Centro de Estudios Económicos
www.colmex.mx
El Colegio de México, A.C.

Serie documentos de trabajo

DYNAMIC PRICE COMPETITION IN INFLATIONARY ENVIRONMENTS WITH FIXED COSTS OF ADJUSTMENT

Alejandro Castañeda

DOCUMENTO DE TRABAJO

Núm. VII - 1993

"Dynamic Price Competition in Inflationary Environments

with Fixed Costs of Adjustment"

Alejandro Castañeda
El Colegio de México.

I am mostly grateful to Timothy Bresnahan and Ken Judd for very helpful comments and strong support. All errors are solely mine.

INTRODUCTION.

In markets with few sellers and inflationary environments, firms do not adjust their prices very frequently[1]. In some cases firms adjust their prices synchronically, and in others they do not. If firms face fixed costs of adjustment, Sheshinski and Weiss (1978) showed that a monopoly will not adjust its prices continuously. The optimal policy will follow an Ss rule. The extension of the analysys of this problem to a market with few sellers has posed major difficulties for economic research. I am not aware of any theoretical development that studies the behavior of oligopolistic firms whenever they face an inflationary process and have to pay a fixed cost each time they want to adjust their prices.

In fact, there are very few examples of models with fixed cost of adjustments and changing environments in the operation research literature, even for the single decision case. In economics, there has been two main approaches to model dynamic interaction with perfect information. On one hand we have the supergame literature, a modelling technique which studies a game that repeats itself each period and therefore assumes an unchanging environment. Because inflation changes the conditions of the game continuously, this technique is useless to model these kinds of problems. The second approach has been the Markov Perfect Equilibrium method of solution. Within this latter approach, the most commonly used technique assumes that firms alternate in their moves. Maskin and Tirole (1988 (a)) have argued that the aim of this method is to capture the idea of short run inaction. Of course, the alternating approach cannot model synchronous adjustments. Therefore, this technique is ineffective in trying to capture the

[1] See Carlton (1989).

stylized facts of the real world.

The aim of this paper is to characterize dynamic oligopolistic interaction with fixed costs of adjustments in inflationary environments. The main procedure for characterizing this rivalry is by solving the game in the computer. If the solution of the game exists[2], it lies in a functional space of infinite dimension. Because the computer cannot approximate the solution in an infinite dimensional space, I project the solution in a subspace of finite dimensions. As far as I am aware, this model constitutes the first dynamic theoretical approach which explains the rigidity of prices in markets with few sellers and inflationary environments. Other explanations of the rigidity of prices undermine the allocational role of prices, and consider that firms will use other mechanisms to distribute goods (see Carlton (1989)). I am not aware of a fully developed theoretical model along these lines.

I will study a dynamic price competition model with differentiated goods and a constant rate of inflation. The firms maximize the present value of profits and have to decide when to adjust their prices, as well as, the amount of the change. Their strategies are assumed to depend on the state of the system, the real prices of the firms. In other words, I am looking for the Markov Perfect Equilibrium solution.

Maskin and Tirole (1988 (a)) have studied dynamic price competition in a duopoly setting with perfect substitutes. They analyze an alternating move game, and

[2] I am not aware of any existing proof in these kind of games, see Chapter two for a discussion of existing proofs. In this chapter we check whether the approximate solution actually converges in the computer.

conclude that, in equilibrium, firms will either follow an Edgeworth cycle or keep their relative price unchanged. Both equilibria are possible in their game. Furthermore, they show the existence of trigger strategies that can sustain the monopoly outcome. Gertner (1986) extended Maskin and Tirole's study to an alternating game in which firms have to pay a fixed cost (menú cost) each time they want to change their prices. He proves that Markov Strategies can sustain collusion. Eaton and Engers (1991) analyze the Maskin and Tirole (1988 (a)) model with differentiated goods. They show that equilibria depends upon the degree of interaction. For low levels of interaction they show the existence of a "spontaneous equilibria", in this equilibria the firms will have a steady state price that they will maintain even if it is undercut by the rival. For high levels of interaction they found a "disciplined equilibria", the firms attain a certain level of collusion which is supported by severe undercutting by the other firm if it were to break.

In my model, alternating is not exogenously imposed; rather, the timing of adjustment is endogenously determined by modelling the source of inaction as a fixed cost. Inflation erodes real prices and firms pay a fixed cost every time they want to adjust their prices.

I will study the asymptotic behavior of the game by making simulations for various initial conditions. We will notice that synchronized behavior will be the most frequent outcome[3]. Nonetheless, alternating behavior cannot be ruled

[3] A property, ruled out by assumption in the alternating move approach.

out[4]. I will find out which of the different initial conditions will maximize the sum of the value functions of the players. Hence, if firms can choose initial conditions, they may choose one pattern over the other.

Sheshinski and Weiss (1990) have studied a decision theoretic approach to this problem in a continuous time environment. They study the two product monopoly[5]. They conclude that depending upon the cost structure and the behavior of the cross partial derivative in the one period return function, the monopoly may want to synchronize or not his two goods.

There are important issues related with dynamic price competition in inflationary environments and menú costs:

Many economies in the real world experience inflationary environments. In turn, these countries face non-trivial stabilization policies. The level of pain that these policies can cause depends upon whether firms want to synchronize their prices. If firms choose not to synchronize prices, exogenous shocks may propagate over time (Blanchard (1983)). When firms stagger their decisions there are two contrasting approaches to stabilization. In one school, if firms stagger prices, stabilization policy is possible (Taylor (1980)). In contrast, in the second tradition, Caplin and Spulber (1987) have maintained that even if firms stagger their decisions uniformly and follow (S,s) rules, money may be neutral. A

[4] These results are reminiscent of those obtained by Maskin and Tirole (1988 (a)). When firms have a kinked demand equilibrium in the Maskin and Tirole paper, they are in fact having a constant relative price. In this paper when firms choose to synchronized their adjustments, they are implictly choosing a constant relative price.

[5] The collusion case.

crucial assumption in the latter proof is that firms stagger uniformly. Yet, Caplin and Spulber ignored the issue of strategic interaction. They do not derive the pricing behavior of firms from first principles; they assumed an Ss pricing rule for each firm with bandwiths invariant to changes in the rate of inflation. In a strategic environment, the inflation level and the behavior of the competitor will have an impact on the level at which firms want to adjust their prices. If any of these two conditions change, the conclusion by Caplin and Spulber will not follow anymore[6]. When we take into account strategic considerations and calculate the behavior of the firm from first principles, these two factors change.

For policy making it is important to address the impact of inflationary environments for the welfare of both consumers and producers in the industry. I will try to deal with this topic in a partial equilibrium context, by using the consumer surplus and producer surplus. I will calculate the highest level of consumer surplus over all the states and see how it varies when the rate of inflation changes. This is, I believe, an "interesting measure of the costs of inflation"(Sheshinski and Weiss (1978)). Additionally, I will compare the game outcome with the one given by the collusion case and the social planner solution. The results of this comparison will furnish us with an idea of the impact on the producer surplus of the costs of inflation. Finally, given that the synchronized outcome appears to be the more robust result, I analyze how producer surplus

[6] More recently Caballero and Engel (1991), have extended the analysis of Caplin and Spulber to a monopolistic competitive environment with strategic complementarities. However, they do not calculate the behavior of these firms from first principles. Rather, they use suboptimal rules, as Caplin and Spulber they assume that the bandwiths in the Ss rules are invariant to changes in the rate of inflation.

changes under the synchronized outcome, as the rate of inflation rises.

The firms face an intertemporal maximization problem in which they have to decide when to change their variables and the amount of the change. Since firms are competing with each other, today's decision taken by a firm will produce a response by a competitor tomorrow, the firm making the choice today takes into account this fact.

The following sections are organized in the following way:

In section 1 I state the model. In section 2 I present the the pair of functional equations that has to be satisfied in order for the model to be a Markov Perfect Equilibria, I also sketch the main computational approach. In section 3 I discuss the functional form and the parameter values chosen to solve the game. We will see that the functional form is simple enough to allow me to speed up the calculations, as well as, flexible enough to permit me to vary degree of interaction between the firms. Since the utility function is known, I can calculate directly the objective function of the social planner.

In section 4 I present the results of the game. First, I characterize the shape of the set in which the game will be played after the first move (the play set). Then, I analyze whether there is a stable orbit in which the game converges after simulating the game from various initial conditions. We will see that for most initial conditions and for most parameter values the game converges to a cycle of price changes in which firms change their prices at equidistant moments of time, always to the same level and in a synchronized fashion. In other words, the firms will follow Ss policies, as in Sheshinski and Weiss (1990). This result appears to be contingent on the property of strategic complementarities in the one period return function[7]. Finally, I answer the question of what sort of initial conditions will the players choose if there are preplay negotiations, again the most reasonable outcome is that the firms choose to follow Ss policies in a synchronized fashion.

In section 5 I make comparative statics with respect to changes in the parameters of the model. Most of the results are congruent with those obtained by Sheshinski and Weiss (1978) and (1990).

In section 6 I make comparisons with the collusion case, the most distinguishing feature is that the collusion solution is more able to exploit the strategic complementarities between the two goods and therefore it allows the single firm to save in fixed costs.

In section 7 I make welfare analysys from a partial equilibrium point of view. The overwhelming and not at all surprising conclusion is that inflation reduces social welfare from the point of view of both the consumer surplus and the producer surplus.

---

[7] In Castañeda (1992) it is shown that the firms will never synchronized for the strategic substitutes case.

1. THE MODEL.

Consider an economy subject to an inflationary process where the price level grows at rate g. I assume two symmetric firms with the one period return function expressed in terms of real prices. In the following section we will see that the one period return for each firm $F^i(\cdot,\cdot)$ $i=1,2$ expressed in terms of real prices (x_1,x_2) is strictly concave. Furthermore, I will assume that demand function is linear in both prices so that given a finite price of the competitor (firm j), the maximum price for firm i is well defined, and finite quantities are demanded for all prices. Finally, I assume that $F^i_{12}(x_1,x_2)>0$ $i=1,2$, i.e. the one period return function exhibits the property of strategic complementarities, consequently one would expect that "raising x_1 , increases the marginal profits of x_2 , making synchronization more likely (Sheshinski and Weiss 1992 p.335)". Indeed, this is one of the main results of the paper.

Each time that the firms want to adjust their prices, they have to pay a fixed cost K , which is larger than the returns obtained by the firms at the static Nash[8], the fixed cost will force the firms to be inactive for some amount of time. A salient feature of this model is that it determines endogenously the time of adjustment by the firms, as well as, the size of the adjustment.

[8] For smaller fixed costs I was not able to reach convergence.

