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Abstract

We document important differences between developed and emerging market economies relating to international relative prices and quantities. In emerging economies, imports are more volatile than exports, the terms of trade is less volatile, and net exports are strongly countercyclical. Moreover, the terms of trade is acyclical or weakly countercyclical, while it is procyclical in developed countries. We compare three mechanisms which could account for the empirical evidence: trend productivity shocks, interest-rate shocks in the presence of financial frictions, and informality. We find that trend productivity shocks are necessary to replicate the observed behaviour of the open-economy variables.

JEL Classification: E32; F41; F44

Keywords: Emerging Markets; Business Cycles; Terms of Trade; Net Exports; Informality.

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1 Introduction

Business cycles of emerging market economies differ in a number of important aspects from their developed country counterparts. In emerging economies, consumption is more volatile than output, the real interest rate is significantly more variable over the cycle, and both the trade balance and the real interest rate are strongly countercyclical.[1] However, to date, the existing literature has ignored the cyclical behaviour of the terms of trade and its relationship with net exports, which is of crucial importance in understanding international business cycle fluctuations.[2] The goal of our paper is to fill this gap.

We first provide novel evidence documenting important differences in the behaviour of international relative price and quantities for 40 emerging and developed countries. Imports are shown to be significantly more volatile than exports in emerging economies, whereas the terms of trade (measured as the ratio of the import price and export price deflators) is found to be relatively more volatile in developed countries. While the terms of trade is procyclical for developed countries, it is generally acyclical or weakly countercyclical for emerging economies. These findings differ from what we observe for the behaviour of net exports, which is strongly countercyclical for emerging economies and acyclical for developed countries.

In order to account for these empirical facts, we develop a two-good, two-sector, small open economy model where the terms of trade is endogenous. The model incorporates three features commonly advocated in the literature as key for understanding the aggregate fluctuations of emerging market economies. First, following Aguiar and Gopinath (2007), we incorporate both (stationary) transitory and (nonstationary) trend shocks to productivity. Second, following Neumeyer and Perri (2005) and Uribe and Yue (2006), we introduce a risk premium (i.e., interest rate) shock and (reduced-form) financial frictions. Third, our model includes both a formal sector that produces tradeable goods and an informal sector that produces non-tradeable goods consistent with a recent literature that advocates the importance of informality in understanding the cyclical fluctuations of emerging economies. The model is calibrated for Mexico, a representative emerging market economy with a large informal sector, commonly used in the small open economy real-business-cycle literature. We evaluate the importance of transitory formal and informal productivity shocks, trend productivity shocks, risk premium shocks and financial frictions, foreign demand shocks, and

[1] See, e.g., Neumeyer and Perri (2005), Uribe and Yue (2006), Aguiar and Gopinath (2007), and Fernández and Gulan (2015).

[2] See Mendoza (1995), Kose (2002), and Ben Zeev et al. (2017).

the informal economy in replicating the major business cycle properties for Mexico.

Our main findings are summarized as follows. Nonstationary shocks to productivity are crucial for the model to replicate the cyclical behaviour observed for international relative prices and quantities. Specifically, trend productivity shocks are important in generating sufficient volatility of net exports and to match the larger volatility of imports relative to exports found in the data. Furthermore, trend productivity shocks are shown to be critical for the model to correctly generate the countercyclicality of both the terms of trade and net exports. In contrast, financial frictions in the form of countercyclical risk premium shocks play little role in the model's ability to match the behavior of the key open-economy variables including the terms of trade. Similar to Fernández and Meza (2015), the informal economy is found to amplify the response of the formal sector to stationary productivity shocks. However, while informality has a minor effect on the propagation of nonstationary productivity shocks, it significantly reduces the response of the formal sector to risk premium shocks.

The key transmission mechanisms behind these results are as follows. In response to a positive trend shock to productivity, domestic absorption exceeds output as permanent income increases, resulting in higher imports and lower exports. The resulting deterioration of the trade balance is accompanied by an improvement (i.e., decrease) of the terms of trade. Consequently, nonstationary productivity shocks generate a negative output correlation for both net exports and the terms of trade, as in the data. In contrast, we show that transitory productivity shocks result in an improvement in the trade balance and generate a counterfactual output correlation for both imports and the terms of trade. While the transmission mechanism for risk premium shocks is also found to generate behaviour for international relative prices and quantities that is consistent with the data, such shocks play a minor role in explaining the moments of the data. In our analysis, risk premium shocks cannot simultaneously match the observed volatility for the real interest rate and the key open-economy moments.

This paper contributes to a growing literature that aims to understand the business cycle behaviour of emerging market economies. The influential contribution of Aguiar and Gopinath (2007) showed that nonstationary productivity shocks can resolve the puzzle of the excess volatility of consumption observed for emerging economies. In contrast, Neumeyer and Perri (2005), Uribe and Yue (2006), Fernández-Villaverde et al. (2011), Álvarez-Parra et al. (2013), and Fernández and Gulan (2015) have shown that countercyclical real interest-rate shocks in the presence of financial frictions play a central role in driving the business cycles of emerging countries. Using Bayesian methods, García-Cicco et al. (2010) and Chang and Fernández (2013) estimate the popular one-good, small open economy model and find that the data assigns a dominant role to risk premium shocks and financial frictions in accounting for aggregate fluctuations with trend productivity shocks playing a minor role. However, none of the existing literature have investigated the behaviour of the terms of trade. We rectify this gap in the literature by first documenting the empirical differences between developed and emerging economies for international relative prices and quantities. We then develop a model economy that incorporates a number of emerging market features in an attempt to explain and understand the stylized facts. Our results suggest that interest-rate shocks and financial frictions play a less dominant role in two-good models with an endogenous terms of trade, offering some support to the argument of Aguiar and Gopinath (2007) that trend productivity shocks are important to understanding emerging market business cycles.

Our paper also contributes to a recent literature that investigates the implications of informality for aggregate fluctuations.[3] Restrepo-Echavarria (2014) finds that the excess volatility of consumption puzzle can be accounted for by the mis-measurement of the informal economy. Horvath and Yang (2022) investigate the role of informality in explaining the dynamics of unemployment. Fernández and Meza (2015) show that informal employment amplifies the effects of productivity shocks, whereas Horvath (2018) considers the implications of informality for the transmission of interest-rate shocks. In this paper, we instead consider the role the informal sector plays in explaining the cyclical behavior of the terms of trade and net exports.

Finally, this paper is related to Rothert (2020), who investigates the cyclical behavior of real exchange rates in emerging market economies. While there are some similarities with our approach, there are a number of significant differences. Since Rothert's focus is on the real exchange rate, he develops a two-country model in the spirit of Chen and Crucini (2016), whereas we build upon the small open economy literature and include an informal sector to examine the behaviour of the terms of trade. In order to match the observed behaviour of real exchange rates, Rothert (2020) shows that interest-rate shocks, in the absence of financial frictions, account for most of the observed fluctuations in output. We find that Rothert's results do not generalize to a small open economy with an endogenous terms of trade. In our model, trend productivity shocks are needed to account for the behavior of the terms of trade and net exports.

[3] In a related contribution, Boz et al. (2015) investigate the role of search-matching labor market frictions for understanding emerging market business cycles.

The remainder of the paper is organized as follows. Section 2 presents our empirical findings on the dynamics of exports, imports, and the terms of trade highlighting the differences between developed and emerging market and economies. Section 3 outlines the model economy. The calibration of the model and its performance relative to the data are discussed in Section 4. Section 5 investigates the transmission mechanisms behind our results. Finally, Section 6 concludes.

2 Empirical evidence

This section documents the business cycle statistics for 40 small open economies over the period 1993Q1–2019Q4 (or the earliest date possible if 1993 is not available). We used the classification of the IMF to split the countries into developed countries and emerging market economies at the start of the sample period. This yielded 20 developed countries and 20 emerging economies listed in Tables 1 and 2, respectively. Similar to the existing literature, we report unconditional second moments for output (y), private final consumption expenditures (c), investment (i) defined as gross fixed capital formation, and the real interest rate (r). The real interest rate for developed countries is constructed by deflating the three-month money market short-term interest rate by the average CPI inflation rate in the current and previous three quarters. Similar to Fernández and Gulan (2015), the real interest rate for emerging economies is defined as the sum of the U.S. real interest rate and the country spread measured using J.P. Morgan EMBI+ data.[4] Our dataset differs from the existing literature in three ways. First, we provide information on the dynamics of the key open-economy variables: real exports (x), real imports (m), nominal net exports over nominal GDP (nx), and the terms of trade (tot), which is constructed as the ratio of the import price and export price deflators. Second, where available, our dataset includes information on total employment (h). Finally, for the case of Mexico, we also document unconditional second moments for both formal and informal employment, and the informality rate.

The data is taken from the Quarterly National Accounts of the OECD and the International Financial Statistics database of the IMF, with the exception of the J.P. Morgan EMBI+ spread data, which is available from the World Bank. All data is quarterly, seasonally adjusted, and in units of domestic currency. All series are logged (except net exports and the real interest rate) and HP-filtered with a smoothing parameter of 1600.

[4]The U.S. real interest rate is obtained by deflating the three-month nominal T-bill rate by the average CPI inflation rate in the current and previous three quarters.

Table 1: International business cycle statistics, 1993Q1–2019Q4: Developed countries

	Relative standard deviation					Correlation with output			
	$\frac{\sigma(x)}{\sigma(y)}$	$\frac{\sigma(m)}{\sigma(y)}$	$\frac{\sigma(m)}{\sigma(x)}$	$\sigma(nx)$	$\frac{\sigma(tot)}{\sigma(y)}$	x	m	nx	tot
Australia	4.08	7.22	1.77	1.04	8.97	0.06	0.45	-0.37	0.04
Austria	3.23	2.83	0.87	0.63	0.76	0.85	0.78	0.24	0.52
Belgium	3.49	3.51	1.01	0.89	0.91	0.84	0.80	-0.12	0.46
Canada	3.18	3.58	1.12	0.95	2.72	0.86	0.77	0.49	-0.47
Denmark	2.52	2.84	1.13	0.97	0.68	0.76	0.79	-0.15	0.23
Finland	2.96	2.36	0.80	1.13	0.95	0.77	0.79	0.16	0.32
France	3.34	3.46	1.04	0.41	1.29	0.87	0.89	-0.47	0.51
Germany	2.77	2.36	0.85	0.72	0.98	0.90	0.79	0.23	0.64
Ireland	1.53	2.97	1.94	8.81	0.66	0.69	0.25	0.20	-0.24
Italy	3.82	3.44	0.90	0.62	1.71	0.84	0.87	-0.34	0.55
Japan	4.22	3.15	0.75	0.70	2.42	0.80	0.79	-0.01	0.55
Luxembourg	2.17	2.39	1.10	2.42	0.69	0.66	0.50	0.41	0.01
Netherlands	2.37	2.99	1.26	1.88	0.58	0.70	0.63	-0.06	0.18
New Zealand	2.02	4.11	2.04	0.96	2.98	0.57	0.43	-0.10	-0.13
Norway	2.06	3.20	1.55	2.61	6.49	0.57	0.50	0.23	-0.25
Portugal	2.68	3.06	1.14	1.18	1.15	0.35	0.76	-0.63	0.15
Spain	2.57	3.63	1.41	0.81	1.32	0.53	0.71	-0.64	0.19
Sweden	2.53	2.68	1.06	0.60	0.55	0.86	0.86	0.01	0.07
Switzerland	4.15	3.36	0.81	2.04	1.02	0.62	0.38	0.32	0.49
U.K.	3.32	2.96	0.89	0.67	1.22	0.56	0.61	-0.12	0.24
Mean	2.95	3.31	1.17	1.50	1.90	0.68	0.67	-0.03	0.20
Median	2.86	3.11	1.08	0.96	1.09	0.73	0.76	-0.03	0.21

Notes: y, x, m, nx, tot denote, respectively, real output, real exports, real imports, net exports, and the terms of trade. nx is the ratio of nominal net exports to nominal GDP, and tot is the ratio of the import price and export price deflators. Data is quarterly, seasonally adjusted, and in units of domestic currency. All data is from the Quarterly National Accounts database of the OECD. All series are logged (except nx) and HP-filtered with a smoothing parameter of 1600. Standard errors are available upon request.

