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A Note on Hedonic Prices in Cost/Benefit Analysis*

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Abstract
Recent arguments suggest that the hedonic land price will accurately measure the net benefits of a 'small' public project going to a homogeneous population in an open economy, while the price will overstate the value of a 'large' project. This conclusion does not hold generally. We show that unless finance is lump sum, net project benefits may be either under- or overstated in land prices in the case of either small or large projects.

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1. Introduction

Recent examinations of the shadow value of public goods financed with distortionary taxes have recognized the need to account for the excess burden of taxation, associated tax revenue changes, and related general equilibrium effects.[1] Pigou's (1947) original conjecture that a first best level of public spending will be too high in the presence of tax distortions is now understood to be correct only if such spending does not induce much new consumption of taxed goods, or increases in their tax rates, thus reducing the need for new revenue. Making use of this insight in applied project evaluation requires, however, that the benefits of public goods be observable. Research on optimal tax and spending policies has devoted little if any space to this important consideration.

The hedonic pricing literature has, on the other hand, ignored Pigou's insight to focus exclusively on the observability of public goods benefits in a first best fiscal environment. Based on the empirical methodology developed by Rosen (1974), the theory of hedonic pricing attributes spatial price variation to variation in commodity characteristics.[2] In the small economy context, the argument is that a competitive Tiebout (1956) sorting process will generate bids for sites within and between communities that tend to reflect the willingness to pay for the comparative advantages of the characteristics of those sites, including differences in public services. Most recent work has been concerned with complications due to heterogeneity in the population, but Scotchmer (1985) and Kanemoto (1988) have lately argued that the equilibrium hedonic price function will exactly reflect the willingness to pay for improvements if (i) households are homogeneous, (ii) improvements are infinitesimally small, (iii) households are mobile, and (iv) the economy is in long-run competitive equilibrium. If the project is large, but these conditions otherwise apply, they show that the hedonic price will overstate the value of the project. Intuitively, a large project will induce an increase in land consumption per household, such that the unit land price will rise by more than the project benefit per household *per unit of land*. A small project, they argue, does not induce a change in lot size, and therefore the unit hedonic land price does not have to be adjusted for the per household land consumption.

[1]See, e.g., Diamond and Mirrlees (1971), Atkinson and Stern (1974) and Triest (1987). Examples in the local public finance context include Epple and Zelenitz (1981), Henderson (1985), Crane (1987) and Zodrow and Mieszkowski (1986).

[2]See Bartik and Smith (1987) and Starrett (1988) for recent surveys and a discussion of methodological issues not covered here.

We demonstrate that this is generally not the case. In particular, by extending the hedonic pricing story to the realistic and important context of distortionary public finance we show that if the project is financed within the economy, equilibrium tax rates may change and thus induce land consumption changes and excess burden costs, even for small projects, which land prices will then reflect. Expressions for the welfare effects of marginal land improvements are derived for a homogeneous economy that give past results as very special cases, and which indicate the adjustments to observed land values that correct for tax effects. The effects of larger projects are also considered, and the excess burden of taxation is shown to act to offset, and possibly dominate, the bias identified by Scotchmer and Kanemoto.

The next section introduces a simple model, where the basic issues are presented. Expressions are derived that show how land rent measures incorporate fiscal distortions in the case, in section 3, of small projects and, in section 4, of large projects. The conclusion discusses some implications of this research.

2. Land Prices in an Open Economy

Our argument proceeds in several steps. We begin by examining the effects of a local improvement, or "project," on the relevant equilibrium conditions that characterize the land market in a small economy. The next step is to investigate how project costs will be financed, and in particular how tax rates will need to adjust to maintain the public budget and land market equilibrium.[3] If taxes are distortionary, in the usual sense that they induce changes in compensated demands, we will find that the simple relationship between prices and net project benefits vanishes.

Consider a local economy[4] made up of identical households, each of whom consumes land, h, a numeraire composite good, x, and a public good, g.[5] Household behavior within a given community is

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[3]The small economy assumption does not require us to specify in any detail either the nature of interjurisdictional residential equilibrium or the mechanisms that characterize household and capital migration, as we do not solve explicitly for migratory behavior. The analysis therefore runs somewhat orthogonal to studies concerned with the existence and efficiency of Tiebout equilibria, which typically assume lump sum finance: see, e.g., Arnott and Stiglitz (1979), Stiglitz (1977, 1983) or Scotchmer (1985).

