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Children and Non-Participation in a Model of Collective Household Labor Supply*

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Abstract

The collective model of household behavior is extended to consider the existence of public consumption, like expenditures on children, together with the possibility of non-participation in the labor market of one partner of the adult couple. This model argues that structural elements of the decision process, such as individual preferences and the intra-household distribution rule of non-public expenditure, can be identified by observing labor supply of each individual and total expenditures on the public good. The identification rests on the existence of a variable that affects household behavior only through its impact on the decision process, i.e. a distribution factor, and the existence and uniqueness of a reservation wage for each household member at which both members are indifferent to whether a member participates or not. This setting provides a conceptual framework for addressing issues related to the impact of the potential wage of a non-participating member on household allocations and the targeting of specific benefits or taxes.

Keywords: Collective household models · Children · Labor supply · Non-participation
JEL classification: D11 · J13 · J22

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1 Introduction

The collective approach, unlike the unitary, provides an adequate theoretical background to analyze intra-household allocations; how individual preferences and the decision process can be recovered from household members' aggregate behavior. This model draws upon the idea that an increment on the decision power of one household member changes household behavior in his or her favor, even though total household resources are kept constant. In that sense, the collective approach can be used to analyze targeting of programs. This is the case of conditional cash transfer (CCT) public programs, in which a household receives a monetary compensation for the fulfillment of certain requirements that are positively related with the household welfare, its goal in many cases is to foment human capital of children. However, some programs give the cash transfer to a particular household member (generally the mother[1]) instead of the intended recipient (children). Therefore, the impact of the cash transfer on a child's consumption depends on how the intra-household allocation processes distributes this additional income; it could mitigate or enhance child's consumption. Furthermore, these programs could influence also other household outcomes, such as the labor status of the cash transfer receiver.

The labor supply model of Chiappori (1992) has been extensively used for empirical applications (see for example Fortin and Lacroix 1997, Canada; Chiappori, Fortin, and Lacroix 2002, the USA); however this framework considers the simplest possible case of household structure (childless households with two working members). This paper extends Chiappori (1992)'s model to incorporate both the presence of children and the decision to participate in the labor market. In the setting of Blundell, Chiappori, and Meghir (2005) that takes into account the presence of children in a household, the model employs the method used by Donni (2003) to address with the possibility of non-participation. The proposed model generalizes the identification results of Chiappori (1992); individual preferences and the sharing rule can be recovered form observed behavior. Identification requires the knowledge of a distribution factor and the existence of a unique reservation wage for each adult household member at which both members are indifferent between that a member participates or not in the labor market. In contexts where children and a non-working partner are frequently observed in a household, empirical applications of the model should increase the sample size and reduce related selection bias. Also, the model can be used to

[1]Examples of CCT programs that gives the transfer preferably to the mother (or female household head) are: Bono de Desarrollo Humano (Ecuador), Chile Solidario (Chile), Familias en Acción (Colombia), Progresa/Oportunidades (Mexico), Programa de Asistencia Familiar - PRAF (Honduras), Red de Protección Social - RPS (Nicaragua).

analyze the impact of CCT on expenditures on children and on household members' labor supplies.

1.1 The collective approach

Given that a household's demand for goods and the labor supply of its members can be obtained from household survey data, what can be said about the structural components of the decision process behind these data that lead to this household behavior? Because the traditional unitary approach, under certain *ad hoc* assumptions to aggregate individual preferences, considers a household as a single decision-making unit; it remains unexplained how the household reaches its agreement to allocate resources. Furthermore, the distinction between individual and household preferences is irrelevant in this approach, which is unsatisfactory from the perspective of welfare analysis.[2] From a public policy perspective, this framework imposes an empirical strait-jacket on policy analysis with an individual targeting emphasis, since price changes are the only tool available for intra-household reallocations (Quisumbing and McClafferty 2006). This approach has also been criticized for a lack of empirical support of its theoretical implications, such as that total income but not its source matters in household consumption decisions (i.e., household members pool their income),[3] or that cross-price substitution effects are symmetric (e.g., the compensated wage changes of spouses have the same effect on each other's labor supply).[4]

Alternative approaches, such as non-cooperative and cooperative (or collective) models, have tried to take into account the multiplicity and heterogeneity of decision makers in a household.[5] On the one hand, in the absence of binding and enforceable agreements

[2]Neverthless, using a strategy that provides more structure to the household allocation problem or that uses more extensive data, it is possible in this framework to do welfare analysis at the individual level (Donni 2008a).

[3]For the rejection of the income pooling hypothesis see among others Thomas 1990; Bourguignon, Browning, Chiappori, and Lechene 1993; Browning, Bourguignon, Chiappori, and Lechene 1994; Lundberg, Pollak, and Wales 1997; Fortin and Lacroix 1997.

[4]For the rejection of the symmetry of the Slutsky matrix see among others Browning and Meghir 1991; Blundell, Pashardes, and Weber 1993; Fortin and Lacroix 1997; Browning and Chiappori 1998.

[5]Some unitary models propose ways to incorporate multiple individuals in the analysis. Samuelson (1956) considers that a household could act as one individual if its members agree on the way of how aggregate their preferences, as a result they choose to maximize a social welfare function. By the "rotten-kid theorem", Becker (1974; 1991) arrives to a household objective function that converges to the preferences of the "altruistic head" of the household. Suppose that a household consist of two members, an altruistic head and an egotistic member. Trying to avoid retaliations of the head, the other individual would not attempt to increment their consumption (behave rottenly) at the expense of the head's consumption. However, these efforts are supported in strong assumptions (Bergstrom 1989; Haddad, Hoddinott, and Alderman 1997) and its "outcomes are empirically indistinguishable from those of constrained individual utility maximization" (McElroy and Horney 1981, 333).

between household members, non-cooperative models have assumed that household members maximize their utility subject to an individual budget constraint and taking as given each others' behavior. However, the intra-household allocations under this framework are not necessarily Pareto efficient; if we consider deviations of the equilibrium outcome, it is possible to increase the welfare of one household member without reducing that of others. In a household context this result is not very satisfactory, since possibilities for Pareto improvements may arise from daily interaction among their members. On the other hand, the only assumption that household collective models have in common is that household decisions are Pareto efficient,[6] so it is not necessary to specify the actual process that determines the intra-household allocation on the efficiency frontier, but only to assume that it exists.