Table 2: International business cycle statistics: Emerging market economies

	Sample	Relative standard deviation					Correlation with output			
		$\frac{\sigma(x)}{\sigma(y)}$	$\frac{\sigma(m)}{\sigma(y)}$	$\frac{\sigma(m)}{\sigma(x)}$	$\sigma(nx)$	$\frac{\sigma(tot)}{\sigma(y)}$	x	m	nx	tot
Argentina	2004Q1-2019Q4	2.10	3.52	1.68	1.24	1.86	0.59	0.84	-0.45	-0.10
Brazil	1996Q1-2019Q4	2.53	4.73	1.87	1.03	3.55	0.30	0.79	-0.41	-0.21
Chile	1996Q1-2019Q4	1.67	3.81	2.28	2.78	4.12	0.61	0.82	-0.10	-0.31
Colombia	1994Q1-2019Q4	2.31	4.27	1.85	1.42	3.76	0.36	0.86	-0.42	-0.15
Costa Rica	1993Q1-2019Q4	2.49	4.15	1.66	1.73	1.85	0.60	0.63	-0.49	0.18
Czech Rep.	1995Q1-2019Q4	2.96	2.69	0.91	1.14	1.01	0.63	0.72	-0.21	0.40
Ecuador	2000Q1-2019Q4	2.29	3.90	1.70	2.42	3.90	0.60	0.46	0.00	-0.11
Estonia	1995Q1-2019Q4	2.06	2.58	1.26	2.69	0.48	0.71	0.85	-0.65	-0.13
Hungary	1995Q1-2019Q4	3.43	3.34	0.97	1.40	0.69	0.60	0.70	-0.24	0.13
Indonesia	1993Q1-2019Q4	3.01	3.93	1.31	2.02	1.38	0.32	0.51	-0.57	0.10
Korea	1993Q1-2019Q4	1.89	3.73	1.98	2.34	1.41	0.20	0.90	-0.79	-0.17
Latvia	1995Q1-2019Q4	1.41	2.26	1.60	2.83	0.75	0.53	0.73	-0.66	0.03
Lithuania	1995Q1-2019Q4	2.38	2.60	1.09	2.68	0.96	0.52	0.75	-0.55	0.00
Mexico	1993Q1-2019Q4	2.01	2.83	1.41	0.99	1.37	0.40	0.91	-0.66	-0.19
Romania	1995Q1-2019Q4	2.55	3.31	1.30	1.83	1.54	0.30	0.50	-0.37	-0.09
Slovak Rep.	1993Q1-2019Q4	2.75	3.22	1.17	3.28	0.83	0.51	0.53	-0.12	0.03
Slovenia	1995Q1-2019Q4	2.44	2.60	1.07	1.37	0.68	0.81	0.88	-0.42	0.31
South Africa	1993Q1-2019Q4	4.66	5.51	1.18	1.03	2.77	0.70	0.75	-0.33	0.13
Thailand	2003Q1-2019Q4	2.64	3.86	1.46	3.35	1.06	0.77	0.68	-0.14	0.05
Turkey	1993Q1-2019Q4	1.54	3.02	1.96	2.10	1.11	0.52	0.88	-0.69	-0.02
Mean		2.46	3.49	1.49	1.98	1.75	0.53	0.73	-0.41	-0.01
Median		2.41	3.43	1.43	1.93	1.38	0.56	0.75	-0.42	-0.01

Notes: See Table 1. All data is from the Quarterly National Accounts database of the OECD except Ecuador and Thailand, where the data is from the International Financial Statistics database of the IMF.

2.1 The dynamics of exports, imports, and the terms of trade

Tables 1 and 2 report the international business cycle statistics for developed and emerging market economies, respectively. Each table presents relative standard deviations and contemporaneous correlations with output. Several interesting differences emerge. For emerging economies, real imports are more volatile than real exports in all cases except the Czech Republic and Hungary. For an average emerging economy, real imports are nearly 50% more volatile than real exports. In contrast, real imports are only 17% more volatile than real exports for an average developed country. While net exports (over GDP) are typically more volatile in emerging economies, the terms of trade (relative to output) is on average more volatile for developed countries. The mean volatility for net exports is 32% larger for emerging economies, whereas the mean relative volatility for the terms of trade is approximately 10% larger for developed countries. In terms of correlations with output, net exports are countercyclical for 19 of the 20 emerging economies and the terms of trade are procyclical for 16 of the 20 developed countries. For the average emerging economy, net exports are strongly countercyclical (-0.41). This is in stark contrast to developed countries where the mean correlation between net exports and output is only -0.03. For the average developed economy, the terms of trade are procyclical (0.20), whereas for the average emerging economy, the terms of trade is acyclical (-0.01), where for half the sample, the terms of trade is weakly countercyclical.

2.2 Other business cycle moments

In addition to the international business cycle moments, Table 3 also reports additional moments for real consumption (c), real investment (i), total employment (h), and the real interest rate (r), averaged over the developed and emerging economies samples.[5] We also report in Table 3 statistics for Mexico, an emerging market economy commonly used in the literature. Consistent with the findings documented in the existing literature, business cycles in emerging countries are more volatile than developed countries, where the mean volatility for output is 75% larger for emerging economies. While consumption is typically less volatile than output in developed countries, consumption is more volatile than output in emerging economies. On average, real interest rates are 2.8 times more volatile in emerging countries than developed countries, and the real interest rate tends to be countercyclical for emerging markets (-0.20) and procyclical for

[5]Individual business cycle statistics for each country are provided in Tables A.1 and A.2 of the appendix.

Table 3: Business cycle statistics: Developed vs. emerging market economies (Averages)

Moment	Developed	Emerging	Mexico
$\sigma(y)$	1.30	2.27	2.16
$\sigma(c)/\sigma(y)$	0.84	1.12	1.00
$\sigma(i)/\sigma(y)$	3.52	3.42	3.58
$\sigma(h)/\sigma(y)$	0.66	0.71	0.36
$\sigma(x)/\sigma(y)$	2.95	2.46	2.01
$\sigma(m)/\sigma(y)$	3.31	3.49	2.83
$\sigma(m)/\sigma(x)$	1.17	1.49	1.41
$\sigma(nx)$	1.50	1.98	0.99
$\sigma(tot)/\sigma(y)$	1.90	1.75	1.37
$\sigma(r)$	0.21	0.58	0.49
$\rho(y_t, y_{t-1})$	0.78	0.81	0.82
$\rho(c, y)$	0.62	0.76	0.93
$\rho(i, y)$	0.67	0.77	0.90
$\rho(h, y)$	0.51	0.46	0.38
$\rho(x, y)$	0.68	0.53	0.40
$\rho(m, y)$	0.67	0.73	0.91
$\rho(nx, y)$	-0.03	-0.41	-0.66
$\rho(tot, y)$	0.20	-0.01	-0.19
$\rho(r, y)$	0.18	-0.20	-0.55

Notes: The table reports average values of the moments for the group of 20 developed and 20 emerging economies given in Tables 1 and 2 of the main text, and Tables A.1 and A.2 of the Appendix. y, c, i, h, x, m, nx, tot, r denote, respectively, real output, real consumption, real investment, total employment, real exports, real imports, net exports (over GDP), terms of trade, and the real interest rate. $\sigma(z)$ denotes the standard deviation of z and $\rho(z, y)$ denotes the correlation between z and y. Standard deviations are expressed in percent.

developed countries (0.18).[6] The cyclical behaviour of investment and employment is similar (on average) for both emerging and developed economies.

2.3 The dynamics of informality in Mexico

To document the business cycle statistics for formal and informal employment, we use data from the National Survey of Occupation and Employment (ENOE in Spanish) of INEGI (the national statistical agency of Mexico) available for the period 2005Q1–2019Q4. Informality in Mexico is typically measured by either estimating the prevalence of informal firms in the economy using

[6] Similar findings for the real interest rate for a different sample of countries are documented by Fernández and Gulan (2015).

Table 4: Formal vs. informal employment: Second moments for Mexico, 2005Q1–2019Q4

Informal employment (h^I)	h^I/h	$\sigma(h^I)$	$\sigma(h^I/h)$	$\rho(h^I/h,y)$	$\rho(h^I,y)$
Measure 1 (h^I_1)	0.25	1.77	1.90	-0.24	-0.09
Measure 2 (h^I_2)	0.23	2.49	2.18	-0.38	-0.20
Measure 3 (h^I_3)	0.28	2.04	1.61	-0.32	-0.11
Measure 4 (h^I_4)	0.58	1.18	0.82	-0.49	-0.09
Mean	0.33	1.87	1.63	-0.36	-0.12
Median	0.27	1.91	1.75	-0.35	-0.10

Formal employment (h^F)	h^F/h	$\sigma(h^F)$	$\frac{\sigma(h^I)}{\sigma(h^F)}$	$\frac{\sigma(h)}{\sigma(h^F)}$	$\rho(h^F,y)$
Measure 1 (h^F_1)	0.75	1.19	1.49	0.66	0.38
Measure 2 (h^F_2)	0.77	0.87	2.85	0.89	0.61
Measure 3 (h^F_3)	0.72	0.78	2.63	1.00	0.63
Measure 4 (h^F_4)	0.42	1.35	0.87	0.58	0.64
Mean	0.67	1.05	1.96	0.78	0.56
Median	0.74	1.03	2.06	0.77	0.62

Notes: $\sigma(h^i)$ denotes the standard deviation of h^i and $\rho(h^i,y)$ denotes the correlation between h^i and output, y. Standard deviations are expressed in percent. All series are logged, seasonally-adjusted, and HP-filtered with a smoothing parameter of 1600. All data is from INEGI. Informality measures h^I_1, h^I_2, h^I_3, and h^I_4 refer respectively to: employment without benefits provided by labor legislation; self-employment; employment in the informal sector; employment under informality conditions. Formal employment measures h^F_1, h^F_2, h^F_3, h^F_4 are calculated by subtracting each measure of informal employment from total employment, h.

self-employment data or the number of employees in the economy without access to social security benefits.[7] We consider four different measures of informal employment: Employment without benefits provided by labor legislation (h^I_1); self-employment (h^I_2); employment in the informal sector (h^I_3); and employment under informality conditions (h^I_4). Measures h^I_3 and h^I_4 are constructed by INEGI to capture the level of informality in Mexico: h^I_3 measures informal sector employment using unregistered employment in economic units not distinguished from households, whereas h^I_4 also includes economic units outside the informal sector that is not recognized (e.g., employment without access to basic labor guarantees).

Table 4 summarizes the key business cycle properties of formal employment, informal employment, and the informality rate h^R defined as $h^R \equiv h^I/h$. For all four measures of informality, informal employment is found to be mildly countercyclical with a mean contemporaneous output correlation of -0.12, the informality rate is countercyclical with a mean correlation of -0.36, and

[7] For further discussion, see Fernández and Meza (2015) and the references therein.

formal employment is strongly procyclical with a mean correlation of 0.56. In terms of volatility, formal employment is less volatile over the cycle than total employment, whereas informal employment is more volatile over the cycle than formal employment (the exception being measure h_4^I).

In summary, we have shown that there are important differences between developed and emerging economies, particularly in terms of the dynamics of key open-economy variables. Net exports and real imports tend to be more volatile in emerging economies, whereas the terms of trade tends to be more volatile in developed countries. Net exports are found to be significantly more countercyclical in emerging markets, whereas the terms of trade are found to be procyclical in developed countries and generally acyclical or weakly countercyclical for emerging economies. We have also summarized other important differences between developed and emerging countries that have been highlighted elsewhere in the literature, and we have documented the cyclical dynamics of formal and informal employment for Mexico.

3 Model

The model is as follows. Consider a small open economy with incomplete international asset markets. The economy consists of two types of firms: formal firms (F) which produce tradeable goods that are sold to domestic and foreign households, and informal firms (I) that produce non-tradeable goods. All firms are assumed to be competitive. Domestic intermediate goods are combined with imported intermediate goods to make final goods. Households choose how much labor to allocate to the formal and informal sectors. Following the existing literature, our stochastic environment includes both transitory and trend shocks to productivity (e.g., Aguiar and Gopinath, 2007), a foreign demand shock (e.g., Kollmann, 2001; Monacelli, 2005), and a risk premium shock in the presence of (reduced-form) financial frictions (e.g., García-Cicco et al., 2010; Álvarez-Parra et al., 2013). In what follows, upper-case letters denote variables that contain a trend in equilibrium and an asterisk denotes variables for the rest of the world.

3.1 Formal sector

The production technology in the formal sector is given by:

$$Y_t^F = z_t^F \left(K_{t-1}^F\right)^{\alpha} \left(\Gamma_t^F h_t^F\right)^{1-\alpha}, \qquad (1)$$

where K^F_{t-1} and h^F_t represent formal capital and labor usage, respectively, the input share is $\alpha \in (0,1)$, z^F_t denotes transitory productivity shocks, and Γ^F_t denotes trend shocks to productivity. In what follows, $g^F_t \equiv \Gamma^F_t/\Gamma^F_{t-1}$ denotes the growth rate of Γ^F_t.

Letting W_t denote the real wage rate and rr_t denote the rental cost of capital, profit maximization yields the following optimality conditions:

$$W_t = (1-\alpha)\frac{Y^F_t}{h^F_t}, \qquad rr_t = \alpha\frac{Y^F_t}{K^F_{t-1}}. \tag{2}$$

Formal firms sell intermediate goods to domestic and foreign households. Export demand is given by:

$$C^X_t = (1-\kappa^*)\left(\frac{1}{tot_t}\right)^{-\eta^*} Y^*_t, \tag{3}$$

where $\eta^* > 0$ and Y^*_t denotes the (exogenous) output of the rest of the world. Let p^F_t and p^{*F}_t denote the prices of domestic and foreign formal goods, and e_t denote the nominal exchange rate. Assuming that the law of one price holds, the terms of trade tot_t for the small open economy is defined as the relative price of its imports over its exports:

$$tot_t \equiv \frac{e_t p^{*F}_t}{p^F_t}.$$

3.2 Informal sector

The production technology in the informal sector is given by:

$$Y^I_t = z^I_t \left(K^I_{t-1}\right)^{\omega} \left(\Gamma^I_t h^I_t\right)^{1-\omega}, \tag{4}$$

where $\omega \in (0,1)$ and z^I_t and Γ^I_t are exogenous stationary and non-stationary productivity shocks in the informal sector. The growth rate of Γ^I_t is given by $g^I_t \equiv \Gamma^I_t/\Gamma^I_{t-1}$. In order for the model to be consistent with balanced growth, we follow Fernández and Meza (2015) and assume that in the steady state the informal sector grows at the same rate as the formal sector $g^I = g^F = g$, but at a lower level $\tilde{g} = \Gamma^I/\Gamma^F < 1$.