[4]Other than in our treatment of public sector output, we abstract from the production side of the economy. So long as we assumed that the economy was in long run competitive equilibrium, modeling firm behavior would not affect the qualitative nature of our results.

[5]The consumption good and the public good could just as well be represented as vectors of commodities, and the results of this paper would be unchanged.

summarized by the household's *conditional* expenditure function:

$$e(q(z),U,g) \equiv min_{x,h}\{q(z)h(z) + x(z):\ U(h,x,g) \geq U^{o},\ z \in Z\} \tag{1}$$

where U^{o} is the utility available elsewhere,[6] $q \equiv t + p$ is the gross-of-tax unit (annualized) price of land, p is the supply price of land, t is the *ad valorem* land tax rate, the price of good x is set at unity, $z \in Z$ indexes sites within the economy, and Z is the set of available sites.[7]

The solution to (1) are the compensated demands $h(q,g,U)$ and $x(q,g,U)$. The willingness of each household to pay for housing may equivalently be described by the bid rent function

$$p(z,g,t,U) \equiv max_{h}\left\{\frac{y - x(z)}{h(z)} - t : U(h,x,g) \geq U^{o},\ z \in Z\right\} \tag{2}$$

giving the maximum price the household can pay per unit of housing and maintain utility level U. From (2), the maximum quantity of housing purchased will be given by the function $\hat{h}(z,g,t,U)$, and from (1) we then have the identity that land consumption at the maximum bid is identically equal to the compensated demand for land; i.e.,[8]

$$\hat{h}(z,g,t,U) \equiv h(q(z),g,U) \tag{3}$$

The following sections examine the effect of public spending on these equilibrium relationships, where we are especially interested in the connection between changes in land prices and the household value of a public project.

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[6]The utility function is assumed to be quasi-concave, differentiable and increasing in all arguments.

[7]The solution to the *unconditional* expenditure minimization problem is the location j satisfying $e(q(g),U) \equiv min_{\{g\}}\{e(q(g^{j}),U,g^{j}): j \in J\}$, where $q^{j} \equiv p^{j} + t^{j}$ is the gross of tax price of land in jurisdiction j, and J is the choice set of communities. This determines the jurisdiction chosen for residence and the public good demand function. When there are random terms in the model, the solution also provides the probability that each jurisdiction is chosen. This equals the probability that the choice yields the highest utility among all other locations, and can be interpreted as the demand function in a discrete choice model.

[8]See the discussion in Fujita (1989), p. 18.

3. A Small Project

Consider the value of a local project dg. Totally differentiating the expenditure function (1) of a household at some location z with respect to g gives:

$$\frac{de(q(z),U,g)}{dg} = \frac{\partial e(.)}{\partial q}\frac{dq(z)}{dg} + \frac{\partial e(.)}{\partial g}$$
$$= h(z)\frac{dq(z)}{dg} - b(z) \tag{4}$$

by Shephard's lemma, where $b(z) \equiv - \partial e(.)/\partial g = -dx(.)/dg$ is the project benefit, measured in terms of the numeraire good, accruing to a household living at z. If movement in and out of the economy is costless then interjurisdictional equilibrium requires that the cost of obtaining the equilibrium utility level be the same everywhere, and we have from (4):

$$b(z) = h(z)\frac{dq(z)}{dg} \tag{5}$$

as a condition of residential equilibrium. At each site, first-order project benefits are fully reflected in gross-of-tax land prices.

