

THE PRICE SYSTEM IN  
CANTILLON'S FEUDAL-MERCANTILE MODEL

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I. Introduction

A certain number of economists have analyzed Marxian and Ricardian-type models, reformulating them with the help of linear algebra, with the purpose of examining their logical consistency (Morishima, 1973 and 1974; Passinetti, 1975; and Sraffa, 1960).

Cantillon, considered by Jevons as "the founder of Political Economy", and by Spengler as "the first of the moderns" developed an interesting model, particularly well suited to mathematical formalization: in Chapters X and XI of Part One of his "Essai sur la Nature du Commerce en General" he presents a model in which every price is reduced, in the last instance, only to wages and rents, implying a zero rate of profits.

This model is feudal, in the sense that it postulates that all surplus labor is converted into rents. Nevertheless, it does not correspond to the popular image of feudalism as a "natural economy" in which exchange is not well developed. In consequence, the most precise characterization of this model is as a feudal-mercantile model.

Smith, in his model of the "early and rude state of society", assumed zero profits and zero rents. Ricardo, in the first chapter of the Principles of Political Economy and Taxation, deals mainly with a model in which rents are zero and all prices are reduced only to wages and profits. In this context, Cantillon's feudal mercantile model, with its assumption of zero profits and its reduction of prices to wages and rents, represent an important element within the set of simplified classical models.

We must point out that, starting from Chapter XIII of Part One, Cantillon considers the existence of entrepreneurial incomes. Later, in Part Two, he considers the existence of a positive rate of profits, which regulates the rate of interest.

Nevertheless, Cantillon does not modify his model of the determination of equilibrium prices in order to consider the effects of a positive rate of profits.

This is why we prefer to limit ourselves to the feudal-mercantile case, the only one in which Cantillon deals explicitly with the determination of the equilibrium price system. The mathematical formalization of this model will allow us to validate most of Cantillon's assertions.

In section II, we consider the concept of total requirements of labor and land, based on a context of one-sector model with produced means of production (here we differ from Brems (1978), who utilizes a two sector model in which intermediate inputs are not considered) and we demonstrate that, in

this model, every price can be reduced to wages and rents paid over total quantities of land and labor absorbed by the product.

Section III deals with the reduction of all products to land inputs and of all prices to rent payments, by means of the concept of subsistence wage.

Section IV examines Cantillon's model from a Marxian point of view, and we obtain an analog to the Morishima-Seton-Okishio Theorem: rents will arise if and only if there exists exploitation.

Section V summarizes the main conclusions obtained.

## II. Natural Prices.

The concept of natural price in Cantillon is the same as that of the classical economists, which is approximately, the long run competitive equilibrium price. This price is clearly different from the market price:

"Si les Fermiers dans un Etat sement plus de blé qu'à l'ordinaire, c'est-à-dire, beaucoup plus de blé qu'il n'en faut pour la consommation de l'année,... comme il y en a une trop grande abondance, & plus de Vendeurs que d'Acheteurs; le prix du blé au Marché tombera nécessairement au-dessous du prix ou valeur intrinseque. Si au contraire les Fermiers sement moins de blé qu'il ne faut pour la consommation, il y aura plus

d'Acheteurs que de Vendeurs, & le prix du blé au Marché haussera au-dessus de sa valeur intrinseque".

("If the Farmers in a State sow more corn than usual, much more than is needed for the year's consumption, ... but as there is too great an abundance of it and there are more sellers than buyers, the Market Price of the Corn will necessarily fall below the intrinsic price or Value. If, on the contrary, the Farmers sow less corn than is needed for consumption, there will be more buyers than sellers and the Market Price of corn will rise above its intrinsic value"). Cantillon (1931) p. 28 to 31.

Cantillon does not limit himself to define natural prices, but he advances an explanation of the determination of these prices.

"Par ces inductions & exemples, je crois qu'on comprendra que le prix ou la valeur intrinseque d'une chose, est la mesure de la quantité de terre & du travail qui entre dans sa production, eu égard à la bonté ou produit de la terre, & à la qualité de travail".

