The Term Structure of the Futures Exchange Rates for a Fixed Exchange Rate System: the Mexican Case.

Abstract

The purpose of this paper is to model the three, six and nine months futures exchange rates for a fixed exchange rate system. The model is empirically implemented with data for the Mexican peso futures rates in the International Monetary Market of the Chicago Mercantile Exchange for the 1973-1981 period. The empirical results are encouraging since a significant portion of the futures exchange rates variances is explained by the model. The paper extends the model in Blanco-Garber(1986) by computing the time series of the two and three quarters ahead probabilities of devaluation. Additionally, I obtain more efficient estimators by estimating the model as a system of nonlinear seemingly unrelated equations. One of the troublesome results of Blanco-Garber (1986) is that the devaluation probabilities series peak before and not during devaluations. The problem was diagnosed to be the specification of the real exchange rate. These results are significantly improved by using Aizeman(1984) model of the real exchange rate. The present version of the model does explain an important part of the futures exchange rate variances with the added advantage that the time series of probabilities do peak before devaluations.

Resumen

En este documento se modelan los tipos de cambio a futuros de tres, seis y nueve meses para un regímen de tipo de cambio fijo. El modelo fué evaluado empíricamente con los datos para el peso Mexicano en el International Monetary Market del Chicago Mercantile Exchange para el período 1973-1981. Los resultados empíricos son bastante estimulantes ya que el modelo "explica" una proporción importante de la variancia de los tipos de cambio a futuro. El documento extiende el modelo de Blanco-Garber(1986) al obtener las series de tiempo de las probabilidades de devaluación de dos y tres trimestres en el futuro. Como un resultado lateral, se incrementa la eficiencia de los estimadores al estimar un sistema de ecuaciones no-lineales con restricciones sobre los parametros. Uno de los problemas con los resultados de Blanco-Garber(1986) es que la serie de probabilidades de devaluación alcanza su punto máximo durante, y no antes de las devaluaciones. La especificación de la paridad del poder de compra parece ser el origen de este problema. Este resultado fué mejorado significativamente usando el modelo de paridad de compra de Aizeman (1984).

The Term Structure of th Future Exchange Rates for a Fixed Exchange Rate System: the Mexican Case.

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and

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to the President of Mexico

April 1986.
Preliminary Version

The purpose of this paper is to build a model of the three, six and nine months futures exchange rates for a fixed exchange rate system[1]. The model is empirically implemented with data for the Mexican peso futures rates in the International Monetary Market of the Chicago Mercantile Exchange for the 1973-1981 period[2]. As a subproduct of the paper I obtain more efficient estimators of the parameters of the Blanco-Garber(1985) devaluation model. In that paper we generated an empirical method aimed at prediciting the timing and magnitude of devaluations forced by speculative attacks. Here, the empirical content of this model is further explored by producing the time series of the two and three quarters ahead probabilities of devaluation. The initial results were not very encouraging since, although a large proportion of the variance of the futures exchange rates was explained, the devaluation probabilities series peak during devaluations. The problem was diagnosed to be the specification of the real exchange rate in Blanco-Garber (1985). The results were significantly improved by using a version of Aizeman(1984) model of the real exchange rate. This version of the model does explain an important part of the futures exchange rate variances with the added advantage that the time series of probabilities do peak before devaluations.

The paper is organized in five sections. In section 1, I propose a model of the futures exchange rates. Section 2 presents the main building blocks of Blanco-Garber(1986). The probabilities and the expected exchange rates conditional on devaluations involved in the model of the futures exchange rates are developed in section 3. The estimation procedure is discussed in the fourth section. In the last section I analyze the empirical results.

The Futures Exchange Rates.

The three, six and nine months ahead futures exchange rates can be represented, under the assumption of risk neutrality, as

$$f_{jt} = E_t e_{t+j} + \epsilon_{jt} \qquad j=1,2,3 \qquad (1)$$

where f_{jt} is the futures exchange rate traded in perido t for delivery in period t+j, $E_t e_{t+j}$ is the forecast of the exchange rate for the period t+j conditional on the information available up to time t and $\epsilon'_t = [\epsilon_{1t} \ \epsilon_{2t} \ \epsilon_{3t}]$ is assumed to have a normal density function with a zero mean and a variance covariance matrix $\sum$. The assumptions about the density function of the disturbances may be inconsistent with their potential origin[3]. The disturbances are either specification errors (mistaken functional forms or the existance of risk premium) and/or differences in the timing of the data for f_{jt} (e.g. closing rate traded around the 15th of the last month of the quarter) and the timing of the data used to generate the exchange rate forecasts (some of them are averages for the quarter, some others are recorded at the end of the quarter).

The empirical implemention of (1) requires an operational model of the exchange rate forecasts for the different horizons. For a fixed exchange rate system, the expected exchange rates for one, two and three periods ahead are averages of the expected exchange rates conditional on different states of the world, weigthed by the probabilities of their occurrence. Mathematically they are represented by

$$Ee_{t+1} = (1-P_d)\, e_t + P_d\, Ee_{t+1}|_d$$

$$Ee_{t+2} = P_{nn}\, e_t + P_{nd}\, Ee_{t+2}|_{nd} + P_{dn}\, Ee_{t+1}|_{dn} + P_{dd}\, Ee_{t+2}|_{dd} \tag{2}$$

$$\begin{aligned} Ee_{t+3} = {} & P_{nnn}\, e_t + P_{dnn}\, Ee_{t+1}|_{dnn} + P_{ddn}\, Ee_{t+2}|_{ddn} \\ & + P_{ndn}\, Ee_{t+2}|_{ndn} + P_{nnd}\, Ee_{t+3}|_{nnd} + P_{ndd} Ee_{t+3}|_{ndd} \\ & + P_{dnd} Ee_{t+3}|_{dnd} + P_{ddd}\, Ee_{t+3}|_{ddd} \end{aligned}$$

where P_i is the probability of state i in t+1, P_{ij} is the joint probability of states i and j in periods t+1 and t+2 and P_{ijk} is the joint probability of states i,j and k in periods t+1, t+2 and t+3, respectively. The indexes i,j and k take a value of d if a devaluation occurs and a value of n if a devaluation does not happen during the appropriate period. The other expressions on the right hand side of these equations are forecasts of the exchange rates conditional on their respective states of nature, e.g. $Ee_{t+2}|_{nd}$ is the forecast made in period t for the period t+2 exchange rate conditional on the first devaluation occurring in period t+2.

The advantage of the Blanco-Garber model over other models[4] is that it delivers endogenous expressions for the different probabilities and conditional forecasts of the exchange rates. In the next section I present some of the main building blocks and results from Blanco and Garber(1985) and then extend their results to produce all the probabilities and conditional forecasts of the exchange rates in equations (2).