Our objective is to derive a relationship between total project benefits, project costs and land value. The endogenous variables are $p(.)$, $h(.)$ and $t(g)$.[9] Since (5) must hold at each location z, we can aggregate over residents and sites to obtain an expression for aggregate project benefits:

$$B = \sum_z n(z)h(z)\frac{dq(z)}{dg}$$
$$= N\,\bar{h}\left[\sum_z \frac{dp(z)}{dg} + t'(g)\right] \tag{6}$$

where $B \equiv \sum n(z)b(z)$ is the aggregate project benefit, $\bar{h}$ is average land consumption, n is the population at site z, N is the population of the local economy, and we sum over $z \in Z$. Equation (6) says that

[9]The arguments in the functions $p(.)$ and $h(.)$ will often be suppressed in the notation for simplicity.

residential equilibrium requires that gross benefits equal land costs plus tax costs. The first term on the right-hand side (RHS) of (6) is the change in land expenditures for current residents due entirely to a change in unit prices, while the second term on the RHS is the change in tax revenues.

We may evaluate $t'(g)$ by differentiating the government's budget constraint:

$$c(g,N) = t\sum_{z} n(.)\,h(.) \tag{7}$$

where $c(g,N)$ is the cost of providing a project of size g to N households: i.e., the public good is subject to congestion. The change in tax rate necessary to finance the project revenues is therefore

$$t'(g) = \frac{1}{N\bar{h}}\left\{\frac{\partial c(.)}{\partial g} + \frac{dN}{dg}\left[\frac{\partial c(.)}{\partial N} - t\,\bar{h}\right] - t\,N\sum_{z}\frac{dh()}{dg}\right\} \tag{8}$$

Tax revenues will adjust to cover marginal project costs, less new tax revenues collected due to expansion or contraction of the tax base.

Earlier studies have assumed that finance is essentially lump sum, such that $dh(.)/dg = 0$. However, even if we take the simple case where the local economy is sufficiently small such that the project has no effect on the equilibrium utility level, the derivative of the compensated demand for land at any site is, using (3):

$$\frac{dh(q,g,U)}{dg} = \frac{\partial h(.)}{\partial q}\left[\frac{dp(.)}{dg} + t'(g)\right] + \frac{\partial h(.)}{\partial g} \tag{9}$$

The direct effect of the project on $h(.)$ includes the direct effect of the project on $p(.)$ and $t(g)$, as well as the direct effect of the project on land consumption via $\partial h(.)/\partial g$ (if land consumption is a complement to, or substitute for, public services) even though the project is so small that the envelope theorem gives $\partial p(.)/\partial h = 0$, by (2).

As an example, let preferences take the form $U(x,h,g) = x^{\sigma}h^{\zeta}g^{\theta}$, where σ, ζ and θ are positive constants. The corresponding expenditure function (1) is

$$e(q,g,U) = \left(\sigma^{-\sigma}\zeta^{-\zeta}\right)^{\gamma} q^{\zeta\gamma} g^{\theta\gamma} U^{\gamma}$$

where $\gamma \equiv 1/(\sigma + \zeta)$ and the compensated demand for land is therefore

$$h(q,g,U) = \zeta\gamma\left(\sigma^{-\sigma}\zeta^{-\zeta}\right)^{\gamma} q^{\sigma\gamma} g^{\theta\gamma} U^{\gamma}$$

so that equation (9) is in this case

$$\frac{dh(q,g,U)}{dg} = KU^{\gamma}\left\{\theta g^{\theta\gamma - 1} + \sigma q^{\zeta\gamma}\left[\frac{dp(.)}{dg} + t'(g)\right]\right\}$$

with $K \equiv \zeta\gamma^2 (\sigma^{-\sigma}\zeta^{-\zeta})^{\gamma}$.

Moreover, Scotchmer (1985) has pointed out that the change in population can have no first-order welfare effect if the initial allocation of households represented an efficient equilibrium. We have made no claim that our initial equilibrium is efficient, but a similar result can be obtained in this context by specifying the economy population solely as a function of the utility level: i.e., $N = N(U)$, so that in equilibrium the cost of land (rationally) adjusts such that $dU/dg = dN/dg = 0$. Making this assumption, and substituting from (8) and (9) into (6) gives the equilibrium condition that relates project benefits to project costs:

$$B - \frac{c'(g)}{D} = \frac{\sum_z nh\frac{dp}{dg}}{D} - \frac{t\sum_z n\frac{\partial h()}{\partial g}}{D} \qquad (10)$$

where

$$D \equiv 1 + \sum_z \frac{t}{p}\varepsilon_{hp} \leq 1$$

represents the tax distortion effect, $\varepsilon_{hp} \leq 0$ denotes the compensated own-price elasticity of land demand. New tax collections are generated if land is, on average, a complement to or a substitute for the public service, and/or if the quantity of land available in the economy changes.