Efficiency means that household allocations are optimal; no other consumption bundle could provide more utility for household members at the same cost. In this sense, an equivalent interpretation of Pareto efficiency is that household members initially reach an agreement on the respective amount each is allowed to spend, a "sharing rule." Then, all members independently choose their consumption subject to their respective share. The approach does not impose a particular form on the rule; it only requires that it exists.

While assuming efficiency of household decisions reduces the set of possible allocations, there could exist a continuum of different structural models that generates the same observable behavior (Chiappori and Ekeland 2009). It is in this sense that particular hypotheses over goods or preferences have been made within the collective framework to recover preferences and decision making from household aggregate demand. The main identification results have been made for the case where all goods consumed in a household are private (i.e., they are consumed non-jointly and exclusively by each member); where one member's consumption does not have a direct effect on another member's well being; and where it is considered an interior solution for household demands. Intuitively, the quantities consumed by each member are a guide of the intra-household bargaining power distribution: the consumption of a good associated with a particular individual will be greater as her or his decision power increases.

Applying the collective framework to the case of household labor supply, the seminal col-

[6] This Pareto efficiency assumption can be justified if all household members are aware of the preferences and actions of each other (there is symmetric information, possibly due to proximity and durability of the household), so they can decide to cooperate to make all better off by means of a binding agreement. Alternatively, this agreement can emerge if the relations between household members can be represented as a repeated game. For a more detailed discussion about assuming efficiency see Browning, Chiappori, and Weiss (2011).

lective model proposed by Chiappori (1988; 1992) allows, under certain assumptions, the recovering of some elements of the decision process from the observed labor supply of their members. Since these results are derived from the simplest possible case, empirical applications based on this model have been used as an observation unit: childless households composed of two adult members who participate in the labor market. However, estimates obtained with this type of sample could be imprecise due to small sample size and may be subject to selection biases if only households with positive hours of work for their members are considered (Fortin and Lacroix 1997).

To properly assess the collective framework as a useful tool for welfare evaluation and policy analysis on an intra-household level, it is necessary not to limit the analysis to childless households with members who participate in the labor market. The objective of this research is to develop a collective theoretical framework that simultaneously takes into account the presence of children and the decision to participate in the labor market.

Although the paper is essentially theoretical, it refers to empirically testable restrictions on household labor supply and to obtain information about aspects of the intra-household decision process, which can be used for individual welfare analysis and policy evaluation. Indeed, analysis using the collective approach would have limited empirical content if its concepts could not be recovered from observed behavior. For example, the proposed model provides an adequate framework for the analysis of a social program targeted at a particular household member (say, a female member). If the policy increased woman's influence in the household decision process, what would be the impact on household demand? In particular, what would the impact be on household expenditures on children? Could it be that this female "empowerment" would also lead to woman's non-participation in the labor market, or could it be the case that she puts so much emphasis on spending on children that she decides to work more hours when she has more power?

The paper is organized as follows. Section 2 discusses the current literature on collective household labor supply models that includes the possibility of labor force participation and public consumption (like expenses on children). Section 3 presents the theoretical framework that incorporates both the decision to participate in the labor market and public goods consumption. Using Donni (2003)'s approach, the possibility of non-participation is introduced in the framework of Blundell, Chiappori, and Meghir (2005) where parents care about children's welfare. When children are present in a household, the main conclusions of Chiappori (1992) can be extended: individual preferences and the (conditional) sharing rule can be recovered if one or both household couple works. Furthermore, individual labor supplies have to satisfy certain testable restrictions. Some final remarks are made in Section 4.

2 Labor market participation and public goods

An application that has been analyzed to a great extent in the collective literature is the household member's supply of positive hours of work with private consumption. However, this setting is too simple to describe other household compositions and dynamics that are observed in real life. When labor market participation and public goods are considered under a collective framework, there are some aspects to take into account. First, the non-participation decision in the labor market may have an influence on outcomes even for individuals who are not directly affected by this decision. If a member's threat point involves participation in the labor market (e.g., because woman's or man's participation involves credible outside options for her or him), (potential) wages could affect bargaining positions within a household. This result is opposite to the one obtained within the unitary model, where potential wages of non-working members are independent of household allocations (only wages of working members matter due to their effect on the budget opportunities). Second, children are likely to be an important source of preference interdependence between parents, since it is reasonable to think that both parents could derive utility from their children's well-being (although not necessary at the same degree). Furthermore, the presence of children could generate non-separabilities on parents' commodities demand and labor supply, say child care may affect the tradeoff between consumption and labor force participation and hours of work at the individual level. Finally, household production could take more relevance when children are present.

Some advances have been made in the collective literature to include the possibility of participation and public consumption (i.e., goods that both spouses derive utility from - it is consumed jointly and non-exclusively by each member - such as the amount spent on children) but in separate branches. On the one hand, Donni (2003) and Blundell, Chiappori, Magnac, and Meghir (2007) have constructed theoretical frameworks to consider non-participation in the labor market. The former work assumes that in a household composed by two adults, both members can freely choose their working hours, while the second assumes that one member can only decide to participate or not.

Donni's work extends the results of Chiappori (1988; 1992), who implicitly considered identification in the two adult members' participation set, to take also into account the case in which one of the two members does not work. Until the moment, the only empirical application of Donni's framework has been made by Bloemen (2010, the Netherlands). In the case of Blundell et al.'s work, it is not nested in Chiappori's model since the choice sets are different (two positive continuous labor supplies versus one discrete and another non-negative continuous labor supply). Donni (2007) develops a similar model to the one of Blundell et al.; they differ in that the former fixes male member's labor supply at full-time instead of the possibility of choosing between working full-time or not at all. Structural elements of the decision process can be identified from Donni's model if female member's labor supply together with at least one household commodity demand are observed.