3.3 Households

3.3.1 Consumption indices

Households combine domestic and imported formal goods to create final consumption goods C_t^T according to the following production technology:

$$C_t^T = \left[\kappa^{\frac{1}{\eta}} \left(C_t^F \right)^{\frac{\eta-1}{\eta}} + (1-\kappa)^{\frac{1}{\eta}} \left(C_t^M \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{5}$$

where C_t^F and C_t^M are the consumption demands of domestic and imported intermediate formal goods, $\kappa \in (0,1)$ is the relative share of intermediate inputs used in the production process, and $\eta > 0$ is the constant elasticity of substitution between domestic and foreign intermediate formal goods. The zero-profit condition is given by:

$$\frac{p_t^T}{p_t^F} C_t^T = C_t^F + tot_t C_t^M, \tag{6}$$

where p_t^T / p_t^F is the relative price of final consumption goods in terms of formal goods. Profit maximization yields the following demand conditions:

$$C_t^F = \kappa \left(\frac{p_t^T}{p_t^F} \right)^{\eta} C_t^T, \qquad C_t^M = (1-\kappa) \left(tot_t \right)^{-\eta} \left(\frac{p_t^T}{p_t^F} \right)^{\eta} C_t^T, \tag{7}$$

and the corresponding relative price index is given by:

$$\frac{p_t^T}{p_t^F} = \left[\kappa + (1-\kappa) \left(tot_t \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{8}$$

Aggregate consumption C_t is defined as:

$$C_t = \left[a^{\frac{1}{\theta}} \left(C_t^T \right)^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} \left(C_t^I \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{9}$$

where $a \in (0,1]$ is the share of formal goods in consumption and $\theta > 0$ is the elasticity of substitution between formal and informal goods. The zero-profit condition for aggregate consumption is given by:

$$\frac{p_t}{p_t^F} C_t = \frac{p_t^T}{p_t^F} C_t^T + \frac{p_t^I}{p_t^F} C_t^I, \tag{10}$$

where p_t / p_t^F denotes the relative price of aggregate consumption goods in terms of formal goods and p_t^I/p_t^F denotes the relative price of informal consumption goods in terms of formal goods. Profit maximization yields the following demand conditions:

$$C_t^T = a\left(\frac{p_t^T}{p_t}\right)^{-\theta} C_t, \qquad C_t^I = (1-a)\left(\frac{p_t^I}{p_t}\right)^{-\theta} C_t, \tag{11}$$

along with the aggregate consumption price index:

$$p_t = \left[a\left(p_t^T\right)^{1-\theta} + (1-a)\left(p_t^I\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{12}$$

We measure net exports nx_t as the difference between exports and imports divided by formal output:

$$nx_t = \frac{C_t^X - tot_t C_t^M}{Y_t^F}. \tag{13}$$

3.3.2 The representative household

The representative household is infinitely lived and chooses aggregate consumption C_t and labor h_t to maximize expected discounted lifetime utility:

$$\max \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, h_t\right), \tag{14}$$

$$h_t = h_t^F + h_t^I, \tag{15}$$

where $\beta \in (0,1)$ denotes the subjective discount factor and formal h_t^F and informal h_t^I labor are assumed to be perfect substitutes. Following Greenwood et al. (1988), household preferences are given by:

$$U = \frac{1}{1-\sigma}\left[\left(C_t - \frac{\psi}{1+\nu}\Gamma_{t-1}^F h_t^{1+\nu}\right)^{1-\sigma} - 1\right],$$

where $\sigma > 0$ is the relative risk aversion coefficient in consumption, $\nu \geq 0$ is the inverse of the Frisch elasticity of labor supply, and $\psi > 0$.

The government raises revenue by levying a proportional income tax τ^T on the household's total formal income: $W_t h_t^F + rr_t K_{t-1}^F$. The household's after-tax resources can be used to finance consumption and investment. The laws of motion for the capital stock in each sector are given by:

$$K_t^F = I_t^F + (1-\delta_F)K_{t-1}^F - \frac{\phi}{2}\left(\frac{K_t^F}{K_{t-1}^F} - g\right)^2 K_{t-1}^F, \tag{16}$$

$$K_t^I = I_t^I + (1-\delta_I)K_{t-1}^I - \frac{\phi}{2}\left(\frac{K_t^I}{K_{t-1}^I} - g\right)^2 K_{t-1}^I, \tag{17}$$

where $\delta_F, \delta_I \in (0,1)$ denote the respective depreciation rates of capital for the formal and informal sectors and $\phi \geq 0$ determines the degree of (quadratic) adjustment costs to capital.

Letting D_{t-1} denote the stock of debt in period t expressed in units of foreign formal goods, the period budget constraint of the representative household is given by:

$$\frac{tot_t D_t}{1+r_t} = tot_t D_{t-1} + \frac{p_t^T}{p_t^F}C_t^T + \frac{p_t^I}{p_t^F}C_t^I + I_t^F - \frac{p_t^I}{p_t^F}Y_t^I + \frac{p_t^I}{p_t^F}I_t^I - (1-\tau^T)\left(W_t h_t^F + rr_t K_{t-1}^F\right). \tag{18}$$

The household maximizes (14) subject to (15)–(18) and (4).

Let $\lambda_t \left(\Gamma_{t-1}^F\right)^{-\sigma}$ denote the Lagrange multiplier. The first-order conditions associated with the household maximization problem are:

$$\left(C_t - \frac{\psi}{1+\nu}\Gamma_{t-1}^F h_t^{1+\nu}\right)^{-\sigma} = \lambda_t \left(\Gamma_{t-1}^F\right)^{-\sigma} \frac{p_t}{p_t^F}, \tag{19}$$

$$\psi \Gamma_{t-1}^F h_t^\nu = (1-\tau^T) W_t \frac{p_t^F}{p_t}, \tag{20}$$

$$\psi \Gamma_{t-1}^F h_t^\nu = (1-\omega) z_t^I \left(\frac{K_{t-1}^I}{h_t^I}\right)^\omega \left(\Gamma_t^I\right)^{1-\omega} \frac{p_t^I}{p_t}, \tag{21}$$

$$\lambda_t \left(\Gamma_{t-1}^F\right)^{-\sigma}\left[1 + \phi\left(\frac{K_t^F}{K_{t-1}^F} - g\right)\right] =$$

$$\beta \mathrm{E}_t \left\{\lambda_{t+1}\left(\Gamma_t^F\right)^{-\sigma}\left[1 - \delta_F + (1-\tau^T) rr_{t+1} + \phi\left(\frac{K_{t+1}^F}{K_t^F} - g\right)\left(\frac{K_{t+1}^F}{K_t^F}\right) - \frac{\phi}{2}\left(\frac{K_{t+1}^F}{K_t^F} - g\right)^2\right]\right\}, \tag{22}$$

$$\lambda_t \left(\Gamma_{t-1}^F\right)^{-\sigma} \frac{p_t^I}{p_t^F}\left[1 + \phi\left(\frac{K_t^I}{K_{t-1}^I} - g\right)\right] = \beta \mathrm{E}_t \left\{\lambda_{t+1}\left(\Gamma_t^F\right)^{-\sigma} \frac{p_{t+1}^I}{p_{t+1}^F} z_{t+1}^I \omega \left(K_t^I\right)^{\omega-1}\left(\Gamma_{t+1}^I h_{t+1}^I\right)^{1-\omega}\right\}$$

$$+\beta \mathrm{E}_t \left\{\lambda_{t+1}\left(\Gamma_t^F\right)^{-\sigma} \frac{p_{t+1}^I}{p_{t+1}^F}\left[1 - \delta_I + \phi\left(\frac{K_{t+1}^I}{K_t^I} - g\right)\left(\frac{K_{t+1}^I}{K_t^I}\right) - \frac{\phi}{2}\left(\frac{K_{t+1}^I}{K_t^I} - g\right)^2\right]\right\}, \tag{23}$$

$$\frac{\lambda_t \left(\Gamma_{t-1}^F\right)^{-\sigma} tot_t}{1+r_t} = \beta \mathrm{E}_t \left\{\lambda_{t+1}\left(\Gamma_t^F\right)^{-\sigma} tot_{t+1}\right\}. \tag{24}$$

The transversality condition is given by:

$$\lim_{l\to\infty} \mathrm{E}_t \left\{ \frac{D_{t+l}}{\prod_{m=0}^{l}(1+r_m)} \right\} = 0. \tag{25}$$

3.3.3 International asset market

The international asset market structure is assumed to be incomplete. The household can purchase international risk-free bonds at a price that costs the inverse of the gross domestic interest rate $1+r_t$. Similar to García-Cicco et al. (2010) and Álvarez-Parra et al. (2013), the country faces a debt-elastic interest-rate premium:

$$r_t = r^* + \chi_d \left[\exp\left(\frac{\tilde{D}_t}{\Gamma_t^F} - d \right) - 1 \right] + \chi_y \left(\mathrm{E}_t \left\{ \frac{Y_{t+1}^F}{\Gamma_t^F} \right\} - y^F \right) + \exp\left(\varrho_t\right) - 1, \tag{26}$$

where $\chi_d > 0$, $\chi_y \le 0$, $r^* > 0$ is the exogenous world interest rate, d and y^F are the steady-state levels of detrended aggregate debt and formal output, and $\tilde{D}_t$ denotes the exogenous aggregate debt level, where in equilibrium $D_t = \tilde{D}_t$. In (26), the debt-elastic interest-rate premium $\chi_d \left[\exp\left(\frac{\tilde{D}_t}{\Gamma_t^F} - d \right) \right]$ is sufficient to ensure that debt holdings are stationary.[8] The risk premium shock ϱ_t and the term $\chi_y \left(\mathrm{E}_t \left\{ \frac{Y_{t+1}^F}{\Gamma_t^F} \right\} - y^F \right)$ capture financial frictions in the economy.

3.3.4 Government

Government consumes only domestically-produced formal goods GS_t and raises revenue from taxing the formal income of households. It is assumed that the government must balance its budget in every period:

$$GS_t = \tau^T \left(W_t h_t^F + rr_t K_{t-1}^F \right). \tag{27}$$

3.3.5 Market clearing

Market clearing in the formal sector requires:

$$Y_t^F = C_t^F + C_t^X + GS_t + I_t^F + \frac{\phi}{2} \left(\frac{K_t^F}{K_{t-1}^F} - g \right)^2 K_{t-1}^F, \tag{28}$$

[8] For an in-depth discussion of the stationary problem of small open economy models, see Schmitt-Grohé and Uribe (2003).

and market clearing in the informal sector requires:

$$Y_t^I = C_t^I + I_t^I + \frac{\phi}{2}\left(\frac{K_t^I}{K_{t-1}^I} - g\right)^2 K_{t-1}^I. \tag{29}$$

Combining the government (27) and household (18) budget constraints, the market-clearing conditions (28) and (29), the zero profit-conditions (6) and (10), and noting that formal firms make zero profit: $Y_t^F = W_t h_t^F + rr_t K_{t-1}^F$, yields the following economy-wide resource constraint:

$$\frac{D_t}{1+r_t} = D_{t-1} + C_t^M - \frac{C_t^X}{tot_t}. \tag{30}$$

The small open economy model is closed by specifying an exogenous process for both the world interest rate r^* and export demand C_t^X.

3.3.6 Equilibrium

Given the initial conditions K_0^F, K_0^I, D_0, and the exogenous shock processes g_t^F, g_t^I, z_t^F, z_t^I, ϱ_t, r^*, and C_t^X, an equilibrium for the small open economy consists of a set of real prices r_t, W_t, rr_t, λ_t, a set of relative prices p_t/p_t^F, p_t^T/p_t^F, p_t^I/p_t^F, tot_t, and a collection of allocations C_t, C_t^T, C_t^F, C_t^I, C_t^M, h_t, h_t^F, h_t^I, K_t^F, K_t^I, I_t^F, I_t^I, Y_t^F, Y_t^I, GS_t, D_t, nx_t, satisfying (i) the optimality conditions of households (19)–(24) and the transversality condition (25); (ii) the optimality conditions of formal and informal firms (1), (2), (4); (iii) the government budget constraint (27); (iv) the laws of motion for capital (16)–(17); (v) the international asset market condition (26); (vi) the aggregate labor constraint (15); (vii) the aggregation conditions for aggregate consumption (10)–(11) and tradeable consumption goods (7)–(8); (viii) all markets clear (28)–(29); (ix) the economy-wide resource constraint (30); and (x) the definition of net exports (13).

3.4 Exogenous shock processes

As is standard in the literature, the stationary productivity shocks are assumed to follow an independent AR(1) process:

$$\ln z_t^j = \rho_{z_j} \ln z_{t-1}^j + \epsilon_t^{z_j},$$

where $\rho_{z_j} \in (0,1)$ and $\epsilon_t^{z_j} \sim N(0, \sigma_{z_j}^2)$ with $\sigma_{z_j}^2 > 0$ for $j \in [F, I]$.

For the nonstationary shocks to productivity, we assume that g_t^F, the growth rate of Γ_t^F, follows an AR(1) process:

$$\ln\left(g_t^F/g\right) = \rho_g \ln\left(g_{t-1}^F/g\right) + \epsilon_t^g,$$

where $\rho_g \in (0,1)$ and $\epsilon_t^g \sim N(0,\sigma_g^2)$ with $\sigma_g^2 > 0$. Following Fernández and Meza (2015), we assume that g_t^I, the growth rate of Γ_t^I, follows:

$$g_t^I = \left(g_t^F\right)^{\varphi}\left(g_{t-1}^I\right)^{1-\varphi},$$

where $\varphi \in (0,1)$ denotes the degree of pass-through of growth shocks from the formal sector to the informal sector.

The risk premium shock ϱ_t and the (detrended) foreign demand shock y_t^* are both assumed to follow a standard AR(1) process:

$$\ln\varrho_t = \rho_\varrho \ln\varrho_{t-1} + \epsilon_t^\varrho, \qquad \ln y_t^* = \rho_{y^*}\ln y_{t-1}^* + \epsilon_t^{y^*},$$

where $\rho_\varrho, \rho_{y^*} \in (0,1)$, $\epsilon_t^\varrho \sim N(0,\sigma_\varrho^2)$, and $\epsilon_t^{y^*} \sim N(0,\sigma_{y^*}^2)$ with $\sigma_\varrho^2, \sigma_{y^*}^2 > 0$.

