < -1$ requires $\mu_y > -\delta\sigma(1+\beta m_\alpha^s m_\alpha^f)$ and $\delta < 0$. Then $|\mathrm{tr}\mathbf{A_5}| < -1 - \det \mathbf{A_5}$ gives (57) and (58). This completes the proof. ■

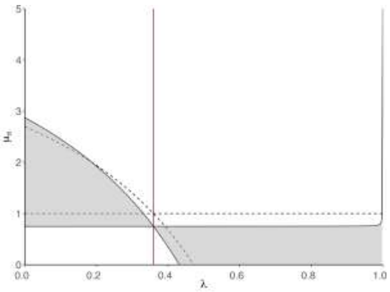
D.2 Determinacy under a strict inflation-targeting rule

Under a strict inflation-targeting rule, the necessary and sufficient conditions for determinacy with imperfect common knowledge are given by:

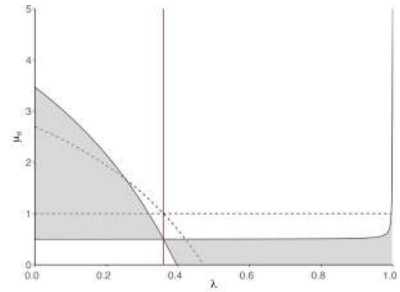
$$\text{Case I:} \quad \delta > 0, \quad \textit{and} \quad \alpha - \frac{\delta\sigma(1-m_\alpha^s)(1-\beta m_\alpha^f)}{\kappa\chi\alpha} < \mu_\pi < \alpha + \frac{\delta\sigma(1+m_\alpha^s)(1+\beta m_\alpha^f)}{\kappa\chi\alpha}; \tag{76}$$

$$\text{Case II:} \quad \delta < 0, \quad \textit{and} \quad \alpha + \frac{\delta\sigma(1+m_\alpha^s)(1+\beta m_\alpha^f)}{\kappa\chi\alpha} < \mu_\pi < \alpha - \frac{\delta\sigma(1-m_\alpha^s)(1-\beta m_\alpha^f)}{\kappa\chi\alpha}. \tag{77}$$

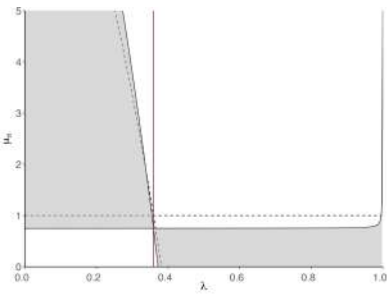
Proof. After setting $\mu_y = 0$, the two conditions (57) and (58) of Case I of Proposition 5 collapse to (76). For Case IIA, condition (59) is always satisfed and the conditions (60) and (61) collapse to (77). Since condition (62) can never be satisfied, Case IIB does not apply. ■



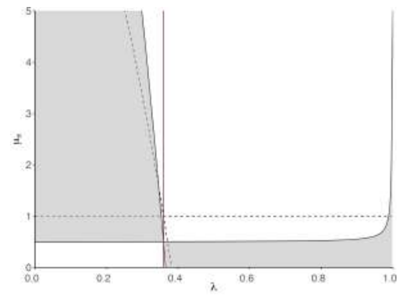
(a) $\theta = 0.5$; $\alpha = 0.75$



(b) $\theta = 0.5$; $\alpha = 0.5$

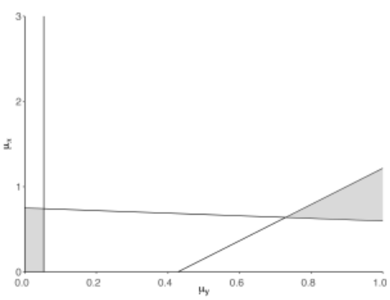


(c) $\theta = 0.75$; $\alpha = 0.75$

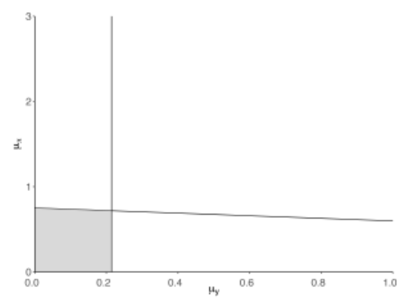


(d) $\theta = 0.75$; $\alpha = 0.5$

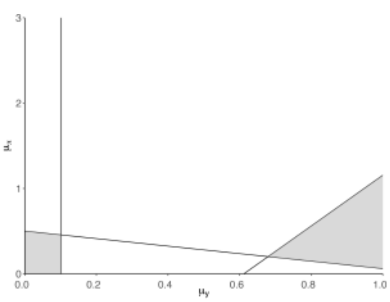
Figure A.2: Determinacy regions (grey areas) under a strict inflation-targeting rule and imperfect common knowledge. Dashed lines illustrate the determinacy regions under rational expectations.