("By these examples and inductions it will, I think, be understood that the Price or instrinsic value of a thing is the measure of the quantity of Land and of Labour entering into its production, having regard to the fertility or produce of the Land and to the quality of the Labour"). Cantillon (1931),

p. 28 and 29.

Hence, to formalize this idea, we must establish a model in which competitive prices will depend upon the quantities of labor and land.

These quantities of labor and land are, obviously, total quantities, including direct and indirect requirements.

The aggregation of the quantities of land and of the quantities of labor, incorporated to the product in different stages of production, requires the postulation of the homogeneity of labor and land or alternatively, the possibility of standardizing the different types of labor and land.

In fact, Cantillon considers the possibility of reducing these heterogeneous quantities of labor (and land) to units of standard labor (and land):

"Un Arpent de terre produit plus de blé, ou nourrit plus de Moutons, qu'un autre Arpent: le travail d'un homme est plus cher que celui d'un autre homme, suivant l'art & les occurrences, comme on l'a déjà expliqué..."

("One Acre of Land produces more Corn or feeds more Sheep than another. The work of one man is dearer than that of another, as I have already explained, according to the superior Skill and Occurrences of the Times...") Cantillon (1931) p. 26 and 27.

As we can see, Cantillon works with the restrictive assumption that a certain acre of land is superior to another, independently of the use given to it, this is, that there are not lands particularly well suited for the cultivation of wheat, but not for the feeding of sheep (or viceversa).

In the formalization of the model, we prefer the stronger assumption of homogeneous labor and land. Also, we will assume absence of durable produced means of production and will let the underlying idea of constant technical coefficients be explicit.

Technical coefficients will be represented by row vector  $\lambda$  for direct labor, ( $\lambda \geq 0$ ), by row vector  $\tau$  for direct land ( $\tau \geq 0$ ), and by matrix  $A$  for non durable produced means of production ( $A \geq 0$ ).

Total labor by unit of product, i.e., the sum of direct labor and total labor embodied in the means of production, will be represented by row vector  $\omega$ . Total land by unit of production, which is the sum of direct land and total land embodied in the means of production will be represented by row vector  $\theta$ . Consequently:

$$(1) \quad \omega = \lambda + \omega A$$

$$(2) \quad \omega = \lambda (I-A)^{-1}$$

$$(3) \quad 0 = r + \theta A$$

$$(4) \quad 0 = r(I-A)^{-1}$$

We shall assume the technical productivity of the economy, the existence of basic products in the sense of Sraffa (Sraffa, 1960, § 6) and also, that at least one basic product requires direct labor, and that at least one of them requires direct land. These assumptions are sufficient to guarantee that total labor and total land will be positive for each product.<sup>1/</sup>

Up to this point, we have established a model in which the concepts of total labor and total land are perfectly meaningful. Our next step is to demonstrate the relationship among relative prices and total quantities of labour and land.

We shall call  $p$  the vector of natural prices, normalized in rent units by taking the rent by period of a unit of land, as the numeraire of the system.

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<sup>1/</sup> The positiveness of total labour under the same assumptions of this model was established by Morishima (Morishima, 1973). For total land, the proof is strictly analogous. Essentially, the idea is that, as basic products enter directly or indirectly into the production of every product, if one of them requires direct labor (land) this primary input will enter directly or indirectly into the production of every product.



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Because of the absence of profits, long run equilibrium prices will equal costs, given by the price of the non durable produced means of production, the wages paid by direct labor and the rents paid over direct land.

$$(5) \quad \pi = \pi A + s\lambda + \tau$$

Where  $s$  is the wage of a unit of labor measured in rent units.

In consequence:

$$(6) \quad \pi = s\lambda(I-A)^{-1} + \tau(I-A)^{-1}$$

and by substitution of (2) and (4) in (6) we obtain

$$(7) \quad \pi = s\omega + 0$$

Equation (7) expresses simply that prices are reduced to the sum of wages and rents paid in all the stages of the elaboration of the product, thus confirming Cantillon's assertion.