3.5 Solution method

To solve the model, we first detrend the equilibrium system of equations and then log-linearize the detrended equilibrium conditions around the deterministic steady state. Appendix 1 lists the stationary system of model equations, the steady state, and complete log-linearized model.

4 Results

4.1 Calibration

The model is calibrated at a quarterly frequency for Mexico, a representative emerging market economy with a large informal sector, which is commonly used in the literature. The parameter values used to compute the equilibrium are summarized in Table 5. For the case of Mexico, the parameters β, σ, ν, α, and δ_F have a standard calibration in the literature (see, e.g., Aguiar and Gopinath, 2007; García-Cicco et al., 2010; Álvarez-Parra et al., 2013, among others). We set the discount factor $\beta = 0.98$, the risk aversion parameter $\sigma = 2$, and the inverse of the Frisch elasticity of labor supply $\nu = 0.6$. For the technology parameters, we set the capital share in formal production $\alpha = 0.32$ and the depreciation rate of formal capital $\delta_F = 0.05$.

Given the lack of data for the informal sector, it is not possible to directly calibrate the parameters δ_I and ω. We assume that the depreciation rate of capital in the informal sector is the same as the formal sector: $\delta_I = \delta_F = 0.05$. It is generally assumed in the literature that the informal sector is more labor intensive than the formal sector. We follow Fernández and Meza (2015) and set the informal capital share equal to $\omega = 0.2$.

Using data from the OECD for the period 1993–2019, we calibrate the steady-state trend growth rate to be $g = 1.0057$ and the income tax on formal income $\tau^T = 0.1056$. In the steady state, the (exogenous) world interest rate r^* is set to satisfy the steady state condition $\beta(1 + r^*) = g^\sigma$. To ensure stationarity of the steady state, we set the debt-elastic interest rate premium $\chi_d = 0.001$, a standard value in the literature (see, e.g., García-Cicco et al., 2010). We set the financial frictions parameter $\chi_y = -0.031$ consistent with the estimates for Mexico of Álvarez-Parra et al. (2013).

The parameters determining the degree of home bias κ, the share of informal goods in consumption $1 - a$, the ratio of formal to informal total factor productivity levels TFP_I^F, and the weight of (aggregate) labor in household preferences ψ are jointly calibrated to satisfy the following four steady state targets for Mexico: (i) households allocate one-third of their time to formal work $h^F = 0.33$ (Boz et al., 2015); (ii) an informal labor share of $h^I/h = 0.334$, which is equal to the average of the four measures of informality given in Section 2.3; (iii) a formal-sector wage premium of 23.5% (Esteban-Pretel and Kitao, 2021); (iv) the share of imports in GDP calibrated to be 29%. Additionally, we ensure that the steady state replicates the external debt to GDP ratio $d/y^F = 0.333$, as calibrated by Boz et al. (2015), and the ratio of consumption to GDP $c^T/y^F = 0.66$, consistent with the average estimate for Mexico over the sample period.

With regard to the elasticity of substitution between formal and informal goods θ and the pass-through of growth shocks from the formal to informal sector φ, we investigated how these two parameters affect the model-implied moments. We find that these two parameters mainly affect the (formal) output correlation of both formal and informal labor. We obtain the best results setting $\theta = 2$ and $\varphi = 0.77$. Therefore, we calibrate θ and φ at these values in the benchmark model, and keep them fixed in all simulations.[9] We ignore capital adjustment costs by setting $\phi = 0$, since the standard deviation for investment generated by the model is relatively low compared to the data.

Following Aguiar and Gopinath (2007), we set the autocorrelation of growth shocks $\rho_g = 0.72$

[9]Note that a value of $\theta = 2$ is consistent with estimates from the household production literature (see, e.g., Chang and Schorfheide, 2003) for the elasticity of substitution between market and non-market consumption.

Table 5: Baseline calibrated parameters

Parameter	Value	Description	Source
Values taken from the literature			
β	0.98	Discount factor	Aguiar and Gopinath (2007)
σ	2	Relative risk aversion	Aguiar and Gopinath (2007)
$\delta_F = \delta_I$	0.05	Depreciation rate of capital	Aguiar and Gopinath (2007)
α	0.32	Formal capital share	Aguiar and Gopinath (2007)
ω	0.20	Capital share in informal production	Fernández and Meza (2015)
ν	0.6	Inverse of the Frisch labor supply elasticity	García-Cicco et al. (2010)
r^*	0.0347	World interest rate	Satisfies $\beta(1+r^*) = g^\sigma$
d/y^F	0.333	Steady state external debt to GDP ratio	Boz et al. (2015)
χ_d	0.001	Debt-elastic interest rate premium	García-Cicco et al. (2010)
χ_y	-0.031	Financial frictions parameter	Álvarez-Parra et al. (2013)
ρ_g	0.72	Autocorrelation of growth shocks	Aguiar and Gopinath (2007)
$\rho_z^F = \rho_z^I$	0.94	Autocorrelation of technology shocks	Aguiar and Gopinath (2007)
ρ_ϱ	0.69	Autocorrelation of risk premium shocks	Álvarez-Parra et al. (2013)
Values estimated from data			
g	1.0057	Steady state trend growth rate	Data: OECD
τ^T	0.1056	Tax on formal income	Data: OECD
c^T/y^F	0.66	Steady state consumption to GDP ratio	Data: OECD
σ_{y^*}	0.4867	Standard deviation of foreign demand shocks	Data: BEA
ρ_{y^*}	0.88	Autocorrelation of foreign demand shocks	Data: BEA
Calibrated from steady state targets			
ψ	1.57	GHH utility labor weight parameter	
$1-a$	0.33	Share of informal goods in consumption	
κ	0.59	Degree of home bias	
TFP_I^F	1.0062	Ratio of formal to informal TFP levels	
Calibrated to match business cycle moments			
θ	2	Elasticity of substitution between formal & informal goods	
φ	0.77	Pass-through of growth shocks from formal to informal sector	

and the autocorrelation of technology shocks $\rho_z^F = \rho_z^I = 0.94$. For the autocorrelation of risk premium shocks, we choose a value of $\rho_\varrho = 0.69$, consistent with estimates for Mexico by Álvarez-Parra et al. (2013). Using U.S. GDP data from the Bureau of Economic Analysis for the period 1993–2019, the foreign demand shock process parameters are estimated as $\sigma_{y^*} = 0.4867$ and $\rho_{y^*} = 0.88$.

We choose the standard deviations of the shocks, σ_g, σ_z^F, σ_z^I, σ_ϱ, and the elasticity of substitution between domestic and foreign tradeable goods, $\eta = \eta^*$, so as to reproduce key business cycle moments of the data, using a simulated method of moments approach. Specifically, we calibrate the volatility and cross-correlation of the shocks so as to minimize the distance between selected model moments and data moments.[10] We include the following five moments in the objective function: the standard deviations of formal output, informal hours, the terms of trade and the real interest rate, plus the correlation between formal hours and formal output, $\rho\left(h^F, y^F\right)$. These moments were selected because they are all influenced by the estimated parameters. In particular, the standard deviation of informal hours is strongly affected by σ_z^I, while the standard deviation of the terms of trade is strongly affected by the trade elasticities η and η^*. We include the standard deviations of the real interest rate in the set of targeted moments to ensure that the risk premium shocks in our model are consistent with the data. Similar to Fernandez and Meza (2015), we find that permanent technology shocks strongly affect the cyclical properties of the labour market. For this reason, we include the correlation $\rho\left(h^F, y^F\right)$ in the set of targeted moments.

4.2 Business cycle moments

We now evaluate the ability of the model to replicate the business cycle properties for Mexico. Similar to Restrepo-Echavarria (2014) and Fernández and Meza (2015), we assume that informal output is unmeasured, such that only formal output is captured in the national accounts data. We compute the second moments for formal and informal employment using the measure of employment in the informal sector h_3^I outlined in Section 2.3. The moments targeted in the calibration are denoted by $\star$. The estimated moments for the Mexican data presented in Section 2 are summarized in column two of Table 6, whereas the third column of Table 6 gives the model-generated moments for the benchmark model.

The benchmark model can successfully replicate several business cycle moments of the data. First, the model matches well the second moments for the open economy variables. In terms of (formal) output correlations, the model correctly predicts the procyclicality of both exports (a correlation of 0.16 compared to 0.40 in the data) and imports (0.83 vs 0.91 in the data), and the strong countercyclicality of net exports (-0.50 vs -0.66 in the data) and the terms of trade (-0.54 vs -0.19 in the data). In terms of volatilities, the model can generate 70% of the observed standard deviation for net exports. The model also correctly predicts that imports are more volatile than

[10] The length of the simulated series is $5,000$ periods after discarding the first 500 observations.

Table 6: Business cycle moments for Mexico: Data and model

	Data (Mexico)	Benchmark model	No risk prem. shocks ϱ_t	No trend shocks g_t	No informality	No EME features
Standard deviations						
$\sigma\left(y^F\right)$	2.16	2.16*	2.14*	2.16*	2.21*	2.15*
$\sigma\left(c^T\right)$	2.15	2.90	2.79	0.96	2.79	0.46
$\sigma\left(i^F\right)$	7.72	5.55	5.30	6.18	5.77	5.29
$\sigma(h)$	0.78	1.37	1.29	0.74	1.64	0.64
$\sigma\left(h^F\right)$	0.78	1.53	1.47	1.03	1.64	0.64
$\sigma\left(h^I\right)$	2.04	2.03*	2.04*	2.05*	–	–
$\sigma\left(h^I/h\right)$	1.61	1.46	1.53	1.77	–	–
$\sigma\left(tot\right)$	2.96	2.94*	2.95*	2.96*	2.88*	2.95*
$\sigma\left(c^M\right)/\sigma\left(c^X\right)$	1.41	1.69	1.70	0.83	1.61	0.42
$\sigma\left(nx\right)$	0.99	0.69	0.62	0.35	0.64	0.26
$\sigma\left(r\right)$	0.49	0.48*	0.00	0.49*	0.49*	0.00
First-order autocorrelations						
$\rho\left(y^F_t, y^F_{t-1}\right)$	0.82	0.78	0.78	0.71	0.78	0.72
Correlations with output						
$\rho\left(c^T, y^F\right)$	0.93	0.89	0.87	0.54	0.94	0.89
$\rho\left(i^F, y^F\right)$	0.90	0.87	0.87	0.97	0.86	0.98
$\rho\left(h, y^F\right)$	0.38	0.48	0.47	0.49	0.65	0.93
$\rho\left(h^F, y^F\right)$	0.63	0.64*	0.64*	0.85	0.65*	0.93
$\rho\left(h^I, y^F\right)$	-0.11	0.02	-0.02	-0.32	–	–
$\rho\left(h^I/h, y^F\right)$	-0.32	-0.43	-0.43	-0.58	–	–
$\rho\left(tot, y^F\right)$	-0.19	-0.54	-0.44	0.76	-0.60	0.96
$\rho\left(c^X, y^F\right)$	0.40	0.16	0.25	0.76	0.17	0.95
$\rho\left(c^M, y^F\right)$	0.91	0.83	0.79	-0.13	0.88	-0.72
$\rho\left(nx, y^F\right)$	-0.66	-0.50	-0.47	-0.73	-0.50	-0.82
$\rho\left(r, y^F\right)$	-0.55	-0.16	0.54	-0.21	-0.19	0.07
Estimated parameter values						
σ_g		0.66	0.63	–	0.66	–
σ_z^F		0.51	0.59	1.17	0.49	1.30
σ_z^I		0.76	0.79	0.84	–	–
σ_ϱ		0.42	–	0.42	0.42	–
η, η^*		0.55	0.51	0.53	0.52	0.56

Notes: y^F, c^T, i^F, h, h^F, h^I, h^I/h, tot, c^X, c^M, nx, r denote, respectively, output, consumption, investment, total employment, formal employment, informal employment, informality rate, terms of trade, exports, imports, net exports, and the real interest rate. $\sigma(z)$ denotes the standard deviation of z and $\rho(z,y)$ denotes the correlation between z and y. Moments targeted are denoted by $\star$. The second moments for h^F, h^I, and h^I/h are computed using the informality measure h^I_3 of Section 2.3.

exports generating a relative volatility $\frac{\sigma(c^M)}{\sigma(c^X)} = 1.69$ close to the value found in the data (1.41).

Second, for the employment variables, the model correctly predicts the procyclicality of aggregate employment (0.48 vs 0.38 in the data) and the strong countercyclicality of the informality rate (-0.43 vs -0.32 in the data). The model predicts that informal employment is acyclical (0.02) rather than the very mild countercyclicality found in the data (-0.11). Moreover, the model generates sufficient volatility for aggregate and formal employment, and correctly predicts that formal employment is less volatile than informal employment, generating an informality rate volatility of 1.46 close to the data (1.61).

Third, the model reproduces the strong procyclicality of both consumption and investment, can generate over 70% of the observed standard deviation for investment, and predicts that consumption is more volatile than output, which as discussed in Section 2, is an important business-cycle feature of several emerging market economies. Finally, while the model can simultaneously match the volatility of the real interest rate and the countercyclicality of the real interest rate, the correlation of -0.16 is lower than found in the data (-0.55).