<u>The Devaluation Model.</u>

A money market provides the central component of this model:

$$m_t - p_t = \beta + \Omega y_t - \alpha i_t + w_t \tag{3}$$

where m_t, p_t and y_t are the logarithms of the money stock, the domestic price level and the aggregate output level, respectively. i_t is the domestic interest rate and w_t is a stochastic disturbance to the money demand. We further assumed the interest rate and the price level are determined by

$$i_t = i_t^* + E_t e_{t+1} - e_t \qquad (4)$$

$$p_t = p^*_t + e_t + u_t \qquad (5)$$

where an asterisk signifies an exogenous foreign variable and e_t and u_t are the logarithms of the nominal and real exchange rate, respectively. For reasons that will be explained below, in this paper I assume that the price level is determined by

$$p_t = E_{t-1}(p^*_t + e_t) \qquad (5')$$

A version of this equation was used by Aizeman(1984) in analyzing the impact of purchasing power parity deviations on the behavior of the flexible exchange rate[5]. Equation (5') implies that prices are set before period t so that, on the average, movements on the foreign price level and the exchange rate are reflected on domestic prices. Alternatively, it is assumed that deviations from purchasing power parity are a function of the forecasting errors of the exchange rate $(e_t - E_{t-1} e_t)$ and that of the foreign price level. Since forecasting errors are not autocorrelated, equation (5') suggests that the real exchange rate is white noise. This result seems to lack empirical content. However, as has been shown by Krasker(1980), for a fixed exchange rate regime the forecasting error of the exchange rate is not white noise. On the contrary, it would be highly autocorrelated been negative for periods when no devaluation occurred and positive for devaluation periods. As can be seen in Figure 1, this pattern of behavior seems to replicate the behavior of the time series of the real exchange rate for the sample period.

Changes in the variables of equation (3) and movements of domestic credit determine the evolution of net foreign reserves. The central bank, having fixed the exchange rate at $\bar{e}$, stops intervening in the foreign exchange market when net reserves reach a critical level $\bar{R}$, measured in foreign currency units. It was then shown that, when this event materializes, the flexible exchange rate constitutes the minimum feasiable exchange rate set by any devaluation rule. The flexible exchange rate is determined by the money market clearing condition which, substituting (4) and (5') into (3), can be represented by the following stochastic difference equation:

$$h_t = -\alpha E_t \tilde{e}_{t+1} + \alpha \tilde{e}_t + E_{t-1} \tilde{e}_t \qquad (6)$$

where $h_t \equiv \log[D_t + \bar{R}\exp(\bar{e})] - \beta - \Omega\, y_t + \alpha i^*_t - p^*_t - w_t + s_t$, D_t is the domestic credit component of the monetary base at time t, s_t is the one-step ahead forecasting error for the foreign price level, and $\tilde{e}_t$ represents the permanently floating exchange rate[6]. We convert $\bar{R}$ into domestic currency using the fixed exchange rate $\bar{e}$ prevailing at the time the switch to floating rates. This follows from our assumption that the government does not repudiate its fixed exchange rate until reserves reach $\bar{R}$.

The stochastic process that drives h_t is assumed to be exogenous to the exchange rate. This assumption is violated by (5) because u_t, which would appear in h_t, is indeed not exogenous to $\tilde{e}_t$.[7] Equation (5') solves this problem since in this specification u_t is not in h_t. The structure of the h_t process[8] is

$$h_t = \theta_1 + \theta_2 h_{t-1} + \theta_3 h_{t-2} + v_t \qquad (7)$$

where v_t is a white-noise process with a normal density function g(v) with zero mean and standard deviation σ.

Ruling out the existence of "bubbles", the solution to the difference equations in (6) and (7) delivers the permanently flexible exchange rate,

$$\tilde{e}_t = \Phi_1 + \Phi_2 h_t + \Phi_3 h_{t-1} + \Phi_4 h_{t-2} \qquad (8)$$

where

$$\Phi_1 \equiv -(1-\alpha)\,\theta_1 \Phi_2,$$

$$\Phi_2 \equiv -(1+\alpha)\Phi_4/\theta_3,$$

$$\Phi_3 \equiv [(1-\theta_2)\alpha\,\Phi_2 - 1]/\alpha \quad \text{and}$$

$$\Phi_4 \equiv -(1+\alpha)\theta_3/\{\alpha[1+\alpha(2-\theta_2)+\alpha^2(1-\theta_2-\theta_3)]\}.$$

If R attains its critical level $\bar{R}$ at time t, we assumed that for simplicity the central bank establishes a new fixed exchange rate $\hat{e}_t$ using the following time invariant linear policy rule:

$$\hat{e}_t = \tilde{e}_t + \delta v_t \qquad (9)$$

where $\tilde{e}_t$ is the permanently flexible exchange rate, and δ is a nonnegative parameter.

Probabilities of Devaluations and the Conditional Exchange Rates.

In Blanco-Garber we showed the equivalence between $\hat{e}_t$ exceeding the current exchange rate and a devaluation at time t. Therefore, from (8) and (9), the probability of a devaluation at time t+1 based on information available at t is

$$P_d \equiv Pr(\Phi_1+\Phi_2 h_{t+1}+\Phi_3 h_t+\Phi_4 h_{t-1}+\delta v_{t+1} > \bar{e})$$

where e is the time t value of the fixed rate. Alternatively, the one-step ahead probability of devaluation is

$$1-F(K_{1t}) \equiv Pr(v_{t+1} > K_{1t}) \qquad (10)$$

where $K_{1t} \equiv (\bar{e} - \mu_1 - \mu_2 h_t - \mu_3 h_{t-1})/\mu_4$

$\mu_1 \equiv \Phi_1+\Phi_2 \theta_1$

$\mu_2 \equiv \Phi_2 \theta_2 + \Phi_3$

$\mu_3 \equiv \Phi_2 \theta_3 + \Phi_4$

$\mu_4 \equiv \Phi_2 + \delta$

and $F(K_{1t})$ is the cumulative distribution function associated with g(v).