2. VALUE FUNCTION.

Let $V^i(x_1,x_2)$ $i=1,2$ be the value function associated with a Markov Perfect Equilibria solution, starting at time 0 with prices (x_1,x_2) . The value function is defined by the solution of the following pair of functional equations:

$$
\begin{aligned}
V^1(x_1,x_2) = \underset{p,}{Max}\, p[q\underset{S^1}{Max}[F^1(S^1,S^2)+BV^1(S^1-g,S^2-g)] \\
+(1-q)\underset{S^1}{Max}[F^1(S^1,x_2)+BV^1(S^1-g,x_2-g)-K]] \\
+(1-p)[q[F^1(x_1,S^2)+BV^1(x_1-g,S^2-g)]+ \\
(1-q)[F^1(x_1,x_2)+BV^1(x_1-g,x_2-g)]]=TV^1
\end{aligned}
$$

(1)

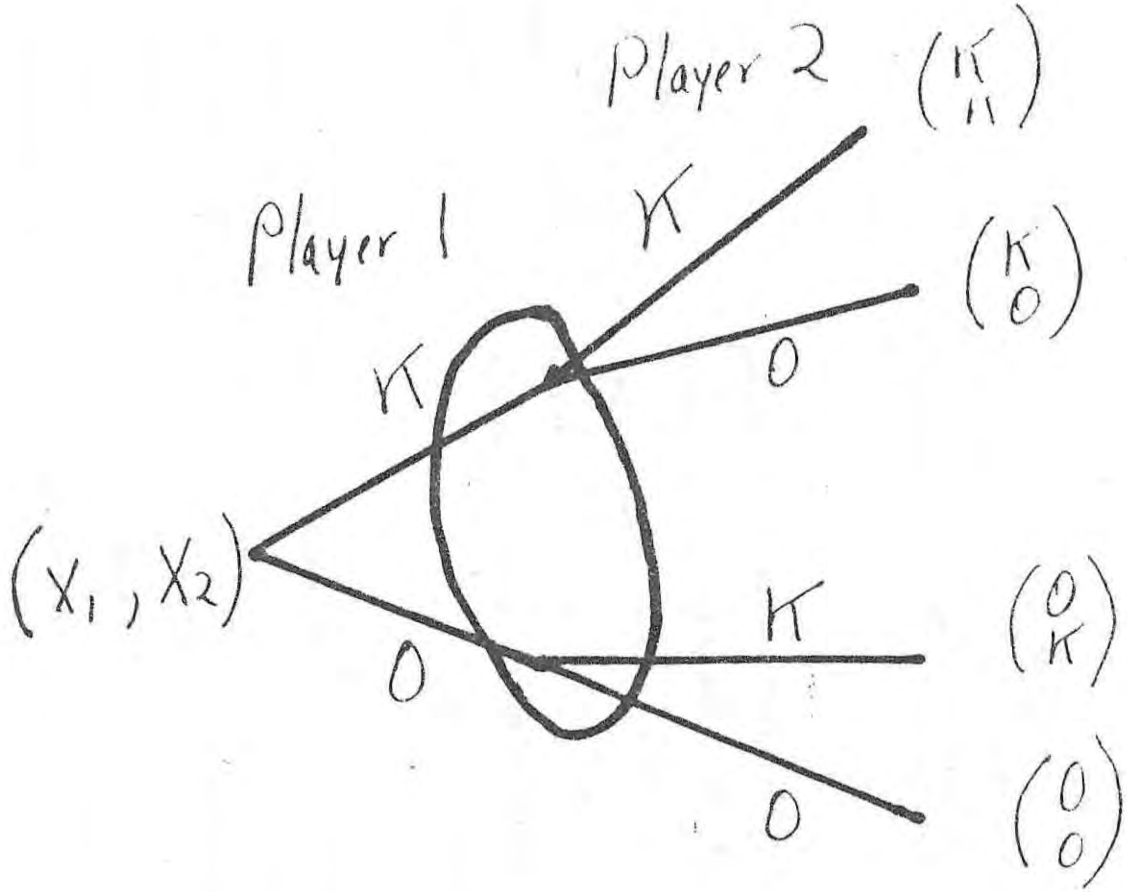
$$
\begin{aligned}
V^2(x_1,x_2) = \underset{q,}{Max}\, q[p\underset{S^2}{Max}[F^2(S^1,S^2)+BV^2(S^1-g,S^2-g)] \\
+(1-p)\underset{S^2}{Max}[F^2(x_1,S^2)+BV^2(x_1-g,S^2-g)-K]] \\
+(1-q)[p[F^2(S^1,x_2)+BV^2(S^1-g,x_2-g)]+ \\
(1-p)[F^2(x_1,x_2)+BV^2(x_1-g,x_2-g)]]=TV^2
\end{aligned}
$$

The timing of decisions within each period is as follows:

At the beginning of the period, firms observe their current state, once they know their current state they decide whether they want to pay the fixed cost or not. They both observe the decision made by both firms with regard to paying the fixed cost. Then they decide where to move. At the end of the period, the exogenous process erodes their state variables.

Within each period, firms play a two stage game. This game's extensive form is depicted in Graph 1 and the corresponding normal form is expressed in Graph 2. As we can see from the extensive form, there may be mixed strategies involved in the decision of whether to pay the fixed cost or not. The p in the above functional equation, represents the probability that player one pays the fixed

Extensive Form



$V^1(S,S) - K$
$V^2(S,S) - K$

$V^1(S^1(x_2), x_2) - K$
$V^2(S^1(x_2), x_2)$

$V^1(x_1, S^2(x_1))$
$V^2(x_1, S^2(x_1)) - K$

$V^1(x_1, x_2)$
$V^2(x_1, x_2)$

GRAPH 2.2
NORMAL FORM

	FIRM 2 PAYS FIXED COST.	FIRM 2 DOES NOT PAY FIXED COST
FIRM 1 PAYS FIXED COST.	$V^1(S,S)-K$ $V^2(S,S)-K$	$V^1(S^1(x_2),x_2)-K$ $V^2(S^1(x_2),x_2)$
FIRM 1 DOES NOT PAY FIXED COST.	$V^1(x_1,S^2(x_1))$ $V^2(x_1,S^2(x_1))-K$	$V^1(x_1,x_2)$ $V^2(x_1,x_2)$

cost in that period. q represents the probability that player two pays the fixed cost. S^1 in the equation above is the optimal response function of player one, due to linearity of the adjustment cost function, it is function of the current state for player two (x_2) if two does not move. If two does move, then S^1 corresponds to the intersection of the optimal response functions. A similar explanation holds for player two.

I can divide the state space in the following way: Let me define Ω_c as the set of x_1 and x_2 in which it is an equilibria that neither of the firms wants to move, p and q are equal to zero in the above equations. Alternatively, Ω_1 is the set of states in which only firm one wants to move, p is equal to one and q is equal to zero in the equations above. When p is equal to zero and q is equal to one then the firms are in the set Ω_2 . If p is equal to one and q is equal to one, then the firms are in the trigger set for both firms Ω_m , whenever firms are in this set they move to the intersection of the optimal response functions $S^i(x_j)$. Finally, I get the set in which p is between one and zero and q is between one and zero, in this set I have a mixed strategy equilibria. Let me call this set Ω_{mx} . $s^1(x_2)$ represents the low boundary between the trigger set for firm one and the continuation set when the state of firm two is at x_2 [9] . Finally, s represents the point at which firm one (and firm two) decides to move whenever both firms are moving together.

If the extensive form of the game for each period is not defined in this way, every time in which there is not a consistent outcome in pure strategies, I would

[9] Since prices can be reduced and increased, there is a high boundary between the continuation set and the trigger set for each of the two firms. But, because inflation is a one sided process these will never be attained after the second movement.

have had a very difficult task in calculating the mixed strategy equilibria. This is so, because the action space is continuous[10]. The fact that firms observe whether they have paid the fixed cost in the first stage of the period, allows me to restrict the strategies upon which firms can randomize. In this setting, the mixed strategy equilibria occurs only when firms have not a consistent decision of whether to move and if so where to move.

We can view the right hand side of the equations in (1) , as an operator which maps tomorrow's value function into today's value function. **If the solution exists,** (see the paragraph below) the standard procedure for solving the pair of functional equations expressed above, is to start with an arbitrary function and to iterate in the following map until convergence is reached.

$$V^{i,n} = TV^{i,n-1} \quad i=1,2$$

In the single decision case with standard assumptions on the one period return function $F^1(\cdot,\cdot)$ and the discount factor B , the contraction mapping theorem guarantees the convergence of the above iteration. Obviously, existence for strategic environments is not a simple extrapolation of this argument.

I am not aware of an existence proof for this type of games. Dutta and Rustichini (Dutta and Rustichini 1991) have proved existence in a game similar

10 Even in the case of discrete action spaces it would have been impossible to calculate the mixed strategy equilibria in the computer, whenever the firms were mixing between several possible choices.

to this one, but with one state variable[11] [12]. If the state space is discrete and the controls that the firms can choose are such that they always fall in the discrete grid then existence can be guaranteed by the proof in the book by Fudenberg and Tirole (see Fudenberg and Tirole (1991)).

Existence from a computational perspective is verified by seeing whether our approximation in the computer is actually converging. As Judd (Judd 1990) have suggested, the computational approach to these models, may be viewed as a parallel procedure towards proving existence in this kind of problems. The computational solution may be contemplated as an epsilon equilibria of the original problem.

Given the fact that I solve the model in the computer in discrete time, differentiability cannot be guaranted for the whole state space. In particular, the value function is not differentiable at the boundary between the continuation set and the trigger set. The fact that a firm can either decide to wait for a period to adjust or rather to adjust right now, implies two different control choices for that firm. It can wait one period to exert its control, thereby choosing implicitly a discretely lower state variable for tomorrow. Or it can exert its control without delay and choose a higher value of the state variable

[11] Since the optimal response function is non-convex, i.e. for some states the player does not move and for others she does. Traditional existence results which rely on the Kakutani Fixed Point Theorem, cannot be used. The Dutta and Rustichini's method uses some kind of monotonicity in the optimal response function and then they use Tarski fixed point theorem to prove the existence, I suspect that this approach may work for this model.

[12] Existence of the open loop equilibria is no problem. Once the other firms strategy is chosen, the optimization problem becomes a single agent procedure. In continuous time, the proof of a existence of an optimal policy for a single decision maker would apply (Bensoussain).

for tomorrow. Clearly, this implies that the value function is not differentiable at any boundary between the continuation set and any of the trigger sets. Moreover, at all those initial states in the continuation set that hit the boundary exactly, the value function is still nondifferentiable[13]. This last reasoning insinuates that the value function is only piecewise differentiable in discrete time. Consequently, it is important to keep our approximation in a space suitable for these properties of the value function. A similar proof to the one made in Castañeda (1991) in continuous time, will show that the value function is continuous in discrete time.

In appendix two I explain more carefully the computational approach, here I only sketch the main procedure. Given the theoretical restrictions analyzed so far it is natural to look for value functions in the space $\hat{B}$ of piecewise differentiable functions mapping $D \subset \Re^2$ into $\Re^1$. Since, the computer cannot approximate the whole space of piecewise differentiable functions. I look for a finite dimensional representation of the value function [14].

I calculate the mapping T in (10) in two stages.

First, I solve for the Nash equilibria in a square lattice with equally distant points in both dimensions so that T is exactly satisfied in the above pair of equations for any point in the lattice. So that any point in the lattice is given by the following ordered pair: (x_i, x_j). Where $x_i = c_0^1 + ih$ $i=0,99$ and $x_j = c_0^2 + jh$ $j=0,99$. Furthermore, the origin of the lattice is in the forty five

[13] See Lucas and Stokey (1990) page 118 for an example that shows this particular problem.

[14] Judd (1990) is a very good paper on computational approaches in economic analysis. The impact of that paper in this part of the paper is considerable.

degree line so that $c_0^1=c_0^2$ [15]. Second, I look for an interpolation method that best satisfies the theoretical restrictions obtained above, and summarizes better the information gotten in the points of the lattice. By using this procedure, I am replacing the theoretical mapping from the space of continuous functions into continuous functions represented above by T, into a finite dimensional approximation of that map. In the interpolating procedure I choose a finite dimensional basis and represent our approximated value function in this subspace. Then, through the iteration procedure I map this finite dimensional approximation into another finite dimensional approximation until I reach a level in which V^n is reasonable similar to V^{n+1}.

Given the fact that the value function is piecewise differentiable, it appears that the best approach is to use finite element basis with small support. With small support basis, errors in approximation in one part of the state does not affect the interpolation in another part of the state. In finite element approaches the interpolation proceeds locally, subinterval by subinterval. The global approximation is obtained by patching together all the subintervals. The approximated Value Function is then expressed in the following way:

$$(2) \qquad V^i(x_1,x_2)=\sum_{j=0}^{n} a_j\phi_j(x_1,x_2) \qquad i=1,2$$

A suitable basis to implement the above procedure are the bilinear cardinal functions. These cardinal functions span the space of purely continuous functions C^o when the size of the grid tends to zero (the number of points go up to infinity) (Lankaster and Salkauskas 1986). I use this basis to gain

---

[15] In general $C_0^1=c_0^2$ were very close to zero, so that the whole grid was in the positive orthant.

computational speed. Bilinear cardinal functions are easy to construct[16].