In terms of the estimated parameter values given in the bottom panel of Table 6, we find that to best match the data the standard deviation of informal productivity shocks are relatively higher than formal productivity shocks (0.76 vs 0.51) with the standard deviation of nonstationary productivity shocks at 0.66. The risk premium shock has the smallest standard deviation of 0.42. The value of 0.55 calibrated for η and η^* is consistent with recent estimates by Boehm et al. (2019) who estimate the elasticity of substitution between home and foreign tradables for the U.S. within the range $0.42 \leq \eta \leq 0.62$.

Columns four and five of Table 6 investigate the relative importance of financial frictions and trend productivity shocks. We also estimate the properties of the foreign demand shock y_t^* by fitting an AR(1) process to US GDP. In column four we shutoff the risk premium shock ϱ_t and set the financial frictions parameter χ_y equal to zero,[11] whereas in column five the nonstationary productivity shock g_t is shutdown. Consistent with the existing literature (e.g., Neumeyer and Perri, 2005; Uribe and Yue, 2006), risk premium shocks are crucial for the model to reproduce the countercyclicality of the real interest rate (a correlation of -0.16 compared to 0.54 in the absence of financial frictions). In contrast, trend shocks to productivity play no significant role for the behaviour of the real interest rate. However, trend productivity shocks are important in generating

[11] We find that the parameter χ_y plays little role for the second moments generated by the model. Similar to Chang and Fernández (2013), it is the risk premium shock that matters for the performance of the model with financial frictions.

a higher relative volatility of consumption with respect to output (1.34 in the benchmark model versus 1.30 in the absence of financial frictions and 0.44 in the absence of trend shocks), and are crucial for replicating the dynamics of the open-economy variables. In particular, nonstationary productivity shocks are important for the volatility of net exports (0.69 vs 0.35 in the absence of trend shocks) and for the model to replicate the larger volatility of imports relative to exports (1.61 vs 0.83 in the absence of trend shocks). Moreover, trend productivity shocks are key for the model to generate the procyclicality of imports (a correlation of 0.83 compared to -0.13 in the absence of trend shocks) and the countercyclicality of the terms of trade (a correlation of -0.54 compared to 0.76 in the absence of trend shocks). In contrast, financial frictions play a minor role in the model's ability to match the behavior of international relative prices and quantities.

Column six of Table 6 considers a formal-only version of the model by removing the informal sector. Compared to the benchmark model, informality helps increase slightly both the volatility of consumption with respect to output (1.34 vs 1.26 without informality), the relative volatility of the terms of trade (1.36 vs 1.30 without informality), and the volatility of net exports (0.69 vs 0.64 without informality). The remaining second moments remain largely unaffected. The final column of Table 6 considers the predictions of a standard small open economy RBC model with no Emerging Market Economy (EME) features. In this model version, we shutoff the nonstationary productivity shock, omit the informal sector, and remove financial frictions, by eliminating both the risk premium shock and setting $\chi_y = 0$. By inspection, a standard RBC model cannot replicate the observed behaviour of consumption and the real interest rate, nor can it reproduce any major empirical feature of the data in relation to international relative prices and quantities.

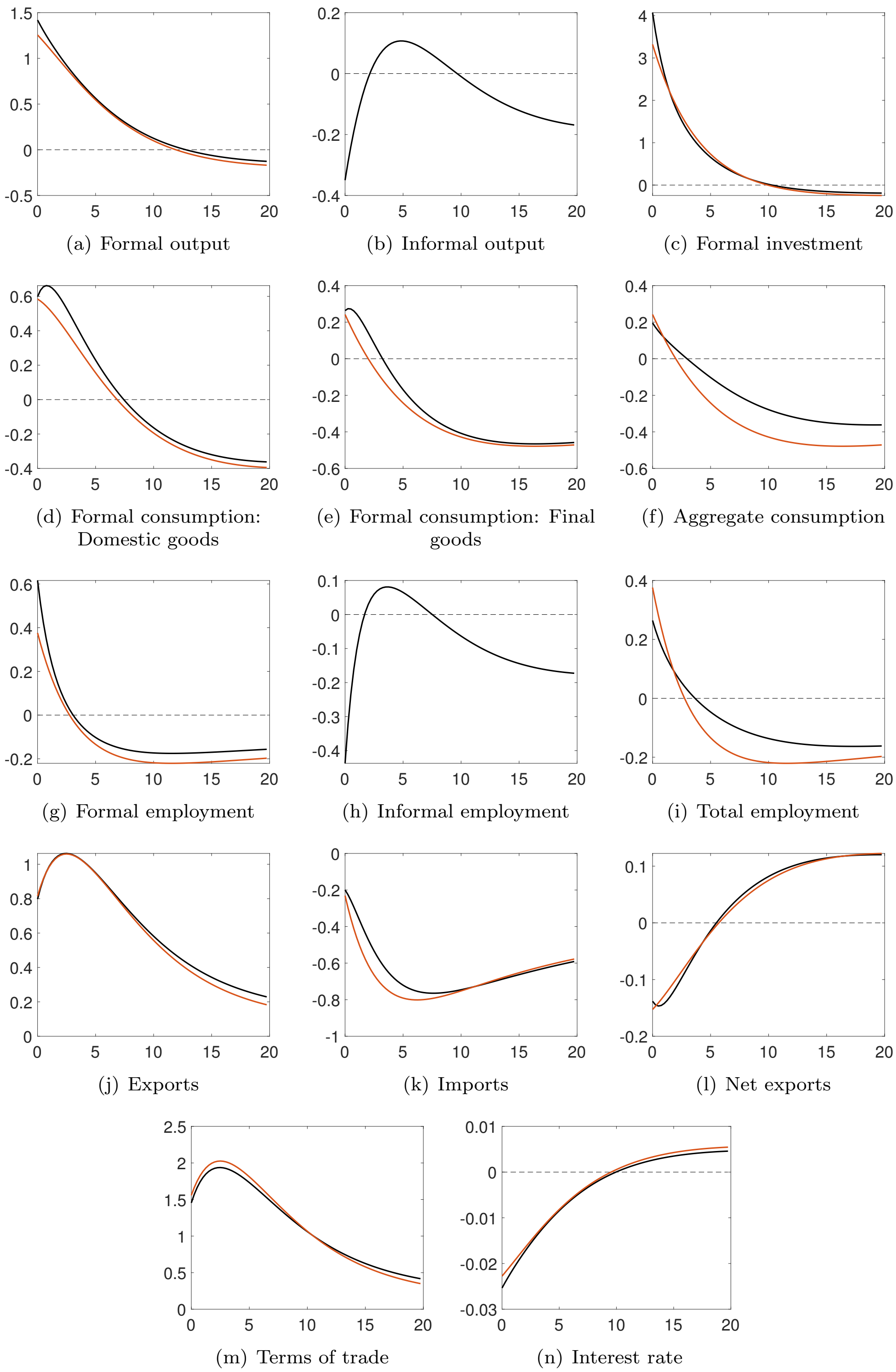
5 Inspecting the mechanism

In this section, we examine how the different shocks are transmitted in both the informal and formal-only versions of the model economy. The key message is that trend productivity shocks are crucial for the model to replicate the key behaviour of international relative prices and quantities, whereas countercyclical risk premium shocks are important for replicating the behaviour of the real exchange rate. While the informal sector is found to amplify the response of the formal sector to transitory productivity shocks, it significantly dampens the transmission of risk premium shocks, while having little effect on the propagation of trend productivity shocks. For completeness, the impulse responses to a foreign demand shock are included in the appendix.

The impulse response functions following a positive formal and informal transitory productivity shock are depicted in Figures 1 and 2. In response to a sector-specific transitory productivity shock, a reallocation occurs towards the relatively more productive sector, resulting in an increase (decrease) in output and employment in the relatively more (less) productive sector. Consequently, since only the formal sector is measured, informal employment is countercyclical under both transitory shocks: a formal (informal) productivity shock results in an increase (decrease) in formal output and a reduction (increase) in informal employment. However, while formal and informal productivity shocks imply a different adjustment mechanism for international relative prices and quantities, the mechanism generated under both shocks is counterfactual with the data. Under a positive formal productivity shock, the increase in formal output results in a fall in its relative price $\widehat{p_t^F/p_t^T}$, and a counterfactual deterioration (i.e., increase) in the terms of trade. While net exports fall below the steady state, as the increase in formal investment is sufficiently large such that domestic absorption initially rises by more than formal output, the fall in imports below the steady state is counterfactual with the data. Under a positive informal productivity shock, the relative price of informal goods $\widehat{p_t^I/p_t^F}$ falls and the relative price of formal goods $\widehat{p_t^F/p_t^T}$ rises resulting in a deterioration in the terms of trade. The economy responds to the fall in production of domestically-produced formal goods (exports), by sucking in more imports. Consequently, informal productivity shocks generate a counterfactual positive correlation between net exports and formal output.

The ability of the model to reproduce the observed behaviour for the terms of trade and net exports rests with the inclusion of nonstationary productivity shocks. The impulse response functions following a positive trend productivity shock are depicted in Figure 3. Since a positive permanent shock to productivity implies a greater increase in permanent income, both formal and aggregate consumption increase significantly compared to transitory productivity shocks. Combined with the large increase in investment in each sector, domestic absorption exceeds formal output, correctly implying that imports are procyclical, and net exports fall below the steady state. However, in stark contrast to transitory formal productivity shocks, $\widehat{C}_t^X - \widehat{C}_t^M$ initially decreases due to an increase in the relative price of formal output $\widehat{p_t^F/p_t^T}$, resulting in an improvement (i.e., decrease) in the terms of trade. Consequently, trend productivity shocks can correctly generate both countercyclical net exports and countercyclical terms of trade.

How important is the informal sector in amplifying stationary and nonstationary productivity



(a) Formal output (b) Informal output (c) Formal investment

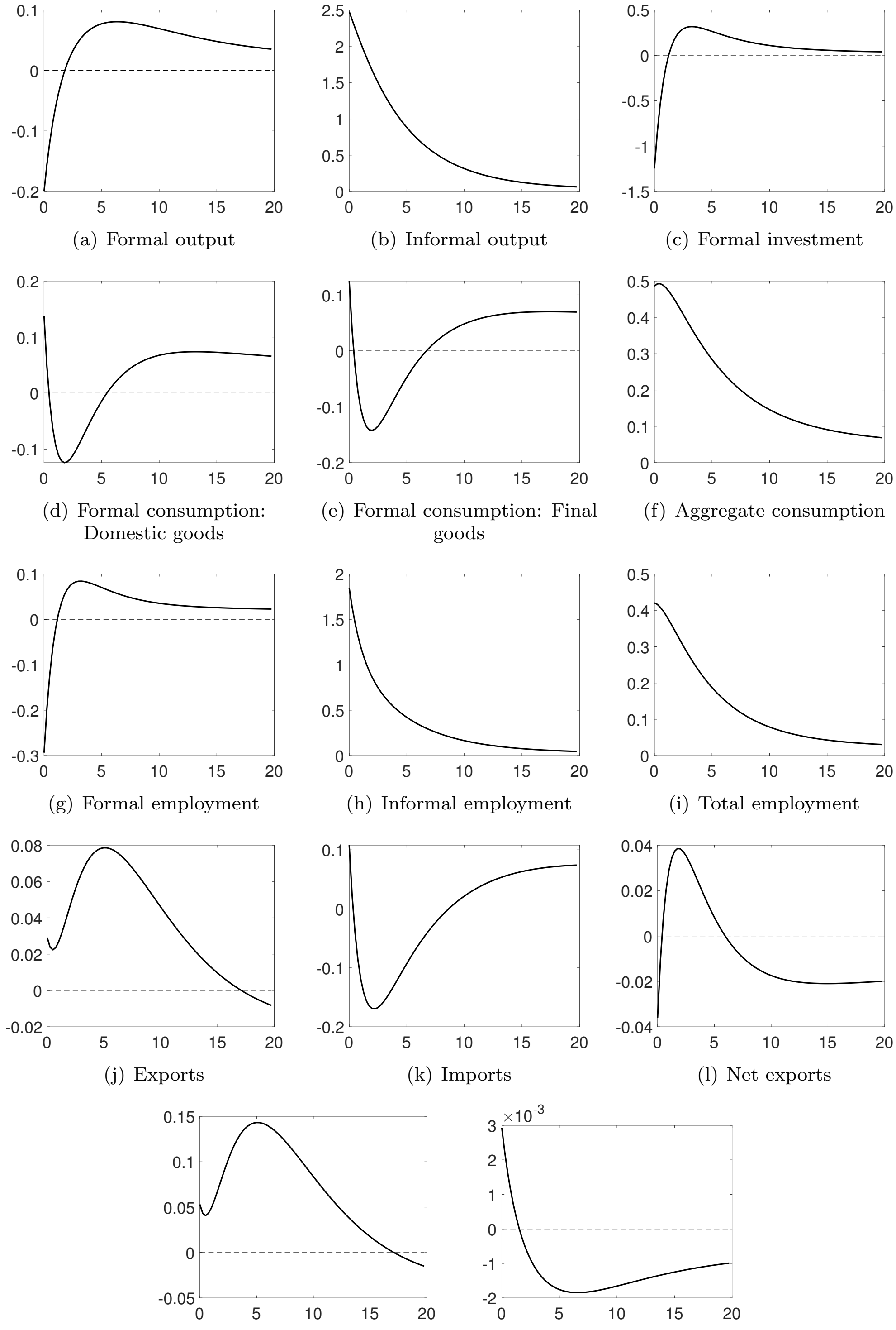
(d) Formal consumption: Domestic goods (e) Formal consumption: Final goods (f) Aggregate consumption

(g) Formal employment (h) Informal employment (i) Total employment

(j) Exports (k) Imports (l) Net exports

(m) Terms of trade (n) Interest rate

Figure 1: Impulse responses for a positive 1% transitory formal productivity shock: Benchmark model with informal sector (–) vs. No informality model (–)



