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ABSTRACT

It is known that the incentive distortions produced by a large inherited debt could be so acute as to render a partial debt forgiveness Pareto improving. This issue has received much attention in the context of international debt. We analyze it in an asymmetric information situation. It has been argued that when the country has private information about relevant parameters the possibility appears of the bank not forgiving debt because it believes the country does not need it. We investigate this problem in a dynamic context. First we do it allowing the bank to give only a temporary relief to the country. We characterize equilibria for different parameter values. The problem gets more complex due to the arising of the ratchet effect. This effect consists, in the context of our work, in the country distorting its effort so as to hide its character and thus avoid the loss of future rents. Next, we study the problem allowing the bank to reduce debt. It is possible that this eliminates the ratchet effect and enhances the welfare of both the bank and the country.

0. INTRODUCTION

It has been argued that the presence of a large inherited debt creates distortions on the actions the debtor undertakes and that these distortions could conceivable render a partial debt forgiveness Pareto improving. This issue has attracted much attention in the context of international debt.

The amount of forgiveness which the creditor finds convenient to grant depends on the country's characteristics. It seems reasonable to assume that the country possesses better information than its creditors about some of these characteristics. Obviously, the country will have incentives to hide the true value of such relevant parameters. It might assert, for instance, that the social costs of further reforms would be too high and any further 'belt tightening' would be unbearable, although it didn't believe so. It has also been argued (Froot, Scharfstein and Stein (1989)) that this asymmetry of information might lead to a stonewalling situation, in that the bank does not reduce debt and the country makes an inefficient action. This problem, as in the case of most incentive problems derived from the presence of a large inherited debt, has been analyzed in a static context. Yet, the relationship between a country and its creditors is not confined to one action from each of them. In a repeated relationship, if it is true that a country has private information about some relevant parameters, then the bank will carefully observe the country's behavior to learn about them. The country, in its turn, will take into account the information revealed through its actions and the consequences of this transmission of information when deciding upon them. Finally the bank, when deciding upon the timing of the debt forgiveness, will take into account its influence upon the potential information revelation. These considerations are important. They are the crucial determinants of equilibrium in the dynamic models we build.

The rest of this paper is organized as follows.

In section 1 we present a static model. We first model a situation in which the amount of optimal debt, from the bank's perspective, depends on one parameter, which we term effort cost, in the country's utility function. Next, we incorporate the assumption that the country has private information about this cost. This leads us to a model similar to that of Froot, Scharfstein and Stein (1989), who make the fundamental point that a stonewalling situation can occur, in which the bank doesn't grant the required forgiveness because it doesn't believe the country needs it and the country, therefore, makes no effort. In the model we present there are two types of countries, a weak one (with a high effort cost) and a strong one (with a low effort cost). We show that if the (prior) probability that the country is weak is low relative to the cost differential, the equilibrium shows stonewalling. The bank doesn't grant an adequate debt reduction. It chooses to renounce to the weak country's repayment by setting the debt at such a level that the county, if weak, makes no effort.

In section 2 we present a dynamic model in which the relationship between the bank and the country lasts for two periods. Each period the bank sets an upper bound to the repayments it will receive and the country decides how much effort to exert. In this section we assume that the bank cannot precommit at the beginning of the game to an upper bound for the second period. The commitment capacity doesn't go beyond the current period. This assumption is consistent with the facts observed during the earliest years of the so-called less developed countries debt problem. In those years a policy of just temporary relief was followed. In this context of temporary relief, we find that if in the one-period model there is no stonewalling, then in the two period model there isn't either. It arises, on the other hand, a new feature which makes the problem more complex: the ratchet effect which means, in our context, that the strong country makes an effort smaller than the first best level so as to hide its type and avoid loosing its future income. To explain our next result let us mention that when the static model exhibits stonewalling, that is, when the bank renounces to the weak country's repayment, the strong country obtains only its reservation utility. For the parameter values that produce stonewalling in the static model, the dynamic model can produce an equilibrium in which the strong country gets more than its reservation utility. Intuitively, this is because the bank is more generous in a dynamic context to induce revelation. When this generosity appears, we have two possible situations. In one of them, it is so intense that even the weak country makes an effort. In the other one, the strong country does not hides itself under the weak country's effort, but the weak country makes no effort at all: we have stonewalling.

In section 3 we present two versions of a model in which the bank has more commitment capacity than in the previous section. We want to improve our understanding of the possible advantages this entails. We assume that the bank can commit, at the beginning of the game, to respect an upper bound for the first *and* the second period. We allow the bank to make its commitment for the second period contingent on the repayment it receives in the first period.

We characterize, in the two versions we present, the situations in which this enhanced commitment capacity allows the bank to increase its expected profits relative to what it obtains in the previous section. To explain the difference between the two versions we present -and the reason to study them-, let us first stress the fact that all along this paper we restrict the bank to choose among debt contracts, i.e., contracts that make the repayment schedule non product-contingent. Yet, when we allow the bank to commit to an upper bound for the second period that depends on the repayment of the first one, we open the possibility to establish an indirect dependence on the first period's product. In other words, when we give the bank more commitment capacity, we might be giving implicitly to it, at the same time, the capacity to design a contract that depends on the first period product. This advantage presents in the first version of the model its polar form. In this version we assume a confiscation technology (which we call gunboat) such that the bank receives the country's product in its entirety, without any cost, if it doesn't receive the payment due. So, the bank can set a repayment so high for the first period that the country never meets it, withdraw the entire first period product, with no cost at all, and give whatever incentives it wants to give to the country through an appropriate second period bound. We compare the equilibrium obtained under this hypothesis with the equilibrium under temporary relief. If the probability of the country being weak is low enough, then both equilibria coincide. Otherwise, the bank can increase its expected utility thanks to the additional instruments it has. If this happens, the country's expected utility will be decreased unless the equilibrium under temporary relief exhibits stonewalling.

The second version of the model studies the other polar case. The bank has no capacity at all to establish an indirect dependence in the first period product. This version is more comparable than the previous one to the one-period model because it preserves untouched the dilemma of the bank: to make profitable for the country to repay its liabilities or else to receive nothing. Likewise, it allows us to isolate the advantages arising from the sheer enhanced commitment capacity. In this version of the model we find that the enhanced commitment capacity never hurts the country. When the equilibrium for this model differs from the one exhibited by the temporary relief model, there are two possible situations. In one of them the only consequence of the commitment capacity is to allow the bank to completely reap the benefits from avoiding the strong country's first period hiding. In the other one, avoiding this hiding also benefits the country.

Section 4 is a technical one and is devoted to exploring a feature of the two-period model under temporary relief that cannot be generalized to models with more than two periods. Namely, the fact that if the one-period model doesn't exhibit stonewalling, the two-period model doesn't exhibit it either. It is possible to have models with more than two periods in which the equilibrium shows stonewalling for parameter values that do not produce stonewalling in the one-period model. A necessary condition for the arising of stonewalling is the presence of the ratchet effect. If the discount factor is low relative to other parameters and the strong country would rather obtain today the fruits of its efficient behavior, in spite of revealing its true character, no stonewalling in one-period implies no stonewalling in T periods. Nevertheless, if the discount factor is big, then the strong country, knowing that if it succeeds in maintaining its reputation of being weak will obtain a generous forgiveness, can find convenient to hide under the weak country first best effort level: the ratchet effect can occur. Therefore, depending on the parameter values we have three possible situations:

i) The welfare losses derived from the ratchet effect are smaller than those derived from the weak country making no effort, i. e., from an initial temporary stonewalling. In equilibrium there is always generous forgiveness and strong country hiding.

ii) The losses from a permanent strong country hiding are higher than those from an initial stonewalling. The bank grants a small forgiveness and thus the weak country does not make an effort, but still it is enough to induce the strong country to make its first best effort and reveal through it its type. Thus, we have a temporary stonewalling: the bank grants a small forgiveness to separate both types of countries and once this is achieved, it grants the weak country a big forgiveness and the strong country a small one, inducing there on a first best effort level from each type of country.

iii) It can happen that in spite of having future welfare losses from the ratchet effect greater than those from a temporary initial stonewalling, the strong country requires an initial compensation so big that even the weak type wants to accept it. Then the instruments we have given the bank do not allow it to separate both types of countries and we will have both generous forgiveness and ratchet effect forever.

One final note. The equilibrium concept we use in our models is that of perfect bayesian equilibrium. The formal structure and the type of features we find are similar to the models of planning under incomplete information as in Freixas, Guesnerie and Tirole (1985). These authors allow a social planner to set payment schedules to a firm that depend linearly on its product. Recently, in the context of regulation problems, the choice set for the planner has been widen to include nonlinear contracts (Laffont and Tirole 1987 and 1988). In these models there is a central planner that maximizes an utilitarian welfare function. What makes the problem non-trivial is the existence of welfare losses from raising funds for the firm. If this were not the case, the planner wouldn't mind paying the firm above its reservation utility level . This interest in reducing the payment to the firm makes the problem similar to that of the bank trying to extract from the country as much as it can, though the initial specification of the model is quite different. At a formal level, besides this difference in the initial specification, it is important to note that we restrict ourselves to payment schedules that do not depend explicitly on the country's product.

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta\,\overline{EU}(\bar{R}_2;\ \lambda_2(\underline{x})) \geq 0 \qquad (\mathrm{VP}\bar{a})$$

As a consequence of lemmas 2.3.2 and 2.3.3, the previous restrictions are necessary and sufficient conditions for the type $\underline{a}$ country to make $\underline{e}_1$ and pay $\underline{z}_1$ and the type $\bar{a}$ country to make $\bar{e}_1$ and pay $\bar{z}_1$. If we want $\underline{a}$ to randomize, then IC$\underline{a}$ must be met with equality.

We now analyze in turn the cases $\lambda > 1-K$ and $\lambda < 1-K$

CASE $\lambda > 1-K$

In this case if $\bar{e}_1$ is observe we will have $\lambda_2 > 1-K$ and if $\underline{e}_1$ is observed we will have $\lambda_2 = 0$. Using $\underline{EU}_2$, $\overline{EU}_2$, and $E\Pi_2$ from Lemma 2.3.1 we find that the bank's problem is

$$\text{Max } (1-\lambda_1)(1-\underline{x}_1)\{\underline{z}_1 + \delta \min[\underline{R}_2, 1/2\underline{a}]\} +$$
$$\{1-(1-\lambda)(1-\underline{x}_1)\}\{\bar{z}_1 + \delta \min[\bar{R}_2, 1/2\bar{a}]\}$$

s.a.

$$\underline{z}_1 = \underline{e}_1, \quad \bar{z}_1 = \bar{e}_1 \qquad \text{(Gunboat condition)}$$

$$\bar{z}_1 = \underline{z}_1 \Rightarrow \bar{R}_2 = \underline{R}_2 \qquad \text{(Non- verifiability of } e_1)$$

$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/2 - \underline{z}_1 - \delta\min\{\underline{R}_2, 1/2\underline{a}\} \geq$$
$$\bar{e}_1 - \underline{a}\,\bar{e}_1^2/2 - \bar{z}_1 - \delta\min\{\bar{R}_2, 1/2\bar{a}\} \qquad (\mathrm{IC}\ \underline{a})$$

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta(1/2\bar{a}) - \delta\min\{\bar{R}_2, 1/2\bar{a}\} \geq$$
$$\underline{e}_1 - \bar{a}\,\underline{e}_1^2/2 - \underline{z}_1 + \delta\,\underline{EU}(\underline{R}_2;\ \lambda_2(\underline{x})) \qquad (\mathrm{IC}\ \bar{a})$$

$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/2 - \underline{z}_1 + \delta(1/2\underline{a}) - \delta\min\{\underline{R}_2, 1/2\underline{a}\} \geq 0 \qquad (\mathrm{VP}\ \underline{a})$$

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta(1/2\bar{a}) - \delta\min\{\bar{R}_2, 1/2\bar{a}\} \geq 0 \qquad (\mathrm{VP}\ \bar{a})$$

To solve the problem we will ignore the gunboat, verifiability, VP$\underline{a}$ and IC$\bar{a}$ conditions. In the end we will check that they are satisfied.

i) VP $\bar{a}$ is binding because $E\Pi$ is strictly increasing in $\bar{z}_1$ and increases in $\bar{z}_1$ make IC$\underline{a}$ slacker.

ii) From (i) and IC$\underline{a}$ it follows that in equilibrium $\underline{EU} > 0$, that is, VP$\underline{a}$ is

1. THE STONEWALLING PROBLEM IN THE STATIC CASE

This section presents two versions of a static model of voluntary debt reduction by a bank. In the first version there is symmetric information. It captures the fact that the amount of forgiveness that the bank chooses to grant depends on one parameter (effort cost) in the country's utility function. In the second version there is asymmetric information about such parameter. It is a model in the spirit of Froot, Scharfstein and Stein (1991), who argue that the bank might not grant a generous debt reduction because it doesn't believe the country needs it.

We begin with the symmetric information version. A country owes an infinite debt to a bank. Its product (y) depends on its effort (e).
We assume $y = e$

We now consider the following two-stage game. In the first stage the bank reduces the debt to a level $\bar{R} \in \mathbb{R}$. In the second stage the country chooses an effort level $e \in \mathbb{R}_+$ and pays the amount
$R = \min \{ e , \bar{R} \}$

Country's utility is: $U = y - a e^2 / 2 - R$
Bank's utility is: $\Pi = R$

That is, the country enjoys keeping product (y - R) and dislikes effort . The bank enjoys receiving more product and has also a linear utility in the product it keeps.

In this game a strategy for the bank is an upper bound $\bar{R} \in \mathbb{R}$.
A strategy for the country is a function assigning to each $\bar{R} \in \mathbb{R}$ an effort level $e \in \mathbb{R}_+$. A subgame perfect equilibrium is a pair of strategies satisfying the following two conditions:
i) The country's strategy maximizes its utility for each upper bound set by the bank.
ii) The bank's strategy maximizes its utility given the country's strategy.

For simplicity we assume that if the country is indifferent between two alternatives, it chooses the one giving the highest expected utility to the bank. This hypothesis will be maintained throughout the paper. In particular, when we speak of uniqueness of equilibrium it should be understood under the qualification "among the class of equilibria that satisfy the condition that the country does not choose a strategy that provides the bank with an expected utility lower than the one resulting from the country choosing an indifferent strategy".

Remark 1

In equilibrium

i) The bank's strategy prescribes an upper bound $1/2a$ that decreases with the parameter a

ii) The country's strategy prescribes the first-best effort $1/a$ when the bank sets the equilibrium bound $1/2a$

iii) The bank obtains its reservation utility (the one it would obtain if it chose an effort level equal to zero)

Proof

In equilibrium the country's strategy solves, for each $\bar{R}$, the problem

$$\max_{e} \begin{cases} e - ae^2/2 - \bar{R} & \text{if } \bar{R} \leq e \\ -a\, e^2/2 & \text{if } 0 \leq e < \bar{R} \end{cases}$$

From where it follows that in equilibrium the bank's strategy solves

$$\max_{\bar{R}} \begin{cases} \bar{R} & \text{if} \quad \max\limits_{e \geq 0} \; e - a\, e^2/2 - \bar{R} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The remark follows then from the function $e - a\, e^2/2$ achieving its maximum $1/2a$ at $e = 1/a$.

∎

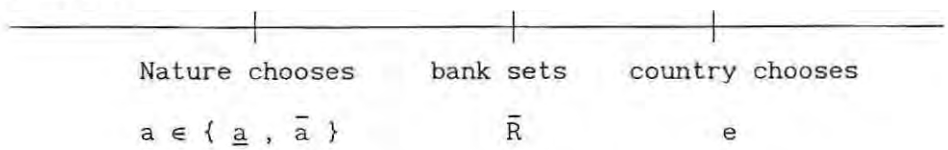
Thus, if starting from an infinite debt the bank entertains the idea of its reduction, it finds that its optimal level depends on a, which reflects the cost of the country's effort. If this parameter is known only to the country we will have an adverse selection problem. It is evident that the country wants the bank to believe it is very high.

We now present a version with asymmetric information. Unlike the previous version, the parameter in the country's utility function is private information. The bank knows, however, that it can take on values in the set $\{ \underline{a} , \bar{a} \}$ and it has beliefs represented by the probability function

$$\lambda = P(\bar{a}) \qquad (1)$$

We consider the following game. In the first stage nature chooses the parameter $a \in \{ \underline{a} , \bar{a} \}$ according to (1). This choice is observed only by the country. In the second stage, without knowing a, the bank sets a debt $\bar{R} \in \mathbb{R}$. Finally, in the third stage, knowing a and $\bar{R}$, the country chooses an effort level $e \in \mathbb{R}_+$ and pays $R = \min \{ e, \bar{R} \}$.
Figure 5 shows the time line for choices.

Figure 5



In this game a strategy for the bank is a debt level $\bar{R} \in \mathbb{R}$. A strategy for the country is a function assigning to each possible pair of effort cost and debt, $(a, \bar{R}) \in \{ \underline{a} , \bar{a} \} \times \mathbb{R}$, an effort level $e \in \mathbb{R}_+$

An equilibrium (A perfect bayesian equilibrium for the one period asymmetric information game) is a pair of strategies such that:
i) The strategy for the country maximizes its utility for each realization of a and for each debt $\bar{R}$
ii) The strategy for the bank maximizes its expected utility (where the expectation is taken with respect to its beliefs about a) given the country's strategy.

The next proposition provides a central benchmark for the rest of the paper. We shall denote by $\underline{e}(\bar{R})$ ($\bar{e}(\bar{R})$) the effort prescribed by the country's strategy if it is type $\underline{a}$ ($\bar{a}$) when it faces a debt $\bar{R}$. We shall omit the argument of $\underline{e}(\cdot)$ (and that of $\bar{e}(\ )$) when the context allows us to do so without causing confusion.

Proposition 1

Let $K = \underline{a} / \bar{a}$

a) If the probability of the country being weak is low relative to K $(\lambda < 1 - K)$ *then, in equilibrium:*
the debt is such $(\bar{R} = 1/\ 2\ \underline{a})$ *that the type* $\bar{a}$ *country makes an effort* $(\bar{e} = 0)$ *lower than the first best level* $(\bar{e}^{fb} = 1/\ \bar{a})$ *and both types of country get its reservation utility (EU = 0)*
(Furthermore, $E\Pi = (1/\ 2\underline{a})\ (1 - \lambda)$, $\underline{e} = \underline{e}^{fb} = 1/\ \underline{a}$*)*

b) If the probability of the country being weak is high relative to K $(\lambda > 1 - K)$ *then, in equilibrium:*
the debt is such $(\bar{R} = 1/\ 2\ \bar{a})$ *that both types of country make its first best effort level* $(\underline{e} = 1/\ \underline{a},\ \bar{e} = 1/\ \bar{a})$*, type* $\bar{a}$ *country gets its reservation utility (EU = 0) and type* $\underline{a}$ *country gets an expected utility strictly greater than its reservation utility* $(EU = 1/\ 2\ \underline{a} - 1/\ 2\ \bar{a})$
(Furthermore, $E\Pi = 1/\ 2\ \bar{a}$*)*

Proof

In equilibrium, for each $\bar{R}$, the bank's strategy solves one of the following problems, depending on the value of a.

$$\max_e \begin{cases} e - \underline{a}\ e^2/\ 2 - \bar{R} & \text{if } \bar{R} \le e \\ -\ \underline{a}\ e^2/\ 2 & \text{if } 0 \le e < \bar{R} \end{cases} \qquad \text{if } a = \underline{a}$$

$$\max_e \begin{cases} e - \bar{a}\ e^2/\ 2 - \bar{R} & \text{if } \bar{R} \le e \\ -\ \bar{a}\ e^2/\ 2 & \text{if } 0 \le e < \bar{R} \end{cases} \qquad \text{if } a = \bar{a}$$

Since the function $e - a\ e^2/\ 2$ reaches its maximum value, $1/\ 2a$, when $e = 1/\ a$, we have that the solution to the country's problem is

$$\underline{e} = \begin{cases} 1/\ \underline{a} & \text{if } \bar{R} \le 1/\ 2\ \underline{a} \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{e} = \begin{cases} 1/\ \bar{a} & \text{if } \bar{R} \le 1/\ 2\ \bar{a} \\ 0 & \text{otherwise} \end{cases}$$

Thus, in equilibrium the bank's strategy solves the problem

$$\text{Max} \begin{cases} \bar{R} & \text{if } \bar{R} \leq 1/\ 2\ \bar{a} \\ (1 - \lambda)\ \bar{R} & \text{if } 1/\ 2\ \bar{a} < \bar{R} \leq 1/\ 2\ \underline{a} \\ 0 & \text{if } 1/\ 2\underline{a} < \bar{R} \end{cases}$$

whose solution is[1]

$$\bar{R} = \begin{cases} 1/\ 2\ \bar{a} & \text{if } (1 - \lambda)(1/\ 2\underline{a}) < 1/\ 2\ \bar{a} \\ 1/\ 2\ \underline{a} & \text{if } 1/\ 2\ \bar{a} < (1 - \lambda)(1/\ 2\underline{a}) \end{cases}$$

from where the proposition follows.

■

The message of the previous model is that it might happen that the bank does not reduce debt to the level that induces $\bar{a}$ to make an effort because it attaches very low probability to the country being type $\bar{a}$. This is the situation termed 'stonewalling' (the bank does not reduce debt and the country makes no effort at all) by Froot, Scharfstein and Stein (1990). These authors suggest that it prevents debt reductions.

2. A DYNAMIC MODEL WITH TEMPORARY RELIEF

In this section we consider a situation in which the relationship between debtor and creditor repeats itself over time. To simplify, we treat the case of two periods. The solution depends crucially on the commitment capacity we assume. In this section the commitment capacity does not go beyond the current period. We study a model in which the bank can commit only not to charge above a specified amount in the current period. This is what we mean by a model with temporary relief. As the debt is infinite, the amount received today cannot influence the debt with which we start tomorrow. We do not allow the bank to reduce debt, but only to grant a temporary relief, that is, not to charge more than a given amount today. Tomorrow it will make a second decision over the bound it has to set.

There are intertemporal features arising in a multiperiod context which a static model cannot capture. If it is true that the country has private

[1] If $(1 - \lambda)\ (1/\ 2\ a) = 1/\ 2\ \bar{a}$, the bank's problem has two solutions. But generically this does not happen and we will ignore it.

information about relevant parameters, the bank will carefully observe the country's behavior to learn about them. The country, in its turn, will take into account the information revealed through its actions and the consequences of this transmission of information when deciding upon them. Finally the bank, when deciding upon the timing of the debt forgiveness, will take into account its influence upon the potential information revelation. The problems appearing in a static setting can be softened or sharpened when put into a dynamic context. We are particularly interested in the stonewalling phenomenon once we take a dynamic perspective.

We now present a broad description about the features arising in a dynamic model. Next we provide the formal specification of the model and its solution.

We know that when a country is most likely weak, the static model exhibits no stonewalling. It turns out that when the static model shows no stonewalling, the dynamic model doesn't exhibit it either. In this case, the bank sets as initial bound the optimal static bound and, therefore, the weak country makes its first-best effort. The strong country, in its turn, benefits from this bank's beliefs. It is because of this that the possibility of the strong country hiding under the weak country's effort appears. This way it can avoid loosing its future rents. This is the ratchet effect. We obtain the following result. The hiding occurs when the discount factor is high (relative to $\underline{a}/\bar{a}$), i.e., when much importance is attached to future.

We know that when the country is (relative to $\underline{a}/\bar{a}$) most likely strong, the bank finds it optimal from a static viewpoint to sacrifice the weak country's payments. There is stonewalling. When the static model exhibits stonewalling, there are three types of equilibria in the dynamic model. We now describe them with the help of figures 2 and 3.