The exchange rate forecast for period t+1 conditional on devaluation is

$$E\hat{e}_{t+1}|_d = E\,[\,\Phi_1 + \Phi_2 h_{t+1} + \Phi_3 h_t + \Phi_4 h_{t-1} + \delta v_{t+1}|_d)$$

$$= \mu_1 + \mu_2 h_t + \mu_3 h_{t-1} + \mu_4 Ev_{t+1}|_d \qquad (11)$$

where

$$Ev_{t+1}|_d = \int_{K_{1t}}^{\infty} f(v_{t+1})\, v_{t+1} dv_{t+1} / P_d \qquad (12)$$

The expressions for the two and three quarters ahead probabilities and expected exchange rates are quite complicated. Here, I present a couple of these formulae leaving for Appendix II the development of the rest. The probability of the first devaluation taking place in t+2 can be derived from equations (8) and (9) and recalling that a devaluation would occur in period t+j whenever $\hat{e}_{t+j}$ is larger than the fixed exchange rate prevailing in t+j-1. The expression for the probability of such an event is

$$P_{nd} \equiv Pr(\hat{e}_{t+2} > \bar{e}, \hat{e}_{t+1} < \bar{e}) = \int_{-\infty}^{K_{1t}} \int_{K_{2t} - \Pi_1 v_{t+1}}^{\infty} f(v_{t+1}) f(v_{t+2}) dv_{t+1}\, dv_{t+2} \quad (13)$$

where $K_{2t} = [\, e - (\Psi_1 + \Psi_2 h_t + \Psi_3 h_{t-1})] / \mu_4$

$\Psi_1 = \Phi_1 + \Phi_2 \theta_1 (1+\theta_2) + \Phi_3 \theta_1,$

$\Psi_2 = \Phi_2 (\theta_2{}^2 + \theta_3) + \Phi_3 \theta_2 + \Phi_4\,,$

$\Psi_3 = \theta_3 (\Phi_2 \theta_2 + \Phi_3)$ and

$\Pi_1 = \mu_2 / \mu_4.$

The expected exchange rate for t+2 conditional on no devaluation occurring in period t+1, is obtained using (8) and (9):

$$E\, \hat{e}_{t+2} |_{nd} = E\, [\, \Phi_1 + \Phi_2 h_{t+2} + \Phi_3 h_{t+1} + \Phi_4 h_t + \delta v_{t+2} |_{nd})$$

$$= \Psi_1 + \Psi_2 h_t + \Psi_3 h_{t-1} + \mu_2 E v_{t+1}|_{nd} + \mu_4 E v_{t+2}|_{nd} \quad (14)$$

The conditional expectations of v_{t+1} and v_{t+2} are

$$E v_{t+j}|_{nd} = \{ \int_{-\infty}^{K_{1t}} \int_{K_{2t} + \Pi_1 v_{t+1}}^{\infty} f(v_{t+1}) f(v_{t+2})\, v_{t+j} dv_{t+1}\, dv_{t+2} \} / P_{nd} \quad (15)$$

for j=1,2.

A non-linear seemingly unrelated equation system is formed when equations (10) to (15) and all the expressions for the rest of the probabilities and the conditional exchange rates[9] appearing in (3) are substituted in (1).

Estimation Procedure.

The parameter are estimated by maximizing the likelihood function for the model in (1)[10]

$$L(\delta,\bar{R},\theta_1,\theta_2,\theta_3,\sigma)= (2\pi)^{-N} \ |\Sigma|^{-N/2} \ \exp[-1/2 \sum_{t=1}^{N} \epsilon_t' \Sigma^{-1} \epsilon_t]$$

where N is the sample size. Wilson(1973) showed that the maximizing likelihood function is equivalent to

$$\min_{[\delta,\bar{R},\theta 1,\theta 2,\theta 3,\sigma]} |\Sigma| = \left|1/N \sum_{t=1}^{N} \epsilon_t \epsilon'_t \right|. \qquad (16)$$

Estimation Results.

In Table 1, I reproduce the Blanco-Garber (1986) estimates of the parameters of the demand for monetary base. I prefered to keep these estimates[11] to make it easier for the reader to trace the changes in the estimates of $\bar{R}$ and δ, in the time series of probabilities and in the expected exchange rates, only to the changes in the real exchange rate model and in the estimation procedure.

The estimates of $\bar{R}$ and δ are presented in Table 2 together with those obtained in Blanco-Garber(1986). The estimate of δ implies that whenever the central bank devalues it sets the exchange rate equal to the permanently floating exchange rate for that period.[12]

Model (1) seems to explain the behavior of the futures exchange rates reasonably well. The R^2 for the three, six and nine months exchange rates are 0.95, 0.92 and 0.90, respectively. However, as can be observed in Figures 2, 3 and 4, the residuals are somewhat pathological, presenting a strong autocorrelation and attaining large values inmediately after devaluations.

One of the most attractive results of this specification is that the time series of one, two and three steps ahead probabilities of devaluation peak before the devaluation periods. As was mentioned above, with the specification in Blanco-Garber, i.e. using equation (5) instead of using equation (5'), these time series peak during the devaluation periods.[13] Furthermore, as can also be observed in Table 3 and in Figures 5 and 6, the probability of devaluation for 1976,2 and 1982,1 seems to be rapidly collapsing towards one as the horizon is extended into the future. In this fashion, the model strongly "predicted" a devaluation within nine months of both dates. Additionally, as "predicted" by the model, devaluations occurred only when

$$\hat{e}_t > e_{t-1}.$$

Appendix I

We derived our data from the following sources:

D : Net financing of the federal government by the Banco de Mexico in millions of pesos. This series is a proxy for the domestic component of the monetary base. Financing of financial intermediaries by the Banco de Mexico and the 'net position: other concepts' figures were not available for the whole sample period: Banco de Mexico.

f : Logarithm of the end-of-quarter rate of pesos for delivery three months forward. Source: International Money Market Yearbook published by the Chicago Mercantile Exchange, various issues.

m : Logarithm of the end of the quarter monetary base in millions of pesos. Source: Banco de Mexico.

i^* : Interest rate on three months Treasury bills in percent per quarter. Source: Federal Reserve Bulletin, Board of Governors of the Federal Reserve System, various issues.

p : Logarithm of the implicit price deflator of the GDP for Mexico. Quarterly data generated by the interpolation method of Ginsburg(1973).

: Logarithm of the implicit price deflator of the U.S. imports of goods and services. Source: Business Statistics, Bureau of Economic Analysis, Departament of Commerce

p^* : Logarithm of the implicit price deflator on the U.S. exports of goods and services. Source: Business Statistics, Bureau of Economic Analysis, Departament of Commerce

y : Logarithm of GDP of Mexico in real terms. Quarterly data generated by the interpolation method of Ginsburg(1973).

Appendix II.

This appendix presents expressions for the two and three periods ahead probabilities of devaluation and for the expected exchange rates conditional on the different states of the world.