(a) Formal output (b) Informal output (c) Formal investment

(d) Formal consumption: Domestic goods (e) Formal consumption: Final goods (f) Aggregate consumption

(g) Formal employment (h) Informal employment (i) Total employment

(j) Exports (k) Imports (l) Net exports

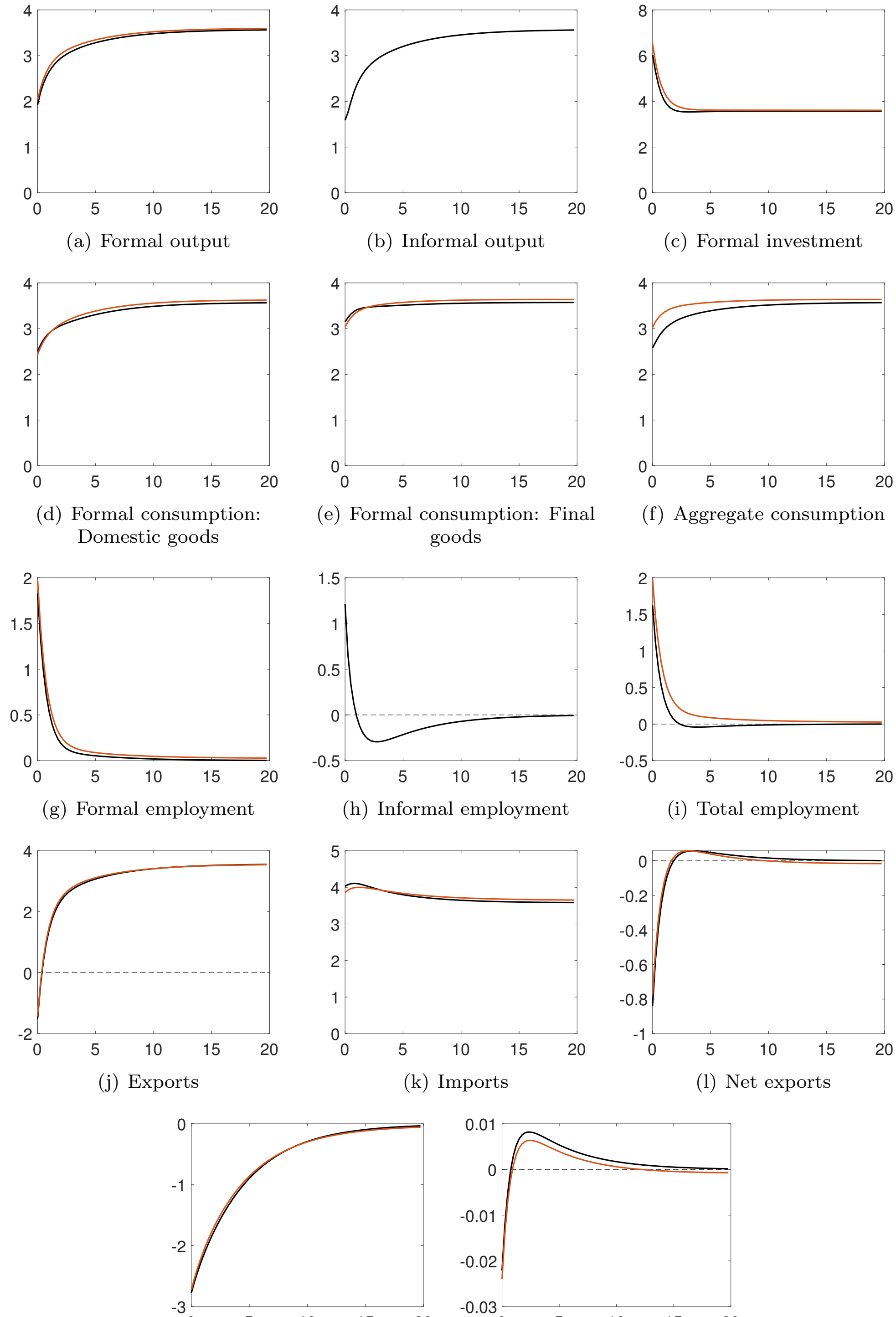
(m) Terms of trade (n) Interest rate

Figure 2: Impulse responses for a positive 1% transitory informal productivity shock

shocks? In Figures 1 and 3, we compare the benchmark informal model (black lines) against the version of the model without informality (red lines). Under transitory formal productivity shocks, the response of formal output, formal employment, and formal investment is larger under informality and the consumption of formal goods is amplified, as households substitute formal for informal consumption. In terms of international relative prices and quantities, informality dampens the response of imports and the terms of trade under transitory productivity shocks, due to the larger increase in formal output. However, while informality magnifies the response of the formal sector to transitory productivity shocks, it has little effect on the propagation of trend productivity shocks, where the behaviour of the terms of trade and net exports are unchanged. Given the importance of trend productivity shocks in helping the model explain the data, this suggests that the model without informality also performs well in explaining the key features of the data.

Figure 4 illustrates the impulse response functions following a rise in the real interest rate caused by an increase in the risk premium. In each sector, a negative risk premium shock results in a contraction in output, investment, and employment, generating a large negative correlation between formal output and the real interest rate. While formal consumption falls, the consumption of informal goods rises as households substitute informal for formal consumption. The increase in savings results in a large increase in net exports and a deterioration (i.e., increase) in the terms of trade. In stark contrast to transitory productivity shocks, informality dampens the response of the formal sector to risk premium shocks, where formal output, formal investment, and formal employment fall less under informality, with minor implications for international relative prices and quantities.

The impulse response analysis suggests that both trend productivity shocks and countercyclical risk premium shocks can generate behaviour for international relative prices and quantities consistent with the data. However, Table 6 of the previous section showed that nonstationary productivity shocks are relatively more important than interest-rate shocks in order for the model to explain the moments of the data. Although risk premium shocks are necessary to match the observed behaviour for the real interest rate, they cannot simultaneously match this moment and the moments relating to exports, imports, the terms of trade, and consumption. In contrast, permanent shocks to productivity can help match the moments of the labor market along with the key open-economy moments.



(a) Formal output (b) Informal output (c) Formal investment

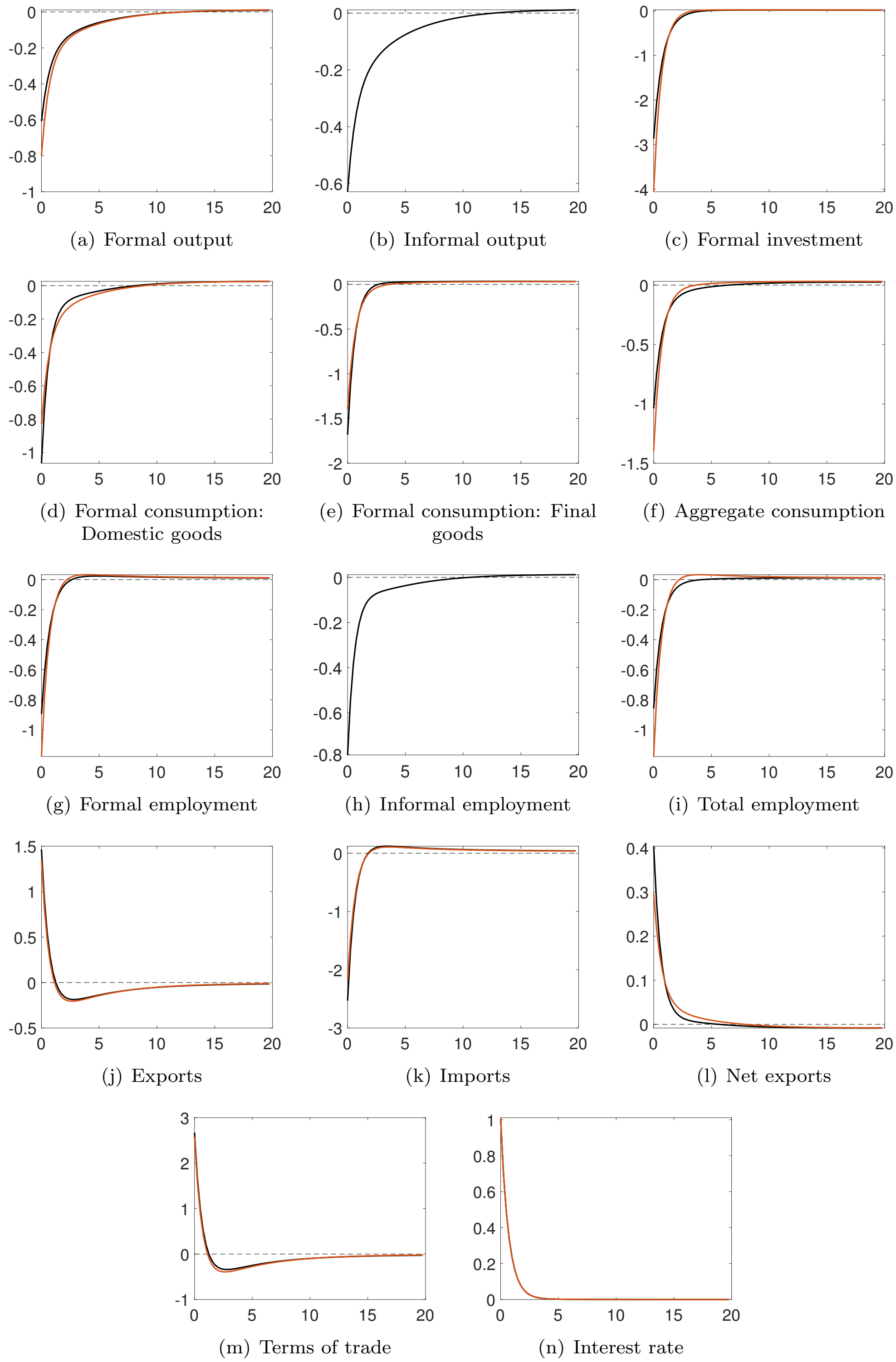
(d) Formal consumption: Domestic goods (e) Formal consumption: Final goods (f) Aggregate consumption

(g) Formal employment (h) Informal employment (i) Total employment

(j) Exports (k) Imports (l) Net exports

(m) Terms of trade (n) Interest rate

Figure 3: Impulse responses for a positive 1% trend productivity shock: Benchmark model with informal sector (–) vs. No informality model (–)



(a) Formal output
(b) Informal output
(c) Formal investment
(d) Formal consumption: Domestic goods
(e) Formal consumption: Final goods
(f) Aggregate consumption
(g) Formal employment
(h) Informal employment
(i) Total employment
(j) Exports
(k) Imports
(l) Net exports
(m) Terms of trade
(n) Interest rate

Figure 4: Impulse responses for a negative 1% risk premium shock: Benchmark model with informal sector (–) vs. No informality model (–)

6 Conclusion

We document important differences between the business cycles of emerging market economies and developed countries relating to the behaviour of exports, imports, and the terms of trade. We develop a two-sector, small open economy model with an endogenous terms of trade in an attempt to replicate the stylized facts. We incorporate three popular features that have been put forward to explaining the aggregate fluctuations of emerging economies using one-good models: trend productivity shocks, interest-rate shocks with financial frictions, and informality. Our analysis offers new insights about the driving forces behind the business cycles of emerging market economies.

Our main findings challenge the conventional view that countercyclical interest-rate shocks and financial frictions are key to understanding the business cycles of emerging economies. We show that trend productivity shocks remain crucial in explaining the cyclical behaviour for international relative prices and quantities. While financial frictions in the form of interest-rate shocks are needed to replicate the empirical evidence observed for the real interest rate, they contribute little in explaining the dynamics of all other variables. Our analysis also disputes the role of informality as a powerful mechanism for amplifying shocks in emerging countries. In our analysis, informality reduces significantly the response of the formal sector to interest-rate shocks. Moreover, in stark contrast to one-good models, informality has little effect on the propagation of trend productivity shocks. These results suggest that furthering our understanding about other market imperfections and frictions in emerging market business cycles, which are captured by trend shocks in our analysis, would be a promising area for future research.