The participation decision is included in the standard unitary model by means of a reservation wage, at which an agent is indifferent between working and not working. Trying to translate this concept to the collective framework, the collective's pillar assumption of Pareto efficiency of the household decision process requires that if one member (say, the wife) is indifferent between working and not working, the other one (say, the husband) must be indifferent as well about the participation decision of the first member (Blundell et al. 2007 called this condition the "double indifference" assumption[7]). Therefore, the participation decision on these two collective models relies on explicitly postulating a reservation wage. Individual preferences and the sharing rule can be recovered for both models.

On the other hand, Blundell, Chiappori, and Meghir (2005, henceforth BCM) introduce children in the model of Chiappori (1988; 1992) assuming that both parents care about their children's welfare (or equivalently, the expenditure on their children is considered a public good for them). In general, the decision process cannot be recovered; a continuum of different structural models can generate the reduced-form of each individual's labor supply and expenditures on children. This result is due to the fact that the level of public consumption influences the analysis of labor supply not only through an income effect but also through its impact on the individual consumption-leisure trade-off. Therefore, identifiability of the intra-household decision-making process under this approach can be obtained under two cases: a) private consumption is separable from expenditures on children, so that the consumption-leisure trade-off effect disappears, or b) introducing a

[7] To see why, Blundell et al. used the following example. Assume that at a wage infinitesimally below the reservation wage of the husband, he is indifferent between working and not working but that his wife experiences a strict loss if he is not working. Now suppose that at the reservation wage he decides to work and he receives ε more to spend on his private consumption that the initially agreed. He is better off since he is indifferent between participating or not and his consumption increases (if the goods consumed are normal). If ε is small enough, the wife is better off too, since the participation of her spouse compensates her more than the reduction on her private consumption.

distribution factor (i.e., a variable that affects the decision process but not the individual preferences or the joint budget set), so that it permits to keep constant the expenditures on children. Empirical applications of BCM's model have been made by Cherchye, de Rock, and Vermeulen (forthcoming, the Netherlands) and Sarmiento (2012, Mexico).

The aim of this paper is to model the decision to participate in the labor market in a single collective framework that considers expenditures on children as a public good. The model simultaneously takes into account the possibility that (potential) wages affect bargaining positions of household members, the utility of each adult member depends of their children's well-being, and individual consumption and labor supply decisions are not separable from expenditure on children. Under these assumptions, the underlying structure (individual preferences and the decision process) can be recovered from observed household behavior. The model extends the results of Chiappori (1992) to a more general context that the one considered in the previous literature.

3 The Framework

In BCM's (2005) framework of household labor supply with expenditures on children, the model incorporates the decision to participate in the labor market. Subsection 3.1 presents the main assumptions of the model. Besides the common collective approach assumptions of individualism and Pareto-efficiency, the model assumes that both adult household members care about their own consumption (they are egoistic) but also care about their children. Subsection 3.2 shows a decentralization procedure of the efficient household allocation under this context. Similar to the case with only private consumption, the decision process can be represented as operating in two phases by the existence of a sharing rule conditional on the residual non-labor income after the public good purchase. Subsection 3.3 shows how the model determines the level of expenditures on children. Here, also the framework tackles the effect of intrahousehold redistribution of power (for example a given policy that "empowers" a specific member of the household, such as the mother) on household expenditures on children. Subsection 3.4 introduces additional assumptions to guarantee the existence of a unique reservation wage for each partner that is consistent with the Pareto-efficiency assumption. The model employs the method used by Donni (2003) to achieve this aim. Finally, subsection 3.5 discusses the identification of the model and the corresponding restrictions on household labor supply. Given a set of (potential) wages, non-labor income, and a distribution factor, the framework can recover individual preferences and the conditional sharing rule if one or both partners works.

3.1 Commodities, preferences and the decision process

The model considers the case of an adult couple ($i = m, f$) in a single period setting. Labor supply of i is denoted by h^i, with market wage w^i. Total time endowment is normalized to one and domestic production is not considered.[8] A Hicksian composite good C is consumed by the household. This good is used for private $\left(C^m, C^f\right)$ and public (K) consumption, with prices set to one (the identifiability results of the model do not require price variation). An interpretation of K could be that it represents the amount spent on children by the household. Non-labor income is denoted by Y.

Each spouse's utility can be written as:

$$U^i = U^i\left(1-h^i, C^i, K\right) \quad , i = m, f \tag{1}$$

where U^i is strongly quasi-concave, infinitely differentiable and strictly increasing in all its arguments. It is also assumed that $\lim_{h^i \to 1} \partial U^i/\partial h^i = \lim_{C^i \to 0} \partial U^i/\partial C^i = \lim_{K \to 0} \partial U^i/\partial K = \infty$, $i = m, f$. These conditions rule out cases where leisure, and individual and public consumption are equal to zero; both members consume strictly positive quantities of these goods. These conditions seem reasonable since leisure is arbitrarily defined and consumption is aggregated.

It is assumed that household decisions generate Pareto efficient outcomes, whatever the mechanism used to reach this agreement. Therefore, there is a function λ such that household allocations $\left(h^{m^*}, h^{f^*}, C^{m^*}, C^{f^*}, K^*\right)$ are the solutions to the program:

$$\max_{h^m, h^f, C^m, C^f, K} \lambda U^m\left(1-h^m, C^m, K\right) + (1-\lambda)\, U^f\left(1-h^f, C^f, K\right) \tag{2}$$

$$s.t. \quad \begin{cases} C^m + C^f + K = w^m h^m + w^f h^f + Y \\ 0 \leq h^i \leq 1, \quad i = m, f \end{cases}$$

[8] The model assumes implicitly that all non-market time corresponds to leisure; it does not consider the division of labor between household and market production. The seminal model of Chiappori (1992) is extended to consider domestic production by Apps and Rees (1997); Chiappori (1997); Donni (2008b). Empirical applications have been made by Apps and Rees (1996, Australia); Donni and Matteazzi (2010b, the USA); Rapoport, Sofer, and Solaz (2011, France); among others. A model that considers that the domestic good is public is developed and estimated with British data by Couprie (2007); and van Klaveren, van Praag, and Maassen van den Brink (2008). Under the collective models with domestic production, the model of Donni and Matteazzi (2010a) is the only one that considers non-participation. A model that considers jointly non-participation, children and household production has not been developed yet.