### III. The Land Theory of Value.

As we have seen in the previous section, Cantillon explains prices in terms of total quantities of labor and total quantities of land required for the elaboration of the product.

This procedure, however, leaves the proportion between the payment for the use of land and the remuneration to the worker undefined. Thus, Cantillon's next step is to establish a long

term necessary relationship between the rent rate and the wage rate.

This is done by assuming the existence of an (long term) equilibrium real wage, which maintains the working population constant. He reaches as far as to establish that real wage must be twice the individual subsistence level, in order to allow the supporting of the worker's children:

"Si le Propriétaire emploie à son travail des Vassaux ou Païsans libres, il les entretiendra probablement un peu mieux qu'il ne feroit des Esclaves, & ce, suivant la coutume du lieu, mais encore dans cette supposition, le travail du Laboureur libre doit correspondre en valeur au double du produit de terre qu'il faut pour son entretien"...

"Les Laboureurs ou Artisans, lorsqu'ils ont leur double portion dan leur propre disposition, s'ils sont mariés emploient une portion pour leur propre entretien, & l'autre pour celui de leurs Enfans".

("If the Proprietor employs the Labour of Vassals or free Peasants he will probably maintain them upon a better foot than Slave according to the custom of the place he lives in, yet in this case also the Labour of a free Labourer ought to correspond in value to double the produce of Land needed for his maintenance")...

("When the Artisans or Labourers have their double portion at their own disposal they employ one part of it for their own upkeep if they are married and the other for their Children"). Cantillon (1931) p. 34 and 35.

Given this "necessary" relationship between the rent and the wage rate, it is possible to explain prices in terms of the quantity of a single input: labor or land,<sup>2/</sup> for this case, Cantillon chooses land.

"Par ces inductions, & autres qu'on pourroit faire dans le meme goût, l'on voit que la valeur du travail journalier a un rapport au produit de la Terre, & que la valeur intrinseque d'une chose peut être mesurée par la quantité de Terre qui est employée pour sa production & par la quantité du travail qui y entre, c'est-à-dire' encore par la quantité de Terre dont on attribue le produit à ceux qui y on travaillé".

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<sup>2/</sup> In fact, the wage-population theory of Cantillon is a little more complicated than the one depicted here: he assumes that women also work, receiving an inferior wage to that of men, but that women's incomes are scarcely equal to the individual subsistence level because of the time they spent in the care of their children (Cantillon 1931 p. 33 to 37). So we can say that, according to Cantillon, men spend actual income and women sacrifice potential income in order to feed and look after their children.

("By these examples and other which might be added in the same sense, it is seen that the value of the day's work has a relation to the produce of the soil, and that the intrinsic value of any thing may be measured by the quantity of Land used in its production and the quantity of Labour which enters into it, in other words by the quantity of Land of which the produce is allotted to those who have worked upon it"). Cantillon (1931) p. 40 and 41.

The election of land as the fundamental input is very sound in terms of Cantillon's analysis: land is, by hypothesis, a non produced input; labor, in turn, can be viewed as a produced input, given that workers need to be provided with a basket of consumption goods. These consumption goods require, directly or indirectly, land for their production and we can say, in these terms, that land is required in order to produce labor.<sup>3/</sup>

This way, total requirements of land,  $\theta$  , are not exhaustive, because they exclude the land vinculated to the supporting of the laborers. Only as a matter of terminology,

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3/ In fact, land is required in order to produce the labor force, not the labor itself, but the distinction between labor and labor force was only made clear later by Marx.

let us call global requirements of land those which include the land for feeding (and dressing) the workers.

In order to demonstrate Cantillon's assertions, we must show that a given real wage is sufficient to close the model determining natural prices completely. Additionally, we must show that natural prices are proportional to global requirements of land (this last point requires a formal definition of global quantities of land).

Since we assume the existence of a given real wage (let us say the row vector  $\sigma, \sigma \geq 0$ ). The price of this real wage must equal the nominal wage,  $s$ .

$$(8) \quad \pi \sigma' = s$$

If we substitute (8) in (7) we obtain:

$$(9) \quad \pi = \theta + \pi \sigma' \omega$$

$$(10) \quad \pi = \theta (I - \sigma' \omega)^{-1}$$

This closes the system, leaving on the right hand side only the real wage and the total quantities of labor and land previously calculated.