The Two Steps Ahead Probabilities:

It is not necessary to compute the expression for P_{nn} since from the definition of probabilities it follows that $P_{nd} + P_{nn} = 1 - P_d$, and P_{nd} was already defined in (11). The probability of a devaluation in t+1 and no devaluation in t+2 is

$$P_{dn} \equiv \Pr\{\hat{e}_{t+2} < \hat{e}_{t+1}, \hat{e}_{t+1} > \bar{e}\} = \int_{K_{1t}}^{\infty} \int_{-\infty}^{K_{3t} - \Pi_2 v_{t+1}} f(v_{t+1}) f(v_{t+2}) dv_{t+1}\, dv_{t+2}$$

where $K_{3t} = (\lambda_1 + \lambda_2 h_t + \lambda_3 h_{t-1}) / \mu_4$,

$\lambda_1 \equiv -\theta_1(\Phi_2(1+\theta_2)+\Phi_3-\Phi_2)$,

$\lambda_2 \equiv \Phi_3-\Phi_4-\Phi_2(\theta_2^2+\theta_3)-\theta_2(\Phi_3-\Phi 2)$,

$\lambda_3 \equiv \Phi_4-\Phi_2\theta_2\theta_3-(\Phi_3-\Phi_2)\theta_3$ and

$\Pi_2 = \Phi_2(\theta_2-1)+\Phi_3-6$.

By definition of joint probabilities, it follows that $P_{dd} = P_d - P_{dn}$.

Three Steps Ahead Probabilities:

The probability of no devaluation in t+1, t+2 and t+3 is

$$P_{nnn} \equiv \Pr\{\hat{e}_{t+3} < \bar{e}, \hat{e}_{t+2} < \bar{e}, \hat{e}_{t+1} < \bar{e}\} =$$

$$\int_{-\infty}^{K_{1t}} \int_{-\infty}^{K_{2t} - \Pi_1 v_{t+1}} \int_{-\infty}^{K_{4t}-\Pi_3 v_{t+1}-\Pi_1 v_{t+2}} f(v_{t+1})\, f(v_{t+2})\, f(vt+3)\, dv_{t+1}\, dv_{t+2}\, dv_{t+3}$$

where $K_{4t} = [e_t - \Gamma_1 - \Gamma_2 h_t - \Gamma_3 h_{t-1}] / (6+\Phi_2)$,

$\Gamma_1 = \Phi_1 + \Phi_2\, \theta_1\, (1 + \theta_2 + \theta_2^2 + \theta_3) + \theta_1\, [\Phi_3\, (1+\theta_2) + \Phi_4]$,

$\Gamma_2 = \Phi_2\, \theta_2\, (\theta_2^2 + 2\theta_3) + \Phi_3\, (\theta_2^2 + \theta_3) + \Phi_4\, \theta_2$

$\Gamma_3 = \theta_3 \ \Psi_2,$

$\Pi_3 = \Psi_2 \ / \ \mu_4$ and

The probability of no devaluation in t+1 and devaluations in both t+2 and t+3 is

$P_{ndd} \equiv Pr[\hat{e}_{t+1} < \bar{e}_t, \hat{e}_{t+2} > \hat{e}_{t+1}, \hat{e}_{t+3} > \hat{e}_{t+2}] =$

$$\int_{-\infty}^{K_{1t}} \int_{K_{2t} - \Pi_1 v_{t+1}}^{\infty} \int_{K_{5t}-\Pi_4 v_{t+1}-\Pi_5 v_{t+2}}^{\infty} f(v_{t+1})\, f(v_{t+2})\, f(vt+3)\, dv_{t+1}\, dv_{t+2}\, dv_{t+3}$$

where $K_{5t} = [\Gamma_4 + \Gamma_5 h_t + \Gamma_6 h_{t-1}] / \mu_4,$

$\Gamma_4 = \Psi_1 - \Gamma_1,$

$\Gamma_5 = \Psi_2 - \Gamma_2,$

$\Gamma_6 = \Psi_3 - \Gamma_3,$

$\Pi_4 = \{\Phi_2 \ [\ \theta_2(\theta_2-1) + \theta_3] + \Phi_3 \ (\theta_2-1) + \Phi_4\} / (\Phi_2 + \delta)$ and

$\Pi_5 = [\Phi_2 \ (\ \theta_2-1) + \Phi_3 - \delta] / (\Phi_2 + \delta).$

The probability of no devaluation in t+2 and t+3 after a devaluation occurred in t+1 is

$$Pdnn = \int_{K_{1t}}^{\infty} \int_{-\infty}^{K_{3t} - \Pi_2 v_{t+1}} \int_{-\infty}^{K_{6t}-\Pi_7 v_{t+1}-\Pi_8 v_{t+2}} f(v_{t+1})\, f(v_{t+2})\, f(vt+3)\, dv_{t+1}\, dv_{t+2}\, dv_{t+3}$$

where $K_{6t} = [\ \Gamma_7 + \Gamma_8 h_t + \Gamma_9 h_{t-1}] / \mu_4,$

$\Gamma_7 = \mu_1 - \Gamma_1,$

$\Gamma_8 = \mu_2 - \Gamma_2,$

$\Gamma_9 = \mu_3 - \Gamma_3$,

$\Pi_7 = (\Phi_2\, \theta_2 + \Phi_3) / (\Phi_2 + \delta)$ and

$\Pi_8 = [\Phi_2\, (\theta_2^2 + \theta_3 - 1) + \Phi_3\, \theta_2 + \Phi_4 - \delta] / (\Phi_2 + \delta)$.

The probability of a devaluation occurring in t+1, t+2 and t+3 is

$$P_{ddd} = \int_{K_{1t}}^{\infty} \int_{K_{3t} - \Pi_2 v_{t+1}}^{\infty} \int_{K_{5t} - \Pi_4 v_{t+1} - \Pi_5 v_{t+2}}^{\infty} f(v_{t+1})\, f(v_{t+2})\, f(v_{t+3})\, dv_{t+1}\, dv_{t+2}\, dv_{t+3}\ .$$

The following equations, which are derived from probability theory, define the rest of the three steps ahead probabilities:

$$P_{nnd} = P_{nn} - P_{nnn},$$

$$P_{ndn} = P_{nd} - P_{ndd},$$

$$P_{dnn} = P_{dn} - P_{dnd}, \text{ and}$$

$$P_{ddn} = P_{dd} - P_{ddd}\ .$$

One Step Ahead Forecasts Conditional on Devaluations:

The one step ahead forecasts conditional on devaluation in t+1 and no devaluation in t+2 is

$$E\hat{e}_{t+1}|_{dn} = \mu_1 + \mu_2\, h_t + \mu_3\, h_{t-1} + \mu_4\, E\, v_{t+1}|_{dn}$$

where

$$Ev_{t+1}|_{dn} = \int_{K_{1t}}^{\infty} \int_{-\infty}^{K_{3t} - \Pi_2 v_{t+1}} v_{t+1} f(v_{t+1}) f(v_{t+2}) dv_{t+1}\, dv_{t+2} \,/\, P_{dn}$$