Let us first analyze the (b.1) area. To gain intuition, let us remember that it is the smallness of the probability of the country being weak what induces stonewalling and let us note that for each pair of values δ, K, if we allow λ to become small enough (close enough to zero) we are in this zone. Therefore, we are in a situation prone to stonewalling. In a static model if λ is small the bank sacrifices $\bar{a}$'s repayments -in that it prefers not to induce $\bar{a}$ to exert effort- and obtains the product from $\underline{a}$'s first-best effort. In this zone, the bank sets the static optimal bound both periods and sacrifices thus $\bar{a}$'s repayments both periods. Yet, it cannot reap twice the payments derived from $\underline{a}$'s first-best effort. The reason is that it cannot induce $\underline{a}$ to exert a first-period first-best effort with probability one. To see why let us suppose it could. Then observing no effort would signal that the country is weak and would induce the bank to reduce the second period bound to a level compatible with $\bar{a}$ exerting effort. Then $\underline{a}$ would find it convenient making no effort to produce such situation. Therefore, for $\bar{R}_1 = \bar{R}_2 = 1/ 2\underline{a}$ being part of an equilibrium it is necessary for the $\underline{a}$ country not to exert an initial first-best effort with certainty. We have ratchet effect with positive probability, a semi-separating equilibrium.

b.2) Let us think about this zone as the one containing "median" λ values. As before the bank sacrifices $\bar{a}$'s repayments in the first period. It induces it to make no effort. There is stonewalling. Yet, in contrast with the (b.1) zone, the equilibrium is separating. There is no ratchet effect: the strong country never hides under the weak country's effort. It makes an initial first best effort. The reason is that the bank chooses an initial suboptimal bound from a static viewpoint to induce this revelation. The bank has to be more generous in this dynamic context to obtain information and thus sets a bound lower than the static optimal bound.

b.3) To interpret this zone it is useful to note that, given δ and K fixed with $\delta < 1 - K$, for λ close enough to $1 - K$ we are in this zone. It is a zone close to the no stonewalling zone. In the dynamic model we have a separating equilibrium in which both types of countries choose their first-best effort in the first period. There is no stonewalling. The weak country makes its first-best effort. Furthermore, there is no ratchet effect: the strong country ($\underline{a}$) never hides under the weak country's ($\bar{a}$) effort. As in the previous case the bank chooses an initial bound which is suboptimal from a static point of view. But it is even more generous. It chooses a bound such that even the weak country is induced to exert effort. It chooses the bound it would choose in a static context if it were sure the country is weak.

We now present the formal specification of the model.

The country's product in period t, y_t, is a function of its effort at that same period, e_t. We assume

$y_t = e_t$ in each period $t = 1, 2$

Consider the game whose timing is now explained with the help of figure 4.

Nature plays first choosing the parameter a in the country's utility function according to (1). The value of this variable is known only to the country.

Next, without knowing a's realization, the bank chooses the payment $\bar{R}_1 \in \mathbb{R}$ that the country must meet.

Knowing a and $\bar{R}_1$, the country exerts $e_1 \in \mathbb{R}_+$, which determines the first-period product y_1 and gives to the bank $R_1 = \min \{ y_1 , \bar{R}_1 \}$, ending the first period.

Figure 2

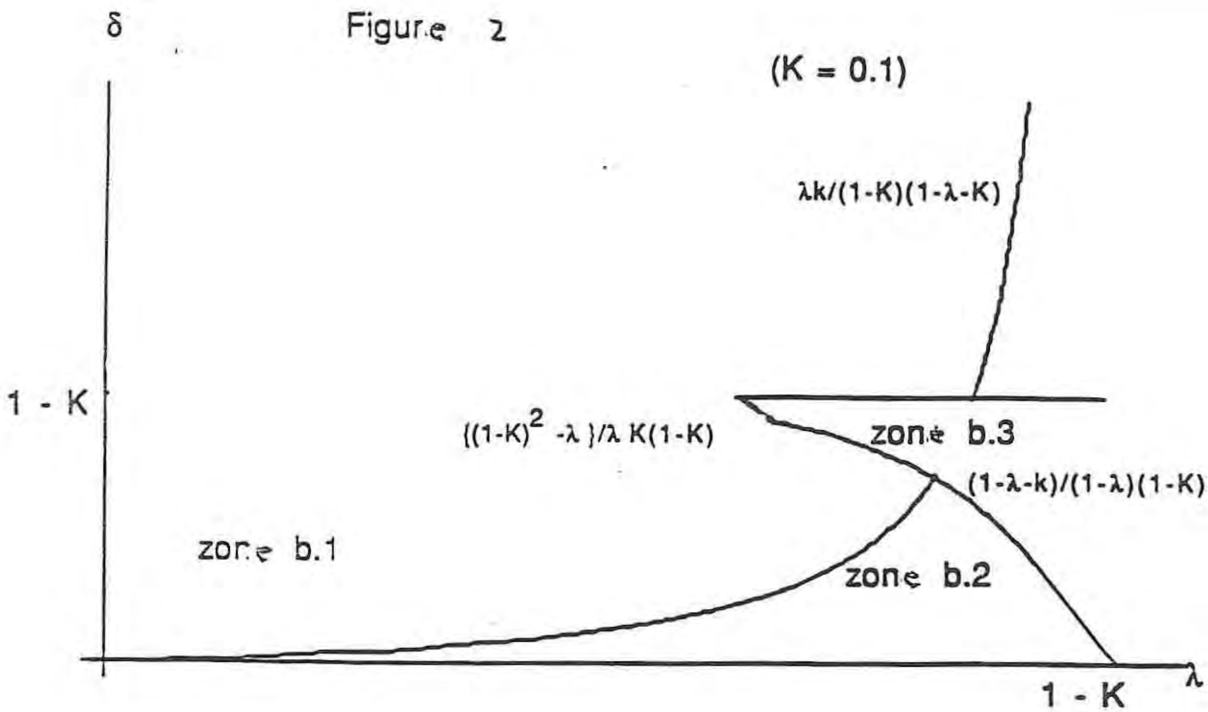
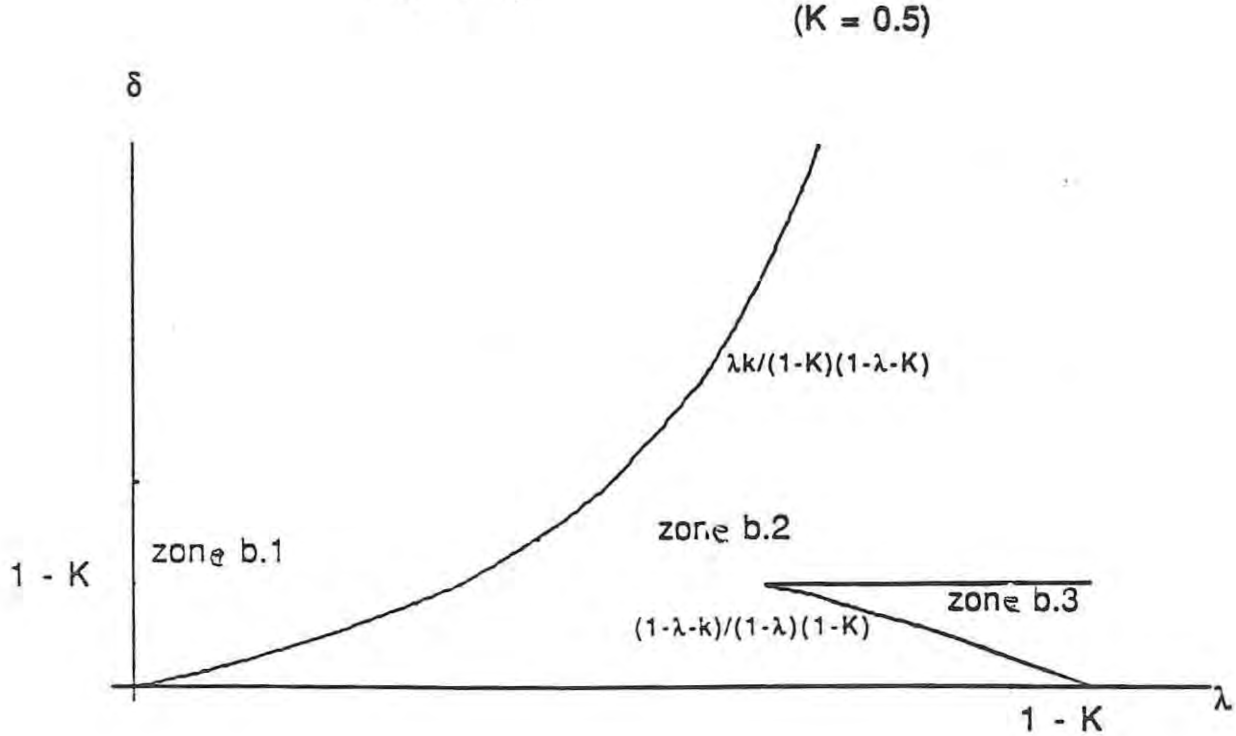


Figure 3



At the beginning of the second period, knowing $\bar{R}_1$ and e_1, the bank sets. $\bar{R}_2 \in \mathbb{R}$ and, finally, knowing $\bar{R}_1$, e_1 and $\bar{R}_2$, the country exerts $e_2 \in \mathbb{R}_+$ and gives to the bank the amount $R_2 = \min \{ y_2, \bar{R}_2 \}$

Figure 4

Nature chooses $a \in \{ \underline{a} , \bar{a} \}$	bank sets $\bar{R}_1$	country chooses e_1	bank sets $\bar{R}_2$	country chooses e_2

Country's utility is

$U = U_1 + \delta U_2$, with $U_t = y_t - R_t - e_t^2 / 2$, $t = 1,2$

Bank's utility is

$\Pi = \Pi_1 + \delta \Pi_2$, with $\Pi_t = R_t$, $t = 1,2$

We define a history observed by the country in period 1 as $h_1^c = (a, \bar{R}_1)$ and, in period 2, as $h_2^c = (a, \bar{R}_1, e_1, \bar{R}_2)$. Likewise, a history observed by the bank in period 2 is $h_2^B = (\bar{R}_1, e_1)$.

In this game a strategy for the country is a function for each period, assigning an effort level e_t to each history observed by the country h_t^c.

A strategy for the bank is an upper bound $\bar{R}_1$ *and* a function assigning an upper bound $\bar{R}_2$ to each history observed by the bank h_2^B.

Equilibrium concept

The equilibrium concept we use is that of perfect bayesian equilibrium. It comprises a strategy for the country, one for the bank and a belief function satisfying the following conditions.

We first describe the strategies. Suppose there is a rule that specifies the belief λ_2 held by the bank about the parameter a as a function of the history $h_2^B = (\bar{R}_1, e_1)$. Then, each time an agent acts, for each history it has observed, it must maximize its expected payoff, given the other player's strategy, and given the bank's belief resulting from the above mentioned rule.

We now explain how beliefs are updated. Suppose we have a pair of strategies. The belief function assigns to each history $h_2^B = (\bar{R}_1, e_1)$, a belief λ_2. This function is simply Bayes' rule when the observed history has positive probability, i.e., when the prescribed pair of strategies produces that history with positive probability. Bayes' rule says nothing about histories observed with probability equal to zero. Unfortunately, the updating of beliefs in such situations can be crucial: in dynamic games with incomplete information it is common to have the beliefs off- the- equilibrium path being relevant to know if the prescribed strategies are best responses one to the other. Thus, it is possible to sustain an equilibrium with unreasonable beliefs. We impose the following condition. Suppose an initial effort is observed that, given the prescribed strategies, has probability equal to zero. If the weak country ($\bar{a}$) can make an initial effort giving it an expected utility strictly greater than the one derived from the observed effort for each (optimal) bank's action $\bar{R}_2^2$ in the last period, then the bank assigns probability zero to the country being weak. Only when such an alternative action for the weak country exists do we restrict out-of-equilibrium beliefs.[2] We thus have the following definition.

A perfect bayesian equilibrium (of the two period game under temporary relief) is a pair of strategies and beliefs $\lambda_2(\bar{R}_1, e_1) = P(\bar{a} \mid \bar{R}_1, e_1)$ such that:

(i) e_2 maximizes EU_2 $\forall$ h_2^c
(ii) $\bar{R}_2$ maximizes $E\Pi_2$ $\forall$ h_2^B, where the expectation is taken with respect to a using $\lambda_2(\bar{R}_1, e_1)$, given the country's strategy in the second period.
(iii) e_1 maximizes EU $\forall$ h_1^C, given the second period strategies
(iv) $\bar{R}_1$ maximizes $E\Pi$ given the posterior strategies.
(B) $\lambda_2(\bar{R}_1, e_1)$ is derived from the prior probability λ_1 and from the country's strategy given by (iii) using Bayes'rule, as long as a history with positive probability is observed.
We ask out off equilibrium beliefs to satisfy the following condition
(B') If a history $h_2^B = (\bar{R}_1, e_1)$ with probability of zero is observed, then $\lambda_2(\bar{R}_1, e_1) = 0$ if there is e_1' s.t. the country $\bar{a}$ obtains an expected utility strictly lower by choosing e_1 than by choosing e_1' $\forall$ $\bar{R}_2^2$ in the set of optimal bank's actions for the second period for some λ_2.

We now define two features appearing in the literature in similar situations.

The equilibrium exhibits stonewalling it type $\bar{a}$ exerts an initial effort equal to zero.

The equilibrium exhibits ratchet effect if type $\underline{a}$ makes the same initial

[2] In our models, imposing this condition implies that the equilibrium we find satisfies the intuitive criterion of Cho-Kreps. On the other hand, abandoning this condition does not increase the number of equilibria.

effort as type $\bar{a}$.

We denote by $\bar{e}_1$ ($\underline{e}_1$) the effort prescribed by the country's strategy in case it is type $\bar{a}$ ($\underline{a}$) and by $\overline{EU}(e_1)$ ($\underline{EU}(e_1)$) the expected utility obtained by the type $\bar{a}$ ($\underline{a}$) country if it exerts an effort e_1.

Proposition 2

Let $K = \underline{a} / \bar{a}$. In the two-period model under temporary relief there is a unique equilibrium[3]. It has the following characteristics.

a) If in the one-period model equilibrium there is no stonewalling (the probability of the country being weak is high relative to K, $1 - \lambda < K$) then in the two-period model equilibrium the bank sets an initial upper bound ($\bar{R}_1 = 1/ 2\ \bar{a}$) equal to the optimal static upper bound and there is no stonewalling ($\bar{e}_1 = \bar{e}^{fb}$). Furthermore, $\underline{a}$ obtains above its reservation utility. If, in addition,

a.1) the discount factor is high relative to K ($\delta > 1 - K$), then there is no ratchet effect: type $\underline{a}$ mimics type $\bar{a}$ (pooling equilibrium)

a.2) the discount factor is low relative to K ($\delta < 1 - K$), then in equilibrium there is no ratchet effect ($\underline{e}_1 = \underline{e}^{fb}$, separating equilibrium)

b) If the one-period model equilibrium exhibits stonewalling (probability of the country being weak is low relative to K : $1- \lambda > K$) then in the two-period model equilibrium one of the three following situations occurs (and each of them occurs for some parameter configuration) (See figures 6 and 7)

b.1) The bank sets an initial upper bound equal to the optimal static upper bound ($\bar{R}_1 = 1/ 2\ \underline{a}$) and there is stonewalling ($\bar{e}_1 = 0$). There is also ratchet effect with positive but lower than one probability (semi-separating equilibrium) and $\underline{a}$ obtains no more than its reservation utility. This occurs when (and only when)

$\delta \geq \lambda K / (1-K)(1-\lambda-K)$, *and*

$\lambda \leq (1-K)^2$ *and* $[\ \delta > (1-K)$ *or* $\delta \leq [(1-K)^2-\lambda]/\lambda K(1-K)\ ]$

---

[3]Without the condition that the country chooses the strategy preferred by the bank, the equilibrium is not unique. Furthermore, uniqueness should be understood generically, i.e., except in the frontier of the different types of equilibria.

b.2) The bank sets an initial upper bound suboptimal from a static viewpoint ($1/ 2\bar{a} < R_1 < 1/ 2\underline{a}$), there is stonewalling ($\bar{e}_1 = 0$) and there is not ratchet effect (separating equilibrium). Furthermore, $\underline{a}$ obtains above its reservation utility. This occurs if and only if

$[\ \delta < (1-\lambda-K)/ (1-\lambda)(1-K) \quad \text{or} \quad \delta > (1-K)\ ]$, and

$\delta \leq \lambda K / (1-K)(1-\lambda-K)$

b.3) The bank sets an initial upper bound suboptimal from a static viewpoint ($\bar{R}_1 = 1/ 2\bar{a}$) and there is no stonewalling ($\bar{e}_1 = 1/ \bar{a}$). Furthermore, there is no ratchet effect (separating equilibrium) and $\underline{a}$ obtains above its reservation utility. This occurs if and only if:

$[(1-K)^2 \leq \lambda \leq (1 - K) \quad \text{or} \quad \delta > \{\ (1-K)^2 - \lambda\ \} / \lambda K(1-K)\ ]$, and

$(1-\lambda-K)/ (1-\lambda)(1-K) < \delta < (1-K)$

3. DEBT-REDUCTION CONTRACTS

This section presents two versions of a model in which a bank has more commitment capacity than in the previous section. We now explain the purpose of doing so and relate this section to the rest of the paper. It is known that during the earliest years of the so-called less-developed-country debt crisis the policy to face it was one of temporary relief, i.e., one of postponing repayments and waiting for things to improve. In the previous section we model such a situation. We also know that this policy of wait and see gradually lost ground to a policy containing longer commitments, in particular partial debt write-off. In this section we model a relationship in which the bank can assume long-term commitments to improve our understanding of the advantages this entails. Let us first explain that we find two types of advantages. The first type will most likely appear only when we restrict the bank, as we do, to repayment schedules in form of debt, i.e., not explicitly product contingent. They arise because allowing the bank to set a second-period upper bound as a function of the first-period repayment opens the possibility of setting an indirect dependence on the first-period product. This cannot be done when a payment is asked for today and, no matter the amount actually repaid today, we start the second period with an infinite debt. The second type of advantages arises from the sheer enhanced commitment capacity. To emphasize the distinction between both types of advantages two versions of the model are presented. In the first version we assume that, if the first-period repayment asked by the bank is higher than the country's product, this can be confiscated in its entirety without any cost. This is the gunboat hypothesis. Under this hypothesis the first- type advantages are as high as possible. In the second version we are interested in studying a situation with no first-type advantages at all. One way to do it is under the hypothesis that if the country's product is smaller than the repayment due, the confiscation of the product leads to its total loss. We call this technology product destruction.

The model

We now consider the previous-section game enlarging the first-period bank's action set. We allow it to commit to begin the last period with a finite debt. Furthermore, we allow it to set this finite debt as a function of the amount repaid in the first period. In the second period it can reduce debt again if it wishes to do so.

Formally, in the first period the bank sets a repayment due in this period, $\bar{R}^1_1$, and an upper bound $\bar{R}^1_2$, legally binding, to the amount due in the last period. We allow this bound to be a function of the amount actually paid in the first period.

We assume the same production technology and utility functions as in the previous section. The timing of the model is now described. (See figure 5)

First, nature plays choosing the parameter a according to (1). The value of this parameter is known only to the country.

Next, without knowing a's realization, the bank makes its first action. Let $\bar{R}^2_1(\cdot)$ be a function from $\mathbb{R}$ into $\mathbb{R}$ assigning to each amount actually repaid in the first period an upper bound with which the second period starts. The bank's first action is choosing a bound $\bar{R}^1_1 \in \mathbb{R}$ for the first period and two ordered pairs of the function[1] $\bar{R}^2_1(\cdot)$, the pair $(\bar{z}_1, \bar{R}^2_1(\bar{z}_1)) \in \mathbb{R}^2$ 'designed' for $\bar{a}$ and the pair $(\underline{z}_1, \bar{R}^2_1(\underline{z}_1)) \in \mathbb{R}^2$ 'designed' for $\underline{a}$, satisfying $\bar{R}^1_1 \in \{ \bar{z}_1, \underline{z}_1 \}$. We assume[2] $\bar{R}^2_1(R_1) = 1/ 2\underline{a}$ if $R_1 \notin \{ \bar{z}_1, \underline{z}_1 \}$. This is interpreted as allowing the bank to make the following commitment: if it receives $\bar{z}_1$, the second period will start with a debt equal to $\bar{R}^2_1(\bar{z}_1)$. If it receives $\underline{z}_1$, the second-period debt will be $\bar{R}^2_1(\underline{z}_1)$. For any other repayment, it is as if the bank committed to nothing.

Knowing a and the announcement made by the bank, the country makes its first action. The difference between the two versions of the model we present lays in the choice set for this country's action.

Under the gunboat technology, the first country's action is to choose an effort level $e_1 \in \mathbb{R}_+$ which determines the first-period product y_1 and, if $y_1 \geq \bar{R}^1_1$, an amount $R_1 \in [\bar{R}^1_1, y_1]$ repaid to the bank. If $y_1 < \bar{R}^1_1$, an amount $R_1 = y_1$ is repaid. That is, the country chooses how much effort to exert (and thus how much to produce). If it decides to produce above $\bar{R}^1_1$, it must also decides whether to pay above $\bar{R}^1_1$ (and how much).

Under the product-destruction technology[3] The country's first action is

[1] We can see the function $\bar{R}^2_1(\ )$ as the set $\{ (x,y) \in \mathbb{R}^2 \mid \bar{R}^2_1(x) = y \}$

[2] This assumption is not essential. It greatly simplifies the proof. There are others specifications for the function $\bar{R}^2_1(\ )$ outside the set $\{\bar{z}_1, \underline{z}_1\}$ compatible with our results.

[3] In an earlier version of this model we define the product- d e struction technology as one in which, if $y_1 < \bar{R}^1_1$, the amount received by the bank is zero, possibly different from the amount withdrawn from the country, y_1. To economize in notation, we prefer not to distinguish between this two variables. We model this technology by introducing as a restriction in the first country's action a characteristic that the equilibrium would exhibit if

to choose an effort level $e_1 \in \{0\} \cup [\bar{R}_1^1, \infty)$ and an amount to be delivered to the bank
$R_1 \in [\bar{R}_1^1, y_1]$ if $y_1 \geq \bar{R}_1^1$. If $y_1 = 0$ then $R_1 = 0$. This ends the first period.

At the beginning of the second period, knowing $\bar{R}_1^1$, $\bar{R}_1^2(\ )$, e_1 and R_1, the bank sets $\bar{R}_2^2 \leq \bar{R}_1^2(R_1)$ and finally, knowing $\bar{R}_1^1$, $\bar{R}_1^2(\ )$, e_1, R_1 and $\bar{R}_2^2$, the country chooses[4] $e_2 \in \mathbb{R}+$ and pays $R_2 = \min\{ y_2, \bar{R}_2^2 \}$

Figure 5

Nature chooses	bank sets	country makes	bank sets	country chooses
$a \in \{ \underline{a}, \bar{a} \}$	$\bar{R}_1^1$, $\bar{R}_1^2(\ )$	e_1 and pays R_1	$\bar{R}_2^2$	e_2 and pays R_2

We define a history observed by the country up to period 1 as
$rh_1^c = (a, \bar{R}_1^1, \bar{R}_1^2(\ ))$ and, up to period 2, as $rh_2^c = (a, \bar{R}_1^1, \bar{R}_1^2(\ ), e_1, R_1, \bar{R}_2^2)$
We define a history observed by the bank up to period 2 as
$rh_2^B = (\bar{R}_1^1, \bar{R}_1^2(\ ), e_1, R_1)$

A strategy for the country consists of two functions. The first one assigns to each history rh_1^C an effort level e_1 and an amount[5] R_1 if $e_1 \geq \bar{R}_1^1$. The second one assigns to each history rh_2^C an effort level e_2.
A strategy for the bank specifies, for the first period, an upper bound $\bar{R}_1^1$ and a function $\bar{R}_1^2(\ )$ and, for the second period, a function that assigns to each rh_2^B an upper bound $\bar{R}_2^2$ satisfying[6] $\bar{R}_2^2(rh_2^B) \leq \bar{R}_1^2(R_1)$

we assumed the earlier condition: $e_1 = y_1 = 0$ if $e_1 < \bar{R}_1^1$. To the ends of characterizing the equilibrium, both assumptions are equivalent.