With the aid of some algebraic manipulations, we can go further and obtain natural prices as a function of the real wage and the technical coefficients:

If we substitute (4) in (10)

$$(11) \quad \pi = \tau (I-A)^{-1} (I-\sigma'\omega)^{-1}$$

$$(12) \quad \pi = \tau ((I-\sigma'\omega) (I-A))^{-1}$$

$$(13) \quad \pi = \tau (I-A-\sigma'\omega+\sigma'\omega A)^{-1}$$

$$(14) \quad \pi = \tau (I-A-\sigma'(\omega-\omega A))^{-1}$$

If we solve from (1) and substitute in (14) we obtain:

$$(15) \quad \pi = \tau (I-A-\sigma'\lambda)^{-1}$$

where (15) is an expression that depends only on the technical coefficients ( $A$ ,  $\tau$ , and  $\lambda$ ) and the real wage ( $\sigma$ ).

The formalization of the concept of global requirements of land is easy and straightforward: global requirements of land ( $\gamma$ ) are obtained by adding to direct land ( $\tau$ ) the global requirements of land for the production of the means of production ( $\gamma A$ ) and the global requirements of land for the production of the wage goods for the direct laborers ( $\gamma \sigma' \lambda$ ):

$$(16) \quad \gamma = \tau + \gamma A + \gamma \sigma' \lambda$$

in consequence,

$$(17) \quad \gamma = \tau (I - A - \sigma' \lambda)^{-1}$$

And we obtain immediately, from (15)

$$(18) \quad \pi = \gamma$$

This shows the proportionality between the natural prices and the global quantities of land. In fact, we obtain complete equality between the two sets of variables, thanks to the normalization adopted.

The formalization of the concept of global quantities of land was made in a form that shows no immediate relationship with the explanation of the concept given by Cantillon. In consequence, it is interesting to try to define the global requirements of land following more closely Cantillon's exposition.

Cantillon starts from the point where all product has



been reduced to a total quantity of land plus a total quantity of labor and then he argues that the total quantity of labor can be reduced to land.

Fortunately, this is not far from our formalization: from (18) and (10) we obtain:

$$(19) \quad \gamma = \theta (I - \sigma' \omega)^{-1}$$

and, rewriting, we have

$$(20) \quad \gamma = \theta + \gamma \sigma' \omega$$

This seems quite close to Cantillon's starting point: global quantities of land are obtained adding to the total quantities of land the global quantities of land contained in the real wage basket paid over the total labor.

The only remaining doubt is if Cantillon thought about converting the total quantities of labor into land in a similar form to the one shown in (20) or only by obtaining the total content of land in the real wage basket paid over the total labor ( $\theta \sigma' \omega$ ). In this case, we will obtain a different vector, let us say  $\bar{\gamma}$ , given by

$$(21) \quad \bar{\gamma} = \theta + \theta \sigma' \omega$$

Even this possible failure is only a matter of incompleteness of the analysis, because, if we develop (19) as a power series we obtain:

$$(22) \quad \gamma = \theta(I + \sigma'\omega + (\sigma'\omega)^2 + (\sigma'\omega)^3 + \dots)$$

$$(23) \quad \gamma = \theta + \theta\sigma'\omega + \theta(\sigma'\omega)^2 + \theta(\sigma'\omega)^3$$

In consequence, global requirements of land includes:

- a) The total requirements of land for the product ( $\theta$ ), i.e., the land required to cultivate the product and all their direct and indirect means of production.
- b) The total requirements of land to produce the wage of the total labor demanded by the product ( $\theta\sigma'\omega$ ).

Up to this point, we have obtained  $\gamma$ , but this is not the end of the story, because wage goods require labor to be produced, and therefore, we must add:

- c) The total land required in order to feed (and dress) the workers occupied on the production of the wage goods for the laborers engaged directly or indirectly in the elaboration of the product ( $\theta(\sigma'\omega)^2$ ).