The forecast conditional on devaluation in t+1 but no devaluation neither in t+2 nor in t+3 is

$$E\hat{e}_{t+1}|_{dnn} = \mu_1 + \mu_2 h_t + \mu_3 h_{t-1} + \mu_4 E\, v_{t+1}|_{dnn}$$

where

$$Ev_{t+1}|_{dnn} = \int_{K_{1t}}^{\infty} \int_{-\infty}^{K_{3t} - \Pi_2 v_{t+1}} \int_{-\infty}^{K_{6t} - \Pi_6 v_{t+1} - \Pi_7 v_{t+2}} v_{t+1}\, f(v_{t+1})\, f(v_{t+2})\, f(vt+3\; dv_{t+1} dv_{t+2} dv_{t+3} / P_{dnn}$$

<u>Two Steps Ahead Forecasts Conditional on Devaluations:</u>

The forecast conditional on devaluations occurring in t+1 and t+2 is

$$E\,\hat{e}_{t+2}\,|_{dd} = \Psi_1 + \Psi_2 h_t + \Psi_3 h_{t-1} + \Psi_4 Ev_{t+1}|_{dd} + \mu_4 Ev_{t+2}|_{dd}$$

where

$$Ev_{t+j}\,|_{dd} = \int_{K_{1t}}^{\infty} \int_{K_{3t} - \Pi_2 v_{t+1}}^{\infty} v_{t+j} f(v_{t+1}) f(v_{t+2}) dv_{t+1}\, dv_{t+2} / P_{dd}$$

for j=1,2.

The following two step ahead forecasts are conditional on the different states of the world as explained in the text:

$$E\,\hat{e}_{t+2}\,|_{ddn} = \Psi_1 + \Psi_2 h_t + \Psi_3 h_{t-1} + \Psi_4 Ev_{t+1}|_{ddn} + \mu_4 Ev_{t+2}|_{ddn}$$

where

$$Ev_{t+j}|_{ddn} = \int_{K_{1t}}^{\infty} \int_{K_{3t} - \Pi_2 v_{t+1}}^{\infty} \int_{-\infty}^{K_{5t} - \Pi_4 v_{t+1} - \Pi_5 v_{t+2}} v_{t+j} f(v_{t+1})\, f(v_{t+2})\, f(v_{t+3})\, dv_{t+1}\, dv_{t+2}\, dv_{t+3} / P_{ddn}$$

for j=1,2.

The exchange rate forecast conditional on devaluation only during the second period ahead is

$$E\,\hat{e}_{t+2}\,|_{ndn} = \Psi_1 + \Psi_2\, h_t + \Psi_3\, h_{t-1} + \Psi_4\, Ev_{t+1}|_{ndn} + \mu_4\, Ev_{t+2}|_{ndn}$$

where

$$Ev_{t+j}|_{ndn} =$$

$$\int_{-\infty}^{K_{1t}} \int_{K_{2t}-\Pi_1 v_{t+1}}^{\infty} \int_{-\infty}^{K_{5t}-\Pi_4 v_{t+1}-\Pi_5 v_{t+2}} v_{t+j}\, f(v_{t+1})\, f(v_{t+2})\, f(v_{t+3})\, dv_{t+1}\, dv_{t+2}\, dv_{t+3}/P_{ndn}$$

for j=1,2.

<u>Three Steps Ahead Forecasts Conditional on Devaluations:</u>

The forecast conditional on devaluations occurring during t+1, t+2 and t+3 is

$$E\hat{e}_{t+3}\,|_{ddd} = \Gamma_1 + \Gamma_2\, h_t + \Gamma_3\, h_{t-1} + \Pi_3\, \mu_4\, Ev_{t+1}|_{ddd} + \Pi_1\, \mu_4 Ev_{t+2}|_{ddd} + \mu_4 Ev_{t+3}|_{ddd}$$

where

$$Ev_{t+j}|_{ddd} = \int_{K_{1t}}^{\infty} \int_{K_{3t}-\Pi_2 v_{t+1}}^{\infty} \int_{K_{5t}-\Pi_4 v_{t+1}-\Pi_5 v_{t+2}}^{\infty} v_{t+j} f(v_{t+1}) f(v_{t+2}) f(v_{t+3}) dv_{t+1} dv_{t+2} dv_{t+3}/P_{ddd}$$

for j=1,2,3 .

The expression for the forecast conditional on devaluation in t+2 and t+3 is

$$E\hat{e}_{t+3}\,|_{ndd} = \Gamma_1 + \Gamma_2\, h_t + \Gamma_3\, h_{t-1} + \Pi_3\, \mu_4 Ev_{t+1}|_{ndd} + \Pi_1\, \mu_4 Ev_{t+2}|_{ndd} + \mu_4 Ev_{t+3}|_{ndd}$$

where

$E\, v_{t+j}|_{ndd} =$

$$\int_{-\infty}^{K_{1t}} \int_{K_{2t} - \Pi_1 v_{t+1}}^{\infty} \int_{K_{5t} - \Pi_4 v_{t+1} - \Pi_5 v_{t+2}}^{\infty} v_{t+j} f(v_{t+1}) f(v_{t+2}) f(v_{t+3}) dv_{t+1} dv_{t+2} dv_{t+3} / P_{ndd}$$

for j=1,2,3 .

The forecast conditional on the first devaluation occurring in t+3 is

$$E\hat{e}_{t+3}|_{nnd} = \Gamma_1 + \Gamma_2 h_t + \Gamma_3 h_{t-1} + \Pi_3\, \mu_4 E v_{t+1}|_{nnd} + \Pi_1\, \mu_4 E v_{t+2}|_{nnd} + \mu_4 E v_{t+2}|_{nnd}$$

where

$E v_{t+j}\, |_{nnd} =$

$$\int_{-\infty}^{K_{1t}} \int_{-\infty}^{K_{2t} - \Pi_1 v_{t+1}} \int_{K_{4t} - \Pi_3 v_{t+1} - \Pi_1 v_{t+2}}^{\infty} v_{t+j} f(v_{t+1}) f(v_{t+2}) f(v_{t+3}) dv_{t+1} dv_{t+2} dv_{t+3} / P_{nnd}$$

for j=1,2,3 .

And, finally, the forecast conditional on devaluations in t+1 and t+3 is

$$E\hat{e}_{t+3}|_{dnd} = \Gamma_1 + \Gamma_2 h_t + \Gamma_3 h_{t-1} + \Pi_3\, \mu_4 E v_{t+1}|_{dnd} + \Pi_1\, \mu_4 E v_{t+2}|_{dnd} + \mu_4 E v_{t+2}|_{dnd}$$

where

$E v_{t+j}|_{dnd} =$

$$\int_{K_{1t}}^{\infty} \int_{-\infty}^{K_{3t} - \Pi_2 v_{t+1}} \int_{K_{6t} - \Pi_7 v_{t+1} - \Pi_8 v_{t+2}}^{\infty} v_{t+j}\, f(v_{t+1})\, f(v_{t+2})\, f(v_{t+3}) dv_{t+1} dv_{t+2} dv_{t+3} / P_{dnd}$$

for j=1,2,3 .