[4] In the final period it is irrelevant the confiscation technology we assume. On the other hand, allowing the country to choose $R_2 \in [\bar{R}_2^2, e_2]$ if $e_2 \geq \bar{R}_2^2$ doesn't change the results either. If $e_2 > \bar{R}_2^2$ the country will always choose $R_2 = \bar{R}_2^2$. If it chose to pay a bigger amount there are no further periods at which it can be granted better treatment in exchange.

[5] If $e_1 < \bar{R}_1^1$, the confiscation technology determines the value of R_1. In this case R_1 is not chosen by the country.

[6] Note that $rh_2^B = (\bar{R}_1^1, \bar{R}_1^2(\ ), e_1, R_1)$

Equilibrium concept

A perfect bayesian equilibrium with debt reduction is a pair of strategies and beliefs $\lambda_2(\bar{R}^1_1, \bar{R}^2_1(\ ), e_1, R_1) = P(\bar{a}\ |\bar{R}^1_1, \bar{R}^2_1(\ ), e_1, R_1)$ such that:

(i) e_2 maximizes EU_2 $\forall$ rh^c_2

(ii) $\bar{R}^2_2$ maximizes $E\Pi_2$ subject to $\bar{R}^2_2 \leq \bar{R}^2_1(R_1)$ $\forall$ h^B_2, where the expectation is taken with respect to a using $\lambda_2(\bar{R}^1_1, \bar{R}^2_1(\ ), e_1, R_1)$, given the second-period country's strategy.

(iii) (e_1, R_1) maximizes EU $\forall$ h^C_1, given the second-period strategies.

(iv) The pair $\bar{R}^1_1$, $\bar{R}^2_1(\ )$ maximizes $E\Pi$ given the rest of the strategies.

(B) $\lambda_2(\bar{R}^1_1, \bar{R}^2_1(\ ), e_1, R_1)$ is obtained from the prior probability λ_1 and the country's strategy specified in (iii) using Bayes' rule, whenever a positive probability event is observed.

We assume the off-equilibrium updated beliefs satisfy the following condition

(B') If a history $rh^B_2 = (\bar{R}^1_1, \bar{R}^2_1(\ ), e_1, R_1)$ with probability zero is observed, then $\lambda_2(\bar{R}^1_1, \bar{R}^2_1(\ ), e_1, R_1) = 0$ if there exist e_1', R_1' s.t. the type $\bar{a}$ country gets an expected utility strictly lower by choosing e_1, R_1 than by choosing e_1', R_1' $\forall$ $\bar{R}^2_2$ in the optimal second-period bank's actions set for some λ_2.

Solution under gunboat technology

In this section we characterize the equilibrium under the gunboat hypothesis. We obtain the following result.

Proposition 3

There exists λ^* *(which depends on* δ, K *and* $\underline{a}$*) such that* $0 < \lambda^* < 1-K$ *and*

If $\lambda > \lambda^*$ *(in particular* λ *such that in the one-period model there is no stonewalling) in every equilibrium of the two- period model with debt reduction:*

i) $\underline{e}_1 = \underline{e}_2 = \underline{e}^{fb}$

ii) $0 < \bar{e}_1 < \bar{e}^{fb}$; $\bar{e}_2 = \bar{e}^{fb}$

iii) The type $\underline{a}$ *country gets an expected utility strictly higher than its reservation utility.*

iv) Type $\bar{a}$ *country gets its reservation utility.*

If $\lambda < \lambda^*$*, then the expected utilities obtained by both types of countries and the bank and the equilibrium efforts are the same under the debt reduction equilibrium than under the temporary relief equilibrium.*

Proof (See appendix)

We now comment on this proposition.

First notice that starting with an infinite debt is equivalent to starting with a debt of $1/2\underline{a}$. Let us think of this value as a "big" debt. In this section we allow the bank to set a debt lower than $1/2\underline{a}$. The importance of this enhanced commitment capacity depends on the parameter values.

If there is little likelihood of the country being weak ($\lambda < \lambda^*$) then this additional commitment capacity does not alter the equilibrium. This result is intuitively reasonable. If the bank believes the country to be strong, it prefers to withdraw large amounts of product both today and tomorrow. It is not interested in starting tomorrow with a small debt. What it would really like (but it can't) is to commit itself not to reduce debt bellow $1/2\underline{a}$ even if it observes no effort today. Therefore, allowing the bank to commit itself to start tomorrow with a small debt does not alter the equilibrium at all.

To explain the result about λ values above λ^*, it is convenient to first comment on the $\lambda > (1 - K)$ case. The equilibrium we obtain is equivalent (in expected utilities for both agents as well as observed efforts) to the bank committing itself in the first period to respect a contract which can be decomposed into two parts, one for each period, as follows. For the first period it proposes the same contract it would propose in a one-period model if we allow it to specify the amount to be repaid as a function of the country's product, instead of a debt (a fix payment, non-product dependent). It has the following characteristics. It offers the country between two pairs of payment-effort.[7]. One of these pairs is designed for the strong country and the other one for the weak country. It is a contract such that each country finds convenient to take the pair designed for it. It gives the strong country more than its reservation utility and the weak country its reservation utility. It prescribes a strong country first best effort level and a strictly positive but bellow the first best level effort for the weak country. The intuition for this first part of the contract is as follows. Instead of setting a unique payment, two pairs of payment-effort are offered. It is not possible to set the optimal complete information pairs, because the pair offered to type $\bar{a}$ would also be taken by type $\underline{a}$. To avoid this, both pairs must give type $\underline{a}$ the same utility. This is why the first-period contract specifies, on the one hand, a payment for $\underline{a}$ lower than the optimal complete information payment and, on the other hand, to discourage it to choose the pair designed for type $\bar{a}$, it reduces the effort prescribed for $\bar{a}$ bellow its first best level (type $\underline{a}$ looses more than type $\bar{a}$ from reducing effort bellow the first best level).

For the second period, the same contract is offered as in the one-period model when $\lambda > (1 - K)$, that is, a "small" payment is set. Let us remark that, since in the first period each country chooses the pair designed for it, at the beginning of the second period the bank knows which type of country it faces. Nevertheless, even though the bank knows that at the beginning of the second period it might have learned it faces a strong country, it sets at the first period a small debt for the second. The bank takes advantage in this way of its commitment capacity.

So, when $\lambda > (1 - K)$ the optimal two-period contract is equivalent to an optimal product-dependent static contract in the first period followed by an optimal debt contract in the second period.[8]. It is worth comparing this

[7] Remember that in our model effort equals product.

[8] Contingency in the first period product is due to the possibility of making tomorrow's debt a function of today's product along with gunboat technology.

version with the one-period model. In this static model, if the country makes an effort and a product lower than the repayment due, its entire product is withdrawn. So in such a a situation it will make no effort at all. In contrast, in this version of the two-period model it is possible to induce the country to make an effort which is positive and at the same time lower than $\bar{R}_1^1$. This is because the contract specifies the future debt as a function of today's actual repayment. Thus, although the first-period bound is not achieved, if the initial repayment is positive a future debt can be set which is lower than the one would prevail if no repayment were made. This allows the contract to be contingent in the first-period product. This is why a contract equivalent to the one we have described is achieved.

When $\lambda < (1 - K)$, it is not possible to establish a contract similar to the one just mentioned. This is because the bank cannot commit to a second period optimal static contract when $\lambda < (1 - K)$. If at the beginning of the second period it knows with certainty that the country is weak, it will reduce the debt if it finds a high debt. It cannot commit to set a "high" repayment (to set the static optimal debt when $\lambda < (1 - K)$) to a country it knows with certainty is weak. This is why the bank chooses one of the two options just mentioned. The frontier between both options is λ^*.

In the following proposition we compare the equilibrium under temporary relief with the equilibrium under debt reduction and gunboat technology.

Proposition 4

Assume the gunboat technology

1. If in the one-period model there is no stonewalling ($\lambda > 1-K$), then:

i.i) The expected utility for the bank is strictly higher under debt reduction than under temporary relief. Nevertheless

i.ii) The expected utility for the type $\underline{a}$ county is strictly higher under temporary relief than under debt reduction.

i.iii) If $\delta < 1-K$, the equilibrium effort under temporary relief is always the first best effort. The equilibrium effort for the type $\bar{a}$ country under debt reduction is strictly lower than the first best level.

More generally, in a two-period model a similar characteristic will arise as long as the actual repayment when the product is lower than the repayment due depends monotonically on the product.

If $\delta > 1-K$, type $\underline{a}$ ($\bar{a}$) country makes a first best effort under debt reduction (temporary relief) and the type $\bar{a}$ ($\underline{a}$) country makes an effort which is lower than first best. The expected value for the equilibrium effort is higher under debt reduction than under temporary relief.
2. If in the one-period model there is stonewalling ($\lambda < 1-K$) and if the equilibria under debt reduction and temporary relief are different from each other ($\lambda > \lambda^*$), then:

2.i) The expected utility for the bank is strictly higher under debt reduction than under temporary relief.
2.ii) The expected utility for the type $\underline{a}$ country is higher under temporary relief than under debt reduction if and only if the equilibrium under temporary relief exhibits no stonewalling.
2.iii) The expected value of the equilibrium effort is higher under temporary relief than under debt reduction if and only if the equilibrium under temporary relief exhibits no stonewalling.

Proof.
It follows from the comparison of equilibria in both cases.

■

The interpretation of this result is similar to the one we have made after proposition 2.3. The bank can increase its profits due to the enhanced commitment capacity it has. This however, can harm the country if it is type $\underline{a}$. The intuition (as proposition 2.5 will confirm) is that the bank can be more selective in its forgiveness to type $\underline{a}$ because it can make the initial repayment contingent on the product.

Solution under technology product destruction

In this section we study the advantages from being able to set today a finite tomorrow's debt that derive from the sheer enhanced commitment capacity. In the previous section we have seen that these advantages can appear mixed with those derived from being able to set a first-period-product-contingent contract. Now we want to isolate them. One way to do it is by assuming that if the payment made by the country is lower than the payment due, the bank receives no payment at all, that is, the confiscation process produces its total loss[9]. This assumption makes the

[9] In equilibrium this will never occur, but the knowledge that it would happen

two-period model more comparable to the one-period model because it faces the bank with the same dilemma it faces under the one-period model: to make attractive to the type $\bar{a}$ country to repay the amount due or else to receive nothing from it. It discards the possibility of the bank inducing effort levels which are positive but smaller than the repayment due. The result is as follows.

Proposition 5

Assume the technology product destruction

a) Assume that in the one-period model there is no stonewalling ($\lambda > 1 - K$) The equilibrium under temporary relief differs (in expected utilities) from the equilibrium under debt reduction and product destruction if and only if the first one exhibits ratchet effect ($\delta > 1-K$). In such a case, the second one exhibits a higher effort level -the first best level- than the one under temporary relief, the same expected utility for the country and higher expected utility for the bank.

b) Assume that the one-period model exhibits stonewalling ($\lambda < 1 - K$) If the equilibrium under temporary relief differs from the equilibrium under debt reduction and product destruction, then the last one shows higher -the first best level- equilibrium effort level and higher expected utilities for both the bank and the country than the former. This situation appears for parameter values that produce type (b.1) and (b.2) (but not (b.3)) equilibria under the temporary relief model

Proof (See appendix)

The bank's capacity to commit itself to start tomorrow with a finite debt does not harm the country.

In the first case its only effect is to allow the bank to reap the benefits derived from avoiding the hiding of the type $\underline{a}$ country in the first period. It achieves this by making the commitment of charging both countries the same amount in the last period. In exchange it charges the type $\underline{a}$ country today an amount equivalent to the loss it would incur into if it had to hide.

if the product were lower than the bound set by the bank will cause the country to make no effort at all in case it decides to produce bellow the repayment due.

In the second case this same commitment also benefits the country. The reason is that without commitment capacity the equilibrium would be either a separating equilibrium in which $\underline{a}$ would have to produce zero to hide itself (instead of the cheaper $\bar{e}^{fb}$- hiding), or else a semi-separating equilibrium in which $\underline{a}$ obtains only its reservation utility because the bank decides to renounce to $\bar{a}$'s repayments both periods.

4. ARISING OF STONEWALLING IN A T-PERIOD MODEL

We have seen that, under the temporary relief scheme, if in the one-period model there is no stonewalling, then in the two-period model there isn't either. Yet, this feature cannot be generalized to models with more periods. This is due to the arising of the ratchet effect.

If the discount factor is small relative to K ($\delta < 1 - K$), then the ratchet effect never appears. Type $\underline{a}$ country prefers to get today the benefits of its efficient behavior even though this gives itself away and it never again obtains a generous forgiveness. In this case stonewalling will never appear as we increase the number of periods in the model: no stonewalling in a one-period model implies no stonewalling in a T-period model.

If the discount factor is big relative to K ($\delta > 1 - K$), then the type $\underline{a}$ country, knowing that if it manages to keep its reputation of being weak it will obtain a generous forgiveness, will hide under the type $\bar{a}$ first-best effort level: there is ratchet effect. Thus we have three possible scenarios:

The bank knows that if it grants a forgiveness big enough to induce type $\bar{a}$ country to make its first-best effort, then type $\underline{a}$ will mimic it: there will be ratchet effect. Yet, it prefers this situation to one in which it has to give up type $\bar{a}$'s repayments. This happens if δ is medium and the probability of the country being weak not too close to one. Then we always have generous forgiveness as well as ratchet effect. The bank does not separate both types of countries because the discounted losses from the ratchet effect are not too big.

For higher discount factors, the permanent hiding of $\underline{a}$ (except for the last period) under type $\bar{a}$'s first-best level (which is suboptimal from $\underline{a}$'s perspective), if the time horizon is long enough, produces welfare losses higher than those resulting from a present stonewalling. Thus, the bank grants a small forgiveness, such that type $\bar{a}$ makes no effort, but attractive enough to induce type $\underline{a}$ to make its first-best effort level and thus reveal its type. Then we will have a transitory stonewalling: the bank will grant a small forgiveness to separate both types of countries and once it achieves this it will grant a generous forgiveness to the weak type and a small one to the strong type, so that thereon each type will make its first-best effort level.

Finally, it can happen that although the welfare future losses from the ratchet effect are higher than those resulting from an initial transitory stonewalling, type $\underline{a}$ requires an initial forgiveness so big that even type $\bar{a}$ wants to take it. Then the instruments with which we have endowed the bank are not sufficient to separate both types of countries and we will have permanent generous forgiveness and ratchet effect.

Now we specify the formal T-period model. The product of the country in period t, y^T_t, is a function of its effort in this same period, e^T_t. We assume $y^T_t = e^T_t$ in each period t, $t = 1, 2, \dots, T$.

The extensive form of the game is as follows. First nature plays choosing the parameter a in the country's utility function according to the probability function (1). The value taken by this variable is known only to the country. Next, without knowing a's realization, the bank chooses the payment $\bar{R}^T_1 \in \mathbb{R}$ which the country must meet. Knowing a and $\bar{R}^T_1$, the country chooses $e^T_1 \in \mathbb{R}+$, which determines the first period product y^T_1 and gives to the bank the amount $R^T_1 = \min \{ y^T_1 , \bar{R}^T_1 \}$, ending the first period. Next we describe the actions in the p^{th} period, $2 \leq p \leq T$. At the beginning of the p^{th} period the bank sets the bound for that period, $\bar{R}^T_p \in \mathbb{R}$, knowing the bounds it has set and the efforts the country has made all the previous periods. Then, having the same information as the bank and, in addition, knowing a and $\bar{R}^T_p$, the country makes $e^T_p \in \mathbb{R}+$ and gives to the bank $R^T_p = \min \{ y^T_p, \bar{R}^T_p \}$.

Figure 6

Nature chooses $a \in \{ \underline{a} , \bar{a} \}$	bank sets $\bar{R}^T_1$	country makes e^T_1	...	bank sets $\bar{R}^T_P$	country makes e^T_p	...	country makes e^T_T

Country's utility is

$$\sum_{t=1}^{T} \delta\, U^T_t \qquad \text{with } U^T_t = y^T_t - R^T_t - (e^T_t)^2/2 \qquad \text{for } t = 1, 2, \dots, T$$

Bank's utility is

$$\sum_{t=1}^{T} \delta\, \Pi^T_t \qquad \text{with } \Pi^T_t = R^T_t \qquad \text{for } t = 1, 2, \dots, T$$

We define a history observed by the country in period 1 as $Th^c_1 = (a, \bar{R}^T_1)$ and, in period p, $2 \leq p \leq T$, as $Th^c_p = (Th^c_{p-1}, e_{p-1}, \bar{R}^T_p)$. Likewise, a history observed by the bank in period 1 is $Th^B_1 = (0)$, in period 2 is $Th^B_2 = (\bar{R}^T_1, e^T_1)$ and, in period p, $2 \leq p \leq T$, $Th^B_p = (Th^B_{p-1}, \bar{R}^T_{p-1}, e^T_{p-1})$.

A strategy for the country specifies a function for each period p = 1, 2,...,T, which assigns an effort level e^T_p to each history observed Th^c_p.

A strategy for the bank specifies an upper bound $\bar{R}^T_1$ for the first period and, for the p^{th} period, $2 \leq p \leq T$, a function which assigns an upper bound $\bar{R}^T_p$ to each observed history Th^B_p.

Equilibrium concept

A perfect bayesian equilibrium for the T-period model is a pair of strategies and beliefs $\lambda^T_p(Th^B_p) = P(\bar{a}|Th^B_p)$ for each period p, $2 \leq p \leq T$ such that:

i) e^T_p maximizes $E\sum_{m=p}^{T} U^T_m \ \forall\ Th^c_p$, given the bank's strategy,

for p = 1, 2, ..., T.

ii) $\bar{R}^T_p$ maximizes $E\sum_{m=p}^{T} R^T_m \ \forall\ Th^B_p$, given the country's strategy,

for p = 1, 2, ..., T.

(B) $\lambda^T_p(Th^B_p)$ is derived from the probability λ^T_{p-1} and the country's strategy using Bayes'rule, whenever a positive probability event is observed.
We ask the off-equilibrium updated beliefs to satisfy the following condition

(B') If a history $Th^B_p = (Th^B_{p-1}, \bar{R}^T_{p-1}, e^T_{p-1})$ with probability equal to zero is observed, then $\lambda_p(Th^B_p) = 0$ if there exists $e^T_{p-1}{}'$ s.t. type $\bar{a}$ country gets an expected utility strictly lower by choosing e^T_{p-1} than by choosing $e^T_{p-1}{}'$for each optimal bank's strategy restricted to periods p, p + 1,...T, for some λ_p.

Proposition 6

Consider the asymmetric information model under temporary relief and assume that in the one-period model there is no stonewalling ($\lambda > 1 - K$ and $\bar{R} = 1/2\bar{a}$).

a) If $\delta < (1 - K)$ then in the T-period model ($\forall$ T) there is neither ratchet effect nor stonewalling in the first period ($\forall$ T we have $\bar{R}^T_1 = 1/2\bar{a}$ and $\underline{e}^T_1 = \underline{e}^{fb}$, $\bar{e}^T_1 = \bar{e}^{fb}$) and from the second period the bank sets the complete information bounds (if $e^T_1 = \bar{e}^{fb}$, then $\bar{R}_t{}^T = 1/2\bar{a}$ for $2 \leq t \leq T$. If $e_1{}^T = \underline{e}^{fb}$ then $\bar{R}_t{}^T = 1/2\underline{a}$ for $2 \leq t \leq T$)

b) If $\delta > (1 - K)$, one of the following three situations occur (and each one of them occurs for some parameter configuration) *b.i) If* $\delta/(1-\delta) \leq \{K+\lambda-1\}/(1-\lambda)(1-K)^2$*, then the T-period model equilibrium exhibits ratchet effect in all but the last period, but it never exhibits stonewalling (*$\forall$ *T the bank sets* $R_t^T = 1/2\bar{a}$ *for* $1 \leq t \leq T$*, and* $\underline{a}$ *hides itself:* $\underline{e}^T_t = \bar{e}^T_t = \bar{e}^{fb}$ *if* $1 \leq t \leq T-1$*)*

Consider the increasing sequence

$SC(T) = 1 - \lambda - K + (1-\lambda)(1-K)^2(\delta + \delta^2 + \ldots + \delta^T)$

and the decreasing sequence

$SD(T) = 1 - K[\delta + \ldots + \delta^{T-1}] + \delta^T$

Notice that if $\delta/(1-\delta) \geq \{K+\lambda-1\}/(1-\lambda)(1-K)^2$ *then there exists T* s.t.*

$SC(T^*) > 0$ *and* $SC(T) \leq 0$ *if* $T < T^*$

b.ii) If $\delta/(1-\delta) \geq \{K+\lambda-1\}/(1-\lambda)(1-K)^2$ *and* $SD(T^*) \geq 0$*, then there exists T* $(T = T^*)$ *s.t. in the T-period model equilibrium there is transitory stonewalling:* $\bar{R}_1^T > 1/2\bar{a}$, $\underline{e}^T_1 = \underline{e}^{fb}$, $\bar{e}^T_1 = 0$ *and from the second period on the bank sets the optimal complete information bounds (if* $e^T_1 = 0$*, then* $\bar{R}_t^T = 1/2\bar{a}$ *for* $2 \leq t \leq T$*. If* $e^T_1 = \underline{e}^{fb}$ *then* $\bar{R}_t^T = 1/2\underline{a}$ *for* $2 \leq t \leq T$*)*

b.iii) If $\delta/(1-\delta) \geq \{K+\lambda-1\}/(1-\lambda)(1-K)^2$ *y* $SD(T^*) < 0$*, then* $\forall$ *T the T-period model equilibrium exhibits ratchet effect in all but the last period and it never exhibits stonewalling: we have the same result as in b.i.*

Remark. In b.i) the transitory stonewalling does not appear because the benefits from avoiding the type $\underline{a}$ hiding are lower than those resulting from the loss of the initial $\bar{a}$'s product. In b.iii) such inequality is reversed, but the initial bound required for $\underline{a}$ to reveal its character is so small that it does not cause stonewalling. It does not induce $\bar{a}$ to make no effort.

Proof (See appendix)

APPENDIX. PROOFS OMMITED IN THE MAIN TEXT

Proof of proposition 2

We first derive two necessary conditions for equilibrium (Similar to Freixas, Guesnerie and Tirole (1985)) .

Lemma 4.1 $\bar{e}_1 \in \{ 0 , 1/ \bar{a} \}$.

Proof:

In the second period the type $\underline{a}$ country obtains zero expected utility for all the bank's beliefs (Proposition 2.1). Thus, in the first period its choice must maximize its expected utility of only this period.

□(Lemma 4.1)

Lemma 4.2 $\underline{e}_1 \in \{ 0 , 1/\bar{a} , 1/\underline{a} \}$.

Proof:

Let us assume that e_1 is in the support of $\underline{e}_1$ but outside the support of $\bar{e}_1$. We will have $\lambda_2(e_1) = 0$ and, therefore, if e_1 is observed, $\underline{a}$ will have in the second period zero expected utility. Thus e_1 must maximize $\underline{a}$'s first period expected utility and be equal to $1/ \underline{a}$

□(Lemma 4.2)

Lemma 4.2 ' $\bar{e}_1 = 0 \Rightarrow \underline{e}_1 \in \{ 0, 1/ \underline{a} \}$

$\bar{e}_1 = 1/ \bar{a} \Rightarrow \underline{e}_1 \in \{ 1/ \bar{a} , 1/ \underline{a} \}$

(The proof is as in lemma 4.2)

□(Lemma 4.2')

Now we establish a necessary condition for the updating of beliefs in equilibrium.