But these workers also eat (and wear clothes) and, shortly, we must extend the summation to the infinite. Nevertheless, if this series converge,  $\gamma$  will be perfectly defined.

In short, Cantillon's analysis is consistent, and his conclusions are correct in terms of his model, except for one possible failure: the omission of the land required to support

$$(22) \quad \gamma = \theta(I + \sigma'\omega + (\sigma'\omega)^2 + (\sigma'\omega)^3 + \dots)$$

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the workers who produce the wage goods for the workers who produce the products.

#### IV. Exploitation and Rent.

Marx considered Cantillon's model as a especial case in which all surplus value is transformed into rent:

"Petty, Cantillon, and in general the writers who are closer to feudal times, assume that ground rent is the normal form of surplus value, whereas profit to them is still vaguely combined with wages, or at best, looks to them like a portion of surplus-value filched by the capitalist from the landlord" Marx (1909), volume III, p. 910.

To demonstrate the correctness of Marx's opinion we must be able to define, in Cantillon's model, the economic concept of surplus value and to establish a defined relationship between the existence of surplus value and the existence of rents.

We must start from identifying Marxian values, this is, quantities of socially necessary labour required for the production of the goods, in Cantillon's model. Fortunately this is simple, since  $\omega$ , the total quantities of labor, are the Marxian values (cfr. Morishima (1973), Chapter 1).

Surplus value is the difference between the labor performed by the worker and the value of the real wage basket

The definition of the exploitation condition (24) has an interesting corollary, as we shall see immediately.

By definition of the power of a matrix

$$(27) \quad (\sigma'w)^2 = \sigma'w\sigma'w$$

but this can be simplified:

$$(28) \quad (\sigma'w)^2 = (w\sigma')\sigma'w$$

doing this in a recursive form, we conclude that

$$(29) \quad (\sigma'w)^j = (w\sigma')^{j-1}\sigma'w$$

and by substitution in (25) we obtain

$$(30) \quad \gamma = \theta + \theta\sigma'w + (w\sigma')\theta\sigma'w + (w\sigma')^2\theta\sigma'w +$$

If (30) is to converge (24) must hold. In consequence, the existence of surplus value is crucial in order to guarantee the convergence of (25).

So, we confirm the deep insight of Marx in identifying profit and rent as different forms of the same basic fact: the exploitation of laborers. We confirm also Marx's judgement of Cantillon's model as a especial case when all surplus value become rent.

### Concluding Remarks

The feudal-mercantile model proposed by Cantillon can be formalized for the general case of  $n$  products and the results confirm most of Cantillon's assertions:

- Natural Prices can be reduced, in the last instance, only to wages and rents.
- Given the homogeneity of labour and land, these payments are simply the quantities of labour and land multiplied by the equilibrium wage and rent rates.
- The rent and wage rates are not independent and the model is closed by assuming a given real wage.
- Since labour force is "produced" utilizing products of the land, the value of a product can be explained in terms only of the quantities of land required for its production. These quantities comprise the direct and indirect land for the materials as well as the direct and indirect land for the supporting of the workers.

Additionally, we obtained, for this model, an analog to the Morishima-Seton-Okishio Theorem: rents will be positive if and only if exploitation is positive.

The main limitation of this model is its central assumption, the non existence of profits.

It is worthwhile to note that, under this assumption, and if we consider rent as a surplus, the branches of production that generate most of the surplus are the land-intensive ones, mainly agriculture, and we can say that the manufactures are "sterile", meaning that they only cover their costs .

From this angle, the study of Cantillon's feudal-mercantile model becomes a helpful element for the understanding of the theoretical conceptions of the Physiocratic School. This connection is not an artificial one, according to the well documented relationship between Cantillon's "Essai" and the rise of the Physiocratic School (see Jevons (1881) and Walsh & Gram (1980), Ch. 2).

Finally, I hope this paper will serve to emphasize the importance of stating the study of the classical economic theory with Cantillon.

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