---

[16] This argument is not true for higher order approximations. In such case the cardinal functions may have to be calculated. And when we calculate them they may not have the small support property.

3. PARAMETER VALUES AND FUNCTIONAL FORMS.

3.1 ONE PERIOD RETURN FUNCTION.

In the computational approach I will study the following model:

I assume that the representative consumer has a quadratic utility function as follows:

$$U(q_1,q_2)=-(q_1+q_2)^2+(q_1+q_2)-DB(q_1-q_2)^2+M$$

Where M represents money[17] and DB represents the degree of interaction between the two goods. If DB is equal to zero then we have the perfect substitutes case. When DB is equal to one then there is no interaction between the goods. Maximization of the above utility function yields the following demand functions.

$$q_1(p_1,p_2)=-(\frac{1+DB}{8DB})p_1+(\frac{1-DB}{8DB})p_2+\frac{1}{4}$$

A symmetric equation holds for $q_2(p_1,p_2)$.

From the last expression we can get the following profit function:

$$\Pi(p1,p2)=(-(\frac{1+DB}{8DB})p_1+(\frac{1-DB}{8DB})p_2+\frac{1}{4})(p_1-c)$$

[17] This implies a constant marginal utility of money.

3.2 CALIBRATION.

I use as a constant marginal cost $c=0.4$ [18]. In Table 1 I show the values that I choose for each possible parameter: The discount factor, the size of the fixed cost, the rate of inflation and the level of strategic interaction. We notice that when $DB=.1$ we are almost in the perfect substitute case. The rates of inflation are weekly inflation rates that correspond to a yearly inflation rate of 20 percent (0.003 in the table)40 percent (0.006 in the table) and 8 percent (0.0012 in the table). I also have three different discount factors that appear too low to represent any real world real interest rate. The reason for choosing these discount factors is speed. For higher rates of discount the computer time is exponentially higher. Nonetheless, I run other models with lower and more realistic real rates (higher discount factor) to account for this problem. Table 2 shows the values of the parameters for this set of models. As we will see below, the qualitative results are basically the same. Considering all the possible combinations, I will start the analysis with 81 models in the first set of parameters and 8 in the second one. The different values for the parameter that measures the degree of strategic interaction imply the values in parentheses for the cross price elasticity. I evaluate this latter at the static Nash Equilibria. In Table 1 we can see that for $DB=0.1$ the Cross Price Elasticity is extremely high[19].

---

[18] The social planner approximation makes me choose this value for the marginal cost. The reason is that the social planner solution for zero fixed costs is trivially both prices equal to marginal cost. (Which is the same for both firms). If this price is equal to zero then the Ss functions would have the S function very close to zero, leaving no room for comparison.

[19] The fixed cost may appear to big for these models. The income per period at the static Nash is approximately 0.09 so the smallest fixed cost $F=.2$ is roughly two times the income per period, I tried smaller fixed costs =.1.

4. RESULTS.

4.1 PLAY SET.

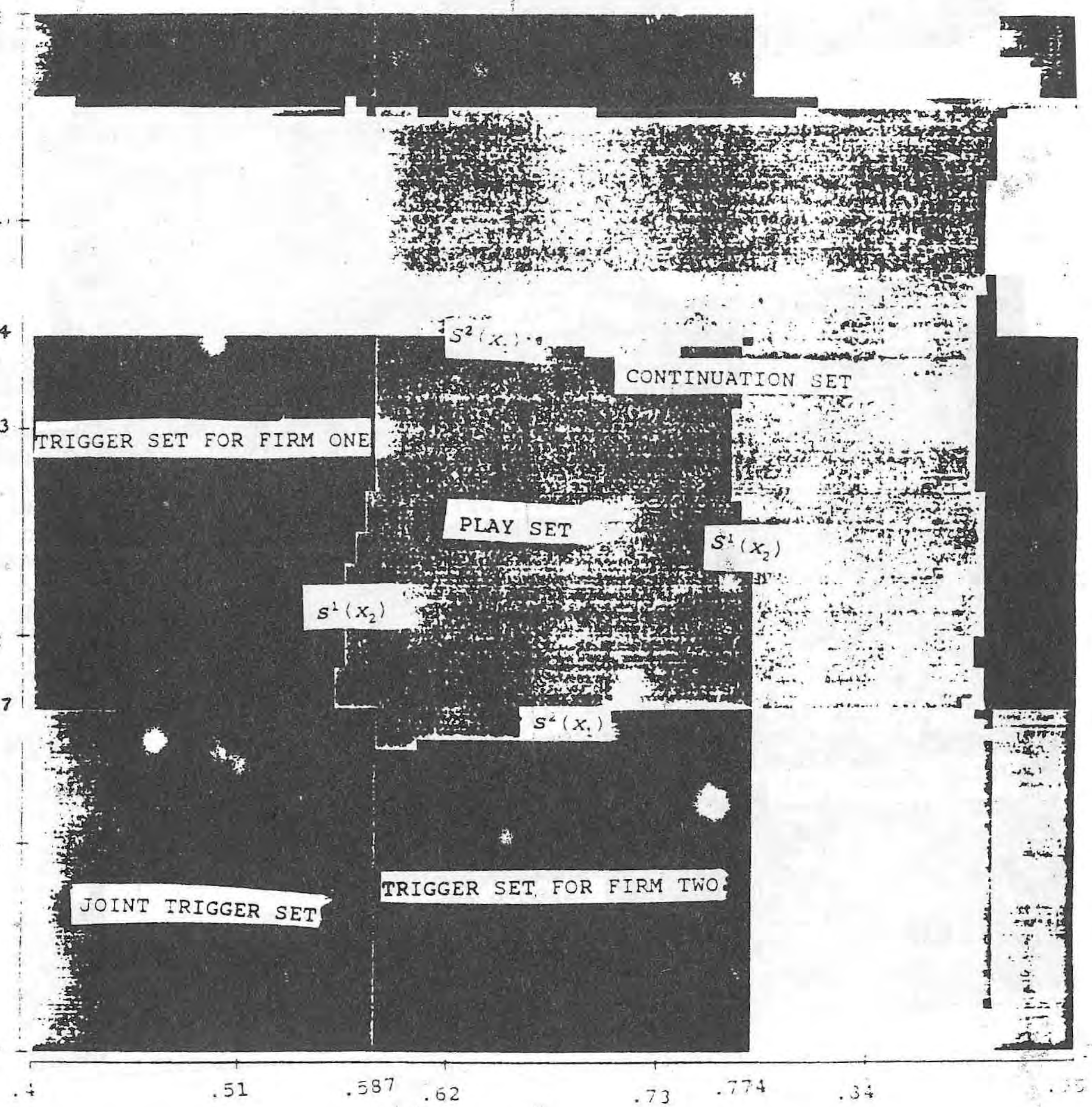
The play set is in graph one; it is defined as the set of states located between the optimal response functions $S^i(x_j)$, $i=1,2$, $j\neq i$ and the boundaries between the continuation set and the trigger sets, the joint trigger set Ω_m, and the trigger set for each firm Ω_i $i=1,2$ $(s^i(x_j)$ $i=1,2$ $j\neq i)$. In other words, the play set comprises the sets of states in which the firms will stay after the initial adjustment of prices. In the interior of the play set, it is an equilibria that both firms stay put. I get this graph from one typical model in the computer. For most of my models with low degrees of interaction $(DB>.5)$, the shape of the play set resembles graph one. However, for high degrees of interaction $(DB=.1)$, I was not able to reach convergence. The value function is cycling for these parameter values. I am not able to account for these results. The only thing that I can say, is that other people have reported similar problems when dealing with these type of dynamic games[20].

We notice that the optimal response function has a v shaped form.
Graph two (not available now) illustrates the optimal response function more clearly. The intuition behind this shape is as follows: We will see later that synchronization is the most reasonable outcome given that both firms will move together. Consequently whenever the firm knows that the competitor will move in a short period, it moves to a point that will bring both firms close to the intersection of the optimal response functions, -as soon as the competitor moves-

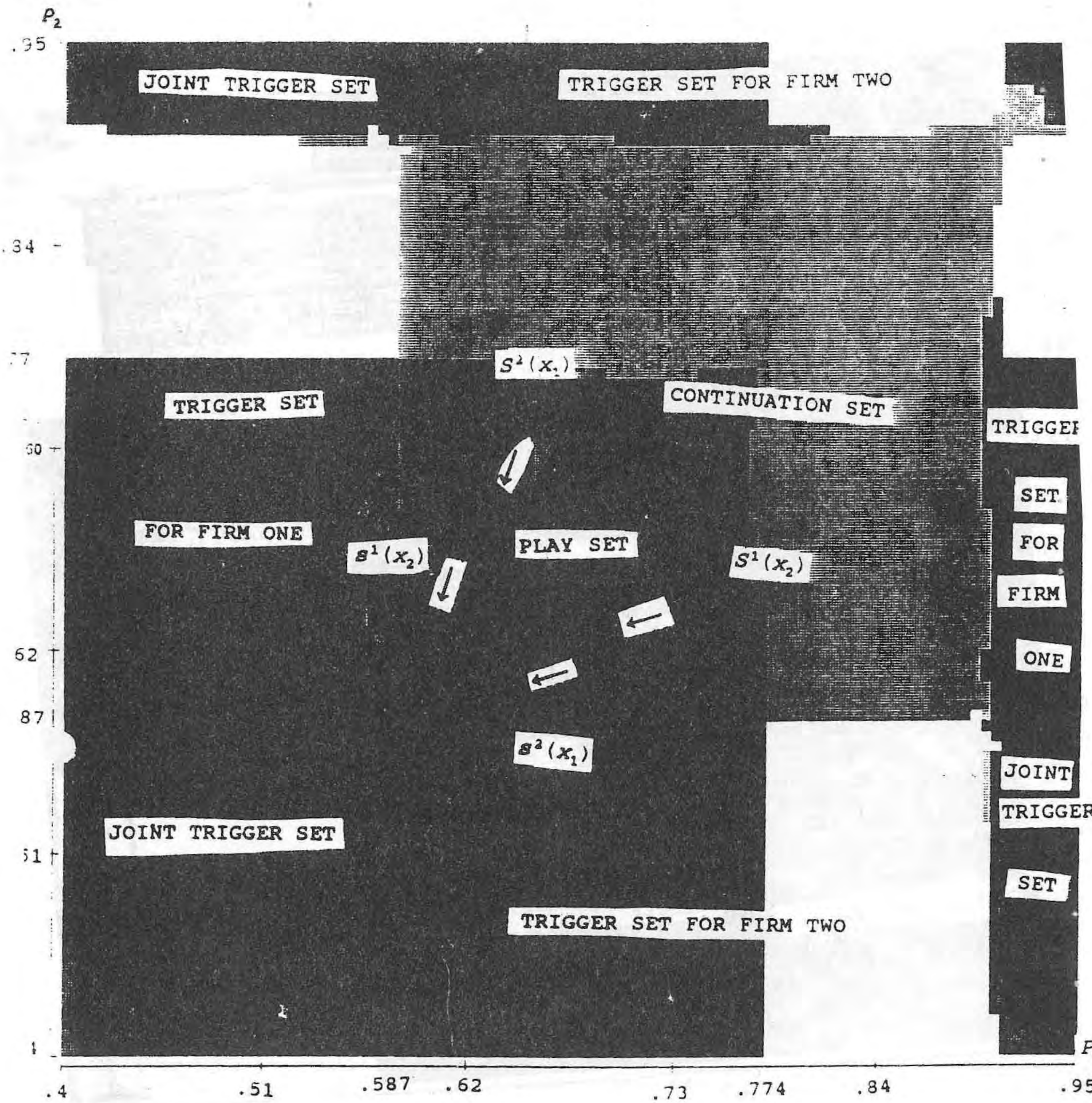
[20] Ariel Pakes have reported these kind of anomalies

GRAPH 1

PLAY SET



GRAPH 3



B =.987 K =.2 G =.003 DB =.8

(See Graph 2).

It is interesting to point out that mixed strategies appeared in few cases. Appendix one discusses this outcome in more detail. As argued there, these results are basically due to the computational calculations. In my view it is not very likely that they will appear in the truly discrete time model.