Lemma 4.3 If $e_1 = 1/ \underline{a}$ is observed, then $\lambda_2 = 0$.

Proof.

It follows from Bayes'rule if $1/\underline{a}$ is in the support of $\underline{e}_1$ and from condition B' otherwise.

□(Lemma 4.3)

Now we will consider in turn the cases in which $(1-\lambda)$ is higher or lower than K

CASE $(1 - \lambda) < K$

Fact 4.1

If $(1 - \lambda) < K$, in equilibrium $\bar{R}_1 \geq 1 / 2\bar{a}$

Proof.

Assume to the contrary that the equilibrium exhibits $\bar{R}_1 < 1/ 2\bar{a}$. The expected utility for the bank is thus

$$E\Pi = \begin{cases} \bar{R}_1 + (1/ 2\bar{a})\,\delta & \text{if } \delta > (1-K) \\ \bar{R}_1 + [\,(1-\lambda)(1/ 2\underline{a}) + \lambda(1/ 2\bar{a})] & \text{if } \delta \leq (1-K) \end{cases}$$

because then the supposed equilibrium must also show:

H4.1.i) $\bar{e}_1 = 1 / \bar{a}$ (in the second period it will obtain zero. This is its optimal first period effort which provides it with a positive expected utility, because $\bar{R}_1 < 1 / 2\bar{a}$)

H4.1.ii) $\underline{e}_1 = 1 / \bar{a}$ if $\delta \geq (1-K)$ (Pooling)

$\underline{e}_1 = 1/ \underline{a}$ if $\delta < (1-K)$

because with $\underline{e}_1 = 1/ \bar{a}$ obtains $1/\bar{a} - (\underline{a}/2)(1/\bar{a}^2) - \bar{R}_1 + [\,1/ 2\underline{a} - 1/ 2\bar{a}]\,\delta$

and with $\underline{e}_1 = 1 / \underline{a}$ obtains $1/ 2\underline{a} - \bar{R}_1 + 0\,\delta$

Thus $E\Pi$ is strictly increasing in $\bar{R}_1$, which is a contradiction.

□ (Fact 4.1)

In the case $\bar{R}_1 = 1 / 2\bar{a}$ it appears a new possibility: that of $\bar{a}$ randomizing. We must make sure that the bank does not want it to do it.

Fact 4.2.

Suppose that in equilibrium $\bar{R}_1 = 1 / 2\bar{a}$. Then the expected utility for the bank is

$$E\Pi = \begin{cases} 1 / 2\bar{a} + \delta\,(1/ 2\bar{a}) & \text{if } \delta > 1\text{-}K \\ 1/ 2\bar{a} + \delta\,[\lambda(1/ 2\bar{a}) + (1-\lambda)(1/ 2\underline{a})] & \text{if } \delta < 1\text{-}K \end{cases}$$

Proof

$\bar{a}$ is indifferent between $\bar{e}_1 = 0$ and $\bar{e}_1 = 1/ \bar{a}$.

Let $\bar{x}_1 = P(\bar{e}_1 = 0)$

Every $\bar{x}_1 \in [\,0,\ 1]$ is optimal. For each $\bar{x}_1$, the optimal reaction on the part of $\underline{a}$ is different. We have multiplicity of equilibria. Yet, under the hipothesis that the chosen $\bar{x}_1$ maximizes the bank's expected profits, we will show that $\bar{R}_1 = 1/ 2\bar{a}$ implies $\bar{x}_1 = 0$ (and thus we obtain the stated profits).

It is clear that $\bar{x}_1 > 0$ has a negative effect on $E\Pi$ by reducing the expected payments from $\bar{a}$. There is, nevertheless, another effect working in the opposite direction: it reduces the incentives for $\underline{a}$ to hide under $\bar{a}$'s product $(1/ 2\ \bar{a})$.

Next we prove that $\bar{x}_1 > 0$ does not maximize the bank's expected profits.

The idea is that if $\bar{x}_1$ is very small, then it does not alter $\underline{a}$'s behavior but it does decrease $\bar{a}$'s repayment (relative to $\bar{x}_1 = 0$). We find that to alter $\underline{a}$'s behavior, $\bar{x}_1$ must be so high that it is disadvantageous to the bank: the benefits from inducing $\underline{a}$ to reveal its type do not make up for the loss of $\underline{a}$'s product.

Let us find the optimal $\underline{e}_1$. Let:

$$P(\underline{e}_1 = 0\,) = \underline{x}_1$$
$$P(\underline{e}_1 = 1/ \bar{a}) = \underline{x}_2$$

The expected utility for $\underline{a}$ under the actions in its support are:

$\underline{EU}(1/ \underline{a}) = 1/ 2\ \underline{a}\ - 1\ /\ 2\ \bar{a} + 0$

(from lemma 4.3 it follows that $1-\ \lambda_2(1/ 2\bar{a},\ 1/ \underline{a}) = 1 > K$ and thus $\underline{a}$'s second period utility is zero)

$\underline{EU}(0) \leq\ 0 + [\ 1/ 2\underline{a} - 1/ 2\bar{a}]\ \delta$

If $\delta < 1$, then $\underline{e}_1 = 0$ is strictly dominated by $\underline{e}_1 = 1/\underline{a}$

Si $\delta = 1$, then $\underline{e}_1 = 0$ is weakly dominated by $\underline{e}_1 = 1/\underline{a}$

$$\underline{EU}(1/\bar{a}) = \begin{cases} '+' + [1/2\underline{a} - 1/2\bar{a}]\delta & \text{if } 1-\lambda_2(e_1 = 1/\bar{a}) < K \\ '+' + 0\,\delta & \text{if } 1-\lambda_2(e_1 = 1/\bar{a}) > K \\ '+' + [1/2\underline{a} - 1/2\bar{a}]\,\delta\, y & \text{if } 1-\lambda_2(e_1 = 1/\bar{a}) = K \end{cases}$$

CASE $\delta < (1-K)$

$\underline{EU}\ (1/\underline{a}) > \underline{EU}\ (1/\bar{a})\ \forall\ \lambda_2$, and thus

$E\Pi = (1/2\bar{a})\ (1 - \lambda\ \bar{x}_1) + \delta\ [\lambda(1/2\bar{a}) + (1-\lambda)(1/2\underline{a})]$

from where it is clear that $\bar{x}_1 = 0$ maximizes the bank's expected profits.

CASE $\delta > (1-K)$ and $\bar{x}_1 < 1 - (1-K)(1-\lambda_1)/K\lambda_1$

In equilibrium:

$\underline{e}_1 = 1/\bar{a}$, $1-\lambda_2(e_1 = 1/\bar{a}) < K$, $\bar{R}_2 = 1/2\bar{a}$, and

$E\Pi = (1-\lambda\ \bar{x}_1)/2\bar{a} + 1/2\bar{a}$ and so

$\bar{x}_1 = 0$ maximizes $E\Pi$, which then attains the value

$E\Pi(\bar{R}_1 = 1/2\bar{a},\ \bar{x}_1 = 0) = 1/2\bar{a} + 1/2\bar{a}$

<u>Proof</u>

By Bayes' rule we have:

$1 - \lambda_2(e_1 = 1/\bar{a}) = (1-\lambda_1)\ \underline{x}_2 / \{(1-\lambda_1)\ \underline{x}_2 + \lambda_1\ (1-\bar{x}_1)\}$

Now,

$(1-\lambda_1)\ \underline{x}_2 / \{(1-\lambda_1)\ \underline{x}_2 + \lambda_1(1-\bar{x}_1)\} < K$ iff

$\underline{x}_2 / (1-\bar{x}_1) < K\lambda_1 / [1-K]\ [1-\lambda]$ (12)

Thus:

$\bar{x}_1 < 1 - (1-K)(1-\lambda_1)/K\lambda_1 \Rightarrow 1 - \lambda_2(e_1 = 1/\bar{a}) < K$

$\Rightarrow \underline{EU}\ (e_1 = 1/\bar{a}\ ;\ \underline{a}) =$ '+' $+ 1/2\underline{a} - 1/2\bar{a}$

Intuitively: $\bar{x}_1$ small $\Rightarrow$ if $e_1 = 1/\bar{a}$ is observed the bank believes the country is weak because $\bar{a}$ chooses $1/\bar{a}$ with a high probability $(1-\bar{x}_1)$.

□(Case $\delta > 1-K$ and $\bar{x}_1 < 1 - (1-K)(1-\lambda)/K\lambda$

CASE $\delta > (1-K)$ and $\bar{x}_1 \geq 1 - (1-K)(1-\lambda_1)/K\lambda_1$

In this case $E\Pi < 1/2\bar{a} + \delta(1/2\bar{a})$

<u>Proof</u>

We will consider all the possible values for $\underline{x}_2$, i.e., for the probability of

$\underline{e}_1 = 1 / \bar{a}$

i) Assume $\underline{x}_2 = 0$, i.e., $\underline{a}$ never chooses $1/\bar{a}$, i.e. $\underline{e}_1 \in \{ 0, 1/ \underline{a} \}$
Then $1-\lambda_2(e_1 = 1/ \bar{a}) = 0 < K$ or $\bar{x}_1 = 1$ (in which case Bayes' rule does not apply)
If $\bar{x}_1 < 1$, (in which case the first of the previous statements apply)
then $\underline{EU}(\underline{e}_1 = 1/ \bar{a}) > \underline{EU}(\underline{e}_1 \in \{ 0, 1/ \underline{a} \})$, and
then the only optimal choice for $\underline{a}$ is $\underline{e}_1 = 1 / \bar{a}$, i.e., $\underline{x}_2 = 1$ #
Thus, $\underline{x}_2 = 0 \Rightarrow \bar{x}_1 = 1$, i.e., $\bar{e}_1 = 0$ with probability one.

ii) Assume $\underline{x}_2 = 1$.
Then $1- \lambda_2(e_1 = 1/ \bar{a}) \leq K$ iff $\bar{x}_1 \leq 1 - (1-K)(1-\lambda_1) / K \lambda_1$
So, $(1 - \lambda_2(e_1 = 1/ \bar{a})) > K \Rightarrow U(e_1 = 1/ \bar{a} ; \underline{a}) < U(e_1 = 0; \underline{a})$
from where $\underline{e}_1 = 1/ \bar{a}$ is not optimal. #
Thus, $\underline{a}$'s optimal action is to randomize, with

$$\underline{x}_2 = (1- \bar{x}_1) K\lambda / (1-K) (1-\lambda)$$

Bank's expected profits are:

$$E\Pi = 1/ 2 \bar{a}[1- \lambda\bar{x}_1] + \delta (1/ 2\bar{a})[1-(1-\lambda)(1-\underline{x}_2] + \delta(1/ 2 \underline{a})(1-\lambda)(1- \underline{x}_2)$$

$$\partial E\Pi / \partial \bar{x}_1 = - \lambda / 2 \bar{a} - \delta \{ K\lambda / (1-K) (1-\lambda)\} \{(1-\lambda)/ 2\bar{a} - (1-\lambda) / 2\underline{a} \}$$

which has the same sign as (dividing by λ and multyplying by 2)

$$- 1/ \bar{a} - \delta \{ K(1- \lambda) / (1-K)(1-\lambda)\} \{ 1/ \bar{a} - 1/ \underline{a} \} = 0,$$

and as (multiplying by $\underline{a}$ and by $K \equiv \underline{a}/ \bar{a}$) :

$$-K - \delta K(-1 + K) / (1-K) = K(\delta-1) \leq 0$$

Thus $\bar{x}_1 = 1 - (1-K)(1-\lambda_1) / K \lambda_1$ maximizes the expected bank's profits in the interval $[1 - (1-K)(1-\lambda_1) / K \lambda_1 , 1]$. Evaluating $E\Pi$ in such value we obtain

$$E\Pi = (1/ 2 \bar{a})(1-\lambda + (1-K)(1-\lambda_1) / K) + \delta(1/ 2\bar{a}) \leq 1/ 2 \bar{a} + \delta (1/ 2 \bar{a})$$

because $\lambda > (1-K)(1-\lambda_1) / K$ iff $(1-\lambda) < K$.
This ends the proof that it is not optimal for the bank to set
$\bar{x}_1 \geq 1 - (1-K)(1-\lambda_1) / K \lambda_1$

□(Fact 4.2)

Fact 4.3

In equilibrium $\bar{R}_1 \leq 1/2\bar{a}$

Proof.

Assume that in equilibrium $\bar{R}_1 > 1/2\bar{a}$. Then it also holds

i) $\bar{e}_1 = 0$ because otherwise $\bar{a}$ obtains a negative EU

ii) $\underline{e}_1 \in \{0, 1/\underline{a}\}$ by lemma 4.2'.

iii)If the support of $\underline{e}_1$ contains as only element 0 , then $E\Pi = 0 + \delta V(\lambda) < (1-\lambda)(1/2\bar{a}) + \delta V(\lambda) < E\Pi(1/2\bar{a})$ which contradicts the optimality of $\bar{R}_1$.

iv) Assume $1/\underline{a}$ belongs to the support of $\underline{e}_1$. Then

Since $\lambda_2(e_1 = 1/2\underline{a}) = 0$ we will have:

$$\underline{EU}(1/2\underline{a}) = 1/2\underline{a} - \bar{R}_1$$

that is, type $\underline{a}$ country doesn't get a positive utility level in the second period.

Moreover, since $\bar{e}_1 = 0$, $\lambda_2(e_1 = 0) \geq \lambda > (1-K)$ (from Bayes' rule)

Thus, if the bank observes $e_1 = 0$, its only optimal answer is to set $\bar{R}_2 = 1/2\bar{a}$, because we will then have $\lambda_2 > (1-K)$ and the country will obtain :

$\underline{EU}(0) = [1/2\underline{a} - 1/2\bar{a}]\,\delta$

Since $\underline{e}_1 = 1/\underline{a}$ is optimal, we have

$\underline{EU}(1/\underline{a}) \geq \underline{EU}(0)$, from where:

$\bar{R}_1 \leq (1/2\underline{a})[1 - \delta(1-K)]$

And thus we have:

$$E\Pi(\bar{R}_1) \leq (1-\lambda)(1/2\underline{a})[1-\delta(1-K)] + \delta[\lambda(1/2\bar{a}) + (1-\lambda)(1/2\underline{a})]$$

$$= (1-\lambda)[1/2\underline{a} + \delta(1/2\bar{a})] + \lambda\delta(1/2\bar{a}) =$$

$$= (1-\lambda)(1/2\underline{a}) + \delta(1/2\bar{a}) < 1/2\bar{a} + \delta(1/2\bar{a}) \leq E\Pi(1/2\bar{a})$$

where the first inequality follows from the bank being unable to get more than the complete information profits in the second period, and the second inequality from the hypothesis that $(1-\lambda)(1/2\underline{a}) < 1/2\bar{a}$ y $\delta < 1$.

□ (Fact 4.3)

Intuitively, if the bank sets $\bar{R}_1 > 1/2\bar{a}$, then $\bar{e}_1 = 0$. If it wants to induce $\underline{a}$ to reveal its character, it has to give to it all that it can obtain tomorrow by hiding its true character. Notice now that in both cases (hiding or revealing its character), the type $\underline{a}$ country will make tomorrow the same effort ($\underline{e}_1 = 1/\underline{a}$) , and this is why if the bank induces it ro reveal its character, the only thing it achieves is to recieve today what it would receive tomorrow. The only relevant change is thus the one from

$1/\ 2\bar{a}$ to $(1-\lambda)(1/\ 2\ \underline{a})$, that is, we face the same comparison as in the one-period model.

CASE $(1-\lambda) > K$

Lemma 4.4.

Let $R_1^{<} = 1/\ 2\ \underline{a} - \delta(1/\ 2\underline{a} - 1/\ 2\ \bar{a})$.

In equilibrium $\bar{R}_1 \in \{\ 1/\ 2\bar{a}\ ,\ R_1^{<}\ ,\ 1/\ 2\ \underline{a}\ \}$

Proof

Assume that in equilibrium $\bar{R}_1 > 1/\ 2\ \bar{a}$. Then in equilibrium we will alsto have the following features

i) $\bar{e}_1 = 0$ (since $\bar{e}_1 = 1/\ \bar{a}$ would yield a negative expected utility)

ii) $\underline{EU}\ (1/\ \underline{a}) = 1/\ 2\underline{a} - \bar{R}_1$

$$\underline{EU}\ (0) = \begin{cases} 0 & \text{if } 1 - \lambda_2(e_1 = 0) > K \\ \delta(1/\ 2\underline{a} - 1/\ 2\ \bar{a}) & \text{if } 1 - \lambda_2(e_1 = 0) < K \end{cases}$$

Now we get rid of the $\bar{R}_1$ values that does not belong to the set mentioned in the lemma.

iii) In equilibrium it cannot be that $1/\ 2\ \bar{a} < \bar{R}_1 < R_1^{<}$

(that is $(1/\ 2\underline{a} - \bar{R}_1) > \delta(1/\ 2\underline{a} - 1/\ 2\bar{a})$)

Proof

Assume it is. Then $\underline{e}_1 = 1\ /\ \underline{a}$ is the only optimal $\underline{a}$'s action, and

$E\Pi(\bar{R}_1) = \bar{R}_1(1-\lambda) + \delta(1-\lambda)(1/\ 2\underline{a}) + \delta\ \lambda\ (1/\ 2\ \bar{a})$

from where

$d\ E\Pi(\bar{R}_1)\ /\ d\ \bar{R}_1 = (1-\lambda) > 0$, which is a contradiction.

□(iii)

iv) In equilibrium it cannot be $R_1^{<} < \bar{R}_1 < 1/\ 2\underline{a}$

(that is $(1/\ 2\underline{a} - \bar{R}_1) < \delta(1/\ 2\underline{a} - 1/\ 2\bar{a})$)

Proof

Assume it is.

iv.i) Assume $\underline{x}_1 = 0$ (Separating equilibrium: $\underline{e}_1 = 0$ never occurs)

Then $1 - \lambda_2(e_1 = 0) = 0$, and

$E\Pi(e_1 = 0\ ;\ \underline{a}) = 1/\ 2\ \underline{a} - 1/\ 2\bar{a} > 1/\ 2\ \underline{a} - \bar{R}_1 = E\Pi\ (e_1 = 1/\ \underline{a}\ ;\ \underline{a})$, and

$\underline{e}^*_1 = 0$ #

iv.ii) Assume $\underline{x}_1 = 1$ (Pooling equilibrium: $\underline{e}_1 = 0$ with certainty)

Then $1 - \lambda_2(e_1 = \quad) = 1 - \lambda_1 > K$, and

$\underline{e}^*_1 = 1/\underline{a}$ #

iv.iii) There exists a semiseparating equilibrium:

$\underline{x}_1 = 0 \Rightarrow 1-\lambda_2 = 0$

$\underline{x}_1 = 1 \Rightarrow 1-\lambda_2 = 1-\lambda_1 > K$

$d(1-\lambda_2) / d\underline{x}_1 = \{ \underline{x}_1(1-\lambda)^2 + \lambda(1-\lambda) - \underline{x}_1(1-\lambda)^2 \} / [\quad]^2 > 0$

and thus

there exists a unique $\underline{x}_1 = K\lambda / (1-\lambda)(1-K) \in (0, 1)$.

Let us now compute $E\Pi$ under this unique semiseparating equilibrium.

$E\Pi = P(e_1 = 0) [0 + V(\lambda(e_1 = 0))] + P(e_1 = 1/\underline{a}) [\bar{R}_1 + V(\lambda(e_1 = 1/\underline{a}))]$

$E\Pi = [(1 - \lambda_1)\underline{x}_1 + \lambda_1] (1/2\bar{a}) + (1-\lambda)(1-\underline{x}_1) [\bar{R}_1 + 1/2\underline{a}]$

The first term tells us that it is possible for the country to make no effort at all in the first period. If this happens, the bank won't receive anything this period and, at the beginning of the second one, its beliefs will be such that $V(\lambda) = 1/2\underline{a}$ (we will have $1-\lambda_2 = K$ and it will be indifferent between setting $\bar{R}_2 = 1/2\bar{a}$ and receiving this amount with certainty, or else setting $\bar{R}_2 = 1/2\underline{a}$ and risking to receive nothing)

This events happens with probability $[(1 - \lambda_1)\underline{x}_1 + \lambda_1]$.

The second term considers the event $e_1 = 1/\underline{a}$. It it happens, the bank receives $\bar{R}_1$ the first period, and its second period beliefs are $\lambda = 1$, so that $V(\lambda) = 1/2\underline{a}$. This occurs with probability $\lambda [1 - \underline{x}_1]$.

We have that $dE\Pi / d\bar{R}_1 = (1-\lambda)(1-\underline{x}_1) > 0$, and so it is not optimal to set $\bar{R}_1$ s.t. $1/2\bar{a} < \bar{R}_1 < R^<_1$

□(iv)

v) In equilibrium it cannot occur $\bar{R}_1 > 1/2\underline{a}$

<u>Proof</u>

If that were the case, then

$\underline{EU}(1/\underline{a}) = 1/2\underline{a} - \bar{R}_1 < 0$

$$\underline{EU}(0) = \begin{cases} 0 + 0 & \text{if } (1 - \lambda_2) > K \\ 0 + [1/2\underline{a} - 1/2\bar{a}]\,\delta & \text{if } (1 - \lambda_2) < K \end{cases}$$

So, $\underline{a}$'s strategt prescribes choosing $\underline{e}_1 = 0$ with certainty which leads to

$1 - \lambda_2 = 1 - \lambda_1 > K$, and

$E\Pi = 0 + \delta(1-\lambda_1)(1/2\underline{a})$

which is lower than the expected utility the bank gets if it sets $\bar{R}_1 = 1/2\bar{a}$ (See fact 4.5), which is a contradiction.

□(v)

Finally we have that

vi) in equilibrium it cannot occur $\bar{R}_1 < 1/2\bar{a}$

(It is evident from the proof of fact 4.5. The only effect of reducing $\bar{R}_1$ below $1/2\bar{a}$ is to reduce the initial payment: the comparison between A and C is not altered)

□(vi)

□(Lemma 4.4)

We will now compute the bank's expected profits under each one of the three possible $\bar{R}_1$ values.