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7 Appendix

7.1 Detrended equilibrium conditions

Let $s_t \equiv \frac{S_t}{\Gamma^F_{t-1}}$, where $S_t = C_t, C^T_t, C^F_t, C^I_t, C^M_t, K^F_{t-1}, K^I_{t-1}, Y^F_t, Y^I_t, D_{t-1}$. De-trending the household optimality conditions (19)–(24) yields (after using (2) to eliminate W_t and rr_{t+1}):

$$\left(c_t - \frac{\psi}{1+\nu} h_t^{1+\nu}\right)^{-\sigma} = \lambda_t \frac{p_t}{p^F_t},$$

$$\psi h_t^{\nu} = (1-\tau^T)(1-\alpha)\frac{y^F_t}{h^F_t}\frac{p^F_t}{p_t},$$

$$\psi h_t^{\nu} = \frac{p^I_t}{p_t} z^I_t (1-\omega)\left(\frac{k^I_{t-1}}{h^I_t}\right)^{\omega}\left(\tilde{g}_t g^F_t\right)^{1-\omega},$$

$$\beta E_t \left\{ \left(g^F_t\right)^{-\sigma} \lambda_{t+1} \left[1 - \delta_F + (1-\tau^T)\alpha \frac{y^F_{t+1}}{k^F_t} + \phi\left(\frac{k^F_{t+1}}{k^F_t} g^F_{t+1} - g\right)\left(\frac{k^F_{t+1}}{k^F_t} g^F_{t+1}\right) - \frac{\phi}{2}\left(\frac{k^F_{t+1}}{k^F_t} g^F_{t+1} - g\right)^2\right]\right\}$$
$$= \lambda_t \left[1 + \phi\left(\frac{k^F_t}{k^F_{t-1}} g^F_t - g\right)\right],$$

$$\lambda_t \frac{p^I_t}{p^F_t}\left[1 + \phi\left(\frac{k^I_t}{k^I_{t-1}} g^F_t - g\right)\right] = \beta E_t \left\{ \left(g^F_t\right)^{-\sigma} \lambda_{t+1} \frac{p^I_{t+1}}{p^F_{t+1}} \omega \left(k^I_t\right)^{\omega-1} z^I_{t+1} \left(\tilde{g}_t g^I_{t+1} h^I_{t+1}\right)^{1-\omega}\right\}$$
$$+\beta E_t \left\{ \left(g^F_t\right)^{-\sigma} \lambda_{t+1} \frac{p^I_{t+1}}{p^F_{t+1}} \left[1 - \delta_I + \phi\left(\frac{k^I_{t+1}}{k^I_t} g^F_{t+1} - g\right)\left(\frac{k^I_{t+1}}{k^I_t} g^F_{t+1}\right) - \frac{\phi}{2}\left(\frac{k^I_{t+1}}{k^I_t} g^F_{t+1} - g\right)^2\right]\right\},$$

$$\frac{\lambda_t tot_t}{1+r_t} = \beta E_t \left\{ \left(g^F_t\right)^{-\sigma} \lambda_{t+1} tot_{t+1}\right\},$$

where $\tilde{g}_{t-1} \equiv \frac{\Gamma^I_{t-1}}{\Gamma^F_{t-1}}$, and from (15):

$$h_t = h^F_t + h^I_t.$$

De-trending the production function of formal (1) and informal firms (4), the aggregation conditions for aggregate consumption goods (9)–(10), and the aggregation conditions for tradeable consumption (6)–(7), and the export demand condition (3) yields:

$$y^F_t = z^F_t \left(k^F_{t-1}\right)^{\alpha}\left(g^F_t h^F_t\right)^{1-\alpha},$$

$$y^I_t = z^I_t \left(k^I_{t-1}\right)^{\omega}\left(\tilde{g}_{t-1} g^I_t h^I_t\right)^{1-\omega},$$

$$\frac{p_t}{p^F_t} c_t = \frac{p^T_t}{p^F_t} c^T_t + \frac{p^I_t}{p^F_t} c^I_t,$$

$$c_t^T = a\left(\frac{p_t^T}{p_t}\right)^{-\theta} c_t,$$

$$c_t^I = (1-a)\left(\frac{p_t^I}{p_t}\right)^{-\theta} c_t,$$

$$c_t^F = \kappa\left(\frac{p_t^T}{p_t^F}\right)^{\eta} c_t^T,$$

$$c_t^M = (1-\kappa)\left(tot_t\right)^{-\eta}\left(\frac{p_t^T}{p_t^F}\right)^{\eta} c_t^T,$$

$$\frac{p_t^T}{p_t^F} = \left[\kappa + (1-\kappa)\left(tot_t\right)^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

$$c_t^X = (1-\kappa^*)\left(\frac{1}{tot_t}\right)^{-\eta^*} y_t^*.$$

De-trending the market-clearing conditions for the formal (28) and informal (29) sectors (after using (16), (17), and (27) to eliminate I_t^F, I_t^I, and GS_t), the economy-wide resource constraint (30), and the international asset market condition (26) yields:

$$(1-\tau^T)y_t^F = c_t^F + c_t^X + k_t^F g_t^F - (1-\delta_F)k_{t-1}^F + \frac{\phi}{2}\left(\frac{k_t^F}{k_{t-1}^F}g_t^F - g\right)^2 k_{t-1}^F,$$

$$y_t^I = c_t^I + k_t^I g_t^F - (1-\delta_I)k_{t-1}^I + \frac{\phi}{2}\left(\frac{k_t^I}{k_{t-1}^I}g_t^F - g\right)^2 k_{t-1}^I,$$

$$\frac{d_t g_t^F}{1+r_t} = d_{t-1} + c_t^M - \frac{c_t^X}{tot_t},$$

$$r_t = r^* + \chi_d\left[\exp\left(d_t - d\right) - 1\right] + \chi_y\left(E_t\left\{y_{t+1}^F\right\} - y^F\right) + \exp\left(\varrho_t - 1\right).$$

Thus, we have a 20 equation system for 20 de-trended variables: λ_t, r_t, p_t/p_t^F, p_t^T/p_t^F, p_t^I/p_t^F, tot_t, c_t, c_t^T, c_t^F, c_t^I, c_t^M, c_t^X, h_t, h_t^F, h_t^I, k_t^F, k_t^I, y_t^F, y_t^I, d_t.

7.2 Deterministic Steady State

Let s denote the steady-state value of s_t. In a balanced-growth path $g^F = g^I = g$. The deterministic steady state is given by the following system of equations:

$$\psi h^{\nu}\left(\frac{p^T}{p}\right)^{-1}\frac{p^T}{p^F} = (1-\tau^T)(1-\alpha)\frac{y^F}{h^F},$$

$$(1-\tau^T)(1-\alpha)\frac{y^F}{h^F} = \frac{p^I}{p^F}(1-\omega)\left(\frac{k^I}{h^I}\right)^{\omega}(\tilde{g}\cdot g)^{1-\omega},$$

$$1 = \beta g^{-\sigma}\left[1-\delta_F+(1-\tau^T)rr\right],$$

$$rr = \alpha\frac{y^F}{k^F},$$

$$\frac{g^\sigma}{\beta} = 1-\delta_I+\omega\left(k^I\right)^{\omega-1}\left(\tilde{g}\cdot g\cdot h^I\right)^{1-\omega},$$

$$\frac{g^\sigma}{\beta} = 1+r,$$

$$h = h^F + h^I,$$

$$\frac{h^I}{h} = 0.335,$$

$$y^F = \left(k^F\right)^{\alpha}\left(g\cdot h^F\right)^{1-\alpha},$$

$$y^I = \left(k^I\right)^{\omega}\left(\tilde{g}\cdot g\cdot h^I\right)^{1-\omega},$$

$$c = \frac{p^T}{p}c^T + \frac{p^I}{p}c^I,$$

$$c^T = a\left(\frac{p^T}{p}\right)^{-\theta}c,$$

$$c^I = (1-a)\left(\frac{p^I}{p}\right)^{-\theta}c,$$

$$c^F = \kappa\left(\frac{p^T}{p^F}\right)^{\eta}c^T,$$

$$c^M = (1-\kappa)\left(\frac{p^T}{p^F}\right)^{\eta}(tot)^{-\eta}c^T,$$

$$\frac{p^T}{p^F} = \left[\kappa+(1-\kappa)\left(tot\right)^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

$$\frac{p^I}{p^F} = \frac{p^I}{p}\left(\frac{p^T}{p}\right)^{-1}\frac{p^T}{p^F},$$

$$g\cdot k^F = \left(1-\delta_F\right)k^F + i^F,$$

$$g\cdot k^I = \left(1-\delta_I\right)k^I + i^I,$$

$$gs = \tau^T y^F,$$

$$\frac{c^T}{y^F} = 0.64,$$

$$y^F = c^F + c^X + i^F + gs,$$

$$y^I = c^I + i^I,$$

$$\frac{d}{y^F}\left(\frac{g}{1+r} - 1\right) = \frac{c^M}{y^F} - \frac{c^X}{y^F}\frac{1}{tot},$$

$$r = r^*,$$

$$TFP_I^F = (g)^{1-\alpha}\frac{k^I}{y^I}\left[\omega + (1-\omega)\left(\frac{h^I}{k^I}\right)^{\rho}\right]^{\frac{1}{\rho}},$$

$$\frac{(1-\alpha)\frac{y^F}{h^F}}{(1-\omega)\frac{y^I}{h^I}} = \Delta_{data}^W,$$

$$tot\left(\frac{c^M}{y^F}\right) = R_{data}^M,$$

$$\omega\left(\frac{y^I}{k^I}\right) = R_{data}^{kI}.$$

7.3 Log-Linearized Model

All variables are % deviations from the steady state except d and r which are the actual deviation.

The complete log-linearized model is given by the following equations:

$$\widehat{\lambda}_t + \widehat{p_t/p_t^F} = \sigma\left(c - \frac{\psi}{1+\nu}h^{1+\nu}\right)^{-1}\left(-c\cdot\widehat{c}_t + \psi h^{1+\nu}\widehat{h}_t\right),$$

$$\nu\widehat{h}_t = \widehat{y}_t^F - \widehat{h}_t^F - \widehat{p_t/p_t^F},$$

$$\nu\widehat{h}_t = \widehat{p_t^I/p_t} + \omega\left(\widehat{k}_{t-1}^I - \widehat{h}_t^I\right) + (1-\omega)\left(\widehat{\widetilde{g}}_t + \widehat{g}_t^F\right) + \widehat{z}_t^I,$$

$$\widehat{\lambda}_t + \phi\cdot g\left(\widehat{k}_t^F - \widehat{k}_{t-1}^F + \widehat{g}_t^F\right) = E_t\widehat{\lambda}_{t+1} - \sigma\widehat{g}_t^F$$
$$+\left[1-\beta(g)^{-\sigma}(1-\delta_F)\right]\left(E_t\widehat{y}_{t+1}^F - \widehat{k}_t^F\right) + \beta\phi(g)^{2-\sigma}\left(\widehat{k}_{t+1}^F - \widehat{k}_t^F + E_t\widehat{g}_{t+1}^F\right),$$

$$\widehat{\lambda}_t + \widehat{p_t^I/p_t^F} + \phi\cdot g\left(\widehat{k}_t^I - \widehat{k}_{t-1}^I + \widehat{g}_t^F\right) - \beta\phi(g)^{2-\sigma}\left(\widehat{k}_{t+1}^I - \widehat{k}_t^I + E_t\widehat{g}_{t+1}^F\right)$$
$$= E_t\widehat{\lambda}_{t+1} + E_t\widehat{p_{t+1}^I/p_{t+1}^F} - \sigma\widehat{g}_t^F + \left[1-\beta(g)^{-\sigma}(1-\delta_I)\right]\left[(1-\omega)\left(\widehat{\widetilde{g}}_t + E_t\widehat{g}_{t+1}^I + E_t\widehat{h}_{t+1}^I - \widehat{k}_t^I\right) + E_t\widehat{z}_{t+1}^I\right],$$

$$\widehat{\lambda}_t + \widehat{tot}_t = -\sigma\widehat{g}_t^F + \beta(g)^{-\sigma}\widehat{r}_t + E_t\widehat{\lambda}_{t+1} + E_t\widehat{tot}_{t+1},$$

$$\widehat{h}_t = \frac{h^F}{h}\widehat{h}_t^F + \frac{h^I}{h}\widehat{h}_t^I,$$

$$\widehat{g}_t^I = (1-\varphi)\widehat{g}_{t-1}^I + \varphi\widehat{g}_t^F,$$

$$\widehat{\tilde{g}}_t = \widehat{\tilde{g}}_{t-1} + (1-\varphi)\widehat{g}_{t-1}^I - (1-\varphi)\widehat{g}_t^F,$$

$$\widehat{y}_t^F = \alpha\widehat{k}_{t-1}^F + (1-\alpha)\left(\widehat{g}_t^F + \widehat{h}_t^F\right) + \widehat{z}_t^F,$$

$$\widehat{y}_t^I = \omega\widehat{k}_{t-1}^I + (1-\omega)\left(\widehat{\tilde{g}}_{t-1} + \widehat{g}_t^I + \widehat{h}_t^I\right) + \widehat{z}_t^I,$$

$$\widehat{p_t/p_t^F} + \widehat{c}_t = \frac{p^T}{p^F}\frac{p^F}{p}\frac{c^T}{c}\left(\widehat{p_t^T/p_t^F} + \widehat{c}_t^T\right) + \frac{p^I}{p^F}\frac{p^F}{p}\frac{c^I}{c}\left(\widehat{p_t^I/p_t^F} + \widehat{c}_t^I\right),$$

$$\widehat{c}_t^T = \theta\left(\widehat{p_t/p_t^F} - \widehat{p_t^T/p_t^F}\right) + \widehat{c}_t,$$

$$\widehat{c}_t^I = \theta\left(\widehat{p_t/p_t^F} - \widehat{p_t^I/p_t^F}\right) + \widehat{c}_t,$$

$$\widehat{c}_t^F = \eta\left(\widehat{p_t^T/p_t^F}\right) + \widehat{c}_t^T,$$

$$\widehat{c}_t^M = \eta\left(\widehat{p_t^T/p_t^F} - \widehat{tot}_t\right) + \widehat{c}_t^T,$$

$$\widehat{p_t^T/p_t^F} = (1-\kappa)\left(tot \cdot \frac{p^F}{p^T}\right)^{1-\eta}\widehat{tot}_t,$$

$$\widehat{c}_t^X = \eta^*\widehat{tot}_t + \widehat{y}_t^*,$$

$$(1-\tau^T)\widehat{y}_t^F = \frac{c^F}{y^F}\widehat{c}_t^F + \frac{c^X}{y^F}\widehat{c}_t^X + \frac{k^F g}{y^F}\left(\widehat{k}_t^F + \widehat{g}_t^F\right) - \frac{k^F}{y^F}(1-\delta_F)\widehat{k}_{t-1}^F,$$

$$\widehat{y}_t^I = \frac{c^I}{y^I}\widehat{c}_t^I + \left[\frac{k^I g}{y^I}\left(\widehat{k}_t^I + \widehat{g}_t^F\right) - \frac{k^I}{y^I}(1-\delta_I)\widehat{k}_{t-1}^I\right],$$

$$d\beta\,(g)^{1-\sigma}\left[\frac{\widehat{d}_t}{d} + \widehat{g}_t^F - \beta\,(g)^{-\sigma}\,\widehat{r}_t\right] = \widehat{d}_{t-1} + c^M\widehat{c}_t^M + \frac{c^X}{tot}\left(\widehat{tot}_t - \widehat{c}_t^X\right),$$

$$\widehat{r}_t = \chi_d\widehat{d}_t + \chi_y y^F E_t\widehat{y}_{t+1}^F + \widehat{\varrho}_t.$$