4.2 *Ss* POLICIES.[21]

The literature has broadly dealt with the issue of whether firms follow a regular pattern of adjustment in their control variables when they are facing a steady process of erosion of their states and fixed costs of adjustment. Caplin and Sheshinski (1987) have proved the emergence of Ss policies as an optimal decision process for a monopoly with non-convex costs in the single product case. Sheshinski and Weiss (1990) show that a two goods monopoly with complementarities, can follow a synchronized pattern of adjustment in its two prices, adjusting both prices at equidistant times always to the same level. In other words the monopoly follows Ss policies in each of its two prices and it adjusts both prices at the same time. Further, they even prove that the policy of following Ss rules in each of the prices of the monopoly at synchronized times is a locally stable process, and that they hold for a variety of initial conditions.

One of the most important results of this paper is to show that the intuition advanced by Sheshinski and Weiss (1990) regarding synchronization and Ss policies

[21] In this section we will make simulations, and see how frequent is the equilibria in which the firms follow a cycle of price changes at equidistant times, always to the same level (an Ss rule) in a synchronized fashion.

is maintained in symmetric Markov games with strategic complementarities between the prices of the firms. For a variety of games and initial conditions, the firms will follow Ss rules in its own price and will adjust their price in a synchronized fashion.

For each model, I pick one hundred equidistant initial conditions[22]. Then, I run 15000 simulations for each initial condition[23]. The aim of this exercise is to find out whether firms will synchronize their adjustment and follow Ss rules in its own price. The results show that synchronization in adjustment and Ss policies is by far the most frequent outcome from most initial conditions. Nonetheless, non-synchronized behavior cannot be ruled out[24].

Table 3 summarizes the results for these models. We can see that synchronization appears as the most frequent outcome. As illustrated in Graph three, the intuitive reason for having synchronization, lies in the fact that for all models the joint trigger set and the continuation set are connected by a continuum of

---

[22] I approximate the game in a grid of ten thousand equidistant points. The distance between each adjacent vertically or horizontally adjacent point is 0.01.

[23] The reader may wonder why I ran so many simulations. The reason comes from the cases in which I have indeterminacy in the choice of the equilibria either in terms of Pareto rankability or either in terms of mixed strategies. In all those cases I have to use the random number generator of the fortran compiler. In order to get a robust result I had to run 15000 simulations. I tried 7000 and I notice that there was a difference between both due to these reasons.

[24] When the firms follow nonsynchronized policies, it will be highly unlikely for them to follow an Ss rule, we can see this by looking at the play set, only in the extreme case in which there is no interaction between the firms *DB*=1.0 will the firms follow Ss rules in a nonsynchronized fashion. When there is positive interaction between the firms, the adjustment in the price by any of the firms is a function of the state of the other firm.

states (the two lines in black). This latter fact makes the synchronized state locally stable. Further, as discussed before, the shape of the optimal response function favors synchronization. Finally, as illustrated by the arrows in the graph, the fact that inflation is an exponential process, forces the state to move towards the 45 degree line in the graph. This latter fact is illustrated by the arrows.

If we look carefully at Table 3, it is interesting to point out that the non-synchronized outcome appears to be more frequent, for low degrees of interaction, *DB*=.9 . This should not surprise us, for low degrees of interaction the play set becomes almost a square. In other words, if we consider the limit in which no interaction exists (*DB*=1.0) , the play set becomes a full square, in which case the firms are perfect monopolies, and each of them follow Ss rules completely independent of the other firm's timing. Synchronization will occur only in the unprovable case of initial conditions with exactly the same initial real prices for both firms. Consequently, we should expect to have more non-synchronized outcomes, as we have a degree of interaction (*DB*) closer to 1.0.

Finally, a word of caution in interpreting the results is in order. We observe in the table that there are several cases in which synchronization was the only asymptotic outcome. But these results do not preclude the existence of non-synchronized outcomes. In fact, what explains this is that the points which I choose to start the simulations, did not happen to be in the right place for the non-synchronized outcome to emerge. Table 3 shows that synchronization is far more frequent than the non-synchronized outcomes.

TABLE 3
SYNCHRONIZATION RESULTS

B	FIXED COST	INFLATION	DEGREE OF INTERACTION	NUMBER OF SYNCHRONIZED OUTCOMES
.992	.9	0.003	.9	98
			.5	100
		0.006	.9	98
			.5	100
		0.0012	.9	88
			.5	100
	.5	0.003	.9	100
			.5	100
		0.006	.9	100
			.5	100
		0.0012	.9	88
			.5	100
	.2	0.003	.9	100
			.5	100
		0.006	.9	100
			.5	100
		0.0012	.9	100
			.5	100
.978	.9	0.003	.9	94
			.5	100
		0.006	.9	82
			.5	100
		0.0012	.9	84
			.5	100
	.5	0.003	.9	100
			.5	100
		0.006	.9	90

CONTINUATION OF TABLE 3

B	FIXED COST	INFLATION	DEGREE OF INTERACTION	NUMBER OF SYNCHRONIZED OUTCOMES
.978	.5	.006	.5	100
		.0012	.9	76
			.5	100
	.2	.003	.9	88
			.5	100
		.006	.9	100
			.5	100
		.0012	.9	96
			.5	100
.987	.9	.003	.9	100
			.5	100
		.006	.9	92
			.5	100
		.0012	.9	100
			.5	100
	.5	.003	.9	98
			.5	100
		.006	.9	98
			.5	100
		.0012	.9	70
			.5	98
	.2	.003	.9	84
			.5	100
		.006	.9	100
			.5	100
		.0012	.9	100
			.5	100

CONTINUATION OF TABLE 3

B	FIXED COST	INFLATION	DEGREE OF INTERACTION	NUMBER OF SYNCHRONIZED OUTCOMES
.998	.4	.003	.8	100
			.5	100
		.006	.8	100
			.5	100
	.2	.003	.8	100
			.5	100
		.006	.8	100
			.5	100

Note: *B* in the first column of the first row corresponds to the discount factor.

4.3 SYNCHRONIZATION IS A REASONABLE OUTCOME IF THERE ARE PREPLAY NEGOTIATIONS.

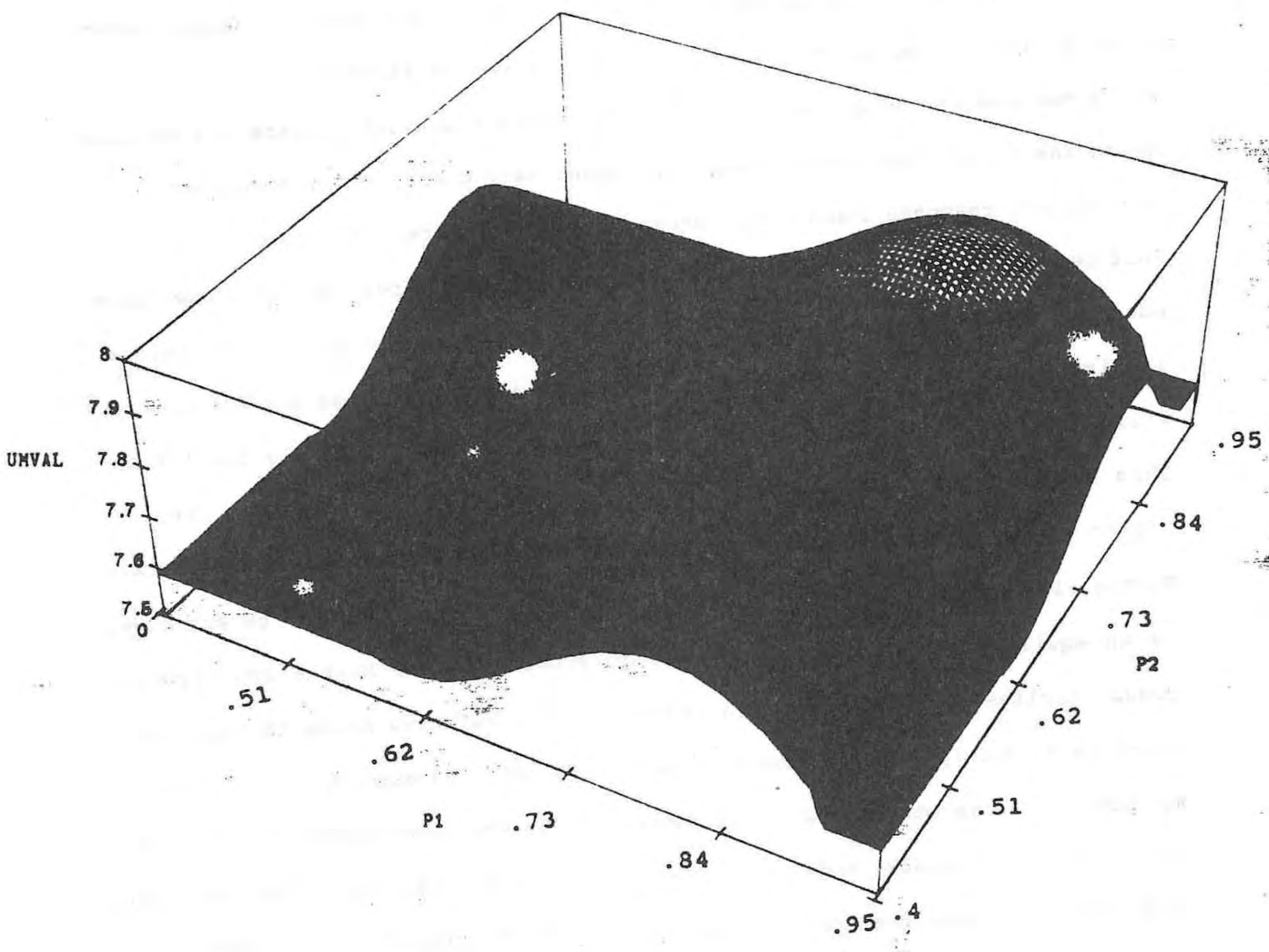
Even though synchronization with Ss policies coexists with non-synchronized results, it is rather interesting to find out that synchronization with Ss policies maximizes the sum of the value functions for the two players. This is true for all 62 parameter models considered for this paper. Graph seven illustrates the sum of the value functions for one of these models.

As we can see in the graph, the sum of the value functions attains its maximum along the forty five degree line, in a point very close to the intersection of the optimal response functions. Graph eight illustrates the contour map.