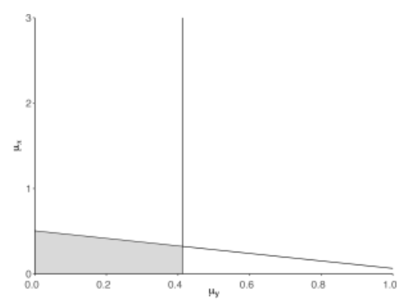
(a) $\lambda = 0.5$; $\alpha = 0.75$



(b) $\lambda = 0.7$; $\alpha = 0.75$



(c) $\lambda = 0.5$; $\alpha = 0.5$



(d) $\lambda = 0.7$; $\alpha = 0.5$

Figure A.3: Determinacy regions (grey areas) under a hybrid Taylor rule and imperfect common knowledge: IADL economy ($\theta = 0.5$).

E Proof of proposition 6

The model equations (16), (23), and (63) can be expressed as:

$$\mathrm{E}_t \mathbf{z}_{t+1} = \mathbf{A_6} \mathbf{z}_t, \qquad \mathbf{z}_t = \left[\widehat{Y}_t \ \widehat{\pi}_t\right]', \qquad \mathbf{A_6} \equiv \begin{bmatrix} \frac{\delta\sigma+\mu_y}{\delta\sigma m^s} - \frac{\kappa(\chi+\mu_y)(\mu_\pi-1)}{\delta\sigma m^s(\beta m^f+\kappa\mu_\pi)} & \frac{\mu_\pi-1}{\delta\sigma m^s[\beta m^f+\kappa\mu_\pi]} \\ -\frac{\kappa(\chi+\mu_y)}{\beta m^f+\kappa\mu_\pi} & \frac{1}{\beta m^f+\kappa\mu_\pi} \end{bmatrix}.$$

The determinate and trace of the coefficient matrix $\mathbf{A_6}$ are:

$$\det \mathbf{A_6} = \frac{1}{m^s\left(\beta m^f + \kappa\mu_\pi\right)}\left(1 + \frac{\mu_y}{\delta\sigma}\right) \qquad \text{and}$$
$$\mathrm{tr}\mathbf{A_6} = \frac{1}{m^s}\left(1 + \frac{\mu_y}{\delta\sigma}\right) + \frac{1}{\beta m^f + \kappa\mu_\pi} - \frac{\kappa\left(\chi + \mu_y\right)\left(\mu_\pi - 1\right)}{\delta\sigma m^s\left(\beta m^f + \kappa\mu_\pi\right)}.$$

Determinacy requires that both eigenvalues are outside the unit circle, which requires that (i) $|\det \mathbf{A_6}| > 1$ and (ii) $|\mathrm{tr}\mathbf{A_6}| < 1 + \det \mathbf{A_6}$. If $\delta > 0$, $\det \mathbf{A_6} < -1$ is not possible, whereas $\det \mathbf{A_6} > 1$ requires (64). It follows that $|\mathrm{tr}\mathbf{A_6}| < 1 + \det \mathbf{A_6}$ can be reduced to:

$$\left[1 - \frac{\delta\sigma\left(1 - m^s\right)}{\chi}\right]\mu_\pi + \left(\frac{1 - \beta m^f - \kappa}{\kappa\chi}\right)\mu_y > 1 - \frac{\delta\sigma(1 - m^s)(1 - \beta m^f)}{\kappa\chi}, \tag{78}$$
$$(1 + m^s)(1 + \beta m^f) + \frac{\kappa\chi}{\delta\sigma} + \left(1 + \beta m^f + \kappa\right)\frac{\mu_y}{\delta\sigma} + \kappa\left(1 + m^s - \frac{\chi}{\delta\sigma}\right)\mu_\pi > 0. \tag{79}$$

Since $\delta \leq 1$, it follows that $\chi - \delta\sigma\left(1 - m^s\right) > 0$, and (78) reduces to (65). If $1 + m^s - \frac{\chi}{\delta\sigma} > 0$, then (79) is always satisfied. Otherwise, an additional condition given by (66) is also required. If $\delta < 0$, $\det \mathbf{A_6} > 1$ requires (64) and condition (ii) implies (67) and (68). Next, $\det \mathbf{A_6} < -1$ requires (69). Then $|\mathrm{tr}\mathbf{A_6}| < -1 - \det \mathbf{A}$ gives (65) and (66). This completes the proof. ■