<u>Fact 4.4</u>

If $\bar{R}_1 = 1/2\underline{a} \Rightarrow E\Pi = (1-\lambda-K)(1/2\underline{a})/(1-K) + \delta(1-\lambda)(1/2\underline{a})$

<u>Proof</u>:

If in equilibrium we have $\bar{R}_1 = 1/2\underline{a}$, then we will also have

H4.4.i) $\bar{e}_1 = 0$

H4.4.ii) $\underline{a}$ randomizes in $\underline{e}_1 \in \{0, 1/\underline{a}\}$ with $\underline{x}_1 = K\lambda / (1-\lambda)(1-K)$

<u>Proof</u> (Of F 4.4.ii)

To find $\underline{a}$'s optimal reaction we first notice that

b.ii) $\underline{EU}(\underline{e}_1 = 1/\underline{a}) = \quad 0 + 0$

$$\underline{EU}(\underline{e}_1 = 0) = \begin{cases} 0 & \text{if } (1-\lambda_2) > K \\ \delta(1/2\underline{a} - 1/2\bar{a}) & \text{if } (1-\lambda_2) < K \\ \delta(1/2\underline{a} - 1/2\bar{a})y & \text{if } (1-\lambda_2) = K \end{cases}$$

where y is the probability of the bank choosing $\bar{R}_2 = 1/2\bar{a}$ if $h_2^B = (1/2\underline{a}, 0)$

Let $\underline{x}_1 = P(e_1 = 0 / \underline{a})$

We have

$(1 - \lambda_2) = \underline{x}_1(1-\lambda) / [\underline{x}_1(1-\lambda) + \lambda]$

$(1 - \lambda_2) \geq K$ iff $\underline{x}_1 \geq K\lambda / (1-\lambda)(1-K)$

i) Assume $\underline{x}_1 = 0$ (i.e. $\underline{e}_1 = 1/\underline{a}$, i.e., separating equilibrium)

Then $1 - \lambda_2(e_1 = 0) = 0$, and

$\underline{EU}(0) = \delta(1/2\ \underline{a} - 1/\ 2\bar{a}) > 0 = \underline{EU}\ (1/\ \underline{a})$ #

ii) Assume $\underline{x}_1 \geq K\lambda\ /\ (1-\lambda)\ (1 - K)$ (a particular case is $\underline{e}_1 = 0$ with certainty, i.e., pooling equilibrium)

Then $1 - \lambda_2(e_1 = 0) \geq K$, and

$\underline{EU}(\underline{e}_1 = 0) = \underline{EU}(\underline{e}_1 = 1/\ \underline{a})$ (if $\underline{x}_1 = K\lambda/(1-\lambda)(1-K)$ it is also necessary to have $y = 0$)

Thus $\underline{e}_1 \in \{ 0,\ 1/\ \underline{a} \}$

We thus have a continuum of semiseparating equilibria. (Moreover, $\underline{e}_1 = 0$ with probability one is also an equilibrium)

To solve this indeterminacy, we assume the equilibrium most preferred by the bank is chosen.

The bank's expected profits are:

$E\Pi = [\ \lambda + (1-\lambda)\ \underline{x}_1\ ]\ 1/\ 2\ \bar{a} + (1-\lambda)(1- \underline{x}_1)\ [\ 1/\ 2\underline{a} + 1/\ 2\underline{a}\ ]$

decreasing in $\underline{x}_1$:

$dE\Pi\ /\ d\ \underline{x}_1 = (1-\lambda)\ (1/\ 2\bar{a}) - (1-\lambda)\ [\ 1/\ 2\underline{a} + 1/2\underline{a}] = (1-\lambda)\ \{\ 1/\ 2\underline{a} -[\ 1/\ 2\underline{a} + 1/\ 2\ \underline{a}\ ]\ \} < 0$

and thus

$\underline{x}_1^* = K\lambda\ /\ (1-\lambda)\ (1-K)$

□(H4.4.ii)

We thus have

$E\Pi(\bar{R}_1 = 1/\ 2\underline{a}) = \delta\ (1-\lambda)\ /\ 2\ \underline{a} + [\ (1-\lambda) - K\ ]\ /\ 2\ \underline{a}\ [\ 1 - K\ ]$

$= (1 - \lambda/(1-K))(1/\ 2\underline{a}) + \delta(1-\lambda)(1/\ 2\underline{a})$

□ (Fact 4.4)

<u>Fact 4.5</u>

If $\bar{R}_1 = 1/\ 2\ \bar{a}$ then

$$E\Pi = \begin{cases} 1/\ 2\bar{a} + \delta\ (1-\lambda)(1/\ 2\underline{a}) & \text{if } \delta > (1-K) \\ 1/\ 2\bar{a} + \delta\ \{\ (1-\lambda)(1/\ 2\underline{a}) + \lambda(1/\ 2\bar{a})\ \} & \text{if } \delta \leq (1-K) \end{cases}$$

<u>Proof</u>

We first see that it is straightforward that

H4.5.i) $\bar{a}$ finds it optimal to set $\bar{e}_1 \in \{ 0,\ 1/\ \bar{a} \}$, that is, $\bar{x}_1 \in [0,1]$.

□

We have multiplicity of equilibria once more. We will select that preferred

by the bank. (We will see later on that $\delta < 1 \Rightarrow \bar{e}_1 = 1/\bar{a}$)
Let us denote by '+' the expression $1/\bar{a} - \underline{a}/2(\bar{a})^2$
$\underline{a}$'s payoff is given by:

$$\underline{EU}(e_1 = 0) = \begin{cases} 0 + 0\,\delta & \text{if } 1 - \lambda(e_1 = 0) > K\lambda/(1-K)(1-\lambda) \\ 0 + (1/2\underline{a} - 1/2\bar{a})\delta & \text{if } 1 - \lambda(e_1 = 0) < K\lambda/(1-K)(1-\lambda) \\ 0 + (1/2\underline{a} - 1/2\bar{a})\delta y & \text{if } 1 - \lambda(e_1 = 0) = K\lambda/(1-K)(1-\lambda) \end{cases}$$

$$\underline{EU}(e_1 = 1/\bar{a}) = \begin{cases} B = \text{'+'} + 0\,\delta & \text{if } 1-\lambda(e_1 = 1/\bar{a}) > K\lambda/(1-K)(1-\lambda) \\ C = \text{'+'} + [1/2\underline{a} - 1/2\bar{a}]\delta & \text{if } 1-\lambda(e_1 = 1/\bar{a}) < K\lambda/(1-K)(1-\lambda) \\ \quad\ \text{'+'} + [1/2\underline{a} - 1/2\bar{a}]\delta y & \text{if } 1-\lambda(e_1 = 1/\bar{a}) = K\lambda/(1-K)(1-\lambda) \end{cases}$$

$$\underline{EU}(e_1 = 1/\underline{a}) = A = 1/2\underline{a} - 1/2\bar{a} + 0$$

Notice that $C > A$ is equivalent to $\delta > 1 - K$ and consider in turn the cases $\delta > 1 - K$ and $\delta < 1 - K$

<u>Caso $\delta > (1-K)$</u>

H4.5.ii) $\underline{a}$ randomizes in $\{1/\bar{a}, 1/\underline{a}\}$, with $\underline{x}^*_2 = K\lambda/(1-K)(1-\lambda)$

<u>Proof</u> (Of Fact 4.5.ii)

By Bayes' rule:

$1-\lambda_2(e_1 = 1/\bar{a}) \leq K$ iff $\underline{x}_2 \leq K\lambda(1-\bar{x}_1)/(1-K)(1-\lambda)$

$1-\lambda_2(e_1 = 0) \leq K$ iff $\underline{x}_1 \leq K\lambda(1-\bar{x}_1)/(1-K)(1-\lambda)$

Suppose $\underline{x}_2 < (1-\bar{x}_1)K\lambda/(1-K)(1-\lambda)$ (For instance $\underline{x}_2 = 0$, i.e., $\underline{e}_1 = 1/\underline{a}$ always)
Then $1-\lambda_2(e_1 = 1/\bar{a}) < K$ and, since $C > A$, the only $\underline{a}$'s best response is $\underline{e}_1 = 1/\bar{a}$ (i.e., $\underline{x}_2 = 1$)##

Suppose $\underline{x}_2 > (1-\bar{x}_1)K\lambda/(1-K)(1-\lambda)$ (For instance $\underline{x}_2 = 1$, i.e., $\underline{e}_1 = 1/\bar{a}$ always)
Then $1-\lambda_2(e_1 = 1/\bar{a}) > K$, and $\underline{e}_1 = 1/\underline{a}$ is the only $\underline{a}$'s best response. (i.e., $\underline{x}_2 = 0$) #
So, in equilibrium,
$\underline{x}_2 = (1-\bar{x}_1)K\lambda/(1-K)(1-\lambda)$

On the other hand,

In equilibrium $\underline{x}_1 = 0$ (if $\delta < 1$)[1] because $\underline{EU}(\underline{e}_1 = 0) < \underline{EU}(\underline{e}_1 = 1/\underline{a})$

Thus

$$E\Pi = (1/2\bar{a})[1-\lambda\bar{x}_1] + (1/2\bar{a})[1-(1-\lambda)(1-\underline{x}_2)]\delta + (1/2\underline{a})(1-\lambda)(1-\underline{x}_2)\delta$$

and we have

$$dE\Pi/d\bar{x}_1 = -\lambda(1/2\bar{a}) + (1/2\bar{a})\delta(1-\lambda)(-K\lambda/(1-K)(1-\lambda)) - \delta(1/2\underline{a})(1-\lambda)(-K\lambda/(1-K)(1-\lambda))$$
$$= -1+\delta$$

Thus,

if $\delta = 1$, $E\Pi$ is constant $\forall\ \bar{x}_1$

if $\delta < 1$, $\bar{x}_1 = 0$ is the unique equilibrium that maximizes the bank's expected profits. Thus, for each of the two possible cases, evaluating $\underline{x}_2$ when $\bar{x}_1 = 0$ we obtain the statement of F4.5.ii

□ (F4.5.ii)

When $\bar{x}_1 = 0$ we have:

$$E\Pi = [\lambda + (1-\lambda)\underline{x}_2][1/2\bar{a} + \delta/2\bar{a}] + (1-\lambda)(1-\underline{x}_2)[1/2\bar{a} + 1/2\underline{a}]$$

from where substituting $\underline{x}_2 = (1-\bar{x}_1)K\lambda/(1-K)(1-\lambda)$ we obtain, after some algebra:

$$E\Pi = 1/2\bar{a} + \delta(1-\lambda)(1/2\underline{a})$$

□(Case $\delta > (1-K)$)

<u>Case $\delta < (1-K)$</u>

If $\delta < (1-K)$ we have $A > C$ and $A > B$, and thus $\underline{e}_1 = 1/\underline{a}$ is the only optimal $\underline{a}$'s strategy, from where it follows the expression for the bank's profits.

□(Case $\delta < 1 - K$)

□(Fact 4.5)

<u>Fact 4.6</u>

If in equilibrium $\bar{R}_1 = R_1^{<}$, then $E\Pi = (1-\lambda)(1/2\underline{a}) + \delta(1/2\bar{a})$

[1] If $\delta = 1$, every $\underline{x}_1 < (1-\bar{x}_1)K\lambda/(1-K)(1-\lambda)$ is a best response for $\underline{a}$. Yet, since the bank's expected profits are then

$E\Pi = 1/2\bar{a}[1-\lambda\bar{x}_1 - (1-\lambda)\underline{x}_1] + \delta(1/2\bar{a})[1-(1-\lambda)(1-\underline{x}_2-\underline{x}_1)] + \delta(1/2\underline{a})(1-\lambda)(1-\underline{x}_1-\underline{x}_2)$

and

$\partial E\Pi/\partial \underline{x}_1 = -(1/2\bar{a})(1-\lambda) + \delta(1/2\bar{a})(1-\lambda) - \delta(1/2\underline{a})(1-\lambda) < 0$, el the bank prefers that type $\underline{a}$ country does not randomize.ce.

(It follows immediatly from points (i), (ii) and (iii) from the proof of lemma 4.4)

□(Fact 4.6)

It turns out that the three $\bar{R}_1$ values are optimal for some parameters configuration. Next we compare $1/2\bar{a}$ with $1/2\underline{a}$

Fact 4.7(1)

Let $K < (1-\lambda)$ and $\delta > (1-K)$

If $\lambda < (1-K)^2$, then $E\Pi(1/2\underline{a}) > E\Pi(1/2\bar{a})$

If $\lambda > (1-K)^2$, then $E\Pi(1/2\underline{a}) < E\Pi(1/2\bar{a})$

Proof:

$E\Pi(\bar{R}_1 = 1/2\bar{a}) - E\Pi(\bar{R}_1 = 1/2\underline{a}) =$

$[\lambda + (1-\lambda)\underline{x}_1](1/2\bar{a}) + (1-\lambda)(1-\underline{x}_1)[1/2\bar{a} - 1/2\underline{a}] \geq 0$ iff

$K \geq (1-\lambda)(1-\underline{x}_1)$ iff $\underline{x}_1 \geq (1-\lambda-K)/(1-\lambda)$

iff $K\lambda/(1-K)(1-\lambda) \geq [1-\lambda-K]/(1-\lambda)$ where the last equivalence follows from $\underline{x}_1 = K\lambda/(1-K)(1-\lambda)$

and it it is equivalent to (after some algebra)

$K^2 - 2K + 1-\lambda \leq 0$

from where the statement of the fact follows.

□(Fact 4.7(1))

Fact 4.7(2)

Let $\lambda < (1-K)$ and $\delta < (1-K)$

If $\lambda > (1-K)^2$ and $\delta > [(1-K)^2-\lambda]/\lambda K(1-K)$, then $E\Pi(1/2\bar{a}) > E\Pi(1/2\underline{a})$

If $\lambda < (1-K)^2$ and $\delta < [(1-K)^2-\lambda]/\lambda K(1-K)$, then $E\Pi(1/2\bar{a}) < E\Pi(1/2\underline{a})$

Proof.

It follows from the comparison of the equilibria.

□(Fact 4.7(2))

Fact 4.8

$E\Pi(\bar{R}_1^<) \geq E\Pi(1/2\underline{a})$ iff $\delta \leq \lambda K/(1-K)(1-\lambda-K)$

Proof

The previous inequality is obtained from

$(1-\lambda)(1/2\underline{a}) + \delta(1/2\bar{a}) \geq (1-\lambda-K)/(1-K)(2\underline{a}) + \delta(1-\lambda)(1/2\underline{a})$

after some algebra.

□(Fact 4.8)

Corolary to Fact 4.8

$$\lambda \geq (1-K)^2 \Rightarrow E\Pi(R_1^<) \geq E\Pi(1/\ 2\underline{a})$$

Proof.

$\lambda \geq (1-K)^2$ iff $\lambda K/(1-K)(1-\lambda K) \geq 1$

Fact 4.9

If $\delta > (1-K)$ or $\delta < (1-\lambda-K)/(1-\lambda)(1-K) \Rightarrow E\Pi(\bar{R}_1^<) > E\Pi(1/\ 2\bar{a})$

If $(1-\lambda-K)/(1-\lambda)(1-K) < \delta < (1-K) \Rightarrow E\Pi(\bar{R}_1^<) < E\Pi(1/\ 2\bar{a})$

Proof.

Assume $\delta > (1-K)$. Then

$E\Pi(R_1^<) = (1-\lambda)(1/\ 2\underline{a}) + \delta(1/\ 2\bar{a}) \quad \geq \quad 1/\ 2\bar{a} + \delta(1-\lambda)(1/\ 2\underline{a}) = E\Pi(1/\ 2\bar{a})$

iff $\delta \leq 1$

Assume $\delta < (1-K)$. Then

i) $\bar{R}_1 = R_1^<$ and $\bar{R}_1 = 1/\ 2\bar{a}$ induce $\underline{e}_1 = 1/\ 2\underline{a}$, $\bar{e}_1 = 1/\ 2\bar{a}$, and thus the bank's second period expected profits are the same.

A comparison of the first period profits yields:

ii) $(1-\lambda)R_1^< < 1/\ 2\ \bar{a}$ iff $(1-\lambda)\ \{\ 1-\ \delta + \delta K\ \} > k\ \delta + K(1-\delta)$ iff $\delta < (1-\lambda-K)/(1-\lambda)(1-K)$

□(Fact 4.9)

We can now make use of the facts 4.7, 4.8 and 4.9 to find the optimal $\bar{R}_1$ value for each configuration of the parameters δ, λ, K . (Results are summed up in figures 2 and 3)

■ (Proposition 2)

Proof of Proposition 3

To solve the model, we first write the expected utilities for the bank and both types of countries as a function of the debt and beliefs with which the final period begins.

Lemma 2.3.1

Under every bayesian perfect equilibrium with debt reduction, $\underline{EU}_2$, $\overline{EU}_2$, and $E\Pi_2$ can be written as a function of λ_2 and the debt with which the second period starts, $\bar{R}_1^2$, and are:

If $\lambda_2 > 1-K$:

$$E\Pi = \min [\bar{R}_1^2, 1/2\bar{a}]$$

$$\underline{EU} = 1/2\underline{a} - \min [\bar{R}_1^2, 1/2\bar{a}]$$

$$\overline{EU} = 1/2\bar{a} - \min [\bar{R}_1^2, 1/2\bar{a}]$$

If $\lambda_2 < 1-K$:

$$E\Pi = \begin{cases} (1-\lambda_2)(1/2\underline{a}) & \text{if } \bar{R}_1^2 \geq 1/2\underline{a} \\ (1-\lambda_2)\bar{R}_1^2 & \text{if } R^*(\lambda) < \bar{R}_1^2 < 1/2\underline{a} \\ 1/2\bar{a} & \text{if } 1/2\bar{a} \leq \bar{R}_1^2 < R^*(\lambda) \\ \bar{R}_1^2 & \text{if } \bar{R}_1^2 \leq 1/2\bar{a} \end{cases}$$

$$\underline{EU} = \begin{cases} 1/2\underline{a} - 1/2\bar{a} & \text{if } \bar{R}_1^2 > 1/2\underline{a} \\ 1/2\underline{a} - \bar{R}_1^2 & \text{if } R^*(\lambda) < \bar{R}_1^2 < 1/2\underline{a} \\ 1/2\underline{a} - 1/2\bar{a} & \text{if } 1/2\bar{a} \leq \bar{R}_1^2 < R^*(\lambda) \\ 1/2\underline{a} - \bar{R}_1^2 & \text{if } \bar{R}_1^2 < 1/2\bar{a} \end{cases}$$

$$\overline{EU} = \begin{cases} 0 & \text{if } \bar{R}_1^2 > 1/\bar{2}a \\ 1/2\bar{a} - R & \text{if } \bar{R}_1^2 < 1/2\bar{a} \end{cases}$$

Proof (Of Lemma)

It is identical to the solution of the one-period model.

□(Lemma 2.3.1)

The previous lemma allows us to assume, without loss of generality, that $\bar{R}^2_1$, $\underline{R}^2_1 \leq 1/2\underline{a}$. To economize in notation, in this proof we will denote them by $\bar{R}_2$ and $\underline{R}_2$, respectively.

If the type $\bar{a}$ country pays[1] the amount $\bar{z}_1$ in the first period and if the type $\underline{a}$ country pays the amount $\underline{z}_1$ with probability $(1 - \underline{x}_1)$ and $\bar{z}_1$ with probability $\underline{x}_1$, we can then write the bank's expected profits at the beginning of the game as:

$$E\Pi_1 = (1-\lambda_1)(1-\underline{x}_1)\{\underline{z}_1 + \delta E\Pi(\lambda_2(\underline{x}_1);\underline{R}_2)\} + \{1-(1-\lambda)(1-\underline{x}_1)\}\{\bar{z}_1 + \delta E\Pi(\lambda_2(\underline{x}_1);\bar{R}_2)\}$$

The bank's problem at the beginning of the game is thus to maximize $E\Pi_1$ having as its instruments $\bar{z}_1$, $\underline{z}_1$, $\bar{R}_2$, $\underline{R}_2$ y $\bar{R}^1_1$.

We now explain the method of solution. We assume that the bank can also choose, besides the above mentioned instruments, the first period effort for each country, that is, it chooses an effort level $\bar{e}_1$ for type $\bar{a}$ country and, for type $\underline{a}$ country, an effort $\underline{e}_1$ and the probability $(1-\underline{x}_1)$ with which it will make it (with probability $\underline{x}_1$ it will make $\bar{e}_1$). The restrictions it faces when making this decision are the necessary and sufficient conditions for these variables to be part of a perfect bayesian equilibrium. That is, it must choose them in such a way that they be optimal actions for each type of country, given the values it will be presented for $\bar{z}_1$, $\underline{z}_1$, $\bar{R}_2$, $\underline{R}_2$ and $\bar{R}^1_1$.

We now establish two lemmas about e_1 in equilibrium.

Lemma 2.3.2

If $\bar{e}_1 \leq \bar{R}^1_1$, then $\bar{e}_1 \in \{0, \bar{z}_1, \underline{z}_1\}$

Proof.

If $\bar{e}_1$ does not belong to the set $\{\bar{z}_1, \underline{z}_1\}$, then the type $\bar{a}$ country obtains a zero expected utility in the second period. Then, in the first period. So, in the first period its choice must maximize its expected utility

1Allowing type $\bar{a}$ to randomize does not help but makes the proof more complex.

of only this period. Since $\bar{e}_1 \le \bar{R}^1_1$, type $\bar{a}$ country receives no product in the first period. Thus it must be $\bar{e}_1 = 0$.

□(Lemma 2.3.2)

Lemma 2.3.3

If $\underline{e}_1 \le \bar{R}^1_1$, then $\underline{e}_1 \in \{ 0, \bar{z}_1, \underline{z}_1 \}$

Proof.

If $e_1 \le \bar{R}^1_1$ and it is in the support of $\underline{e}_1$ but not in the support of $\bar{e}_1$, we will have $\lambda_2(e_1) = 0$. Since $\bar{R}^2_1(R_1) = 1/ 2\underline{a}$ if $R_1 \notin \{ \bar{z}_1, \underline{z}_1 \}$, type $\underline{a}$ country will obtain zero expected utility in the second period. Then e_1 must maximize its expected utility of only the first period. Since $\underline{e}_1 \le \bar{R}^1_1$, type $\underline{a}$ country receives no product at all in the first period. Thus it must be $\underline{e}_1 = 0$
#

□(Lemma 2.3.3)

Notice that $\bar{R}^1_1$ does not appear in the objective function. We will set $\bar{R}^1_1$ big enough as to allow the bank (according to the gunboat technology) to withdraw the entire first period country's product. In the end (Lemma 2.3.3) we will show this is without loss of generality. We allow the bank to choose the pair of triplets ($\bar{e}_1$, $\bar{z}_1$, $\bar{R}_2$), designed for $\bar{a}$, and ($\underline{e}_1$, $\underline{z}_1$, $\underline{R}_2$), designed for $\underline{a}$, as well as $\underline{x}_1$, the probability of $\underline{a}$ hiding. (We will have $\bar{z}_1 = \bar{e}_1$ and $\underline{z}_1 = \underline{e}_1$). We will identify the first country's action as the choice between one of these triplets.

Thus, given $\bar{R}^1_1 \ge \max \{ \bar{e}_1, \underline{e}_1 \}$, the bank must choose ($\bar{e}_1$, $\bar{z}_1$, $\bar{R}_2$), ($\underline{e}_1$, $\underline{z}_1$, $\underline{R}_2$) and $\underline{x}_1$ to maximize $E\Pi_1$ under the following restrictions:

$\underline{z}_1 = \underline{e}_1$, $\bar{z}_1 = \bar{e}_1$ (Gunboat condition when $\bar{R}^1_1 \ge \max\{ \bar{e}_1, \underline{e}_1 \}$)

$\bar{z}_1 = \underline{z}_1 \Rightarrow \bar{R}_2 = \underline{R}_2$ (Non-verifiability of e_1)

$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/ 2 - \underline{z}_1 + \delta\, \underline{EU}(\underline{R}_2; \lambda_2(\underline{x})) \ge \bar{e}_1 - \underline{a}\,\bar{e}_1^2/ 2 - \bar{z}_1 + \delta\, \underline{EU}(\bar{R}_2;\lambda_2(\underline{x})) \qquad (IC\underline{a})$$

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/ 2 - \bar{z}_1 + \delta\, \bar{EU}(\bar{R}_2; \lambda_2(\underline{x})) \ge \underline{e}_1 - \bar{a}\,\underline{e}_1^2/ 2 - \underline{z}_1 + \delta\, \underline{EU}(\underline{R}_2; \lambda_2(\underline{x})) \qquad (IC\bar{a})$$

$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/ 2 - \underline{z}_1 + \delta\, \underline{EU}(\underline{R}_2; \lambda_2(\underline{x})) \ge 0 \qquad (VP\underline{a})$$

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta\,\bar{EU}(\bar{R}_2;\ \lambda_2(\underline{x})) \geq 0 \qquad (VP\bar{a})$$

As a consequence of lemmas 2.3.2 and 2.3.3, the previous restrictions are necessary and sufficient conditions for the type $\underline{a}$ country to make $\underline{e}_1$ and pay $\underline{z}_1$ and the type $\bar{a}$ country to make $\bar{e}_1$ and pay $\bar{z}_1$. If we want $\underline{a}$ to randomize, then IC$\underline{a}$ must be met with equality.