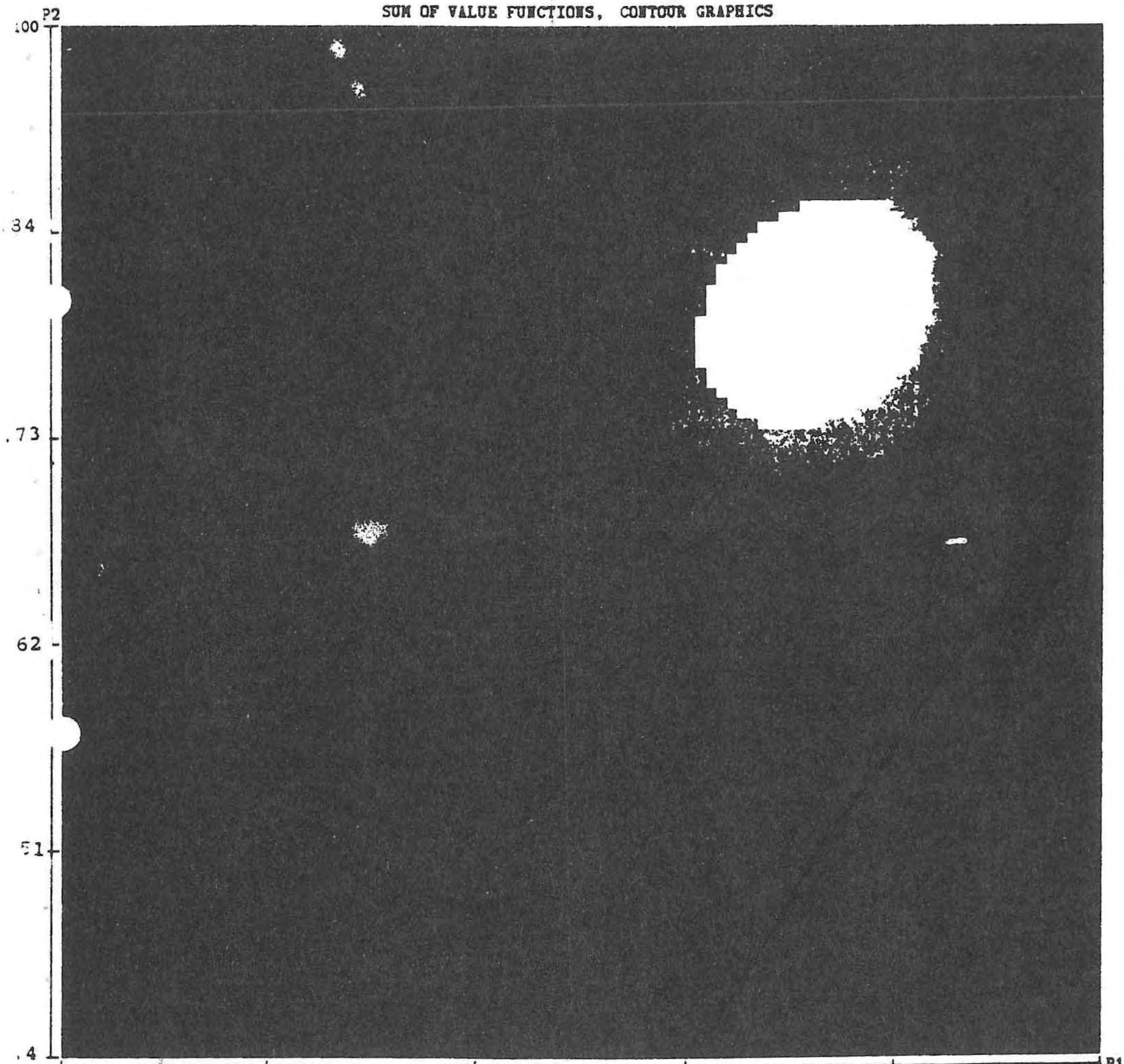
This refinement makes synchronization with Ss policies a possibly more desirable outcome for both firms, whenever they are able to pick up the initial conditions. If firms sit down before the start of the game and look for an equilibria that will satisfy both, synchronization may emerge as the most likely equilibria. This is due to the fact that the sum of the value functions for both firms is higher when they are synchronized. This result is reminiscent of the Maskin and Tirole (1988(a)) result which proves that the monopoly outcome can be sustained as an equilibria and it is in the Pareto Frontier. The Maskin and Tirole's result implies that firms will choose a constant relative price through time, which is precisely what synchronization with Ss policies means in the context of my game. On the other hand, this conclusion is the counterpart for the game case, of the Sheshinski and Weiss (1990) result that says that in the collusion case the monopolist prefers synchronization with Ss policies over staggering.

GRAPH 7

SUM OF VALUE FUNCTIONS



GRAPH 8



SUM OF VALUE FUNCTIONS, CONTOUR GRAPHICS
P2
100
.84
.73
.62
.51
.4
P1
.4
.51
.62
.73
.84
.95

For all cases the frequency of price adjustment[25] decreases. Table 7 summarizes these results. For the synchronized case the results are consistent with those obtained by Sheshinski and Weiss (1978).

5.2 CHANGES IN THE RATE OF INFLATION.

When inflation rises I get the same results as in the fixed cost case with regard to S , s , $S^1(x_2)$ and $s^1(x_2)$. S goes up, s goes down for all the models. The reader can confirm this latter statement by looking at Table 4. $S^1(x_2)$ goes up for $DB=.8$ and $DB=.9$, and is undetermined for $DB=.5$. Finally, $s^1(x_2)$ goes down. A difference with the fixed cost case is that the frequency of price adjustments goes up. Graphs five depict this case and Table 8 summarizes it.

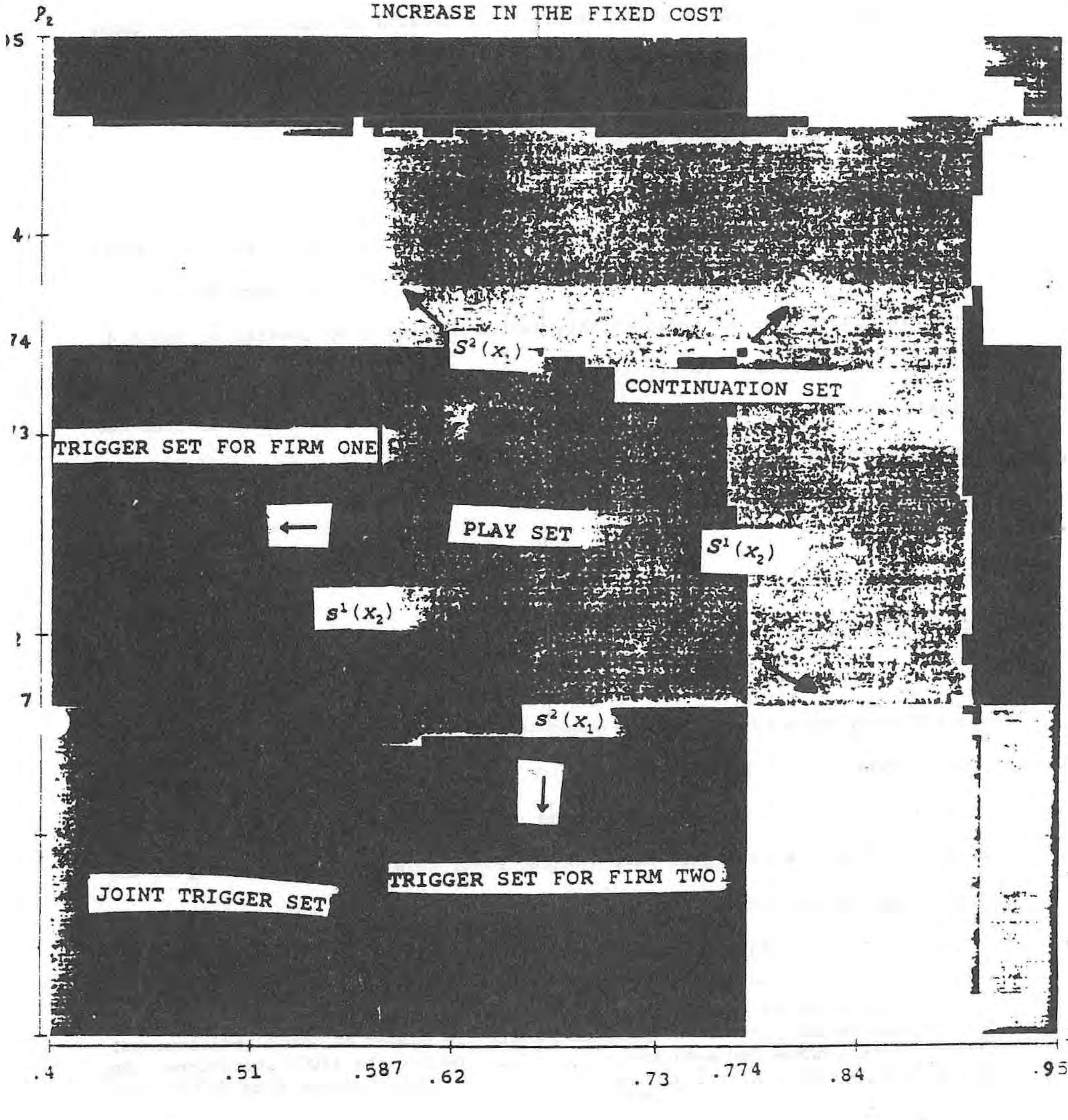
5.3 CHANGES IN THE DISCOUNT FACTOR.

In the synchronized case when the discount factor B goes down we have that the point towards which both firms move (S) , whenever they do so, goes down. In addition, the point at which both firms decide to move (s) declines as well (see Table 4). We can gain some insight about this result by looking at the following first order condition, which have to be satisfied when both firms are moving in a synchronized fashion to the optimal point (S,S) , and have to wait

[25] There are two ways in which I infer these results from the model first in the simulation part of the program I count the number of jumps (adjustments) that the firm makes for each initial condition during the 15000 iterations. An alternative way of inferring these result is by looking at the size of the play set.

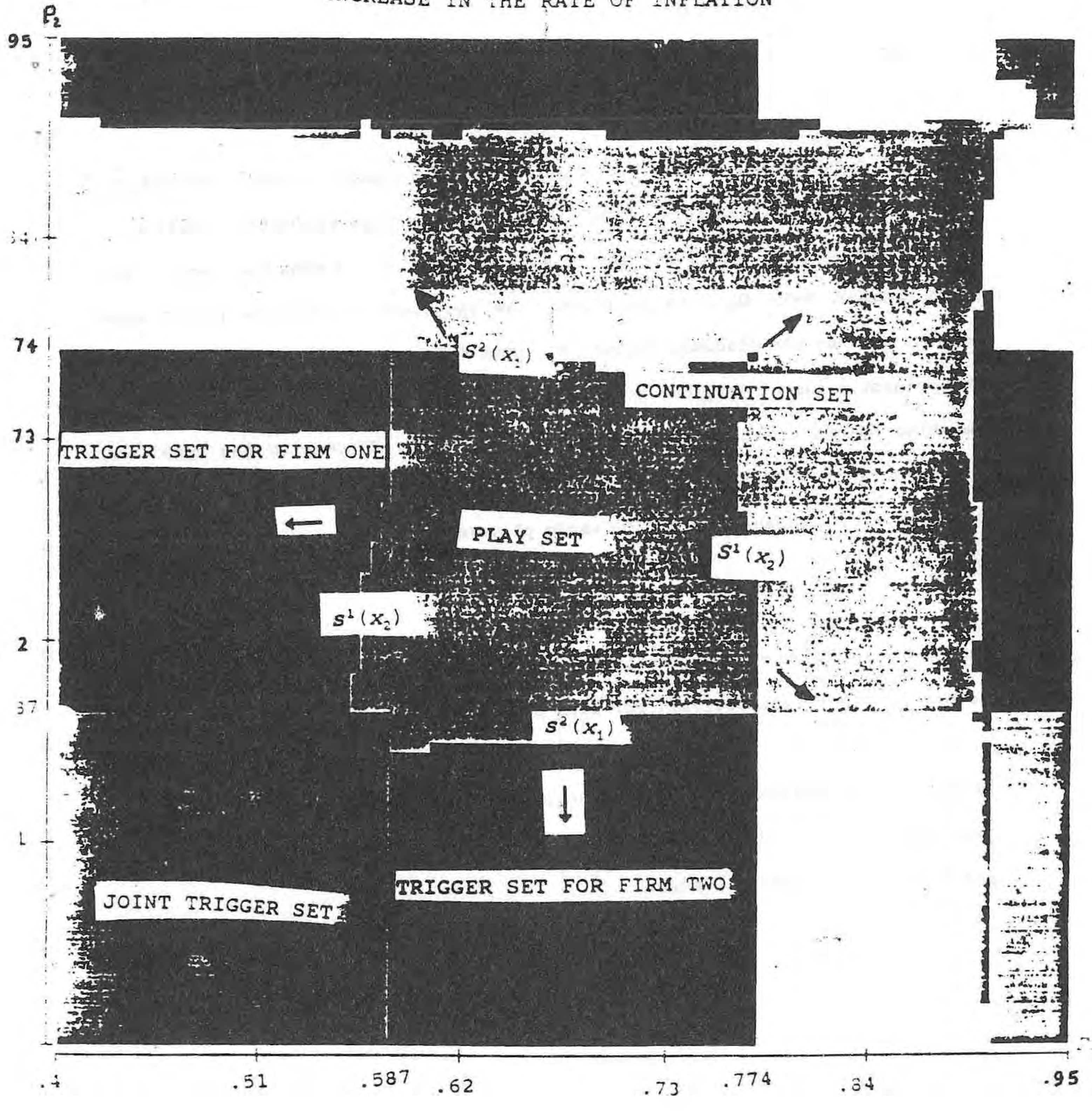
GRAPH 4

INCREASE IN THE FIXED COST



GRAPH 5

INCREASE IN THE RATE OF INFLATION



Θ_1 , to reach their trigger sets:

$$V_1^1(S,S)=\sum_{m=1}^{\Theta_1} e^{-rhm}F_1^1(S-ghm,S-ghm) = 0$$

By strict concavity of $F(\cdot,\cdot)$, if $F_1^1(S,S)>0 \Rightarrow F_1^1(S-gm,S-gm)>0$. Which contradicts the above equation therefore $F_1^1(S,S)\leq 0$. In order for the above equation to be zero the marginal profitability of each firm at S is negative, and at the joint trigger set Ω_m , is positive. As Sheshinski and Weiss (1978) have pointed out: as the Discount Factor B goes down "the net effect is to reduce the present value of marginal profits ..the firms's response is to increase the relative weight of the positive marginal profits by decreasing the upper and lower real prices" (Sheshinski and Weiss (1978) p. 297). This effect on marginal profitability is translated to the whole play set, causing a southwest shift of this set.