In (10), adjusted net benefits are just offset by an adjusted measure of rent changes and the tax revenue effect. (10) also provides the algorithm for reconstructing the correct welfare measure from observable data. The tax revenue effect cannot be signed without more model structure, but note that if land and the project are neither complements nor substitutes, on average, then:

$$B - c'(g) \lesseqgtr B - \frac{c'(g)}{D} = \frac{R}{D} \lesseqgtr R \text{ as } D \lesseqgtr 0$$

while $B - c'(g) = R$ if and only if $D = 1$. That is, observed rent changes tend to under- or overestimate net benefits unless land demand is everywhere perfectly price-inelastic. In summary,

> **Proposition 1:** *In an open economy with identical residents and distortionary taxes, land prices are a biased measure of small project benefits, net of production cost, to an extent that depends on the excess burden of the tax, and induced tax revenues due to the complementarity and substitutability of land and public services.*

This represents a new result in the project evaluation literature in that the separate roles of land value, tax revenue effects and tax distortions are recognized to be explicit components of the cost-benefit calculation.[10] The standard result holds only if city size is fixed and land demand is price inelastic; i.e., if finance is lump sum.

Consider a simple example for a one person economy. Suppose a project induces benefits valued at *\$30*, at a cost of *\$5*, and that tax revenue effects are zero. If the project is small, so that the envelope theorem holds, the traditional argument is that land rents will rise by the full amount of net benefits, *\$25*. However, while the welfare effects of the project are first-order in magnitude, Proposition 1 argues that tax effects will nonetheless generate changes in land consumption. If the tax rate is *5%* and the price elasticity of demand is -0.75, then $D = 0.964$ and $B - c'(g)/D = 24.81$, implying that $R = \$23.92$. Hence, net benefits are underestimated by *4.3%*. The error will be larger the more price elastic land demand and the smaller the net benefit.

[10] See Atkinson and Stern (1974) and Drèze and Stern (1987) for models without hedonic prices.

4. A Large Project

Kanemoto (1988) has shown that with lump sum finance, the hedonic price measure will overstate the value of a large project financed by lump sum taxes. We can illustrate his main point with a much simplified, if somewhat less general, one-person example without production. The land price measure is $R \equiv h(p^* - p)$, where an asterisk denotes the post-project value, so that $h^* \equiv h(p^*,g^*,U)$ and $h \equiv h(p,g,U)$. The value of the project is taken to be the compensating variation of the price change plus the value of the tax expenditure, or $B \equiv e(p^*,g,U) - e(p,g,U) + T$, where T is the lump sum tax.[11] The project cost is denoted by C, which equals T to balance the public budget. The land price measure then tends to overstate net benefits if $B - C \leq R$, where

$$\begin{aligned} B - C &= e(p^*,g,U) - e(p,g,U) \\ &= e(p^*,g,U) - ph - x \\ &= R + e(p^*,g,U) - p^*h - x \\ &\leq R \end{aligned}$$

since $e(p^*,g,U) \equiv p^*h(p^*,g,U) + x(p^*,g,U) \leq p^*h(p,g,U) + x(p,g,U)$, by revealed preference. The bias comes about due to the fact that consumption has been induced to change, so the unit land price adjusts in response to two factors, rather than only one: the value of the project, and the quantity of land consumption per household.[12] If $h(p^*,g,U) = h$ and $x(p^*,g,U) = x$ then the bias, in our example, disappears.

The extension to distortionary taxation is, essentially, quite straightforward and intuitive, and our discussion will be brief. We argued in the previous section that if the tax used to finance a small project was distortionary, and land demand was not too elastic, then land rents would tend to underestimate net project benefits. The Kanemoto result is that, on the other hand, the consumption adjustments brought about by a large project, when finance is nondistortionary, are such that land prices will tend to overestimate net project benefits. We would expect, and indeed show this to be the case below, that the upward bias of large projects and the downward bias of distortionary finance would act to offset one

[11] Note that $e(p,g,U) - e(p^*,g^*,U) = T$ by spatial arbitrage.