The Pareto weight λ reflects the relative power of m in the household and $(1-\lambda)$ the one of f, in the sense that a larger (smaller) λ corresponds to a larger (smaller) weight of m's preferences in the household allocation problem, favoring the outcomes enjoyed by m (f). It is assumed that $\lambda \in [0,1]$ is a continuously differentiable function of wages, non-labor income, as well as at least one distribution factor z, i.e., $\lambda = \lambda\left(w^m, w^f, Y, z\right)$.

There are some remarks to make. First, it is assumed that the bundle $\left(w^f, w^m, Y, z\right)$ varies within a compact subset $\mathcal{K}$ of $\mathbb{R}^3_+ \times \mathbb{R}$. Second, it is also assumed that h^m, h^f, C, and K are observed (as functions of w^m, w^f, Y, and z); whereas the individual consumptions C^m and C^f are unobserved. In general, household surveys do not collect information about intrahousehold allocation of expenditures but about their aggregate consumption C at household level. Third, it is assumed that both partners' wages are observed always, even when a partner does not participate in the labor market. In practice, it is possible to calculate a potential wage for the non-participating member by means of an auxiliary equation.

3.2 The conditional sharing rule

The solution to the household program (2) can be thought of as a two-stage process: 1) the couple agrees on the level of the public expenditure and how to distribute the resulting residual non-labor income between them, 2) conditional on the outcomes of the first stage, the couple decide, independently from each other, their individual consumption and labor supply. Formally, let $h^{m^*}\left(w^m, w^f, Y, z\right)$, $h^{f^*}\left(w^m, w^f, Y, z\right)$, $C^{m^*}\left(w^m, w^f, Y, z\right)$, $C^{f^*}\left(w^m, w^f, Y, z\right)$, and $K^*\left(w^m, w^f, Y, z\right)$ be the solution of program (2); then a function ρ^i exists such that:

$$C^{i^*}\left(w^m, w^f, Y, z\right) = \rho^i\left(w^m, w^f, Y, z\right) + w^i h^{i^*}\left(w^m, w^f, Y, z\right), \quad i = m, f$$

Here ρ^m and ρ^f characterize the *conditional sharing rule*;[9] the portion of non-labor income allocated to each member once spending on the public good has been discounted:

$$\rho^m\left(w^m, w^f, Y, z\right) + \rho^f\left(w^m, w^f, Y, z\right) = Y - K^*\left(w^m, w^f, Y, z\right)$$

[9] When private goods are considered together with public goods, the (conditional) sharing rule is implied by efficiency, but now it is not equivalent to efficiency for a particular level of public expenditure. The level of public consumption depends also on the allocation of private consumption and labor supply, fact that cannot be isolated completely with the two-stage process interpretation of the household problem (see BCM 2005).

Note that ρ^i can be positive or negative, they could agree to spend beyond their non-labor income on the public good and also transfers between spouses are possible.

Fixing $K = K^*\left(w^m, w^f, Y, z\right)$, the second stage of the household program (2), can be represented as:

$$\max_{h^i, C^i} U^i\left(1 - h^i, C^i, K\right) \quad s.t. \quad C^i = w^i h^i + \rho^i, \qquad i = m, f \tag{3}$$

With $h^{i^*}\left(w^m, w^f, Y, z\right)$ and $C^{i^*}\left(w^m, w^f, Y, z\right)$ as interior solutions to the individual problem. The structure of both partners' labor supplies can be described by:

$$h^{m^*}\left(w^m, w^f, Y, z\right) = H^m\left[w^m, \rho\left(w^m, w^f, Y, z\right)\right] \tag{4}$$

$$h^{f^*}\left(w^m, w^f, Y, z\right) = H^f\left[w^f, Y - K - \rho\left(w^m, w^f, Y, z\right)\right] \tag{5}$$

Where $\rho = \rho^m$, and when ρ is fixed, H^m and H^f are Marshallian labor supply functions. With the idea of expressing labor supplies in terms of public expenditures (K) and to mantain the assumption that K is fixed, the following process is used. Let $\mathcal{O}$ be some open subset of $\mathcal{K}$ such that $\partial K/\partial z$ does not vanish on $\mathcal{O}$. The condition $K^*\left(w^m, w^f, Y, z\right) = K$ is used to express z as a function ζ of $\left(w^m, w^f, Y, K\right)$ by the implicit function theorem. Following from this construction, the couples' labor supplies are:

$$\tilde{h}^m\left(w^m, w^f, Y, K\right) = H^m\left[w^m, \rho\left(w^m, w^f, Y, \zeta\left(w^m, w^f, Y, K\right)\right)\right] \tag{6}$$

$$\tilde{h}^f\left(w^m, w^f, Y, K\right) = H^f\left[w^f, Y - K - \rho\left(w^m, w^f, Y, \zeta\left(w^m, w^f, Y, K\right)\right)\right] \tag{7}$$

In this way, i's labor supply is described as a function of wages, non-labor income, and a distribution factor z such that public expenditures are exactly K. Hence, the values of w^m, w^f, and Y are not constrained to assure that $K^*\left(w^m, w^f, Y, z\right) = K$; the key role of z is to guarantee that level of public expenditures is exactly K. This structure generates testable restrictions because the same function $\rho\left(w^m, w^f, Y, z\right)$ enters each member's labor supply (see footnote 12).