The boundary between the continuation set and the trigger set for firm one $s^1(x_2)$ ($s^2(x_1)$ respectively for firm two) decreases.

Finally, the optimal response function $S^1(x_2)$ goes down too (in the continuation set). This happens for all cases. Table 9 summarizes these results and Graph six illustrates them.

GRAPH 6

INCREASE IN THE DISCOUNT FACTOR

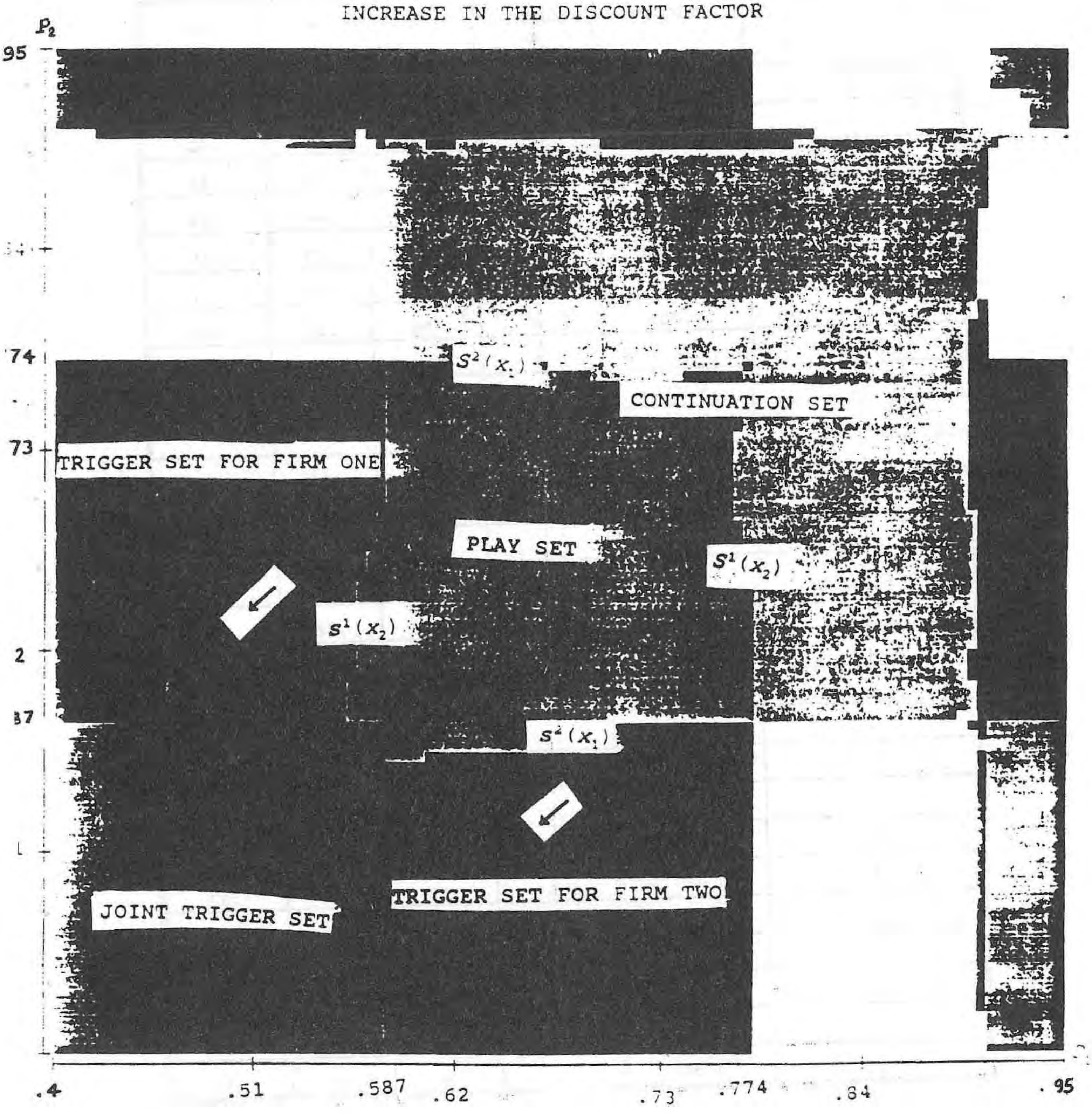


TABLE 4
S-s RESULTS.

B	FIXED COST	INFLATION	DEGREE OF INTERA.	S	s	S-s
.992	.9	0.003	.9	.82	.49	.33
			.5	.75	.47	.28
		0.006	.9	.87	.46	.41
			.5	.79	.44	.35
		0.0012	.9	.77	.52	.25
			.5	.71	.5	.21
	.5	0.003	.9	.80	.53	.27
			.5	.74	.52	.22
		0.006	.9	.84	.5	.34
			.5	.77		
		0.0012	.9	.76	.56	.2
			.5	.70	.53	.17
	.2	0.003	.9	.78	.58	.2
			.5	.71	.55	.16
		0.006	.9	.80	.56	.24
			.5	.73	.53	.2
		0.0012	.9	.75	.60	.15
			.5	.69	.56	.13
.978	.9	0.003	.9	.77	.41	.3
			.5	.71	.41	.36
		0.006	.9	.82	.39	.43
			.5	.75	.38	.37
		0.0012	.9	.73	.42	.31
			.5	.67	.41	.26
	.5	0.003	.9	.76	.48	.28
			.5	.71	.47	.24
		0.006	.9	.81	.46	.35

CONTINUATION OF TABLE 4

B	FIXED COST	INFLATION	DEGREE OF INTERA.	S	s	S-s
.978	.5	.006	.5	.74	.45	.29
		.0012	.9	.73	.49	.24
			.5	.67	.47	.2
	.2	.003	.9	.76	.55	.21
			.5	.70	.54	.16
		.006	.9	.78	.54	.24
			.5	.72	.52	.2
		.0012	.9	.72	.56	.16
			.5	.67	.55	.12
.987	.9	.003	.9	.80	.46	.34
			.5	.74	.45	.29
		.006	.9	.86	.42	.44
			.5	.78	.42	.36
		.0012	.9	.75	.48	.27
			.5	.69	.45	.24
	.5	.003	.9	.79	.51	.28
			.5	.72	.49	.23
		.006	.9	.83	.49	.34
			.5	.76	.48	.28
		.0012	.9	.74	.53	.21
			.5	.69	.51	.18
	.2	.003	.9	.77	.57	.2
			.5	.71	.55	.16
		.006	.9	.80	.55	.25
			.5	.73	.54	.19
		.0012	.9	.74	.59	.15
			.5	.68	.55	.13

CONTINUATION OF TABLE 4.

B	FIXED COST	INFLATION	DEGREE OF INTERA.	S	s	S-s
.998	.4	.003	.8	.80	.56	.24
			.5	.74	.54	.2
		.006	.8	.83	.53	.3
			.5	.77	.43	.34
	.2	.003	.8	.78	.59	.19
			.5	.72	.58	.14
		.006	.8	.80	.56	.24
			.5	.74	.54	.2

Note: *B* in the first column of the first row corresponds to the discount factor.

6. COMPARISON WITH COLLUSION.

Sheshinski and Weiss (1990) have examined the collusion case. They conclude that the synchronized movement is locally stable for the case in which we have strategic complementarities. They also find out that, for the quadratic profit function, an increase in the rate of inflation increases the frequency of price changes. In turn, an increase in the costs of adjustments reduces the frequency of price changes. A " stronger positive price interaction reduces the frequency of price changes in the synchronized steady-state" (Sheshinski and Weiss (1990) p.4). I solved the collusion case in the computer and I confirm all of the Sheshinski and Weiss results.

More interesting than these results are those in which we compare the collusion results with the game results. In the simulations we have for all initial conditions and for all models with the same parameter values[26], the frequency of adjustments (jumps) in prices is higher under the game than under the collusion case; these differences go up as the degree of interaction increases (as *DB* goes down). Table 6 depicts this result. In the case of synchronization the difference between S and s goes up as we switch from the game to collusion (see Table 5). This represent another way to confirm the results regarding the frequency of price adjustments.

Secondly, as the degree of interaction increases, $(S^{co}-s^{co})-(S^{ga}-s^{ga})$ (where the supraindices *co* and *ga* represent the collusion and the game case

[26] I solved the collusion case mainly for the cases written in Table Five and Six. However, I ran a representative sample of the other cases, and the results were exactly the same.

respectively S and s have the same meaning as above) goes up. The monopoly will be more able to exploit the complementarities and hence to maintain a higher price. This property allows it to adjust the prices less frequently and to save fixed costs. Table 5 illustrates this result for a subset of the parameter models considered.

TABLE 5
S-s RESULTS COMPARISON BETWEEN THE COLLUSION OUTCOME AND THE GAME OUTCOME.

B	*K*	INFL	*DB*	S GAME	s GAME	S-s GAME	S COLL	s COLL	S-s COLL
.987	.9	.003	.9	.80	.46	.34	.81	.46	.35
			.5	.74	.45	.29	.81	.46	.35
		.006	.9	.86	.42	.44	.86	.43	.43
			.5	.78	.42	.36	.86	.43	.43
		.0012	.9	.75	.48	.27	.76	.48	.28
			.5	.69	.45	.24	.76	.48	.28
	.5	.003	.9	.79	.51	.28	.80	.51	.29
			.5	.72	.49	.23	.80	.51	.29
		.006	.9	.83	.49	.34	.85	.49	.36
			.5	.76	.48	.28	.85	.49	.36
		.0012	.9	.74	.53	.21	.75	.53	.22
			.5	.69	.51	.18	.75	.53	.22
	.2	.003	.9	.77	.57	.2	.78	.57	.21
			.5	.71	.55	.16	.78	.57	.21
		.006	.9	.80	.55	.25	.81	.55	.26
			.5	.73	.54	.19	.81	.55	.26
		.0012	.9	.74	.59	.15	.75	.59	.16
			.5	.68	.55	.13	.75	.59	.16

Note: The letters in the first row correspond to the following meanings: *B* corresponds to the discount factor, *K* to the size of the fixed cost, *DB* corresponds to the degree of interaction.

7. WELFARE ANALYSIS.

Whether increases in the rate of inflation diminishes the social welfare is an important topic that the literature has tried to address for many years.

In this partial equilibrium model a rationale for affecting the level of inflation would exist if I can show that, for a wide variety of parameters, increases in the rate of inflation lessen the social welfare. Indeed, this is what happens in the context of my model.