We now analyze in turn the cases $\lambda > 1-K$ and $\lambda < 1-K$

CASE $\lambda > 1-K$

In this case if $\bar{e}_1$ is observe we will have $\lambda_2 > 1-K$ and if $\underline{e}_1$ is observed we will have $\lambda_2 = 0$. Using $\underline{EU}_2$, $\bar{EU}_2$, and $E\Pi_2$ from Lemma 2.3.1 we find that the bank's problem is

$$\text{Max } (1-\lambda_1)(1-\underline{x}_1)\{\underline{z}_1 + \delta \min[\underline{R}_2, 1/2\underline{a}]\} +$$
$$\{1-(1-\lambda)(1-\underline{x}_1)\}\{\bar{z}_1 + \delta \min[\bar{R}_2, 1/2\bar{a}]\}$$

s.a.

$$\underline{z}_1 = \underline{e}_1\ , \quad \bar{z}_1 = \bar{e}_1 \qquad \text{(Gunboat condition)}$$
$$\bar{z}_1 = \underline{z}_1 \Rightarrow \bar{R}_2 = \underline{R}_2 \qquad \text{(Non- verifiability of } e_1)$$
$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/2 - \underline{z}_1 - \delta\min\{\underline{R}_2, 1/2\underline{a}\} \geq$$
$$\bar{e}_1 - \underline{a}\,\bar{e}_1^2/2 - \bar{z}_1 - \delta\min\{\bar{R}_2, 1/2\bar{a}\} \qquad (IC\ \underline{a})$$
$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta(1/2\bar{a}) - \delta\min\{\bar{R}_2, 1/2\bar{a}\} \geq$$
$$\underline{e}_1 - \bar{a}\,\underline{e}_1^2/2 - \underline{z}_1 + \delta\,\underline{EU}(\underline{R}_2;\ \lambda_2(\underline{x})) \qquad (IC\ \bar{a})$$
$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/2 - \underline{z}_1 + \delta(1/2\underline{a}) - \delta\min\{\underline{R}_2, 1/2\underline{a}\} \geq 0 \qquad (VP\ \underline{a})$$
$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta(1/2\bar{a}) - \delta\min\{\bar{R}_2, 1/2\bar{a}\} \geq 0 \qquad (VP\ \bar{a})$$

To solve the problem we will ignore the gunboat, verifiability, VP$\underline{a}$ and IC$\bar{a}$ conditions. In the end we will check that they are satisfied.

i) VP $\bar{a}$ is binding because $E\Pi$ is strictly increasing in $\bar{z}_1$ and increases in $\bar{z}_1$ make IC$\underline{a}$ slacker.

ii) From (i) and IC$\underline{a}$ it follows that in equilibrium $\underline{EU} > 0$, that is, VP$\underline{a}$ is not binding

iii) If $\underline{x}_1 < 1$, IC$\underline{a}$ is binding (if $\underline{x}_1 = 1$ we can easily check that the bank's expected profits are not as high as possible)

iv) Substituting $\bar{z}_1 + \delta \min\{ \bar{R}_2, 1/2\bar{a} \}$ from VP$\bar{a}$ and $\underline{z}_1 + \delta\min\{\underline{R}_2, 1/2\underline{a}\}$ from IC$\underline{a}$ we have a problem in $\underline{e}_1$ and $\bar{e}_1$ whose first order conditions are:

v) $\underline{e}_1 = 1/\underline{a}$,

$\bar{e}_1 = 1 / \{ \bar{a} + \Delta a(1-\lambda)/\lambda \}$

Notice that $\underline{z}_1 + \delta \{ \underline{R}_2 \} = 1/2\underline{a} - [\bar{e}_1 - \underline{a}\, \bar{e}_1^2/2] + \bar{z}_1 + \delta \{\bar{R}_2\}$ implies $\underline{z}_1 + \delta\{\underline{R}_2\} > \bar{z}_1 + \delta\{ \bar{R}_2\}$, and so in equilibrium $\underline{x}_1 = 0$

Thus

$\bar{a}$'s payment is: $\bar{z}_1 + \delta \bar{R}_2 = \bar{e}_1 - \bar{a}\, \bar{e}_1^2/2 + \delta\, (1/2\bar{a})$

$\underline{a}$'s payment is: $\underline{z}_1 + \delta \underline{R}_2 = 1/2\underline{a} - \Delta a\, \bar{e}_1^2/2 + \delta(1/2\bar{a})$

and the expected bank's profits are:

$E\Pi^A = (1-\lambda_1) \{ 1/2\underline{a} - \Delta a\, \bar{e}_1^2/2 + \delta(1/2\bar{a})\} + \lambda \{ \bar{e}_1 - \bar{a}\, \bar{e}_1^2/2 + \delta\, (1/2\bar{a})\}$

Consider now the

CASE $\lambda < 1-K$

In this case if $\bar{e}_1$ is observed we cannot guarantee that $\lambda_2 \geq 1-K$.

Fact 2.3.1

In equilibrium $\bar{R}_2 \in \{ (-\infty, 1/2\bar{a}] \cup \{ 1/2\underline{a} \} \}$

Proof.

We will show that if $1/2\bar{a} < \bar{R}_2 < 1/2\underline{a}$, there exists an alternative bank's choice which produces higher expected profits, namely, $1/2\underline{a}$.

i)The objective function is increasing in $\bar{R}_2$.

ii) If $1/2\bar{a} < \bar{R}_2 < 1/2\underline{a}$ then, since $\bar{EU}(\ )$ is constant for $1/2\bar{a} < \bar{R}_2$, $\bar{EU}(\bar{R}_2) = \bar{EU}(1/2\underline{a})$. Thus, by shifting from $\bar{R}_2$ to $1/2\underline{a}$, VP $\bar{a}$ is still satisfied. (VP$\bar{a}$ is binding because $E\Pi$ increases in $\bar{z}_1$ and increases in $\bar{z}_1$ make IC$\underline{a}$ slacker)

iii) IC $\underline{a}$ is made slacker because $\underline{EU}(\bar{R}_2)$ is decreasing if $1/2\bar{a} < \bar{R}_2 < 1/2\underline{a}$

Summing up, if we replace $\bar{R}_2$ by $1/2\underline{a}$ leaving all else equal, we obtain a choice for the bank which is better than the original choice.

□(Fact 2.3.1)

Let us now consider the bank's problem restricted to

Sub case $\bar{R}_2 \leq 1/2\bar{a}$

It is easily verified that we obtain the same solution as in case $\lambda > 1-K$. The reason is that when $\bar{R}_2 \leq 1/2\bar{a}$, $E\Pi$, $\overline{EU}$, $\underline{EU}$, do not depend on λ. It is clear that if the final period begins with $\bar{R}_2 \leq 1/2\bar{a}$, the bank will not reduce debt, no matter what beliefs it has. Thus the problem is the same we face when $\lambda > 1 - K$.

Assume now

Sub case $\bar{R}_2 = 1/2\underline{a}$.

i) If $\underline{x}$ is such that $\lambda_2(\underline{x}, \lambda_1) > 1 - K$, then in the second period the bank will reduce the debt to a level $1/2\bar{a}$, and thus it would be exactly the same as if a debt $\bar{R}_2 = 1/2\bar{a}$ had been initially set. The utility both agents get is the same in both cases. The bank obtains $E\Pi^A$ once more.

ii) If $\underline{x}$ is such that $\lambda_2(\underline{x}, \lambda_1) \leq 1 - K$, the bank's expected profits are (for the $\underline{x}$ value which maximizes them)

$$E\Pi^B = (1-\lambda_1-K)/(1-K)(2\underline{a}) + \delta(1-\lambda_1)/2\underline{a}$$

Proof (of ii)

Substituting $\underline{EU}_2$, $\overline{EU}_2$, and $E\Pi_2$ from Lemma 1 in the case $\lambda_2 \leq 1 - K$ we find the bank's problem is

$$\text{Max} \quad (1-\lambda_1)(1-\underline{x}_1)\{\underline{z}_1 + \delta\min[\underline{R}_2, 1/2\underline{a}]\} + \{1-(1-\lambda_1)(1-\underline{x}_1)\}\{\bar{z}_1 + \delta(1-\lambda_2)(1/2\underline{a})\}$$

s.t.

$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/2 - \underline{z}_1 - \delta\min\{\underline{R}_2, 1/2\underline{a}\} \geq \bar{e}_1 - \underline{a}\,\bar{e}_1^2/2 - \bar{z}_1 - \delta\, 1/2\underline{a} \qquad (\text{IC } \underline{a})$$

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 \geq 0 \qquad (\text{VP } \bar{a})$$

$$\underline{z}_1 = \underline{e}_1, \quad \bar{z}_1 = \bar{e}_1 \qquad (\text{Gunboat})$$

We first notice that, since $\bar{z}_1 = \bar{e}_1$ (from gunboat), restriction (VP $\bar{a}$) is

$- \bar{a}\, \bar{e}_1^2 / 2 \geq 0$, from where $\bar{z}_1 = \bar{e}_1 = 0$. Substituting in (IC $\underline{a}$) we get:

$$\underline{e}_1 - \underline{a}\, \underline{e}_1^2/2 - \underline{z}_1 + \delta\ 1/2\underline{a} - \delta \min \{\underline{R}_2, 1/2\underline{a}\} = 0$$

Thus the bank's expected profits are:

$$E\,\Pi = (1-\lambda_1)(1-\underline{x}_1)\{\underline{z}_1 + \delta \min[\underline{R}_2, 1/2\underline{a}]\} + \{1-(1-\lambda_1)(1-\underline{x}_1)\}$$
$$\{\delta(1-\lambda_2)(1/2\underline{a})\}$$

If $\underline{x}_1 = 1$, then $\lambda_2 = \lambda_1$ and $E\Pi = \delta(1-\lambda_1)(1/2\underline{a})$

If $\underline{x}_1 < 1$, then a necessary condition for optimality is $\underline{e}_1 = 1/\underline{a}$ (otherwise it is possible to make IC $\underline{a}$ slacker and increase $\underline{z}_1$)

Taking into account that $\lambda_2(\underline{x}, \lambda_1) \leq 1-K$ we arrive at the stated result after some algebra.

□(ii)

Thus, when $\lambda < 1-K$, there are two points which satisfied the stated necessary conditions for an optimum, and they produce the following expected profits:

$$E\Pi^A = (1-\lambda_1)\{(1/2\underline{a} - \nabla a\, \bar{e}_1^2/2) + \delta(1/2\bar{a})\} + \lambda\{\bar{e}_1 - \bar{a}\, \bar{e}_1^2/2 + \delta(1/2\bar{a})$$

$$E\Pi^B = (1-\lambda-K)/(1-K)(2\underline{a}) + \delta(1-\lambda)/2\underline{a}$$

We have:

$$d(E\Pi^A - E\Pi^B)/d\lambda > 0$$

$$\lambda = 0 \Rightarrow E\Pi^A - E\Pi^B < 0$$

$$\lambda = 1-K \Rightarrow E\Pi^B = 0 + \delta(1/2\underline{a})K = \delta(1/2\bar{a}) \Rightarrow E\Pi^B < E\Pi^A$$

Therefore, there exists λ^*, $0 < \lambda^* < 1-K$, s. t. $E\Pi^A \leq E\Pi^B$ iff $\lambda \leq \lambda^*$

The following lemma completes the proof of the proposition.

<u>Lemma</u> <u>2.3.4</u>

There is no loss of generality in assuming that in equilibrium $\bar{R}_1^1 \geq \max\{\bar{e}_1, \underline{e}_1\}$

<u>Proof</u>

The proof consists in observing the following three facts

<u>Fact</u> <u>2.3.4</u> <u>a</u>

If the bank wants the country to find it optimal $\bar{e}_1 > \bar{z}_1$ or $\underline{e}_1 > \underline{z}_1$, its

optimization problem must respect all the restrictions present when $\bar{R}^1_1 \geq \max\{ \bar{e}_1, \underline{e}_1 \}$
(except, obviously, $\underline{e}_1 = \underline{z}_1$, $\bar{e}_1 = \bar{z}_1$) and some additional ones.

Proof

The proof consists in rewriting lemmas 2.3.1 and 2.3.2 for the case $\bar{R}^1_1 < \max\{ \bar{e}_1, \underline{e}_1 \}$
and noting that the support of $\bar{e}_1$ can include $1/\bar{a}$ and that of $\underline{e}_1$ $1/\underline{a}$, and so if the bank wants the country to make $\bar{e}_1$ if its type is $\bar{a}$ and $\underline{e}_1$ if its type is $\underline{a}$, it must make sure that these actions provides an expected utility higher than $1/\bar{a}$ and $1/\underline{a}$.
Since $e_1 = 0$, as well as mimic the other country, continue to be feasible actions for the country, restrictions VP$\bar{a}$, VP$\underline{a}$, IC$\bar{a}$ e IC$\underline{a}$ are still present. Finally the non-verifiability of e_1 condition is still present too, because it follows from an assumption we always maintain.

□(Fact 2.3.4 a)

As to the restrictions common to both problems the following can be said.

Fact 2.3.4 b

In the case $\bar{R}^1_1 \geq \max\{ \bar{e}_1, \underline{e}_1 \}$, ignoring restrictions $\underline{e}_1 = \underline{z}_1$, $\bar{e}_1 = \bar{z}_1$ does not increase the bank's expected profits if $\lambda < 1 - K$ or if, having $\lambda > 1-K$ either of the two following inequalities holds.

i) $\bar{R}_2 \leq 1/ 2\bar{a}$

ii) $\underline{x}$ such that $\lambda_2(\bar{e}_1; \underline{x}, \lambda_1) > 1 - K$

Proof

The proof consists in going through the solution under the hypothesis $\bar{R}^1_1 = \max\{ \bar{e}_1, \underline{e}_1 \}$
In the case $\lambda > 1 - K$, we have found that the solution to the bank's problem is to set

$$\bar{z}_1 + \delta \bar{R}_2 = \bar{e}_1 - \bar{a}\, \bar{e}^2_1/ 2 + \delta\, (1/2\bar{a})$$

$$\underline{z}_1 + \delta \underline{R}_2 = 1/ 2\underline{a} - \Delta a\, \bar{e}^2_1/ 2 + \delta(1/ 2\bar{a})$$

that is, what matters is $[\bar{z}_1 + \delta \bar{R}_2]$ and $[\underline{z}_1 + \delta \underline{R}_2]$, but $\bar{z}_1$ and $\underline{z}_1$ are not determined, and so we can set them equal to $\bar{e}_1$ and $\underline{e}_1$, respectively, without reducing the bank's profits.

If $\lambda < 1 - K$, we see that the problem is the same as in the case $\lambda > 1 - K$ if one of the stated inequalities holds.

□(Fact 2.3.4 b)

It remains to be analyzed the case in which $\lambda_2(\bar{e}_1;\ \underline{x},\ \lambda_1) > 1 - K$ and $\bar{R}_2 = 1/2\underline{a}$.

The hypothesis $\bar{R}^1_1 = \max\{\ \bar{e}_1,\ \underline{e}_1\ \}$ allowed us to infer $\bar{z}_1 = \bar{e}_1$. From this last equality it easily follows that the bank's expected profits equal $E\Pi^B$.

But without that hypothesis, the gunboat technology ($R_1 = \min\ \{\ \bar{R}^1_1,\ e_1\}$ unless the country voluntarily gives more than such quantity) does not necessarily imply that $R_1 = e_1$. It is possible that $R_1 < e_1$, in which case the following restrictions arise.

if $\bar{R}^1_1 < \bar{e}_1$, then $\bar{z}_1 \geq \bar{R}^1_1$,

if $\bar{R}^1_1 < \underline{e}_1$, then $\underline{z}_1 \geq \bar{R}^1_1$

Thus, let us assume that $\bar{e}_1 > \bar{z}_1$. We then have

Fact 2.3.4 c

In equilibrium it cannot occur that $\bar{e}_1 > \bar{z}_1$, $\lambda_2(\bar{e}_1;\ \underline{x},\ \lambda_1) \leq 1 - K$ and $\bar{R}_2 = 1/2\underline{a}$

Proof

Let us assume it occurs.

From (VP $\bar{a}$) we obtain that $\bar{z}_1 < 1/2\bar{a}$ and from the gunboat restriction that $\bar{z}_1 \geq \bar{R}^1_1$. From these two inequalities it follows that

$\underline{z}_1 + \delta\ \underline{R}_2 \leq 1/2\bar{a} + \delta\ (1/2\underline{a})$

since, if $\underline{e}_1 \leq \bar{R}^1_1$, then, as $\underline{z}_1 \leq \underline{e}_1$, we will have $\underline{z}_1 \leq \underline{e}_1 < \bar{R}^1_1 \leq 1/2\bar{a}$ and, if $\underline{e}_1 > \bar{R}^1_1$, then, as any payment above $\bar{R}^1_1$ must be voluntary, we will have $\underline{z}_1 + \delta\ \underline{R}_2 \leq \bar{R}^1_1 + \delta(\ 1/2\underline{a}\ ) \leq 1/2\bar{a} + \delta(\ 1/2\underline{a}\ )$.

Then $E\Pi \leq 1/2\bar{a} + \delta(1 - \lambda)(1/2\underline{a})$

Now, since $\delta \leq 1$, it holds:

$1/2\bar{a} + \delta(1 - \lambda)(1/2\underline{a}) \leq (1 - \lambda)(1/2\underline{a}) + \delta\ (1/2\bar{a})$

Finally, notice that

$(1/2\underline{a}) + \delta\ (1/2\bar{a}) < E\Pi^A$

since $E\Pi^A = \max_{\underline{e}_1, \bar{e}_1} \quad (1 - \lambda) \{ \underline{e}_1 - \underline{a}\, \underline{e}_1^2/ 2 - \Delta a\, \bar{e}_1^{\,2}/ 2 + \delta(1/ 2\bar{a}) \} +$

$$\lambda \{ \bar{e}_1 - \bar{a}\, \bar{e}_1^{\,2}/ 2 + \delta (1/ 2\bar{a}) \}$$

and $(1 - \lambda)(1/ 2\underline{a}) + \delta (1/ 2\bar{a})$ is the value of the objective function when the non-optimal pair of values $\underline{e}_1 = 1/ \underline{a}$, $\bar{e}_1 = 0$ is chosen.

□(Fact 2.3.4 c)

This ends the proof of lemma 2.3.4 and proposition 3.

□(Lemma 2.3.4)

■(Proposition 3)

Proof of proposition 5

Now the bank cannot set $\bar{R}_1^1$ so high as to withdraw the entire first period country's product and commit to reward the country in the second period depending on the initial repayment. It cannot do it because if $y_1 < \bar{R}_1^1$, then the repayment equals zero. This causes that we have to add some restrictions to the initial bank's problem.

They are essentially the following.

From the first period type $\bar{a}$'s country problem it follows that if the bank wants $\bar{a}$ to choose $\bar{e}_1$ outside the set $\{ 0, 1/\bar{a} \}$, then $\bar{e}_1 = \bar{z}_1 > 1/ \bar{a}$: if type $\bar{a}$ county makes a payment lower than $1/ \bar{a}$, it prefers to make it with its first best effort (unless it wants to mimic $\underline{a}$, but in equilibrium this will never happen because it would provide this country with a lower expected utility)

The restriction of stopping $\underline{a}$ to mimic $\bar{a}$ remains being binding. There appears a further restriction: if $\bar{R}_1 > 0$, the type $\underline{a}$ country can take the option $(\bar{z}_1, \bar{R}_2)$ without making an effort $\bar{e}_1$. This is so because the bank cannot specify in the contract that to have the right of the bound $\bar{R}_2$ an effort $\bar{e}_1$ has to be made. Yet, if $\underline{a}$ acts in this way, since the bank observes $\underline{e}_1$, it will update its beliefs by setting $\lambda_2 = 0$ (by Bayes'rule if $\underline{a}$ randomizes and by condition B'if it doesn't)

If the type $\bar{a}$ country repays[2] the amount $\bar{z}_1$ in the first period and it the type $\underline{a}$ country repays the amount $\underline{z}_1$ with probability $(1 - \underline{x}_1)$ and the amount $\bar{z}_1$ with probability $\underline{x}_1$, we can then write the bank's expected profits

[2] Allowing $\bar{a}$ to randomize only makes the proof more complex.

ate the beginning of the game as:

$$E\Pi_1 = (1-\lambda_1)(1-\underline{x}_1)\{\underline{z}_1 + \delta\, E\Pi(\lambda_2(\underline{x}_1);\underline{R}_2)\} + \{1-(1-\lambda)(1-\underline{x}_1)\}\{\bar{z}_1 + \delta\, E\Pi(\lambda_2(\underline{x}_1);\bar{R}_2)\}$$

The bank's problem at the beginning of the game is then to maximize $E\Pi_1$ having as instruments $\bar{z}_1$, $\underline{z}_1$, $\bar{R}_2$, $\underline{R}_2$ and $\bar{R}^1_1$.

The method of solution consists of assuming that the bank can also choose, besides the mentioned instruments, the effort each type of country will make in the first period, that is, an effort $\bar{e}_1$ for the type $\bar{a}$ country, and an effort $\underline{e}_1$ and a probability $(1-\underline{x}_1)$ with which it will make it (with a probability $\underline{x}_1$ it will make $\bar{e}_1$). The restrictions it faces when making such decision are the necessary and sufficient conditions for these variables to form part of a perfect bayesian equilibrium. That is, it must choose it in such a way that they are optimal actions for the type of country they are designed, given the accompanying values of $\bar{z}_1$, $\underline{z}_1$, $\bar{R}_2$, $\underline{R}_2$ and $\bar{R}^1_1$.

The bank's problem is thus to maximize $E\Pi_1$ given the following two restrictions.

$$(\underline{e}_1, \underline{z}_1) \in \underset{(e_1, R_1)}{\operatorname{argmax}}\ e_1 - R_1 - \underline{a}\, e_1^2/2 + \delta\, \underline{EU}(R^1_2(R_1), \lambda_2(\ ))$$
$$\text{s.t.} \quad e_1 \geq R_1 \geq \bar{R}^1_1 \text{ " } e_1 = R_1 = 0$$

$$(\bar{e}_1, \bar{z}_1) \in \underset{(e_1, R_1)}{\operatorname{argmax}}\ e_1 - R_1 - \bar{a}\, e_1^2/2 + \delta\, \overline{EU}(R^1_2(R_1), \lambda_2(\ ))$$
$$\text{s.t.} \quad e_1 \geq R_1 \geq \bar{R}^1_1 \text{ " } e_1 = R_1 = 0$$

From these restrictions it follows that the following are necessary conditions for equilibrium

$$\bar{z}_1 < \bar{R}^1_1 \Rightarrow \bar{z}_1 = 0, \qquad \underline{z}_1 < \bar{R}^1_1 \Rightarrow \underline{z}_1 = 0$$

$$\underline{z}_1 \leq \underline{e}_1$$

$$\bar{z}_1 \leq \bar{e}_1$$

$$\underline{e}_1 - \underline{a}\, \underline{e}_1^2/2 - \underline{z}_1 + \delta\, \underline{EU}(\underline{R}_2;\lambda_2(\underline{x})) \geq \bar{e}_1 - \underline{a}\, \bar{e}_1^2/2 - \bar{z}_1 + \delta\, \underline{EU}(\bar{R}_2;\lambda_2(\underline{x})) \quad (IC\underline{a})$$

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta\,\overline{EU}(\bar{R}_2;\lambda_2(\underline{x})) \geq \underline{e}_1 - \bar{a}\,\underline{e}_1^2/2 - \underline{z}_1 + \delta\,\underline{EU}(\underline{R}_2;\lambda_2(\underline{x})) \quad (IC\bar{a})$$

$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/2 - \underline{z}_1 + \delta\,\underline{EU}(\underline{R}_2;\lambda_2(\ )) \geq \max_{e_1 \geq \bar{z}_1} e_1 - \underline{a}\,e_1^2/2 - \bar{z}_1 + \delta\,\underline{EU}(\bar{R}_2;\lambda_2) \quad (IIC\underline{a})$$

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta\,\overline{EU}(\bar{R}_2;\lambda_2(\ )) \geq \max_{e_1 \geq \underline{z}_1} e_1 - \bar{a}\,e_1^2/2 - \underline{z}_1 + \delta\,\underline{EU}(\underline{R}_2;\lambda_2) \quad (IIC\bar{a})$$

$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/2 - \underline{z}_1 + \delta\,\underline{EU}(\underline{R}_2;\lambda_2(\underline{x})) \geq 0 \quad (VP\underline{a})$$

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta\,\overline{EU}(\bar{R}_2;\lambda_2(\underline{x})) \geq 0 \quad (VP\bar{a})$$

Remark 2.5.1

In equilibrium the bank's profits are greater or equal than in the temporary relief model.