3.3 The determination of public expenditures

The Bowen-Lindahl-Samuelson condition characterizes the efficiency for public good expenditures. Formally, the first-order conditions for household program (2), with an interior solution for individual and public consumption, gives:

$$\frac{\partial U^m/\partial K}{\partial U^m/\partial C} + \frac{\partial U^f/\partial K}{\partial U^f/\partial C} = 1 \tag{8}$$

Equivalently, this condition can be expressed in terms of individual indirect utilities. First, let $V^i\left(w^i, \rho^i, K\right)$ denote the value of the second stage of the household program (3) for member i:

$$V^i\left(w^i, \rho^i, K\right) = \max_{h^i, C^i} U^i\left(1 - h^i, C^i, K\right) \quad s.t. \quad C^i = w^i h^i + \rho^i, \qquad i = m, f$$

That is, V^i is called the indirect conditional utility because it is the maximum utility that i can achieve given their wage and conditional on the outcomes (ρ^i, K) of the first stage decision. Next, returning to the first stage, efficiency leads to the following program:

$$\max_{\rho^m, \rho^f, K} \lambda V^m\left(w^m, \rho^m, K\right) + (1-\lambda) V^f\left(w^f, \rho^f, K\right) \quad s.t. \quad \rho^m + \rho^f + K = Y \tag{9}$$

The first order conditions give:

$$\lambda \frac{\partial V^m}{\partial \rho^m} = (1-\lambda)\frac{\partial V^f}{\partial \rho^f} = \lambda \frac{\partial V^m}{\partial K} + (1-\lambda)\frac{\partial V^f}{\partial K}$$

Therefore:

$$\frac{\partial V^m/\partial K}{\partial V^m/\partial \rho^m} + \frac{\partial V^f/\partial K}{\partial V^f/\partial \rho^f} = 1 \tag{10}$$

The ratio $\frac{\partial V^i/\partial K}{\partial V^i/\partial \rho^i}$ is i's marginal willingness to pay (MWP) for the public good, in this case children. Thus the condition (10) states that individual MWPs (or Lindahl prices) must add up to the market price of expenditure on children. From Proposition 1 by BCM, is possible to state that if i's preferences are such that both public and private consumption augment with non-labor income (i.e., K and ρ^i are normal "goods", so i's MWP is decreasing in K and increasing in ρ^i), a marginal increase on i's Pareto weight increases household's expenditure on children if and only if i's MWP is more sensitive to changes in their share than that of the other member. That is, a marginal increment of m's power will augment the amount spent on children if and only if m's MWP is more income sensitive that that of f, and viceversa. Because a positive transfer from one member to the other decreases the MWP for the public good of the transferer and increases the MWP of the transferee, this proposition establishes when the effect on the transferee is more than sufficient to compensate the reduction of the transferer. Hence, the key property for analyzing changes in the distribution of power within a household is not the magnitude of the MWP's (say, who cares more for children), but how the MWPs respond to changes on individual resources for private consumption.

3.4 The participation decision

The standard unitary framework deals with the participation decision of an agent by means of the definition of a reservation wage. At this wage, the agent is indifferent between working and not working. A reasonable generalization of this definition under a collective model with two adult members is that at the reservation wage of one household member, not only this member is indifferent between working and not working but also that the other member is indifferent (Blundell et al. 2007).

To characterize the participation decision of a household member, a procedure similar to the one used by Neary and Roberts (1980) is employed to model household behavior under rationing, or more generally of quantity constraints, which is characterized in terms of its unconstrained behavior when faced with shadow prices. The reservation wage of i (ϖ^i) is defined by:

$$\varpi^i = \frac{U^i_{h^i}\left(1, \rho^i, \bar{K}\right)}{U^i_{C^i}\left(1, \rho^i, \bar{K}\right)}$$

where the notation f_x stands for the partial derivative of function f with respect to variable x (here $f = U^i$ and $x = h^i, C^i$). This equation is the marginal rate of substitution between leisure and private consumption computed along the axis $h^i = 0$ for a given sharing rule ρ^i (and equal to C^i) and a level of public expenditures equal to $\bar{K}$.

To concentrate on the second stage of the household problem (2), particularly on labor supply decisions, public expenditures are fixed to some arbitrary level $\bar{K}$. In this way, the problem is basically reduced to the problem considered by Donni (2003) in which the participation decision is analyzed in a framework with only private goods. As above, let $\mathcal{O}$ be some open subset of $\mathcal{K}$ such that $\partial K/\partial z$ does not vanish on $\mathcal{O}$, and impose the condition $K^*\left(w^m, w^f, Y, z\right) = \bar{K}$, where the latter is equivalent, by the implicit function theorem, to $z = \zeta\left(w^m, w^f, Y, \bar{K}\right)$. Let $y = Y - \bar{K}$ denote the portion of non-labor income not devoted to public expenditures which could be positive or negative (labor income can also be used for public consumption). Therefore, if ϖ^i is a function of $\left(w^m, w^f, Y, \bar{K}\right)$, for notational simplicity, it can be expressed as $\varpi^i\left(w^m, w^f, y\right)$. Then, i's reservation wage is implicitly defined as a function of $\left(w^m, w^f, y\right)$:

$$\begin{aligned} w^i &= \varpi^i\left(w^m, w^f, Y, \zeta\left(w^m, w^f, Y, \bar{K}\right)\right) \\ &= \varpi^i\left(w^m, w^f, Y, \bar{K}\right) \\ &= \varpi^i\left(w^m, w^f, y\right) \end{aligned} \tag{11}$$

Without additional assumptions, equation (11) could have several solutions, i.e., the uniqueness of a reservation wage for member i has to be explicitly postulated under the collective framework. Intuitively, there are two reasons to explain why there can be many wage rates for which i is indifferent between working and not working. The first one comes from the assumption that the sharing rule ρ^i depends on i's wage, so there could be more than one combination of w^i and ρ^i at which i is indifferent. The second one is related to the possibility that the sharing rule itself may depend on the non-participation of household members. As shown later, the existence of a well-behaved participation frontier is needed to recover the decision process when one member of the couple does not participate in the labor market. A sufficient condition to obtain a unique reservation wage (fixed point) for each member is to define that the function ϖ^i is a contraction mapping.