For all parameter values, I calculate the present value of the consumer surplus as the discounted stream of consumer surplus flows that emerge from the Nash outcome at each possible state. For each node in the grid I have the present value of welfare that the consumer will get if the game starts in that node. Once I identify the present value of the consumer surplus for each node, I calculate the maximum of the present value consumer surpluses over all nodes. Not surprisingly, when the rate of inflation rises, the maximum present value of consumer surplus decreases for all models. Let us take the synchronized case to gain some intuition for this result. As we saw in the comparative statics section: As inflation goes up s goes down, and the frequency of price adjustments increases. Therefore, though firms will charge a lower price than before for some time, they spend much less time in the low price section of the Ss band thereby decreasing consumer surplus.

Another interesting exercise is to compare the frequency of price adjustments for the social planner[27] with the one gotten from the oligopoly. For each parameter values[28], I compare the frequency of price adjustments between the social planner and the oligopoly. Since the social planner internalizes the positive interactions in adjusting the prices, it is not surprising to notice that the social planner adjusts a lower number of times than the oligopolistic firms for all models (See Table 6).

We can see in Table 5 -- as we noticed in the collusion comparison above--, that the difference in the frequency of price adjustments goes up as the degree of interaction increases (as *DB* decreases). It is interesting to notice that $(S^{sp}-s^{sp})-(S^{ga}-s^{ga})$ (where the supraindices *so* and *ga* represent the social planner and the game case respectively *S* and *s* have the same meaning as before) increases as the degree of interaction goes up. The social planner intends to maximize social surplus net of production costs with fixed costs of adjustments for the prices. Thus, the social planner will try to save on the fixed costs by exploiting the complementarities in prices. This intuition rationalizes the above results. As complementarities go up (*DB* goes down), the social planner will further exploit these complementarities, sharpening the differences with the game.

It is meaningful to compare how the difference in the frequency of price

---

[27] The social planner maximizes social surplus.

[28] For very large fixed costs the optimal policy of the social planner is never to adjust the prices. Since the prices erode at an exponential rate, they will never become negative. Thus, for a fixed cost equal to .9 this is always the optimal policy. I am not making use these parameter values in this model's comparison.

adjustments for the game vis-á-vis the social planner changes as the rate of inflation goes up. In all the models studied[29] the frequency of price adjustments goes up as the rate of inflation increases. As the rate of inflation goes up, the difference in price adjustments between oligopolist and the social planner increases. Consequently, the oligopolists deviate further from the optimal frequency of expenditures of fixed costs, when I increase the rate of inflation. This measure could be a proxy for the social costs of inflation. It shows that as inflation goes up, oligopolistic organization of markets tend to spend more resources than what it would be optimally desirable. This happens because oligopolies are not able of internalizing the complementarities in deciding how frequent to adjust prices, as well as, the social planner does. This argument might serve to justify public policies that reduce the rate of inflation.

Finally, we need to examine what happens with the producer surplus in the synchronized outcome, when the rate of inflation increases. In fact, for all models, when the rate of inflation goes up the producer surplus goes down. The explanation for this result follows from the arguments above, as the rate of inflation increases, the frequency of price adjustments goes up, thus forcing the oligopolists to spend far more fixed costs.

---

[29] As in the collusion case, I solved the social planner case mainly for the parameter values indicated in Tables five and six. However, I also ran a sample of the other cases, and I got the same results.

TABLE 6
COMPARISON IN THE NUMBER OF ADJUSTMENTS BETWEEN
THE GAME, THE SOCIAL PLANNER AND THE COLLUSION CASE.

B	*K*	INFL	*DB*	**AVERAGE NUMBER ADJUST COLLUS**	**AVERAG NUMBER OF ADJUST SOC PLA**	**AVERAG NUMBER OF ADJUST GAME**	**DIFFEREN IN ADJUST BETWEEN GAME AND SOC PLA**
.987	.9	.003	.9	77	N.A.+	79	N.A.+
			.5	76	N.A.+	92	N.A.+
		.006	.9	125	N.A.+	129	N.A+
			.5	125	N.A.+	156	N.A.+
		.0012	.9	38	N.A.+	39	N.A.+
			.5	38	N.A.+	40	N.A.+
	.5	.003	.9	98	41	100	59
			.5	97	41	100	59
		.006	.9	158	70	165	95
			.5	158	69	198	129
		.0012	.9	51	19	52	33
			.5	50	19	57	38
	.2	.003	.9	138	68	142	74
			.5	138	68	142	74
		.006	.9	221	109	228	119
			.5	221	109	273	169
		.0012	.9	72	34	74	40
			.5	71	34	86	52

Note: The figures in the table above were calculated as an average of the one hundred initial conditions.
+ I do not have figures for these parameter values because *K* is to big, therefore, the optimal policy that the social planner chooses is to wait until the real prices reach the axes of the positive orthant. Since inflation erodes the real prices at an exponential rate, these trigger boundary is in fact never reached.
Note: The letters in the first row correspond to the following meanings: *B* corresponds to the discount factor, *K* to the size of the fixed cost, *DB* corresponds to the degree of interaction.

CONCLUDING REMARKS.

In this paper I have examined a symmetric duopoly model of price competition with a fixed exogenous inflation rate and fixed adjustment costs. I characterized this game by approximating the solution in the computer. What follows, summarizes the insights I was able to obtain from this approach:

Synchronization with Ss policies appears to be the most frequent outcome. The firms follow synchronized adjusment in their prices with each firm following an Ss rule far more frequently than nonsynchronized outcomes, -and this is so from a wide variety of initial conditions and a wide variety of models-. Hence, in dynamic oligopolistic competition with fixed costs of adjustments synchronized Ss rules appear to be the most robust result. Secondly, and not for that less important, synchronization with Ss rules followed by each firm, appears to be the most reasonable outcome for both firms to start the game, if we allow for preplay communication. This result follows from the fact that the sum of the two value functions attains its maximum at a synchronized level.

The comparative statics with respect to a change in the rate of inflation, a change in the discount factor and a change in the costs of adjustments are consistent with the results advanced by Sheshinski and Weiss (1978) for the single good case. As inflation rises, the frequency of price adjustments go up. As the fixed cost goes up, the frequency of price adjustments decreases.

Finally, I was able to draw some comparisons between the outcome of the game with the collusion case, as well as with the social planner case. Not surprisingly, in the collusion case, the frequency of price adjustments decreases, when we compare this with the game solution. Moreover, the difference between the frequency of price adjustments in the collusion case and the frequency of price adjustments in the game case, goes up as the degree of interaction increases. The same property holds for the comparison of the game with the social planner.

Additionally, I try to deal with the issue of the social costs of inflation. I calculated the maximum of the present value of the consumer's surplus and then I analyzed its behavior as the rate of inflation increases. I conclude that as the rate of inflation increases, the present values of the consumer surplus decreases. An alternative proxy to measure how the social costs of inflation behave as the rate of inflation is increased was the difference between the frequency of price adjustments under the social planner solution and the solution of the game. This difference climbs as the rate of inflation increases. Consequently, the oligopoly spends more resources (fixed costs) as the rate of inflation goes up than what it would be socially desirable. Furthermore, for the synchronized case, the producer's surplus drops when the rate of inflation increases. This latter result, confirms in an alternative way, the impact on the frequency of price adjustments, of changes in the rate of inflation.

APPENDIX 1.

Finally, allow me to make a brief comment about mixed strategy equilibria. The only area in the state space in which mixed strategy equilibria appears[30], is in the region highlighted in black in Graph nine. That is the area of the state space in which the continuation set, the trigger set for one of the firms and the joint trigger set are close. This equilibria occurs because of problems approximating the boundaries between the different sets. As mentioned in above, these boundaries are non-differentiable. I was not lucky to put the nodes in the right position. Consequently, my finite element technique has problem of approximating the boundary in this area of the state. For any outcome of the mixed strategy equilibria, the next period both firms will be close to the intersection of the optimal response functions. i.e. either both firms move today or both firms move tomorrow or one of the firms move today and the other moves tomorrow. In the limit (in continuous time), since we have reactions within an instant, the outcome will be that both firms move together to the intersection of the optimal response functions.

---

[30] Mixed Strategy equilibria appear in very few models, most of the models do not have that equilibria.

Appendix 2.

As mentioned in the main text the computational approach works in two stages. First I solve for the Nash equilibria in a square lattice with equally distant points in both dimensions so that T in equation (1) is exactly satisfied for any point in the lattice. Second, I look for an interpolation method that satisfies the piecewise differentiability imposed by the discretization of time. In this second procedure, I choose a finite dimensional basis and represent our approximated value function in this subspace. Then through the iteration procedure I map this finite dimensional approximation into another finite dimensioanl approximation until I reach a level in which V^n is reasonably similar to V^{n+1}. As mentioned in the text I use bilinear cardinal function because they span the space of purely continuous functions C^o when the size of the grid tends to zero (the number of points go up to infinity), and therefore satisfies the property of piecewise differentiability of the theoretical value function. Secondly, this basis gives me computational speed. By definition of the bilinear cardinal functions, the coefficients which accompany this basis in the representation of the value function (equation (2) in the text), are just the value function calculated in the solution of the Nash Equilibria at each one of the nodes that surround the point of interest for evaluating the function. In other words, the difficult task is to calculate the cardinal functions. Once these are calculated, the projection coefficients (the a_j in (2)) are trivially determined. Due to the small support property of the bilinear cardinal functions, the representation of the value function in (2) in the case of bilinear interpolation is reduced to the following expression:

$$\overline{V^1}(x_1,x_2)=\overline{V^1}(x_1^1,x_2^1)\,\phi_{11}(x_1,x_2)+\overline{V^1}(x_1^0,x_2^1)\,\phi_{01}(x_1,x_2)$$
$$+\overline{V^1}(x_1^0,x_2^0)\,\phi_{00}(x_1,x_2)+\overline{V^1}(x_1^1,x_2^0)\,\phi_{10}(x_1,x_2)$$

Where ϕ_{11} is equal to one if $x_1=x_1^1$ and $x_2=x_2^1$, at this point all the other ϕs are equal to zero. Alternatively ϕ_{01} is equal to one if $x_1=x_1^0$ and $x_2=x_2^1$, the remaining ϕs are equal to zero here. A similar reasoning holds for the other two ϕs . All the other terms are zero. Only those cardinal functions which are in the nodes at the corners of the rectangle in which the state falls are non-zero.

Given the properties of this model, it is not advisable to make use of the spectral methods that have been used in economics to approximate the value function (See Judd 1990). The use of Chebychev polynomials for example, impose a degree of smoothness in the solution (C^∞) which is clearly undesirable for this problem.

More explicitly, following the technique of finite element, for each subsquare $[x_1^j, x_1^{j+1}] X [x_2^k, x_2^{k+1}]$, I use the information gotten from the first stage at each one of the nodes of this subsquare[31]. I search for a bilinear function[32] that best approximates the value function in this subsquare. In practice this is usually done by the use of the standard rectangle. The rectangle with vertices in the points (1,1) , (-1,1) , (-1,-1) , (1,-1) . I map the value of x_1 and x_2 to the standard rectangle by the following functions:

The cardinal functions are given by the following expression:

We notice immediately that $\phi_1=1$ when $\zeta=1$ and $\eta=1$. $\phi_2=1$ when $\zeta=-1$ and $\eta=1$. $\phi_3=1$ when $\zeta=-1$ and $\eta=-1$. Finally $\phi_4=1$, when $\zeta=1$ and $\eta=-1$.

[31] In the context of this model, this information comprises only the values of the operator $TV^{1,j}$ at each of the nodes.

[32] The class of bilinear functions is spanned by the monomials 1 , x , y and xy .

$$\zeta = 1 - \frac{2\,(x_1^{j+1} - x_1)}{x_1^{j+1} - x_1}$$
$$\eta = 1 - \frac{2\,(x_2^{j+1} - x_2)}{x_2^{j+1} - x_2}$$

The interpolant to the data $TV^{1,j}(x_1^{j+1}, x_2^{k+1})$, $TV^{1,j}(x_1^{j}, x_2^{k+1})$, $TV^{1,j}(x_1^{j}, x_2^{k})$ and $TV^{1,j}(x_1^{j+1}, x_2^{k})$ is given by the following equation:

$$B(\zeta,\eta) = TV^{1,j}(x_1^{i+1}, x_2^{k+1})\,\phi_1(\zeta,\eta) + TV^{1,j}(x_1^{i}, x_2^{k+1})\,\phi_2(\zeta,\eta)$$
$$+ TV^{1,j}(x_1^{i}, x_2^{k})\,\phi_3(\zeta,\eta) + TV^{1,j}(x_1^{i+1}, x_2^{k})\,\phi_3(\zeta,\eta)$$

It is interesting to notice that due to the linearity of the approach, in addition to the fact that the axes are parallel to the subsquares, I have a unique surface that interpolates the whole grid (see Lankaster and Salkauskas 1986).