[12] Scotchmer (1985) was the first to make this observation in this context.

another, with the net effect being an empirical matter.

The project value is $B \equiv e(q^*,g,U) - e(q,g,U) + T + EB$, where EB is the excess burden of the taxes required to finance the project, $T \equiv t^*h^* - th$, and $R \equiv h(p^* - p)$ as before.[13] In this case, with $q \equiv p + t$, the net of cost project benefit is

$$\begin{aligned} B - C &= e(q^*,g,U) - e(q,g,U) + EB \\ &= e(q^*,g,U) - (p+t)h - x + EB \\ &= R + EB + e(q^*,g,U) - (p^*+t)h - x \\ &\gtreqless R \end{aligned}$$

with $B - C > R$ for EB sufficiently large, while $B - C \leq R$ as in the lump sum case if EB is sufficiently small. In summary, we have:

> **Proposition 2:** *The hedonic measure of a large project may either over- or understate the net of cost value of a large project in an open economy with identical residents and distortionary taxes. This measure is more likely to understate project benefits the more price elastic is land demand.*

Hence, the change in land rents cannot in general be taken as an accurate measure of the value of the project less project costs, nor can it be said to always overstate the net value of a large project. The value of this information depends, as always, on one's needs, but note that if we redefine net project benefits as $B - C - EB$ that Kanemoto's result holds: land rent differentials provide an overstatement of net of economic cost project benefits. The algorithm that adjusts observed prices into their benefit, cost and tax cost components is not available in this case, however, as it was for small projects in Section 3.

[13]The compensating variation measure of the excess burden of the project is the *change* in $e(q^*,g,U) - e(q,g,U) - th$ per dollar of tax revenues, i.e. $-\Delta(th)/\Delta t$, and is zero only if the compensated own-price elasticity of demand for land is zero and $h(q,g^*,U) - h(q,g,U) = 0$. See Topham (1985) for a similar measure as part of a discussion of the relationship between the shadow value of public spending and the excess burden of taxation.

5. Conclusion

By incorporating fundamental behavioral effects that are ordinarily overlooked by the literature, we have shown that even where households are identical that land prices are a biased measure of net benefits for either large or small projects except under restrictive assumptions. The applied use of the corrected benefit measures has not been discussed, but Henderson (1985) has forcefully argued that either profit-maximizing developers or land-value maximizing governments will choose the efficient level of public spending if they understand the capitalization process and properly account for induced migration. This paper has been concerned with the ability of land prices to convey the necessary information to these decision makers.

Remarkably, the empirical literature on the demand for public goods has altogether neglected the influence of either distortionary taxes or hedonic prices on the form of the demand equation. Rather, most studies simply specify the household cost of a marginal change in public spending as the share of local property tax revenues, or "tax-price." This paper has shown that the marginal price paid for public services is endogenous, even in the case of infinitesimally small changes. If a household "sees" the local budget constraint when he or she considers a change in local public spending, it can be shown that a naive tax-price measure of cost will tend to understate true costs, implying that existing estimates of public expenditure demand elasticities are biased.[14] No empirical study has yet incorporated these considerations, or tested for their importance.

Finally, the results of this paper imply that the conventional approach to estimating housing demand yields biased parameter estimates. Because of the effect of tax distortions on the tradeoffs made by households between the characteristics of a house, including public policies at that location, and observed house prices, the price will fail to provide a useful measure of the marginal value of a unit of housing. Traditional studies[15] misspecify these tradeoffs by ignoring tax effects, so that the relevant price elasticities estimates are biased. The implicit price of housing in a housing demand model that incorporates fiscal characteristics of the house is the potential price a household would pay by consuming zero public goods. The estimation of this implicit price requires that the public goods component of the observed price be isolated, much in the manner of this paper.

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[14]See Wildasin (1987) for a discussion.

[15]Such as those discussed in Follain and Jimenez (1985) and Olsen (1987).

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