Footnotes.

[1] The term structure of the forward exchange rates for flexible exchange rate has been studied by Porter(1971), Brillembourg(1978) and Hakkio(1980).

[2] For a descriptive analysis of this period see Ortiz and Solis(1979). Some of the characteristics of the data for the Mexican case are discussed in Blanco-Garber(1986). The efficiency of the futures and forward market in pesos has been documented by Lizondo(1983).

[3] To explore the impact of such problems on the estimation results, in a later version of this paper I will consider the following model of the term structure of the futures exchange rate:

$$f_{jt} - f_{j+1t} = E_t\,(e_{t+j} - e_{t+j+1}) + \xi_{jt}$$

for j=1,2 , where $\xi'_t=[\xi_{1t}\ \ \xi_{2t}]$ has normal density function with a zero mean and a variance covariance matrix $\sum^*$. If the estimates of the parameters of (1) and of the above equation differ significantly, it would imply either that the specification errors were offset or augmented by substracting ϵ_{j+1t} from ϵ_{jt}. In this case there would be some grounds for the rejection of the model. Additionally, it could be that one of theses equations replicates better the behavior of the time series for the sample period.

[4] The expected exchange rate for a fixed exchange rate system was modeled in a similar fashion by Krasker(1980) and Lizondo(1983).However, in these models the probabilities and conditional exchange rates are exogenously imposed.

[5] Aizeman(1984) used this model for a world of only traded goods. The extension to a world of traded and non-traded goods is not by any means straight forward. However, (5') is proposed as a first approximation that replicates the behavior of the real exchange rate for the sample period.

[6] For estimation purposes it was assumed that s_t is zero for all t, i.e. that p^*_t can be perfectly forecasted. What is been assumed is that it is easier to forecast the foreign price level than the exchange rate for next quarter. This assumption is equivalent to ignoring an additional stochastic term, i.e. the forecasting error for the foreign price level. This misspecification could have some impact on the empirical content of the model. Alternatively, I could use a model to forecast p^*_t.

[7] The lack of exogeneity of u_t with respect to e_t can be observed in many theoretical models with sticky prices. Figure 1 shows this phenomenon for the Mexican sample.

[8] In Blanco-Garber we discussed the potential differences between the behavior of h_t during a fixed exchange rate regime and during a permanently floating exchange rate system. However, since the last one is unobservable, we assumed for estimation purposes that both processes are identical. The order of the process was decided by the criteria of statistical significance of the autoregressive coefficients and using the Box-Pierce statistic to detect further autocorrelation in the residuals. However, it is worthwhile noticing that there is a significant spike for the 8th order auto-

correlation coefficient. The different specification of the purchasing parity equation explains the different order of the autoregressive processes between Blanco-Garber (AR1) and the AR2 in this paper.

9 See Appendix II for the expressions not appearing in the text.

10 The estimation procedure is more complicated than it is implied in the text. I use a two-step optimization process, estimating $\theta_1, \theta_2, \theta_3$, and σ by minimizing the sum of squared residuals of (7) for every $\bar{R}$, and estimating R and δ by minimizing (16). Intuitively, the reason for this procedure is that if R is estimated by minimizing some function of the sum of squared residuals of (7), $\bar{R}$ would attain a corner value since it enters in the definition of h_t in the following form: $\log[D_t + \bar{R}\exp(\bar{e})]$.

11 I have developed a model which includes the orthodox partial adjustment specification for the demand for monetary base. The empirical results for this specification will appear in a later version of this paper.

12 For this preliminary version of the paper I have used a grid search method to estimate R and δ. For this reason I have not reported estimates of their standard deviations. In a later version the parameters will be estimated with the Davidson-Fletcher-Powell algorithm. At this point in time, the difference with the estimate of δ from Blanco-Garber is unexplained.

13 When the model in (1) is estimated with the Blanco-Garber specification on the domestic price level i.e. equation (5), I also obtained comparable R^2 for the three, six and nine months futures exchange rates. Likewise, the residuals present high autocorrelation. However, as was mentioned before, the one, two and three step ahead probabilities peak during the devaluations.

Table 1
Estimates of the Demand for Base Money Parameters

Parameter	Estimate
Ω	1.196 (0.051)
α	1.310 (0.627)
March	-5.729 (0.599)
June	-5.765 (0.598)
September	-5.786 (0.601)
December	-5.656 (0.599)
Q(15)	27.021
R^2	0.962

Source.- See Appendix.

Note.- The set of instrumental variables consists of the second, third and fourth lags of the interest rates. Q(n) is the Box-Pierce statistic. The numbers in parentheses are standard errors.

Table 2

Estimates of the Future Exchange Rate Parameters

Parameter	Blanco-Garber	Equation (1)
$\bar{R}$	-3018.068 (657.942)	-3200.0
δ	1.956 (0.547)	0.000
θ_1	0.181 (0.438)	0.142 (0.146)
θ_2	0.929 (0.438)	1.243 (0.176)
θ_3	0.000	-0.292 (0.187)
σ	0.195	0.347

Source: See Appendix I.

Note: The second column refers to the results of Blanco-Garber(1986). The standard errors are reported in parentheses. Those for $\bar{R}$ and δ are conditional on the values of θ_1, θ_2 and σ. The standard errors for θ_1, θ_2 and σ are derived from ordinary least squares estimation of the h_t equation, conditonal on the estimated $\bar{R}$. The value of $\bar{R}$ is in real dollars of 1970.

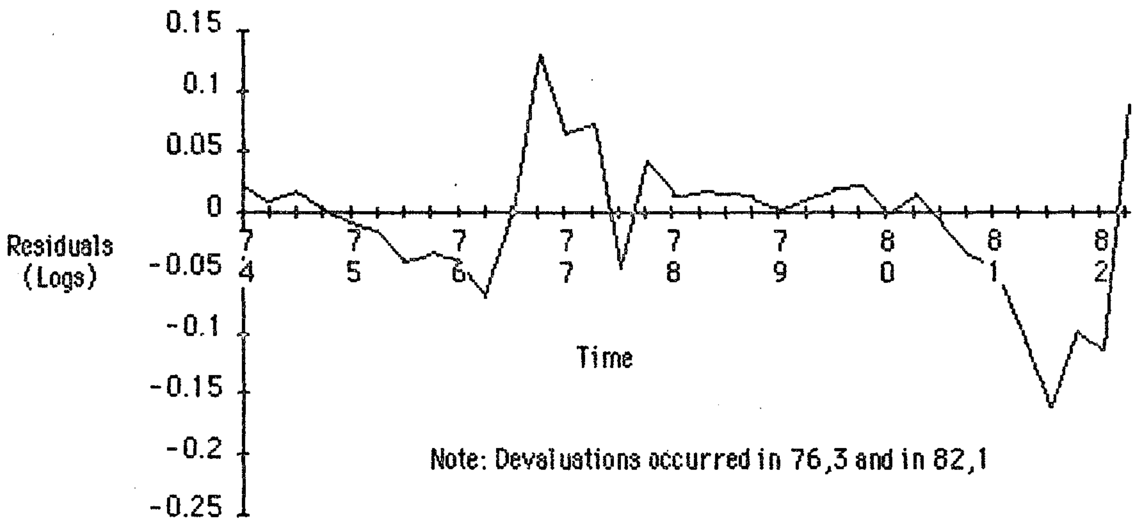
Table No. 3

Probabilities of Devaluation

	One-Step Ahead	Two-Steps Ahead	Three-Steps Ahead
1976,2	0.375	0.555	0.655
1981,4	0.544	0.690	0.752