Proof

Notice first that Proposition 4.2 is also valid under the product destruction technology. To prove it we proceed in the same way we did under the gunboat technology. Then the bank can set $\bar{R}_2 = \underline{R}_2 = 1/2\underline{a}$, that is, promising nothing in the future, and reproduce the temporary relief equilibrium.

□(Remark 2.5.1)

CASE $\lambda > 1 - K$

In this case if $\bar{e}_1$ is observed, $\lambda_2 > 1 - K$ will be set and, therefore, the expected profits for the bank are[3]

$$E\Pi = (1-\lambda)\{\underline{z}_1 + \delta \min[\underline{R}_2, 1/2\underline{a}] + \lambda\{\bar{z}_1 + \delta \min[\bar{R}_2, 1/2\bar{a}]\}$$

Restrictions IC($\underline{a}$) and VP($\bar{a}$) are

$$\underline{e}_1 - \underline{a}\,\underline{e}_1^2/2 - \underline{z}_1 - \delta\,\underline{R}_2 \geq \bar{e}_1 - \underline{a}\,\bar{e}_1^2/2 - \bar{z}_1 - \delta \min[\bar{R}_2, 1/2\bar{a}]\} \quad (IC\underline{a})$$

$$\bar{e}_1 - \bar{a}\,\bar{e}_1^2/2 - \bar{z}_1 + \delta(1/2\bar{a}) - \delta \min[\bar{R}_2, 1/2\bar{a}] \geq 0 \quad (VP\bar{a})$$

From (IIC$\bar{a}$)[4] it follows that:

[3] We omit the randomization in $\underline{a}$'s first action because this only complicates the proof.

[4] (Assuming $\underline{R}_2, \bar{R}_2 \leq 1/2\underline{a}$ without loss of generality)

If $\underline{e}_1 \geq \bar{z}_1 \geq \bar{R}_1^1$ then $\underline{z}_1 + \delta \underline{R}_2 \leq \bar{z}_1 + \delta \bar{R}_2$

When VP$\bar{a}$ is satisfied with equality and is substituted in IC$\underline{a}$, it yields
$\underline{z}_1 + \delta \underline{R}_2 \leq \underline{e}_1 - \underline{a}\, \underline{e}_1^2/ 2 + \delta (1/ 2\bar{a}) - (\bar{a}-\underline{a})\bar{e}_1^{\,2}/ 2$

We will analyze separately the cases $\delta > 1$-K and $\delta < 1$-K

Sub case $\delta > 1$-K

i) $\bar{e}_1 = \bar{z}_1 = 0$ cannot be part of an equilibrium.

Proof

Assume the contrary. We will show that the expected bank's profits are then lower than the ones it gets if the temporary relief solution is reproduced. If the type $\underline{a}$ country effort is also equal to zero with probability one, then $E\,\Pi < \delta\ (1/ 2\bar{a})$.

If $\underline{a}$ finds it optimal to choose $\underline{e}_1$ different from zero, then it must be that:
$\underline{EU}\ (\underline{e}_1) = \underline{e}_1 - \underline{z}_1 - \underline{a}\ \underline{e}_1^2/ 2 + \delta(1/ 2\underline{a}) - \delta\ \underline{R}_2 \geq \underline{EU}(0) \geq \delta\ (1/ 2\underline{a} - 1/ 2\bar{a})$,
where the first inequality follows from IC$\underline{a}$ and the last one to $\lambda_2(\bar{e}_1) > 1$ - K so that the bank will reduce the second period debt to $1/ 2\bar{a}$ if it is greater than this level and observes $\bar{e}_1$.

Thus

$\underline{z}_1 + \delta\ \underline{R}_2 \leq 1/ 2\underline{a} + \delta(1/ 2\bar{a})$, from where

$E\Pi \leq (1-\lambda)\ \{ 1/ 2\underline{a} + \delta(1/ 2\bar{a})\ \} + \lambda\ \delta\ (1/ 2\bar{a}) = (1-\lambda)\ (1/ 2\underline{a}) + \delta\ (1/ 2\bar{a})$

which are smaller than the profits obtained by the bank if it reproduces the solution under temporary relief $(1/ 2\bar{a} + \delta(1/ 2\bar{a}))$, which is a contradiction.

□(i)

Consider now, in turn, what happens if si $\bar{z}_1 \geq 1/\ \bar{a}$ and if $\bar{z}_1 < 1/\ \bar{a}$

ii) The bank can obtain by setting $0 < \bar{z}_1 \leq 1/\ \bar{a}$, greater expected profits than those under temporary relief. If the equilibrium exhibits $0 < \bar{z}_1 \leq 1/\ \bar{a}$, then it also exhibits $\bar{e}_1 = \bar{e}^{fb}$, $\underline{e}_1 = \underline{e}^{fb}$,
$\bar{z}_1 + \delta \min[\bar{R}_2,\ 1/ 2\ \bar{a}] = 1/ 2\ \bar{a} + \delta\ (1/ 2\bar{a})$ and
$\underline{z}_1 + \delta\ \underline{R}_2 = 1/ 2\bar{a} + \delta\ (1/ 2\bar{a}) + [1/ 2\underline{a} - 1/\ \bar{a} + \underline{a}/ 2\ \bar{a}]$

Proof

If $0 < \bar{z}_1 \leq 1/\bar{a}$, then in equilibrium $\bar{e}_1 = \bar{e}^{fb} = 1/\bar{a}$ (if the type $\bar{a}$ country finds it optimal to give the bank $0 < \bar{z}_1 \leq 1/\bar{a}$, the optimal way to do it is by making an effort $\bar{e}_1 = 1/\bar{a}$). Let's assume now that the only restrictions are VP$\bar{a}$ and IC$\underline{a}$ (Later on we will check that the other ones are satisfied)

Now we will see that it is optimal to set $\underline{e}_1 = \underline{e}^{fb} = 1/\underline{a}$. Assume a different value is set. By changing to the alternative value $\underline{e}_1 = 1/\underline{a}$ restriction IC$\underline{a}$ continues to be satisfied, and so does VP$\bar{a}$ (because $\underline{e}_1$ does not appear here). We can then enhance the expected bank's profits by increasing $\underline{z}_1$, which contradicts the optimality of $\underline{e}_1$ being different from $1/\underline{a}$.

Let us now set $\underline{z}_1 + \delta\, \underline{R}_2$ in such a way that IC$\underline{a}$ is satisfied with equality and $\bar{z}_1 + \delta \min\{\bar{R}_2, 1/2\bar{a}\}$ in such a way that VP$\bar{a}$ is also satisfied with equality.

Let us finally check that all the other restrictions are also satisfied. In particular, if we substitute $\bar{e}_1 = 1/\bar{a}$ in IC$\underline{a}$ we find that, when $\delta > 1\text{-}K$, (IC$\underline{a}$) $\Rightarrow$ (IIC$\underline{a}$).

Therefore, in equilibrium, restrictions IC$\underline{a}$ and VP$\bar{a}$ are binding and first best efforts are chosen.. The expected profits are higher than those under temporary relief. In this later case, when $\delta, \lambda > 1 - K$, we have $\bar{e}_1 = \underline{e}_1 = 1/\bar{a}$, $\bar{z}_1 = \underline{z}_1 = \bar{R}^1_1 = 1/2\bar{a}$, $\bar{R}_2 = \underline{R}_2 = 1/2\bar{a}$. It is a feasible choice and it it not chosen. The bank can improve the solution under temporary relief. The improvement comes from eliminating the hiding of $\underline{a}$. The distortion in its effort.

□(ii)

iii) $\bar{z}_1 = \bar{e}_1 > 1/\bar{a}$ cannot be part of an equilibrium.

Proof

From VP$\bar{a}$ we have that $\bar{z}_1 + \delta \min[\bar{R}_2, 1/2\bar{a}]$ is lower than that obtained with $\bar{e}_1 = 1/\bar{a}$. Substituting VP$\bar{a}$ in IC$\underline{a}$ we get that $\underline{z}_1 + \delta\, \underline{R}_2$ is also decreased relative to the one obtains with $\bar{e}_1 = 1/\bar{a}$

□(iii)

Sub case $\delta < 1-K$

i) $\bar{e}_1 = \bar{z}_1 = 0$ cannot be part of an equilibrium.
(the proof is as in the subcase $\delta > 1 - K$)

□(i)

Let us suppose that in equilibrium $\bar{z}_1 > 0$. Then $\bar{z}_1 \geq \bar{R}_1^1$. We will consider, in turn, the cases in which $\bar{z}_1 > 1/\ \underline{a}$ and $\bar{z}_1 \leq 1/\ \underline{a}$

ii) If $0 < \bar{z}_1 \leq 1/\ \underline{a}$ then the expected bank's profits are lower or equal than under temporary relief. If equilibrium exhibits $0 < \bar{z}_1 \leq 1/\ \bar{a}$, then it also exhibits $\bar{e}_1 = \bar{e}^{fb}$, $\underline{e}_1 = \underline{e}^{fb}$,

$\bar{z}_1 + \delta \min\ [\bar{R}_2,\ 1/\ 2\ \bar{a}] = 1/\ 2\ \bar{a} + \delta\ (1/\ 2\bar{a})$ and

$\underline{z}_1 + \delta\ \underline{R}_2 = 1/\ 2\bar{a} + \delta\ (1/\ 2\bar{a}) + [1/\ 2\underline{a} - 1/\ \bar{a} + \underline{a}/\ 2\ \bar{a}]$

Proof

From (VP $\bar{a}$) we obtain:

$\bar{z}_1 + \delta \min[\bar{R}_2,\ 1/\ 2\bar{a}] \leq 1/\ 2\bar{a} + \delta\ (1/\ 2\bar{a})$

from where it follows

$\bar{z}_1 + \delta\ \bar{R}_2 \leq 1/\ 2\bar{a} + \delta\ (1/\ 2\bar{a})$

because

if $\bar{R}_2 \leq 1/\ 2\ \bar{a}$ then $\bar{z}_1 + \delta\ \bar{R}_2 \leq 1/\ 2\ \bar{a} + \delta\ (1/\ 2\ \bar{a})$, and

if $\bar{R}_2 \geq 1/\ 2\ \bar{a}$ then $\bar{z}_1 \leq 1/\ 2\bar{a}$

A feasible action for the $\underline{a}$ country is to make an effort $\underline{e}_1 = 1/\ \underline{a} > \bar{z}_1 \geq \bar{R}_1^1$ and to pay $\bar{z}_1$. Then its first action must satisfy, in equilibrium

$\underline{e}_1 - \underline{a}\ \underline{e}_1^{\ 2} + \delta(1/\ 2\underline{a}) - \underline{z}_1 - \delta\ \underline{R}_2 \geq 1/\ 2\underline{a} - \bar{z}_1 - \delta\ \bar{R}_2 + \delta\ (1/\ 2\underline{a})$,

from where

$\underline{z}_1 + \delta\ \underline{R}_2 \leq \underline{e}_1 - \underline{a}\ \underline{e}_1^{\ 2}/\ 2 - 1/\ 2\ \underline{a} + \bar{z}_1 + \delta\ \bar{R}_2$

and, since $e_1 - \underline{a}\ e_1^{\ 2}/\ 2 \leq 1/\ 2\underline{a}$, we have

$\underline{z}_1 + \delta\ \underline{R}_2 \leq \bar{z}_1 + \delta\ \bar{R}_2$, from where

$\underline{z}_1 + \delta\ \underline{R}_2 \leq 1/\ 2\ \bar{a} + \delta\ (1/\ 2\ \underline{a})$

Thus, $\underline{a}$'s repayment cannot be higher than that under temporary relief, equal to the left hand side of the previous inequality. $\bar{a}$'s repayment under temporary relief cannot be surpassed either because in both periods it makes its first best effort and gets zero expected utility.

□(ii)

iii) If $1/\ \underline{a} < \bar{z}_1 = \bar{e}_1$, the bank cannot get more profits than under the temporary relief solution.

Proof

If $1/\underline{a} < \bar{z}_1 = \bar{e}_1$, using VP$\bar{a}$ we find that $\bar{a}$'s repayment is smaller than the one under temporary relief. Substituting VP$\bar{a}$ in IC$\underline{a}$ we see that $\underline{a}$'s repayment is also smaller than the one under temporary relief.

$$\underline{z}_1 + \delta\, \underline{R}_2 \leq \underline{e}_1 - \underline{a}\, \underline{e}_1^{\,2}/\, 2 + \delta\, (1/\, 2\bar{a}) - (\bar{a} - \underline{a})\bar{e}_1^{\,2}/\, 2$$
$$\leq 1/\, 2\underline{a} + \delta(1/\, 2\bar{a}) - (\bar{a} - \underline{a})\bar{e}_1^{\,2}/\, 2$$
$$\leq 1/\, 2\underline{a} + \delta(1/\, 2\bar{a}) - (\bar{a} - \underline{a})/\, 2\, \underline{a}^2 \text{ since } \bar{e}_1 \geq 1/\, \underline{a}$$

Finally, this last expression is lower than $(1/\, 2\bar{a}) + \delta\, (1/\, 2\underline{a})$ iff $K(1 - \delta) < 1$

(iii)

Thus, if $\delta < 1\text{-}K$, the bank cannot improve the solution under temporary relief

CASE $\lambda < (1 - K)$

In this case if $\bar{e}_1$ is observed we cannot be sure that $\lambda_2(\bar{e}_1;\, \underline{x},\, \lambda_1) > 1 - K$

Notice first that Fact 5.1 of the gunboat technology section also applies:

$\bar{R}_2 \in \{\, (-\infty\, ,\, 1/\, 2\bar{a}] \cup \{\, 1/\, 2\underline{a}\, \}\, \}$ (we can ignore the case $1/\, 2\bar{a} < \bar{R}_2 < 1/\, 2\underline{a}$ because $E\Pi_1$ is increasing in $\bar{R}_2$, $\overline{EU}(\bar{R}_2)$ is constant in this interval and $\underline{EU}(\bar{R}_2)$ decreasing, and thus IC$\underline{a}$ and IIC$\underline{a}$ become slacker)

<u>Sub case</u> $\bar{R}_2 \leq 1/\, 2\bar{a}$

i) If $\bar{e}_1 = \bar{z}_1 = 0$, then we cannot improve the expected profits under temporary relief when $\bar{R}_1 = R_1^{<}$

Proof

$EU(\underline{e}_1) \geq \delta\, (1/\, 2\underline{a} - 1/\, 2\bar{a})$ because $\bar{R}_2 \leq 1/\, 2\bar{a}$, from where

$E\Pi \leq (1\text{-}\lambda)(1/\, 2\underline{a}) + \delta\, (1/\, 2\bar{a}) = E\Pi(R_1^{<})$

□(i)

ii) If $\bar{z}_1 > 0$ we will analyze in turn the cases $\delta > 1 - K$ and $\delta < 1 - K$.

If <u>$\delta > 1 - K$,</u> then:

The bank can obtain, by setting $0 < \bar{z}_1 \leq 1/\, \bar{a}$, greater expected profits than under the temporary relief solution.

Proof

If $0 < \bar{z}_1 \leq 1/\, \bar{a}$, then in equilibrium $\bar{e}_1 = \bar{e}^{fb} = 1/\, \bar{a}$ (if the type $\bar{a}$

country finds it optimal so give to the bank $0 < \bar{z}_1 \leq 1/\bar{a}$, the optimal way to do it is by making an effort $\bar{e}_1 = 1/\bar{a}$). Let us assume now that only the restrictions $VP\bar{a}$ and $IC\underline{a}$ apply (later on we will check the other restrictions are satisfied as well)

We now see it is optimal to set $\underline{e}_1 = \underline{e}^{fb} = 1/\underline{a}$. Assume a different value is set. By changing from this alternative value to $\underline{e}_1 = 1/\underline{a}$ we will still satisfy restrictions $IC\underline{a}$ (which will be satisfied with inequality), and $VP\bar{a}$ (in which $\underline{e}_1$ does not appear). We can then improve the expected bank's profits by increasing $\underline{z}_1$, which contradicts the optimality of $\underline{e}_1$ different from $1/\underline{a}$. Let us now set $\underline{z}_1 + \delta\,\underline{R}_2$ in such a way that $IC\underline{a}$ satisfies with equality and $\bar{z}_1 + \delta \min\{\bar{R}_2, 1/2\bar{a}\}$ in such a way that $VP\bar{a}$ is satisfied with equality. Let us finally check that all the other restrictions are also satisfied. In particular, if we substitute $\bar{e}_1 = 1/\bar{a}$ in $IC\underline{a}$ we find that, when $\delta > 1-K$, $(IC\underline{a}) \Rightarrow (IIC\underline{a})$.

Thus, in equilibrium, the bank satisfies $IC\underline{a}$ and $VP\bar{a}$ with equality and chooses the first best effort levels for both countries. Notice that this could not be done under temporary relief. In the temporary relief scheme, we find that when $\delta > 1 - K$ and $\bar{a}$ chooses $\bar{e}_1 = 1/\bar{a}$ (having been set $\bar{z}_1 = 1/2\bar{a}$), then $\underline{a}$ randomizes between $1/\bar{a}$ and $1/\underline{a}$. With commitment, the bank can avoid this randomization, making more attractive this alternative. Let us see that the expected profits from this proposal are higher than $E\Pi(R_1^<)$ for some parameter values.

We have that

$$E\Pi(\bar{e}_1 = 0) = (1-\lambda)(1/2\underline{a}) + \delta(1/2\bar{a})$$

$$E\Pi(\bar{e}_1 = 1/\bar{a}) = \lambda(1/2\bar{a}) + (1-\lambda)(1/2\underline{a} - 1/2\bar{a} + \underline{a}/2\bar{a}^2) + \delta(1/2\bar{a})$$

$$E\Pi(\bar{e}_1 = 1/\bar{a}) \geq E\Pi(\bar{e}_1 = 0) \text{ iff } \lambda \geq (1-K)/(2-K)$$

Because the equilibrium under temporary relief exhibits $\bar{R}_1 = R_1^<$ if and only if

[$\delta < (1-\lambda-K)/(1-\lambda)(1-K)$ " $\delta > (1-K)$], and

$\delta \leq \lambda K/(1-K)(1-\lambda-K)$

then there exist values for (λ, δ, K) such that the equilibrium under temporary relief exhibits $\bar{R}_1 = R_1^<$ but the equilibrium under debt reduction does not.

Finally, we can check that $1/\bar{a} < \bar{z}_1 = \bar{e}_1$ cannot be part of an equilibrium because then from $VP\bar{a}$ we obtain that $\bar{z}_1 + \delta \min[\bar{R}_2, 1/2\bar{a}]$ is lower than the one obtained with $\bar{e}_1 = 1/\bar{a}$ and, substituting $VP\bar{a}$ in $IC\underline{a}$, we obtain that $\underline{z}_1 + \delta\,\underline{R}_2$ is also reduced relative to that obtains with $\bar{e}_1 = 1/\bar{a}$

If $\delta < 1-K$ we cannot improve the expected profits under temporary relief. (The proof is as in the case $\lambda > 1 - K$)

Sub case $\bar{R}_2 = 1/ 2\underline{a}$

We then obtain the same expected profits than under temporary relief:

i) $\bar{e}_1 > 0$ cannot be part of an equilibrium.

Proof

Assume it is part of an equilibrium. Since $\bar{R}_2 = 1/ 2\underline{a}$, in the second period $\bar{a}$ gets zero expected utility. If $\bar{a}$ makes $\bar{e}_1 > 0$, then it must be $\bar{z}_1 \leq 1/ 2\bar{a}$ (de VP$\bar{a}$), from where

$\underline{z}_1 + \delta\, \underline{R}_2 \leq 1/ 2\bar{a} + \delta(1/ 2\underline{a})$

because a feasible action for $\underline{a}$ is to make an effort $\underline{e}_1 = 1/ \underline{a} > \bar{z}_1$ and repay $\bar{z}_1$. Then its first action must satisfy, in equilibrium,

$\underline{e}_1 - \underline{a}\, \underline{e}_1^{2} + \delta(1/ 2\underline{a}) - \underline{z}_1 - \delta\, \underline{R}_2 \geq 1/ 2\underline{a} - \bar{z}_1 - \delta\, \bar{R}_2 + \delta\, (1/ 2\underline{a})$,

from where

$\underline{z}_1 + \delta\, \underline{R}_2 \leq \underline{e}_1 - \underline{a}\, \underline{e}_1^{2}/ 2 - 1/ 2\, \underline{a} + \bar{z}_1 + \delta\, \bar{R}_2$

and, because $e_1 - \underline{a}\, e_1^{2}/ 2 \leq 1/ 2\underline{a}$, we have

$\underline{z}_1 + \delta\, \underline{R}_2 \leq \bar{z}_1 + \delta\, \bar{R}_2$, from where

$\underline{z}_1 + \delta\, \underline{R}_2 \leq 1/ 2\, \bar{a} + \delta\, (1/ 2\, \underline{a})$

Thus

$E\Pi \leq 1/ 2\bar{a} + \delta(1-\lambda)(1/ 2\underline{a}) < (1-\lambda)(1/ 2\underline{a}) + \delta(1/ 2\bar{a}) = E\Pi(R_1^{<})$, which contradicts the optimality of $\bar{e}_1$

□(i)

ii) If $\bar{e}_1 = 0$, the expected benefits under temporary relief cannot be improved.

Proof

If $\bar{R}_2 \geq 1/ 2\underline{a}$ and $\lambda_2(\bar{e}_1;\ \underline{x},\ \lambda_1) > 1 - K$, at the beginning of the second period debt will be reduced to $1/ 2\bar{a}$, and thus the problem is the same as in the case $\bar{R}_2 = 1/ 2\bar{a}$. If $\bar{R}_2 \geq 1/ 2\underline{a}$ and $\lambda_2(\bar{e}_1;\ \underline{x},\ \lambda_1) \leq 1 - K$ then, from Bayes' rule, $\underline{x} \geq K\lambda/ (1 - K)(1 - \lambda)$, from where

$E\Pi = (1 - \lambda)(1 - \underline{x})(1/ 2\underline{a}) + \delta\, (1-\lambda)(1/ 2\underline{a}) \leq$

$(1 - \lambda - K)(1\, / 2\underline{a})/ (1 - K) + \delta\, (1 - \lambda)(1/ 2\underline{a})$

which are the expected benefits under temporary relief when $\bar{R}_1^{1} = 1/ 2\underline{a}$ is set.

□(ii)

There is only one bank proposal for which we have not proved the existence of an equilibrium under temporary relief yielding higher expected profits. This proposal is setting $\bar{e}_1 = \bar{e}^{fb}$, $\underline{e}_1 = \underline{e}^{fb}$ and satisfying $VP\bar{a}$ and $IC\underline{a}$ with equality when $\delta > 1-K$. Moreover, we have shown the existence of parameter values for which the equilibrium under temporary relief exhibits $\bar{R}^1_1 = R^<_1$ and the equilibrium under debt reduction is the new candidate. We will now show the existence of parameter values for which the equilibrium under temporary relief exhibits $\bar{R}^1_1 = 1/ 2\underline{a}$ and the equilibrium with debt reduction is the new candidate.