An important property of this model is the fact that the optimal response function of firm i S^i is independent of the state in which the firm is located. It only depends on the other firm's state. This property holds because the cost of adjustment function of the state is linear. This characteristic allows me to speed up the calculations in the computer. Since I am assuming that the operator $TV^{1,j}(\cdot,\cdot)$ is C^0 , in order to calculate the optimal response function $S_1^*(\cdot)$ whenever player one is moving, I use a golden section algorithm. A shortcoming of this algorithm is its speed. But it is the only one available for C^0 functions.

It is interesting to point out that the uniqueness of the interpolating polynomial is not maintained when I try to approximate the surface with polynomials of higher degree. For instance, if I use bicubic splines for example, some conditions about the second derivative have to be imposed. Besides, for polynomials of higher degree, the cardinal functions do not posses the small support property. In the linear case the cardinal functions posses the same nice properties as Bsplines functions. This was the rationality for using them. In higher degrees the Cardinal functions may not be easy to construct. The calculation may involve the inversion of a Vandermodian matrix, which we know is a dangerous procedure (see Lankaster and Salkauskas (1986)).

Consequently, if I want to extend the interpolation method to polynomials with second degree,- and hence allow for a higher degree of smoothness in the value function-, I should use as a basis B splines, which maintain the small support property and therefore, allow me to calculate the approximation more efficiently.

Table 7
Changes in the Fixed cost

	Size of the Play Set	S	s	$S^1(x_2)$	$s^1(x_2)$	Frequency of Price Adjustments
DB=5	up	up	down	not clear	down	down
DB=8	up	up	down	up	down	down
DB=9	up	up	down	up	down	down

Note: The entries indicate an increase in the fixed cost.

Table 8
Changes in the Rate of Inflation.

	Size of the Play Set	S	s	$S^1(x_2)$	$s^1(x_2)$	Frequency of Price Adjustments
DB=5	up	up	down	not clear	down	up
DB=8	up	up	down	up	down	up
DB=9	up	up	down	up	down	up

Note: The entries indicate an increase in the rate of inflation.

Table 9
Changes in the Discount Factor.

	S	s	$S^1(x_2)$	$s^1(x_2)$		Play Set shifts southwest
DB=5	down	down	down	down		yes
DB=8	down	down	down	down		yes
DB=9	down	down	down	down		yes

Note: The entries indicate a decrease in the discount factor.

Table 10
Collusion Comparisons.

	S-s	Frequency of Price Adjustments.
DB=5	up	down
DB=8	up	down
DB=9	up	down

	$(S^{co}-s^{co})-(S^{ga}-s^{ga})$ (where the supraindices *co* and *ga* represent the collusion and game case respectively)
Degree of Interaction goes up	up

Note: The results in the tables above indicate a shift from the noncooperative outcome to the cooperative one. The results above are a summary of those gotten in Table six.

Table 11
Welfare Analysis.

	Consumer Surplus
Increase in the Rate of Inflation.	Down

	S-s	Frequency of Price Adjustments.
DB=5	up	down
DB=8	up	down
DB=9	up	down

	$(S^{sp}-s^{sp})-(S^{ga}-s^{ga})$ (where the supraindices *sp* and *ga* represent the social planner and the game case respectively)	Difference in the Frequency of Price Adjustments between the game and the social planner solution
Degree of Interaction goes up	up	up

	Difference in the Frequency of Price Adjustments between the game and the social planner solution.
Increase in the Rate of Inflation.	up

Note: The results in the table above indicate a shift from the noncooperative outcome to the social planner case. These results, are just a summary of the results in Table 5 and 6.

BIBLIOGRAPHY.

Abreu, D., D. Pearce, and E. Stacchetti (1986): "Optimal Cartel Equilibria with Imperfect Monitoring," *Journal of Economic Theory*, 39, 251-269.

Abreu, D. (1986): "Extremal Equilibria of Oligopolistic Supergames. *Journal of Economic Theory*, 39, 191-225.

Bénabou, R. (1985): *Optimal Price Dynamics and Speculation with a Storable Good*. Ph.D. thesis, Department of Economics, Massachusetts Institute of Technology.

Brock, W.A. and J.A. Sheinkman (1985): "Price-Setting Supergames with Capacity Constaints," *Review of Economic Studies*, 52, 371-382.

Caplin, A. and E. Sheshinski (1987): "Optimality of (S,s) Pricing Policies" Working Paper in Economics E-87-43, Hoover Institution.

Castañeda, A. (1992): *Dynamic Oligopolistic Games with Fixed Cost of Adjustments and Changing Environments*. Ph.D. thesis, Department of Economics, Stanford University.

Dutta, P. and A. Rustichini (1990): "s-S Equilibria in Stochastic Games with an Application to Product Innovations," mimeo. Paper Presented in the S.I.T.E. seminar at Stanford University summer 1991.

Eaton, J. and M. Engers (1990): "Intertemporal Price Competition," *Econometrica*, 58, 637-659.

Friedman, J.W. (1971): "A Noncooperative Equilibrium for Supergames," *Review of Economic Studies*, 38, 1-12.

Fudenberg, D. and J. Tirole (1983): "Capital as Commitment: Strategic Investment to Deter Mobility," *Journal of Economic Theory*, 31, 227-250.

Fudenberg, D. and J. Tirole (1991): *Game Theory*. Cambridge: MIT Press.

Gertner, R. (1986): *Dynamic Duopoly with Price Inertia*. Ph.D. thesis, Department of Economics, Massachusetts Institute of Technology.

Green, E. and R. Porter (1984): "Noncooperative Collusion Under Imperfect Price Information," *Econometrica*, 52, 87-100.

Hanig, M. (1986): *Differential Gaming Models of Oligopoly*. Ph.D. thesis, Department of Economics, Massachusetts Institute of Technology.

Judd, K. (1985): "Closed Loop Equilibrium in a Multi-Stage Innovation Race," Discussion Paper 647, Kellog Graduate School of Management, Northwestern University.

Judd, K. (1990): "Cournot vs. Bertrand: A Dynamic Resolution," mimeo, Hoover Institution.

Lee, T. and L. Wilde (1980): "Market Structure and Innovation: A Reformulation," *Quaterly Journal of Economics* 194, 429-436.

Maskin E. and J. Tirole (1988 (b)): "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs," *Econometrica*, 56, ??-570.

Maskin E. and J. Tirole (1988 (a)): "A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles," *Econometrica*, 56, 571-599.

Maskin E. and J. Tirole (1987): "A Theory of Dynamic Oligopoly, III: Cournot Competition," *European Economic Review*, 31, 947-968.

Reinganum, J. (1982): "A Dynamic Game of R&D: Patent Protection and Competitive Behavior," *Econometrica* 50, 671-688.

Reinganum, J. (1981): "Dynamic Games of Innovation," *Journal of Economic Theory*, 25, 21-46.

Rotemberg, J. and G. Saloner (1986): "A Supergame-Theoretic Model of Business Cycles and Price Wars During Booms," *American Economic Review*, 76, 390-407.

Simon, L. and M. Stinchcombe: "Extensive Form Games in Continuous Time: Pure Strategies," *Econometrica* 57, 1171-1214.

Shapiro, C. (1987): "Theories of Oligopoly Behavior," mimeo, Princeton University.

Sheshinski, E. and Y. Weiss (1990) "Staggered and Synchronized Price Policies under Inflation: The Multiproduct Monopoly Case," Working Paper # 234, The Hebrew University of Jerusalem.

Sheshinski, E. and Y. Weiss (1992) "Staggered and Synchronized Price Policies under Inflation: The Multiproduct Monopoly Case," *Review of Economic Studies*,

Sheshinski, E. and Y. Weiss (1978): "Inflation and Cost of Price Adjustment," *Review of Economic Studies* 50: 513-529.

Spence, A.M. (1977): "Entry, Capacity, Investment and Oligopolistic Pricing," *Bell Journal of Economics*, 8, 534-544.

Sulem, A. (1986): "Explicit Solution of a Two Dimensional Deterministic Inventory Problem," *Mathematics of Operations Research*, 11, 134-146.

Tirole, J. (1988): *The Theory of Industrial Organization*. Cambridge: MIT Press.

Villas-Boas, J.M. (1990): "Dynamic Duopolies with Non-Convex Adjustment costs," mimeo, Massachusetts Institute of Technology.

SERIE DOCUMENTOS DE TRABAJO

The following working papers from recent years are still available upon request from:

Rocío Contreras,
Centro de Documentación, Centro De Estudios Económicos, El Colegio de México A.C., Camino al Ajusco # 20 C.P. 01000 México, D.F.

90/I Ize, Alain. "Trade liberalization, stabilization, and growth: some notes on the mexican experience".

90/II Sandoval Musi, Alfredo. "Construction of new monetary aggregates: the case of Mexico".

90/III Fernández, Oscar. "Algunas notas sobre los modelos de Kalecki del ciclo económico".

90/IV Sobarzo, Horacio E. "A consolidated social accounting matrix for input-output analysis".

90/V Urzúa, Carlos M. "El déficit del sector público y la política fiscal en México, 1980 - 1989".

90/VI Romero, José. "Desarrollos recientes en la teoría económica de la unión aduanera".

90/VII García Rocha, Adalberto. "Note on mexican economic development and income distribution".

90/VIII García Rocha, Adalberto. "Distributive effects of financial policies in Mexico".

90/IX Mercado, Alfonso and Taeko Taniura "The mexican automotive export growth: favorable factors, obstacles and policy requirements".

91/I Urzúa, Carlos M. "Resuelve: a Gauss program to solve applied equilibrium and disequilibrium models".

91/II Sobarzo, Horacio E. "A general equilibrium analysis of the gains from trade for the mexican economy of a North American free trade agreement".

91/III Young, Leslie and José Romero. "A dynamic dual model of the North American free trade agreement".

91/IV Yúnez-Naude, Antonio. "Hacia un tratado de libre comercio norteamericano; efectos en los sectores agropecuarios y alimenticios de México".

91/V Esquivel, Hernández Gerardo. "Comercio intraindustrial México-Estados Unidos".

91/VI Márquez, Colín Graciela. "Concentración y estrategias de crecimiento industrial".

92/I Twomey, J. Michael. "Macroeconomic effects of trade liberalization in Canada and Mexico".

92/II Twomey, J. Michael. "Multinational corporations in North America: Free trade intersections".

92/III Izaguirre Navarro, Felipe A. "Un estudio empírico sobre solvencia del sector público: El caso de México".

92/IV Gollás, Manuel y Oscar Fernández. "El subempleo sectorial en México".

92/V Calderón Madrid, Angel. "The dynamics of real exchange rate and financial assets of privately financed current account deficits"

92/VI Esquivel Hernández, Gerardo. "Política comercial bajo competencia imperfecta: Ejercicio de simulación para la industria cervecera mexicana".

93/I Fernández, Jorge. "Debt and incentives in a dynamic context".

93/II Fernández, Jorge. "Voluntary debt reduction under asymmetric information".

93/III Castañeda, Alejandro. "Capital accumulation games".

93/IV Castañeda, Alejandro. "Market structure and innovation a survey of patent races".

93/V Sempere, Jaime. "Limits to the third theorem of welfare economics".

93/VI Sempere, Jaime. "Potential gains from market integration with individual non-convexities".

93/VII Castañeda, Alejandro. "Dynamic price competition in inflationary environments with fixed costs of adjustment".