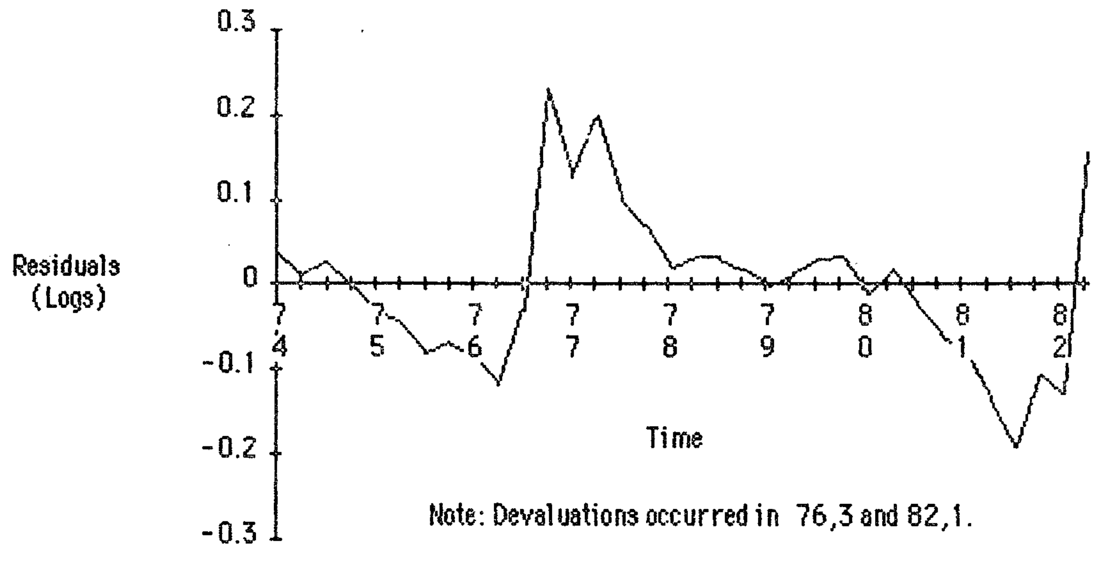
Note: Devaluations occurred in 1976,3 and 1982,1.

Figure No. 2
Residuals from the Three Months
Futures Exchange Rate Equation.



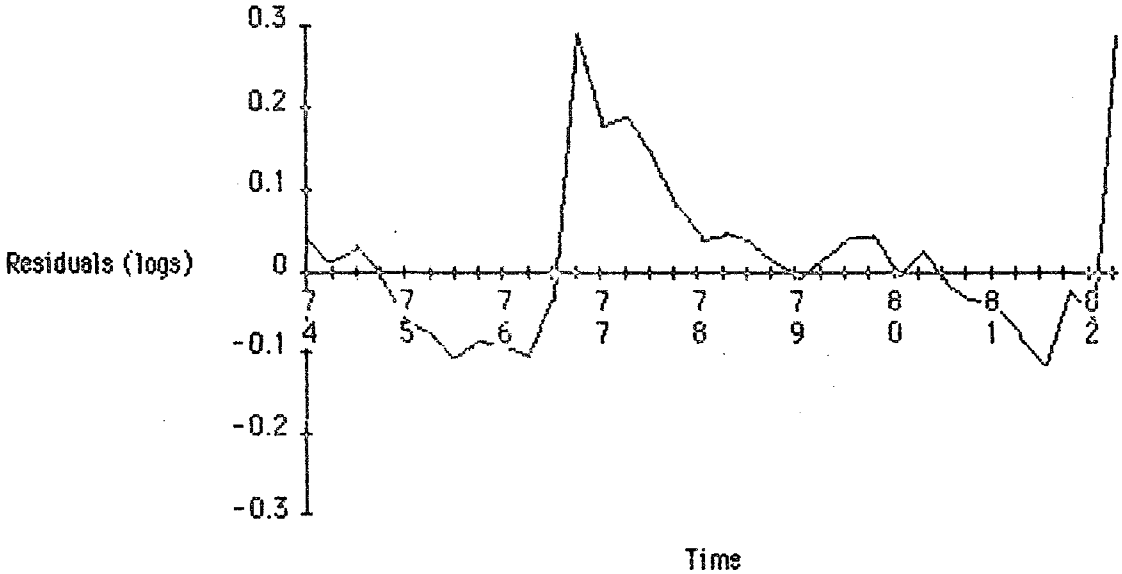
Note: Devaluations occurred in 76,3 and in 82,1

Figure No. 3
Residuals from the Six Months Futures Exchange Rate.



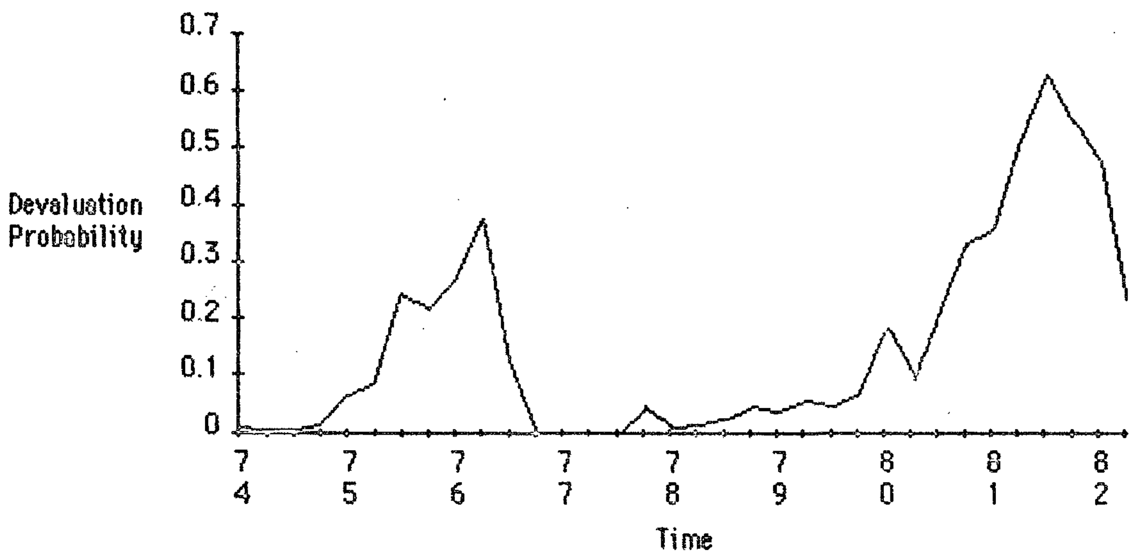
Note: Devaluations occurred in 76,3 and 82,1.