For the new candidate to yield higher expected profits than $\bar{R}^1_1 = 1/ 2\underline{a}$ under temporary relief it is necessary and sufficient that the following three conditions are satisfied:

ci) $\delta \geq 1 - K$

cii) $\lambda \leq 1 - K$

ciii) $\lambda(1/ 2\bar{a}) + (1 - \lambda)(1/ 2\underline{a} - 1/ 2\bar{a} + \underline{a}/ 2\bar{a}^2) + \delta(1/ 2\bar{a}) \geq$

$(1 - \lambda - K)(1/ 2\underline{a})/ (1 - K) + \delta (1 - \lambda)(1 / 2\underline{a})$

Now, $\bar{R}^1_1 = 1/ 2\underline{a}$ is part of equilibrium under temporary relief if and only if, besides i) and ii) being satisfied, it is also satisfied:

civ) $\delta \geq \lambda K / (1 - K)(1 - \lambda - K)$

We claim that

<u>Fact 2.5</u> The set of (δ, λ, K) satisfying (ci) - (civ) is nonempty.

<u>Proof</u>

(ciii) is equivalent to

$\delta \leq \{\lambda K(1 - K) + (1 - \lambda)(1 - K)(1 - K + K^2) - 1 + \lambda + K\} / (1 - K)(1 - \lambda - K)$

and the right-hand side of this inequality is higher than $(1 - K)$ if and only if $\lambda \geq (1 - K)^2/ (1 + K + K^3)$, where $(1 - K)^2/ (1 + K + K^3) \leq (1 - K)\ \forall\ K \leq 1$

Thus, given $K \in (0, 1)$, every λ satisfying

$(1 - K)^2/ (1 + K + K^3) \leq \lambda \leq (1 - K)$

results in an interval

$(1 - K,$

$\{\lambda K(1 - K) + (1 - \lambda)(1 - K)(1 - K + K^2) - 1 + \lambda + K\} / (1 - K)(1 - \lambda - K))$

such that if it contains δ, then ci) cii) and ciii) are satisfied.

Finally we find after some algebra that

$\lambda K / (1 - K)(1 - \lambda - K) \leq$

$\{\lambda K(1 - K) + (1 - \lambda)(1 - K)(1 - K + K^2) - 1 + \lambda + K\} / (1 - K)(1 - \lambda - K)$

is equivalent to

$\lambda \geq (1 - K)^2 / 2 - 3K - K^2$, con $(1 - K)^2 / 2 - 3K - K^2 \leq (1 - K) \Leftrightarrow K \leq (2)^{1/2} - 1$

Thus, if $K \leq (2)^{1/2} - 1$, every λ satisfying

$\max\{ (1 - K)^2 / (2 - 3K - K^2), (1 - K)^2 / (1 + K + K^3) \} \leq \lambda \leq (1 - K)$

results in an interval

$(\max\{ 1 - K, \lambda K/(1 - \lambda - K)(1 - K) \},$

$\{\lambda K(1 - K) + (1 - \lambda)(1 - K)(1 - K + K^2) - 1 + \lambda + K\} / (1 - K)(1 - \lambda - K))$

such that if it contains δ, then (ci)-(civ) are satisfied.

□

Then the only change takes place when, having $\delta > 1-K$, $\bar{R}_1 = 1/ 2\underline{a}$ or $\bar{R}_1 = R_1^{<}$ were optimal under temporary relief and now they are not. Remember that:

$\underline{EU} = (\bar{a} - \underline{a})/ 2\bar{a}^2 + \delta(1/ 2\underline{a} - 1/ 2\bar{a})$

$\underline{EU}(R_1^{<}) = \delta(1/ 2\underline{a} - 1/ 2\bar{a})$

$\underline{EU}(1/ 2\underline{a}) = 0$

So, if $\bar{R}_1 = R_1^{<}$ or $\bar{R}_1 = 1/ 2\underline{a}$ stop being part of the equilibrium, the country will improve.

■(Proposition 5)

Proof of proposition 6

We will first establish the following lemmas

Lemma 6.1

If in the T-period model $\bar{R}_1^T = 1/\ 2\bar{a}$ induces separation, then $\bar{R}_1^T = 1/\ 2\bar{a}$ is optimal.

Proof.

If $\bar{R}_1^T = 1/\ 2\bar{a}$ induces separation, then from the second period on, the bank gets the profits from complete information. On the other hand, there is no other $\bar{R}$ value which can yield higher expected profits in the first period, because $1/\ 2\bar{a}$ is the solution to the one-period model when $(1 - \lambda) < K$. Thus, $1/\ 2\ \bar{a}$ yields higher expected profits in all periods than any other $\bar{R}$.

□(Lemma 6.1)

Lemma 6.2

$$\delta \leq 1 - K \quad \Rightarrow \quad \bar{R}_1^T = 1/\ 2\bar{a} \ \text{ induces } \underline{e}_1 = \underline{e}_1^{fb} \ \forall\ T.$$

Proof.

For T = 2 we have already proved it in the two-period model.

Assume (induction hypothesis) that it is true for $T \leq L - 1$.

Then, by lemma 2, $\bar{R}_1^{L-1} = 1/\ 2\bar{a}$ is optimal when we start from $(1 - \lambda) < K$

Then $\underline{a}$'s repayments in the L-period model are, if $\bar{R}_1^L = 1/\ 2\bar{a}$ is set,

$\underline{EU}(1/\ \underline{a}) = 1/\ 2\underline{a} - 1/\ 2\bar{a}$

$\underline{EU}\ (1/\ \bar{a}) = 1/\ \bar{a} - \underline{a}\ /\ 2\bar{a}^2 - 1/\ 2\bar{a} + \delta[1/2\underline{a} - 1/2\bar{a}]$ (because in the second period, when we are at the beginning of an (L - 1)-period model, the bank will find optimal to set as initial bound $1/\ 2\bar{a}$ which will induce separation: $\underline{a}$ will reveal its type and then will obtain nothing more)

That is, we have the same payments as in a two-period model, and we have that

$1/\ 2\underline{a} - 1/\ 2\bar{a} \geq 1/\ \bar{a} - \underline{a}\ /\ 2\bar{a}^2 - 1/\ 2\bar{a} + \delta[1/2\underline{a} - 1/2\bar{a}]$ iff

$(1 - K) \geq \delta$

□(Lemma 6.2)

Corollary to Lemma 6.2 $\quad \delta \leq (1 - K) \ \Rightarrow$ in equilibrium $\bar{R}_1^T = 1/\ 2\bar{a} \quad \forall\ T$

Let us analyze the case $\delta > 1 - K$. For the two-period model we know that $\bar{R}_1^2 = 1/\ 2\bar{a}$ is the optimal bank's action. ¿Does it hold true for a model containing more periods? We first see that if this is the case, then $\bar{R}_1^T = 1/\ 2\bar{a}$ induces ratcheting (Lemma 6.3). We then see that if, as we keep adding periods there comes a time at which (for some T) $\bar{R}_1^T = 1/2\bar{a}$ is not optimal, then there is only one alternative candidate and it is a number higher than $1/2\bar{a}$ (Lemma 6.4). Next we define necessary and sufficient conditions for such a candidate to actually dominate $1/2\bar{a}$. In this process we consider a λ and we ask the optimality of $\bar{R}_1^T = 1/2\bar{a}$ in all remaining periods, not only for that λ, but also $\forall \lambda' \geq \lambda$. Later on (Lemma 6.6) we check that this hypothesis is without loss of generality.

<u>Lemma 6.3</u>
Let $\delta > (1 - K)$ and let λ s.t. $\bar{R}_1^{T-n} = 1/2\bar{a}$ is optimal for $n = 1,2,\ldots T - 1$ and $\forall \lambda' \geq \lambda$. Then $\bar{R}_1^T = 1/2\bar{a}$ induces $\underline{e} = 1/\bar{a}$ with certainty.

<u>Proof</u>.
We proceed by induction over T.
i) For T = 2 we have that, if $\bar{R}_1^2 = 1/2\bar{a}$ is set, $\underline{a}$'s repayments are:

$$\underline{EU}(1/\underline{a}) = 1/2\underline{a} - 1/2\bar{a}$$

$$\underline{EU}(1/\bar{a}) = 1/\bar{a} - \underline{a}/2\bar{a}^2 - 1/2\bar{a} + \delta[1/2\underline{a} - 1/2\bar{a}]$$

because

$\lambda_2(e_1 = 1/\underline{a}) = 0$, and

$\lambda_2(e_1 = 1/\bar{a}) \geq \lambda \geq (1 - K)$ where the first inequality follows from Bayes' rule and from $\bar{a}$ choosing $1/\bar{a}$ with probability one and the second one from the hypothesis that $\bar{R}_1^1 = 1/2\bar{a}$ is optimal for λ. Notice now that the difference $\underline{EU}(1/\bar{a}) - \underline{EU}(1/\underline{a})$ equals

$$1/\bar{a} - \underline{a}/2\bar{a}^2 - 1/2\bar{a} + \delta[1/2\underline{a} - 1/2\bar{a}] - \{1/2\underline{a} - 1/2\bar{a}\} \geq 0 \text{ iff } \delta \geq (1 - K)$$

Thus, the lemma holds for T = 2.

ii) Assume that for T = (L - 1) the Lemma holds (induction hypothesis) Consider T = L. By hypothesis, for $n = 1,2,\ldots L - 1$ $\bar{R}_1^{L-n} = 1/2\bar{a}$ is optimal. By induction hypothesis, for $n = 1,2,\ldots,L - 1$ the equilibrium exhibits $\underline{e}_1^{L-n} = 1/\bar{a}$ and thus, if $\bar{R}_1^L = 1/2\bar{a}$ is set, $\underline{a}$'s repayments will be:

$\underline{EU}(1/\underline{a}) = 1/2\underline{a} - 1/2\bar{a}$ (because $\lambda_2(e_1 = 1/\underline{a}) = 0$)

$\underline{EU}(1/\bar{a}) = [1/\bar{a} - \underline{a}/2\bar{a}^2 - 1/2\bar{a}][1 + \delta + \delta^2 + \ldots + \delta^{K-1}] + \delta^K[1/2\underline{a} - 1/2\bar{a}]$

(because $\lambda_2(e_1 = 1/\bar{a}) \geq \lambda$)

The difference $\underline{EU}^T(1/\bar{a}) - \underline{EU}^T(1/\underline{a})$ equals

$[1/\bar{a} - \underline{a}/2\bar{a}^2 - 1/2\bar{a}][1 + \delta + \delta^2 + \ldots + \delta^{K-1}] + \delta^K[1/2\underline{a} - 1/2\bar{a}]$, a sequence which is increasing in the number of periods:

$[1/\bar{a} - \underline{a}/2\bar{a}^2 - 1/2\bar{a}][1 + \delta + \delta^2 + \ldots + \delta^{T-1}] + \delta^T[1/2\underline{a} - 1/2\bar{a}] <$

$< [1/\bar{a} - \underline{a}/2\bar{a}^2 - 1/2\bar{a}]\ [1 + \delta + \delta^2 + \ldots + \delta^{T-1}] + \delta^T\{1/\bar{a} - \underline{a}/2\bar{a}^2 - 1/2\bar{a} + \delta[1/ 2\underline{a} - 1/ 2\bar{a}]\} =$

$= [1/\bar{a} - \underline{a}/2\bar{a}^2 - 1/2\bar{a}]\ [1 + \delta + \delta^2 + \ldots + \delta^T] + \delta^{T+1}\ [1/ 2\underline{a} - 1/ 2\bar{a}]$

where the sign < follows from $\delta > (1 - K)$

Thus

If $R_1^T = 1/ 2\bar{a}$ is optimal, $\underline{e}^T_1 = 1/ \bar{a}$ (There is ratcheting)

□(Lemma 6.3)

Corollary to Lemma 6.3

Let $\delta > (1 - K)$. If $\bar{R}_1^{T-n} = 1/ 2\ \bar{a}$ is optimal for n = 0,1,2,....T - 1, then $\underline{e}^{T-n}_1 = 1/ \bar{a}$ for n = 1,2, (T - 1), that is, there is ratcheting (hiding) in (T - n) for n = 0,1,...,(T - 1)

Proof. It is straightforward.

Lemma 6.4

Let $\delta > (1 - K)$ and let λ s.t. $\bar{R}_1^{T-n} = 1/ 2\ \bar{a}$ is optimal for n = 1,2,....T - 1 and $\forall\ \lambda' \geq \lambda$ (λ such that for all the remaining periods $\bar{R}_1 = 1/ 2\bar{a}$ is optimal $\forall\ \lambda' > \lambda$).

Let

$1/ 2\underline{a} - \bar{R}_1^T = A$

$[1/ 2\bar{a} - \underline{a}/ 2\bar{a}^2]\ [1 + \delta + \delta^2 + \ldots + \delta^{T-1}] + \delta^T[1/ 2\underline{a} - 1/ 2\bar{a}] = B$

$\bar{R}_1^{A=B}$ s.t. if $\bar{R}_1^T = \bar{R}_1^{A=B}$ then A = B

Then in the T-period model equilibrium:

i) $\bar{R}_1^T \in \{ 1/ 2\bar{a},\ \bar{R}_1^{A=B} \}$

ii) $\delta /(1 - \delta) \leq (K - 1 + \lambda)/(1 - \lambda)(1 - K)^2 \Rightarrow \bar{R}_1^T = 1/ 2\bar{a}$

iii) If for λ $\bar{R}_1^{A=B} \geq 1/ 2\bar{a}$ and in equilibrium $\bar{R}_1^T = 1/ 2\bar{a}$,

then $\forall\ \lambda' \geq \lambda$ we also have $\bar{R}_1^T = 1/ 2\bar{a}$ in equilibrium

Proof.

We first see that for such a λ there are only two candidates for optimal $\bar{R}_1^T$.

It is immediate that $\bar{R}_1^T < 1/ 2\bar{a}$ is not optimal (it induces both countries to exactly the same behavior in the same period as $\bar{R}_1^T = 1/ 2\bar{a}$ and the bank decreases its initial profits)

If the bank sets $\bar{R}_1^T > 1/\ 2\ \bar{a}$, we have in equilibrium:

i) $\bar{e}_1 = 0$

ii) $\underline{e}_1 \in \{0,\ 1/\ 2\underline{a}\}$

iii) $\lambda_2(e_1 = 1/\ 2\underline{a}) = 0$

iv) $\lambda_2(e_1 = 0) \geq \lambda_1$ (from (i))

Thus, $\underline{a}$'s repayments are:

$\underline{EU}\ (1/\ \underline{a}) = 1/\ 2\underline{a} - \bar{R}_1^T = A$ (from (iii)), and

$\underline{EU}(0) = NR\ [\ 1 + \delta + \delta^2 + \ldots + \delta^{T-1}] + \delta^T Rev = B$ (from iv and the $\lambda' > \lambda$ hypothesis)

$\Rightarrow\ \bar{R}_1^T = 1/\ 2\bar{a}$

We thus have:

v) If $B < A$, $\bar{R}_1^T$ is not optimal.

Proof

$\bar{R}_1^T$ induces $\underline{e}_1 = \bar{e}_1 = 0$. If the bank set $\bar{R}_1^T = 1/\ 2\bar{a}$ it would receive today that amount and would start tomorrow with the same beliefs as when setting the proposed $\bar{R}_1^T$.

vi) If $B > A$, R_1^T is not optimal.

Proof: Take $R_1^T + \varepsilon$. It also yields $B > A$ and, therefore, induces the same $\underline{a}$'s behavior. But it yields higher profits today.

From (v) and (vi) it follows that in equilibrium $R_1^T \in \{\ 1/\ 2\bar{a},\ R^{A=B}\ \}$

Notice now that

$E\Pi\ (1/\ 2\bar{a}) = 1/\ 2\bar{a}\ [\ 1 + \delta + \delta^2 + .. + \delta^T] = \delta\ \Pi_{+1}(\lambda > 1 - K) + 1/\ 2\bar{a}$

$E\Pi\ (\bar{R}_1^{A=B}) = (1 - \lambda)\ \{\ 1/\ 2\underline{a} - [1/2\bar{a} - \underline{a}/2\bar{a}^2]\ [\delta + \delta^2 + .. + \delta^{T-1}] - \delta^T[1/2\underline{a} - 1/2\bar{a}]\ \} + (1 - \lambda)\delta\Pi_{+1}(\lambda=0) + \lambda\delta\ \Pi_{+1}(\lambda = 1)$si $\bar{R}_1^{A=B} > 1/\ 2\bar{a}$

$E\Pi\ (1/\ 2\bar{a})$ does not depend on λ

and after some algebra we can find that

$dE\Pi\ (\bar{R}_1^{A=B})/d\ \lambda < 0$

Thus, if for λ it holds that $E\Pi(1/2\bar{a};\lambda) > E\Pi\ (\bar{R}_1^{A=B},\lambda)$, then it will also hold for $\lambda' > \lambda$ in the T-period model. QED

Let us now find Π_{+1} $(\lambda > 1 - K)$, $\underline{C}_{+1}(\lambda > 1 - K)$ and $\Pi_{+1}(\lambda=1)$.

We have that under $\bar{R}_1^T = 1/ 2\bar{a}$, $\underline{e}_1 = 1/ \bar{a}$ and thus

Π_{+1} $(\lambda > 1 - K) = 1/ 2\bar{a}$ $[1 + \delta + \delta^2 +...+ \delta^T] = 1/ 2\bar{a}$ $[1 - \delta^{T+1}]/ [1 - \delta]$

(notice that it does not depend on λ for $\lambda > 1 - K$ because we have assume $\bar{R}_1 = 1/2\bar{a}$ will be optimal for every remaining period)

Under $\bar{R}_1^T = 1/ 2\bar{a}$, it holds that:

$\underline{C}_{+1}(\lambda > 1 - K) = [1/2\bar{a} - \underline{a}/2\bar{a}^2]$ $[1 + \delta +..+ \delta^{T-1}] + [1/2\underline{a} - 1/2\bar{a}]$ (δ^T)

$\Pi_{+1}(\lambda=1) = 1/ 2\underline{a}$ $[1 + \delta +...+ \delta^T]$

Thus, under the hypothesis that $\bar{R}_1^{A=B} \geq 1/ 2\bar{a}$, we have:

$E\Pi$ $(\bar{R}_1^{A=B}) \geq E\Pi(1/ 2\bar{a})$ iff

$(1 - \lambda)[1/ 2\underline{a} - \delta$ $\underline{C}_{+1}(\lambda > 1 - K) + \delta$ $\Pi_{+1}(\lambda=0) \geq 1/2\bar{a} + \delta$ $(1 - \lambda)$ $\Pi_{+1}(\lambda > 1 - K)$ iff

(after some algebra)

$1 - \lambda - K + (1 - \lambda)\delta(1 - K)^2(1 + \delta +..\delta^{T-1}) \geq 0$

The left-hand side is a sequence which increases in T and converges, when $T \to \infty$, to

$1 - \lambda - K + (1 - \lambda)\delta(1 - K)^2/(1 - \delta) \geq 0$ iff

$\delta/ (1 - \delta) \geq (K - 1 + \lambda) / (1 - \lambda)(1 - K)^2$

□(Lemma 6.4)

Lemma 6.5

The condition $R_1^{A=B} = 1/ 2\underline{a} - \delta$ $\underline{C}_{+1}(\lambda > 1 - K) \geq 1/ 2\bar{a}$ ("feasibility of $R^{A=B}$") stops holding from some T iff $\delta > 1/ (1 - K)$.

Proof.

$R_1^{A=B}$ is feasible iff $1/ 2\underline{a} - 1/ 2\bar{a} \geq \delta$ $\underline{C}_{+1}(\lambda > 1 - K)$

(the right-hand side is what $\underline{a}$ obtains if, playing $\underline{e}_1 = 1/ \bar{a}$ with probability one, "makes a mistake" and plays instead $\underline{e}_1 = 1/\underline{a}$, thus inducing $\lambda = 1$)

iff

$$1/2\underline{a} - 1/2\bar{a} - [\ 1/\bar{a} - 1/2\bar{a} - \underline{a}/2\bar{a}^2\ ]\ [\ \delta + \delta^2 + \ldots \delta^{T-1}\ ] - [1/\ 2\underline{a} - 1/2\bar{a}]\delta^T \geq 0$$

the left-hand side is a decreasing (in T) sequence which, as $T \longrightarrow \infty$, converges to

$$1/2\underline{a} - 1/2\bar{a} - [\ 1/\bar{a} - 1/2\bar{a} - \underline{a}/2\bar{a}^2\ ]\ \delta/(1 - \delta) \geq 0 \text{ iff}$$

$$\delta \leq 1/\ (1 + K)$$

□(Lemma 6.5)

Thus, if $\delta > 1/\ (1 + K)$, from some T, in the T-period model, $\bar{R}_1^{A=B} < 1/\ 2\bar{a}$.

From lemma 6.4 and condition $\bar{R}_1^{A=B} > 1/\ 2\bar{a}$ we obtain that if $\delta > 1 - K$ transitory stonewalling arises iff

$\delta/(1 - \delta) > (K - 1 + \lambda)/(1 - \lambda)(1 - K)^2$ and the first T value for which

$SC(T) = 1 - \lambda - K + (1 - \lambda)(1 - K)^2(\delta + \delta^2 + \ldots + \delta^T) > 0$ (Condition $R_1^{A=B} > R_1 = 1/2\bar{a}$)

(increasing in T sequence)

satisfies

$SD(T) = 1 - K[\ \delta + \ldots + \delta^{T-1}] + \delta^T \geq 0$ (feasibility of $R_1^{A=B}$)

(decreasing in T sequence)

Finally we have:

Lemma 6.6

If for λ it is the case that in equilibrium $\bar{R}_1^T = 1/\ 2\bar{a}\quad \forall\ T$, then if $\lambda' \geq \lambda$ it is also the case that the equilibrium exhibits $\bar{R}_1^T = 1/\ 2\bar{a}\quad \forall\ T$

Proof.

If T = 2, from the two-period model solution we know that the lemma holds. If we assume that it holds for T = L - 1, then from lemma 6.4 - iii and from the fact that SD(T) does not depend on λ it follows that it also holds for T = L)

□(Lemma 6.6)

■(Proposition 6)

Finally, the following corollary is easily obtained:

Corollary

If K produces transitory stonewalling and $(1 - \delta) < K' < K$, then K' produces transitory stonewalling.

If λ produces transitory stonewalling and $\lambda' < \lambda$, then λ' produces transitory stonewalling

(A change in δ has an ambiguous effect in the zone we are placed)

Proof

$\partial\, SC(T) / \partial \lambda = - 1 - (1 - K)^2(\delta + \delta^2 + \ldots + \delta^T) < 0 \; \forall\, T$

$\partial\, SC(T) / \partial K = - 1 - 2(1 - K)(1 - \lambda)(\delta + \delta^2 + \ldots + \delta^T) < 0 \quad \forall\, T$

$\partial\, SD(T) / \partial K = - [\, \delta + \ldots + \delta^{T-1}] < 0 \quad \forall\, T$

$\partial\, SD(\lambda) / \partial \lambda = 0 \quad \forall\, T$

SERIE DOCUMENTOS DE TRABAJO

The following working papers from recent years are still available upon request from:

Rocío Contreras,
Centro de Documentación, Centro De Estudios Económicos, El Colegio de México A.C., Camino al Ajusco # 20 C.P. 01000 México, D.F.

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90/II Sandoval Musi, Alfredo. "Construction of new monetary aggregates: the case of Mexico".

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