Assumption R. *For any $\left(w^{m^*}, w^{f^*}, y\right)$ and $\left(w^{m^o}, w^{f^o}, y\right) \in \mathbb{R}_+^2 \times \mathbb{R}$, preferences and the sharing rule are such that there is some non-negative real number $r < 1$ for which the following condition is satisfied*

$$\max_{i=m,f}\left[\left|\varpi^i\left(w^{m^*}, w^{f^*}, y\right) - \varpi^i\left(w^{m^o}, w^{f^o}, y\right)\right|\right] \leq r \max_{i=m,f}\left(\left|w^{i^*} - w^{i^o}\right|\right)$$

Two remarks can be made at this point. First, this condition does not affect the level of public expenditures; z varies to guarantee that public expenditures is exactly $\bar{K}$. Consequently, the distribution factor allows that w^m, w^f and Y can vary freely, and therefore ϖ^i, whereas K is kept constant. Second, the assumption only holds in the neighborhood of the participation frontier; in the interior of other household participation sets the allocation of additional income stemming from the participation of one member could be more complex.

In essence, Assumption R restricts the impact on both individual shares (and hence individual consumption) of a change in one household member's wage. This amounts to assuming that the Pareto weights are smooth functions of both wages and non-labor income, and therefore the individual utilities' smoothness is preserved at the participation frontier of each individual.[10]

Assumption R is not expected to be very restrictive and it simplifies the analysis by not having to use more restrictive fixed point theorems to ensure the existence of a well-behaved participation frontier. Under this assumption, the system of equations ϖ^m and ϖ^f is a contraction with respect to w^m and w^f for any y. Using the Banach contraction principle,[11] two corollaries of this assumption are:

1. For any y, the functions ϖ^m and ϖ^f have a unique fixed point. Then, there exists a unique pair of wages, $\hat{w}^m(y)$ and $\hat{w}^f(y)$, such that both adult members are indiferent between working and not working.

2. For any w^j $(j \neq i)$ and y, each ϖ^i has a unique fixed point with respect to w^i. Then, there exists a function $\gamma^i(w^j, y)$ such that member i participates in the labor market if and only if $w^i > \gamma^i(w^j, y)$, $i = m, f$.

Therefore, considering the possible interactions of household members' participation decision, four connected sets can be defined:

[10] In order to understand in greater detail the intuition behind Assumption R, the effect on m's private consumption is going to be analyzed at m's participation frontier first when there is an infinitesimal increase in m's wage, and second when there is an infinitesimal increase in f's wage. When m's wage increases, the magnitude of the increase in m's private consumption depends on whether m is participating or not. When m is not participating, an increase in m's wage probably has a positive impact on m's bargaining power, both m's reservation wage and consumption share increase. When m is participating, an increase on m's wage has also a positive effect on household income, m's consumption share increases more.

When f's wage increases, the effect on m's private consumption depends also on whether f is participating or not. When f is not participating, the increase in f's wage reduce m's bargaining power. Since the sharing rule reflects the distribution of power between household members, if individual leisure is a normal good, it is expected that the decrease of m's share is associated with a reduction of m's reservation wage. When f is participating, an increase on f's wage has also a positive effect on household income which may compensate m's share for the increase on f's bargaining power.

Then, the condition that the difference in m's reservation wage can not be more in absolute value than the initial increase in m (f)'s wage is satisfied when m's consumption share responds less, in absolute value, to changes on m (f)'s wage when m (f) is not participating than when m (f) is participating.

[11] See Green and Heller (1981) for a definition of contraction and of the Banach contraction principle (contraction mapping theorem).

- Participation set (P): The set of $\left(w^m, w^f, y\right)$ is such that both household members choose to work.
- f's non-participation set (N^f): The set of $\left(w^m, w^f, y\right)$ is such that f chooses not to work and m chooses to work.
- m's non-participation set (N^m): Vice versa of N^f.
- Non-participation set (N): The set of $\left(w^m, w^f, y\right)$ is such that both household members choose not to work.

3.5 Identification

This section discusses the empirical restrictions on both household member's labor supply implied by the collective setting with children and non-participation. Also, it shows that is possible to recover the structural model (preferences and the sharing rule) simply by observing the labor supplies and the household expenditure on children.

The non-participation set N is not taken into account to identify individual utilities and the decision process given the lack of information for this purpose (the hours of work for both partners are zero and, consequently, the sharing rule within the household cannot be deduced from the labor supply of both individuals and hence individual utilities cannot be recovered). Therefore, it is assumed that at least one of the partners' supply is an interior solution to (2). The following theorem establishes the identification and testability results.

Theorem 1. *Let $\left(\tilde{h}^m, \tilde{h}^f\right)$ be a pair of labor supplies, satisfying the regularity conditions listed in Lemmas 1-3 (below). Under Assumption R:*

1. *Both labor supplies have to satisfy some testable restrictions under the form of partial equations on the participation set P.*
2. *Individual preferences and the sharing rule are identified up to some additive constant $D\left(\bar{K}\right)$ when at least one of the partners works. Moreover, for each choice of $D\left(\bar{K}\right)$, preferences are exactly identified.*

The proof of this theorem is developed in the next subsections. First, subsection 3.5.1 identifies the sharing rule in the participation set in which both household members choose to work (P). The knowledge of the two labor supplies in the set P allows to recover ρ simply by applying a theorem in Chiappori (1992). Next, subsection 3.5.2 identifies ρ in the set in which one of the couple does not work (N^f and N^m). The recovering of ρ on the set P can be extended to the set in which one of the couple does not work by the knowledge of the sharing rule along the participation frontier.

3.5.1 Identification in the partners' participation set

This case considers only a positive labor supply for both adults. This is the only situation implicitly considered by BCM (2005). For any $\left(w^m, w^f, y\right) \in P$ such that $\tilde{h}_y^m \cdot \tilde{h}_y^f \neq 0$, the following definitions are introduced:

$$A\left(w^m, w^f, y\right) = \frac{\tilde{h}_{w^f}^m\left(w^m, w^f, y\right)}{\tilde{h}_y^m\left(w^m, w^f, y\right)}, \qquad B\left(w^m, w^f, y\right) = \frac{\tilde{h}_{w^m}^f\left(w^m, w^f, y\right)}{\tilde{h}_y^f\left(w^m, w^f, y\right)}$$

Note that A and B are indeed the marginal rates of substitution of the sharing rule $\left(\frac{\rho_{w^f}}{\rho_y} = \frac{\tilde{h}_{w^f}^m\left(w^m, w^f, y\right)}{\tilde{h}_y^m\left(w^m, w^f, y\right)} \text{ and } \frac{\rho_{w^m}}{\rho_y} = \frac{\tilde{h}_{w^m}^f\left(w^m, w^f, y\right)}{\tilde{h}_y^f\left(w^m, w^f, y\right)}\right)$, which can be identified in terms of the observable labor supplies of m and f.