Figure No. 4
Residuals for the Nine Months Futures
EXchange Rates.



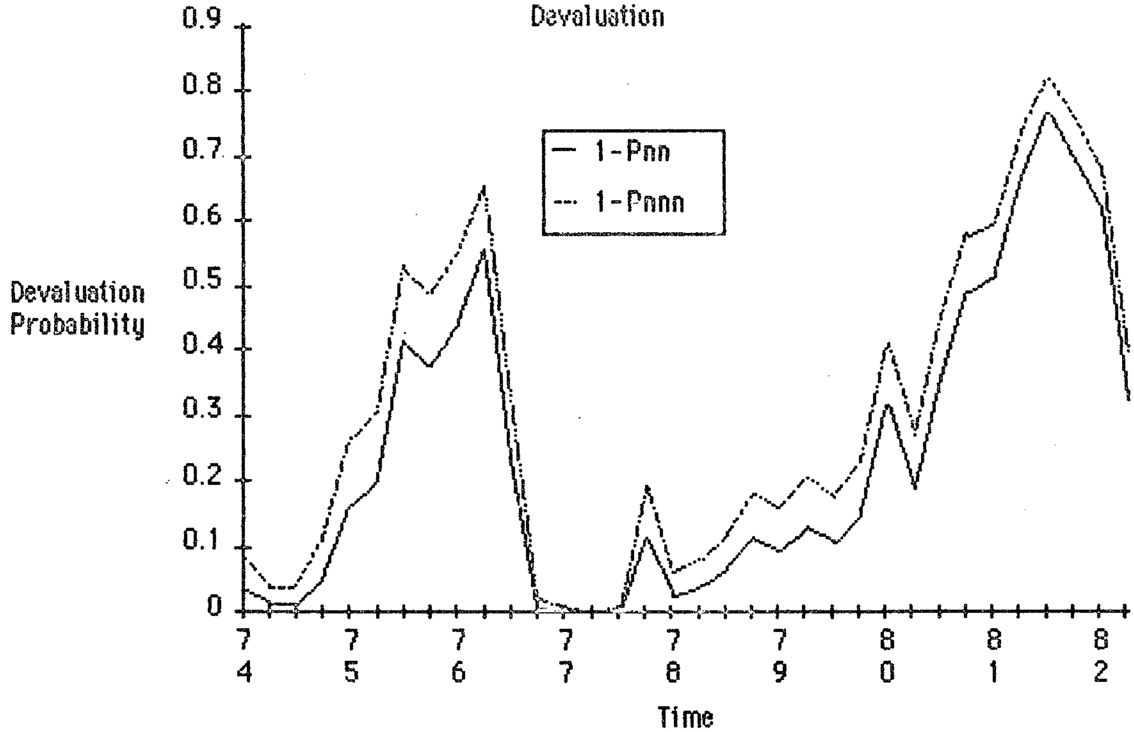
Note: Devaluations occurred in 76,3 and 82,1.

Figure No. 5
One-Step Ahead Probability of
Devaluation



Note: Devaluations occured in 76,3 and 82,1.
Source: See Appendix I.

Figure No. 6
Two and Three Periods Ahead Probabilities of Devaluation



Note: (1-Pnn) and (1-Pnnn) are the probabilities of a devaluation in two and three quarters ahead, respectively; devaluations occurred in 76,3 and in 82,1.

Source: See Appendix I

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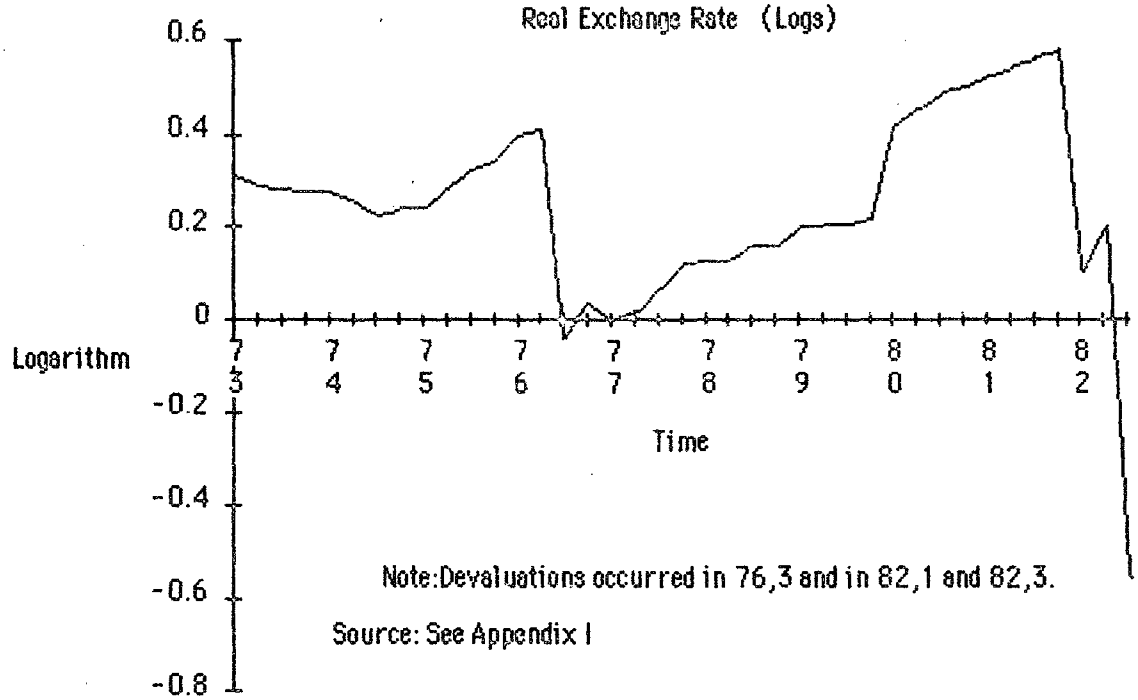
Figure No. 1

Real Exchange Rate (Logs)



Note: Devaluations occurred in 76,3 and in 82,1 and 82,3.

Source: See Appendix I

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