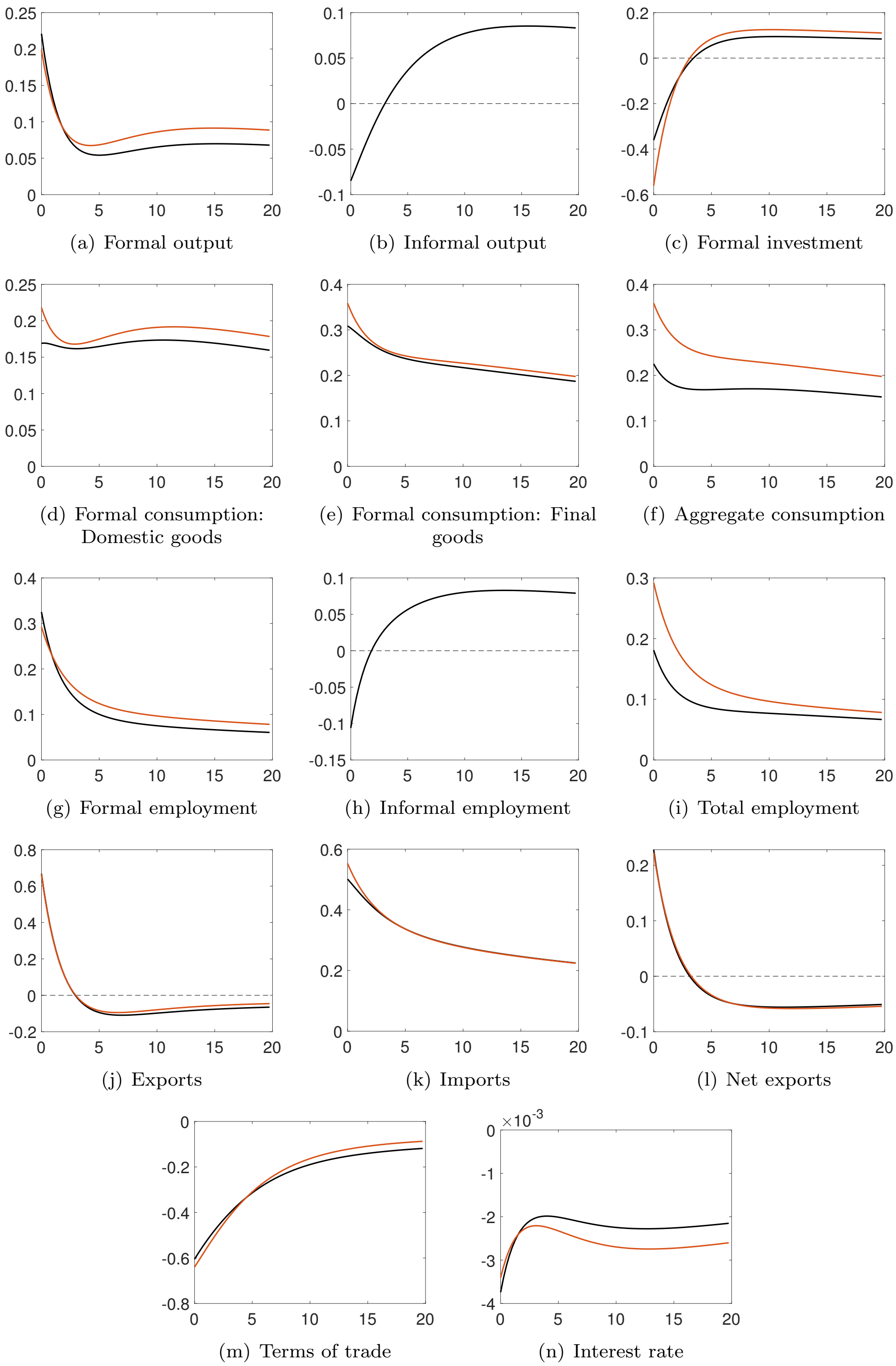

Figure A.1: Impulse responses for a positive 1% foreign demand shock: Benchmark model with informal sector (–) vs. No informality model (–)

Table A.1: Additional business cycle statistics: Developed countries

	Standard deviations					Correlation with output y_t				
	$\sigma(y)$	$\frac{\sigma(c)}{\sigma(y)}$	$\frac{\sigma(i)}{\sigma(y)}$	$\frac{\sigma(h)}{\sigma(y)}$	$\sigma(r)$	y_{t-1}	c_t	i_t	h_t	r_t
Australia	0.56	1.36	6.50	1.07	0.21	0.59	0.57	0.65	-0.06	0.43
Austria*	1.10	0.66	1.89	0.42	0.16	0.87	0.49	0.71	0.80	0.23
Belgium*	0.89	0.65	3.21	0.55	0.22	0.85	0.56	0.64	0.43	0.09
Canada	1.05	0.65	3.29	0.54	0.24	0.87	0.58	0.74	0.70	0.33
Denmark*	1.29	1.04	3.62	0.69	0.24	0.79	0.61	0.70	0.52	0.10
Finland	1.77	0.82	2.03	0.57	0.15	0.81	0.61	0.79	0.53	-0.26
France	0.89	0.74	2.55	0.59	0.20	0.88	0.61	0.92	0.76	0.44
Germany	1.42	0.45	2.10	0.40	0.14	0.84	0.48	0.87	0.45	0.32
Ireland*	3.08	0.61	6.77	0.64	0.36	0.60	0.48	0.18	0.55	-0.10
Italy*	1.19	0.87	2.11	0.52	0.19	0.86	0.67	0.84	0.41	0.25
Japan	1.41	0.74	1.91	0.56	0.20	0.74	0.69	0.78	0.32	-0.27
Luxembourg*	2.00	0.79	3.63	0.28	0.18	0.65	0.44	0.22	0.16	0.27
Netherlands*	1.20	0.79	6.93	0.70	0.17	0.88	0.76	0.41	0.72	0.57
New Zealand	1.06	1.10	4.62	1.06	0.27	0.64	0.57	0.65	0.41	0.32
Norway*	1.10	0.96	4.99	0.84	0.29	0.45	0.57	0.45	0.39	0.11
Portugal*	1.19	1.26	3.83	0.90	0.22	0.86	0.87	0.85	0.78	-0.06
Spain*	1.18	1.19	2.90	1.23	0.26	0.93	0.88	0.81	0.93	0.08
Sweden	1.46	0.68	2.42	0.68	0.19	0.86	0.69	0.80	0.39	-0.09
Switzerland*	1.12	0.39	2.07	0.58	0.17	0.84	0.53	0.73	0.32	0.29
U.K.	1.04	1.00	2.98	0.45	0.21	0.89	0.80	0.71	0.73	0.64
Mean	1.30	0.84	3.52	0.66	0.21	0.78	0.62	0.67	0.51	0.18
Median	1.18	0.79	3.10	0.58	0.21	0.85	0.59	0.72	0.49	0.24

Notes: y, c, i, h, r denote, respectively, real output, real consumption, real investment, total employment, and the real interest rate. r is defined as the short-term interest rate based on three-month money market rates, deflated by the average CPI inflation rate in the current and previous three quarters. Data is quarterly, seasonally adjusted, and in units of domestic currency. All data is from the Quarterly National Accounts database of the OECD except for the CPI inflation rate, which is taken from the International Financial Statistics (IFS) database of the IMF, and the employment series for Japan and the U.K., which are taken from the IFS database and the Office of National Statistics, respectively. The sample period is 1993Q1–2019Q4, except the employment series for countries denoted by $*$, which is for the period 1995Q1–2019Q4, and the real interest rate series for Japan and Switzerland, which is for the periods 2002Q2–2019Q4 and 1999Q3-2019Q4, respectively. All series are logged (except r) and HP-filtered with a smoothing parameter of 1600. Standard deviations are expressed in percent. Standard errors are available upon request.

Table A.2: Additional business cycle statistics: Emerging market economies

	Standard Deviations					Correlation with output y_t				
	$\sigma(y)$	$\frac{\sigma(c)}{\sigma(y)}$	$\frac{\sigma(i)}{\sigma(y)}$	$\frac{\sigma(h)}{\sigma(y)}$	$\sigma(r)$	y_{t-1}	c_t	i_t	h_t	r_t
Argentina	2.76	1.24	3.20	0.30	2.32	0.79	0.87	0.92	0.29	-0.39
Brazil	1.65	1.11	3.21	-	0.53	0.77	0.80	0.90	-	-0.30
Chile	1.76	1.31	3.32	0.81	0.29	0.80	0.90	0.80	0.56	0.02
Colombia	1.57	0.96	5.59	0.90	0.34	0.81	0.90	0.81	-0.26	-0.23
Costa Rica	1.39	0.99	4.55	-	-	0.76	0.71	0.72	-	-
Czech Rep.	1.74	0.74	2.24	0.40	-	0.90	0.59	0.79	0.55	-
Ecuador	1.66	0.97	3.25	-	1.60	0.82	0.65	0.61	-	-0.06
Estonia	3.90	1.05	3.17	0.62	-	0.90	0.90	0.84	0.82	-
Hungary	1.43	1.20	3.32	0.60	0.32	0.83	0.62	0.45	0.14	-0.20
Indonesia	2.89	0.88	3.04	-	0.38	0.85	0.67	0.90	-	-0.42
Korea	2.03	1.57	2.56	0.70	0.20	0.82	0.93	0.87	0.85	-0.25
Latvia	4.21	1.21	2.87	0.75	-	0.89	0.82	0.74	0.87	-
Lithuania	3.46	1.20	3.22	0.62	0.17	0.88	0.85	0.90	0.73	-0.17
Mexico	2.16	1.00	3.58	0.36	0.49	0.82	0.93	0.90	0.38	-0.55
Romania	2.40	1.56	4.53	0.85	-	0.82	0.80	0.60	0.25	-
Slovak Rep.	2.10	0.97	4.23	0.65	-	0.72	0.50	0.60	0.62	-
Slovenia	1.88	0.89	3.17	0.57	-	0.85	0.28	0.77	0.46	-
South Africa	1.01	1.55	3.74	2.45	0.32	0.88	0.82	0.59	0.28	-0.01
Thailand	1.86	0.93	2.59	0.38	0.27	0.51	0.74	0.72	0.17	0.36
Turkey	3.62	1.02	3.09	0.43	0.34	0.79	0.90	0.93	0.66	-0.32
Mean	2.27	1.12	3.42	0.71	0.58	0.81	0.76	0.77	0.46	-0.20
Median	1.96	1.04	3.21	0.62	0.34	0.82	0.81	0.80	0.51	-0.23

Notes: See Table A.1. The real interest rate r is defined as the sum of the U.S. real interest rate and the EMBI country spread. The country spread is measured using J.P. Morgan EMBI+ data available from the World Bank. The U.S. real interest rate is defined as the three-month nominal T-bill rate deflated by the average CPI inflation rate in the current and previous three quarters. To construct the U.S. real interest rate we used inflation data from the International Financial Statistics (IFS) database of the IMF, while the U.S. T-bill rate data is taken from FRED. Table A.3 summarizes the sample period of each country for the series r. Due to insufficient data, the series r was dropped for Costa Rica, Czech Republic, Estonia, Latvia, Romania, Slovak Republic, and Slovenia. The data for y, c, and i is from the Quarterly National Accounts database of the OECD except Ecuador and Thailand, where the data is from the IFS database of the IMF. The sample period is the same as Table 2. For the total employment series h, Table A.4 summarizes the sample period and the data sources for each country. Due to insufficient data, the series h was dropped for Brazil, Costa Rica, Ecuador, and Indonesia.

Table A.3: Real interest rate series for emerging market economies: Sample periods

Country	Sample Period	Country	Sample Period
Argentina	1993Q4–2019Q4	Korea	1997Q4–2004Q2
Brazil	1994Q2–2019Q4	Lithuania	2009Q4–2019Q4
Chile	1999Q2–2019Q4	Mexico	1994Q1–2019Q4
Colombia	1997Q1–2019Q4	South Africa	1997Q4–2019Q4
Ecuador	1995Q1–2019Q4	Thailand	1997Q2–2006Q1
Hungary	1999Q1–2019Q4	Turkey	1996Q2–2019Q4
Indonesia	2004Q2–2019Q4		

Table A.4: Total employment series for emerging market economies: Sample periods and data sources

Country	Sample Period	Data	Country	Sample Period	Data
Argentina	2003Q4–2019Q4	IMF	Lithuania	1995Q1–2019Q4	IMF
Chile	1993Q1–2019Q4	IMF	Mexico	2005Q1–2019Q4	INEGI
Colombia	2001Q1–2019Q4	OECD	Romania	1995Q1–2019Q4	OECD
Czech Rep.	1995Q1–2019Q4	OECD	Slovakia	1994Q1–2019Q4	IMF
Estonia	1995Q1–2019Q4	OECD	Slovenia	1995Q1–2019Q4	OECD
Hungary	1995Q1–2019Q4	IMF	South Africa	2008Q1–2019Q4	IMF
Korea	1993Q1–2019Q4	IMF	Thailand	2001Q1–2019Q4	IMF
Latvia	1995Q1–2019Q4	OECD	Turkey	2006Q1–2019Q4	IMF