Lemma 1. *It is assumed that $\tilde{h}_y^m \cdot \tilde{h}_y^f \neq 0$, and $AB_y - B_{w^f} \neq BA_y - A_{w^m}$ for any $\left(w^m, w^f, y\right) \in P$. Then for any given $\bar{K}$, the individual preferences and the sharing rule are identified on P up to an increasing function of $\bar{K}$.*

Proof. See Lemma 1 in BCM (2005) and proposition 4 in Chiappori (1992). □

The sketch of the proof goes as follows. The idea under a collective framework is that the labor supply of spouse i is affected by changes either in the non-labor income or j's wage by means of their effects on the sharing rule. Therefore, from (6) and (7) it is possible to obtain a system of two partial differential equations in ρ:

$$\rho_{w^f} - A\rho_y = 0 \quad \text{and } \rho_{w^m} - B\rho_y = -B$$

The indifference surfaces of i's share can be derived in the space (w^j, y) from noting that if there is a simultaneous change in non-labor income and j's wage that maintain at the same level i's labor supply, then i's share also remains constant. In addition, j's share can also be derived from the fact that both shares must add up to the non-labor income devoted to non-public consumption. The system of partial differential equations can be solved if it is differentiated again and if the symmetry of cross-partial derivatives is taken into account.[12]

The sharing rule and couples' preferences have to be adjusted to consider the presence of public expenditures. For the sharing rule ρ and the pair of utilities U^m and U^f there exists a constant $D\left(\bar{K}\right)$ such that, for all $\left(w^m, w^f, y\right) \in P$

$$\begin{aligned} \tilde{\rho}\left(w^m, w^f, y\right) &= \rho\left(w^m, w^f, y\right) + D\left(\bar{K}\right) \\ \tilde{U}^m\left(h^m, C^m, \bar{K}\right) &= g^m\left[U^m\left(h^m, C^m - D\left(\bar{K}\right), \bar{K}\right), \bar{K}\right] \\ \tilde{U}^f\left(h^f, C^f, \bar{K}\right) &= g^f\left[U^f\left(h^f, C^f + D\left(\bar{K}\right), \bar{K}\right), \bar{K}\right] \end{aligned}$$

where g^m and g^f are twice continuously differentiable mappings, increasing in their first argument. The functions $\tilde{U}^i$ and U^i are different, although impossible to distinguish between them from the sole observation of labor supplies,[13] but once $D\left(\bar{K}\right)$ has been chosen, $\tilde{U}^i$ and g^i coincide up to an increasing function of $\bar{K}$.

3.5.2 Identification when one member of the couple does not participate

In the case where only one of the adult household members works ($w^i > \gamma^i\left(w^j, y\right)$ and $w^j \leq \gamma^j\left(w^i, y\right)$), the observation of i's labor supply characterizes the sharing rule on the set N^j. In addition, the values of the partial derivatives of the sharing rule are identified on j's frontier by Lemma 1, providing boundary conditions for the identification of the sharing rule on N^j. Indeed, by continuity of $\tilde{h}^i$ and ρ,[14] the recovering of the sharing rule

[12]The solution consists of partial derivatives of the sharing rule that can be deduced from observed labor supplies. Assuming that $AB_y - B_{w^f} \neq BA_y - A_{w^m}$, let $\alpha = \left(1 - \frac{BA_y - A_{w^m}}{AB_y - B_{w^f}}\right)^{-1}$ and $\beta = 1 - \alpha$. The partial derivatives are given by $\rho_y = \alpha$, $\rho_{w^f} = A\alpha$, and $\rho_{w^m} = B\left(\alpha - 1\right) = -B\beta$. In words, α (β) is the share of marginal non-labor income not devoted to public expenditures received by m (f).

[13]The intuition in the case of member m is the following. Switching from ρ and U^m to $\tilde{\rho}$ and $\tilde{U}^m$ affects: 1) the budget constraint of m, there is a vertical translation of magnitude $D\left(\bar{K}\right)$; 2) all m's indifference curves are also shifted downward by $D\left(\bar{K}\right)$, so m's labor supply does not change. Because m's consumption, C^m, cannot be observed, $\left(\rho, U^m\right)$ is empirically indistinguishable from $\left(\tilde{\rho}, \tilde{U}^m\right)$.

[14]Although $\tilde{h}^m$, $\tilde{h}^f$ and ρ are generally nondifferentiable along the participation frontiers, it can be shown that couples' labor supplies and also the sharing rule are infinitely differentiable in all their arguments on P, $int\left(N^f\right)$ and $int\left(N^m\right)$ (for an appropiate proof of this result see Theorem A.3 of Magnus and Neudecker 2007, 163).

on P can be extended to the frontier between P and N^j if w^j approaches the participation frontier $\gamma^j(w^i, y)$.

To understand more the technique employed, the participation set N^f in which only member m works and f does not (i.e., $w^f \leq \gamma^f(w^m, y)$) is initially considered. For any $(w^m, w^f, y) \in int(N^f)$ such that $\tilde{h}_y^m \neq 0$, it is defined:

$$A(w^m, w^f, y) = \frac{\tilde{h}_{w^f}^m(w^m, w^f, y)}{\tilde{h}_y^m(w^m, w^f, y)}$$

Along f's participation frontier, for any set I^f of (w^m, y) such that $w^m \geq \hat{w}^m(y)$, the following definition is made by continuity, if $\lim_{w^f \uparrow \gamma^f} \tilde{h}_y^m \neq 0$:

$$a(w^m, y) = A(w^m, \gamma^f(w^m, y), y)$$

Lemma 2. *It is assumed that* $\lim_{w^f \uparrow \gamma^f} \tilde{h}_y^m \neq 0$ *and* $1 + a \cdot \gamma_y^f \neq 0$ *for any* $(w^m, y) \in I^f$ *and* $\tilde{h}_y^m \neq 0$ *for any* $(w^m, w^f, y) \in int(N^f)$*. Then the sharing rule is identified on* N^f *up to some additive constant* $D(\bar{K})$*.*

Proof. The same technique used by Donni (2003) can be applied, the only adjustment that must be made is that the additive constant is indexed by the level of public expenditures. From Lemma 1, it is known that ρ must satisfy the partial differential equation

$$\rho_{w^f} - A\rho_y = 0 \tag{12}$$

which characterizes the sharing rule on N^f. Additionaly, the sharing rule along the participation frontier $(w^f - \gamma^f(w^m, y) = 0)$ gives a boundary condition for the partial differential equation. From standard theorems in partial differential equations theory, the identification of the sharing rule (up to an additive constant) is achieved if the following condition is fulfilled. First, (12) can be written as $\bigtriangledown\boldsymbol{\rho}\mathbf{u} = 0$, where $\bigtriangledown\boldsymbol{\rho}$ denotes the gradient of ρ and $\mathbf{u}$ is the vector $(0, 1, -A)$. Now, the condition is that $\mathbf{u}$ is not tangent to f's participation frontier. The intuition of this condition is the following, (12) defines the indifference surfaces of the sharing rule (the values of w^f, w^m, and y that keep constant the sharing rule at some level) that pass through f's participation frontier. Since $\bigtriangledown\boldsymbol{\rho}$ is a vector normal to surfaces of constant ρ and $\mathbf{u}$ indicates the direction in which the sharing rule is constant, (12) states that $\mathbf{u}$ is everywhere perpendicular to $\bigtriangledown\boldsymbol{\rho}$. Therefore, $\mathbf{u}$ is a vector that is

tangent to the surfaces of constant ρ at every point and, in particular, is a tangent vector to the surface in the participation frontier of f.
Given that, on the frontier, A coincides with a, this condition states that, for all $(w^m, y) \in I^f$:

$$1 + a \cdot \gamma_y^f \neq 0 \tag{13}$$

If this condition is fulfilled on the frontier, then the partial differential equation (12) together with the boundary condition defines ρ up to an additive constant, $D\left(\bar{K}\right)$ in the context analyzed. □

Now, the participation set N^m in which only member f works (i.e., $w^m \leq \gamma^m\left(w^f, y\right)$) is considered. The approach is the same that on N^f. For any $\left(w^m, w^f, y\right) \in int\left(N^m\right)$ such that $\tilde{h}_y^f \neq 0$, it is defined:

$$B\left(w^m, w^f, y\right) = \frac{\tilde{h}_{w^m}^f\left(w^m, w^f, y\right)}{\tilde{h}_y^f\left(w^m, w^f, y\right)}$$

Along m's participation frontier, for any set I^m of $\left(w^f, y\right)$ such that $w^f \geq \hat{w}^f(y)$, the following definition is made by continuity, if $\lim_{w^m \uparrow \gamma^m} \tilde{h}_y^f \neq 0$:

$$b\left(w^f, y\right) = B\left(\gamma^m\left(w^f, y\right), w^f, y\right)$$

Lemma 3. *It is assumed that* $\lim_{w^m \uparrow \gamma^m} \tilde{h}_y^f \neq 0$ *and* $1 + b \cdot \gamma_y^m \neq 0$ *for any* $\left(w^f, y\right) \in I^m$ *and* $\tilde{h}_y^f \neq 0$ *for any* $\left(w^m, w^f, y\right) \in int\left(N^m\right)$. *Then the sharing rule is identified on* N^m *up to some additive constant* $D\left(\bar{K}\right)$.

Proof. As above, using the partial differential equation

$$\rho_{w^m} - B\rho_y = -B \tag{14}$$

and the boundary condition $w^m - \gamma^m\left(w^f, y\right) = 0$. □

4 Final remarks

The richness of collective models comes from the opportunities that this framework provides to consider the theoretical foundations of how individuals share resources within a basic unit of analysis in an intragroup decision making process such as a household. In this sense, this approach could serve as an empirical tool for understanding intrahousehold allocations, particularly when policies with a targeting purpose are evaluated. However, the literature on the possibility of identifying the structural elements of household behavior in a more general case than private consumption with interior solutions is relatively recent. In particular the literature has provided some results based on the separate consideration of the presence of children and non-working individuals within a household.

This paper extends Chiappori's (1992) model of collective labor supply to bring together the decision to participate in the labor market and expenditures on public goods, say expenditures on children. The paper unites in a single framework the works of Blundell, Chiappori, and Meghir (2005) for children and Donni (2003) for non-participation. The model generates testable restrictions on household labor supply behavior. In particular, labor supply functions have to satisfy certain structural conditions under the form of partial differential equations. Moreover, the model can recover individual preferences and the sharing rule from the simply observation of adult members' labor supply and expenditures on children. Identifiability when at least one of the partners works requires i) the knowledge of a distribution factor to control for the effect of public consumption on the optimal individual choice of consumption and labor supply, and ii) the explicit postulation of a unique reservation wage to identify the structure in the non-participation sets of each household member.

Two topics for future research are to consider also household production and to apply empirically the model. Welfare comparisons at the individual level can be biased if household production is not taken into account. For example, the specialization of a woman in domestic activities is interpreted as an increase in her individual leisure consumption; her share of household non-labor income is interpreted as a lump-sum transfer from her partner instead of the exchange of her domestic production for market goods. The data necessary for future applications of the model can be basically obtained from household income and expenditure surveys. This type of surveys has information of household composition, household income sources, labor situation of individual members, and expenditures on children (like for example education, food, health). The stochastic specification of the model has to take into account that wages are not observed for non-participants members, and particularly the assumption that both the labor supply of the participating member and the sharing rule have to be continuous at the participation frontier of the other member.

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