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Abstract

We introduce equilibrium indeterminacy into a two-country incomplete asset model with imperfect competition to analyze the role of self-fulfilling expectations or beliefs in explaining international business cycles. We find that when self-fulfilling beliefs are correlated with technology shocks, the model can account for the counter-cyclical behavior observed for the terms of trade and real net exports, while simultaneously generating higher volatilities relative to output, as in the data. The choice of the labor supply elasticity is shown to be critical for generating a negative correlation between the real exchange rate and relative consumption, thereby resolving the *Backus-Smith* puzzle.

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Keywords: Indeterminacy; Sunspots and Self-Fulfilling Expectations; International Business Cycles; Net Exports; Terms of Trade; Consumption-Real Exchange Rate Anomaly; Combined Impulse Responses.

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1 Introduction

Since the pioneering work of Benhabib and Farmer (1994), there now exists a large literature exploring the role of equilibrium indeterminacy and self-fulfilling beliefs in explaining business cycle fluctuations.[1] While the endogenous-business-cycle approach has been successful in explaining a number of features of closed-economy business cycles, the importance of endogenous sunspot fluctuations in understanding international business cycles remains largely unexplored quantitatively. Previous studies have restricted their attention to explaining one specific feature of the open-economy data, namely the cross-country correlation between consumption and output.[2] Instead, this paper takes a broader look at international business cycle fluctuations. We show that the indeterminacy model does not suffer from volatility anomalies and counterfactual output-correlations for international relative prices and quantities. However, in order to generate a negative correlation between relative consumption and the real exchange rate, the model requires that the inverse labor-supply elasticity is set close to zero.

The model economy we consider is a two-country, two-good, incomplete-asset market economy with imperfect competition. Similar to the closed-economy studies of Farmer and Guo (1994) and Schmitt-Grohe (1997), among others, we assume increasing returns to scale technology. As a result, indeterminacy is generated via an upward-sloping aggregate labor demand schedule, which is a common feature of many indeterminacy models.[3] Under indeterminacy, the forecast errors can be redefined as a fundamental disturbance, namely the belief shocks or sunspot shocks.[4] We consider two alternative assumptions. First, we assume that the forecast errors to the terms of trade are the only source of business cycle fluctuations (*autonomous beliefs*). Secondly, we assume that the forecast errors to the terms of trade are correlated with fundamental shocks (*correlated beliefs*).[5]

[1] See, e.g., Farmer and Guo (1994), Schmitt-Grohé (1997, 2000), Benhabib and Wen (2004), Jaimovich (2007), Guo and Harrison (2010), Benhabib and Wang (2013), Dufourt et al. (2015), Pintus et al. (2016), and Pavlov and Weder (2017).

[2] See Guo and Sturzenegger (1998) and Xiao (2004).

[3] The upward-sloping aggregate labor demand schedule is common to many indeterminacy models because it arises under a wide set of modeling assumptions. Models with increasing returns to scale (e.g. Benhabib and Wen, 2004), models with positive externalities in production (e.g. Benhabib and Farmer, 1994), and models with firm entry under monopoly power (e.g. Jaimovich, 2007, and Pavlov and Weder, 2017) all feature an upward-sloping aggregate labor-demand schedule.

[4] A recent literature has considered the effects of changes in agent beliefs on economic fluctuations. Huo and Takayama (2015) and Angeletos et al. (2018) model beliefs as shocks to higher-order expectations consistent with equilibrium uniqueness. This paper takes a different approach by modeling beliefs as in the indeterminacy literature initiated by Benhabib and Farmer (1994), where belief shocks are the mechanism for selecting an equilibrium path.

[5] In what follows we use the terms forecast errors, expectational errors, sunspots, self-fulfilling expectations or beliefs interchangeably.

Our main findings are summarized as follows. We first show that international business cycle fluctuations driven solely by autonomous beliefs cannot replicate the major features of the data. This finding is in stark contrast to Guo and Sturzenegger (1998) and Xiao (2004) who find that self-fulfilling expectations can help explain the positive cross-country correlations observed for consumption and output. However, both Guo and Sturzenegger (1998) and Xiao (2004) introduce indeterminacy into a two-country, one-good model, while we generate indeterminacy in a two-good framework, in order to look at a wider set of puzzles related to international relative prices and quantities.

In one-good indeterminacy models self-fulfilling expectations stimulate world demand and generate positive cross-country correlations for consumption and output, as in the data. However, in our two-good model revisions to the terms of trade forecasts are the source of endogenous fluctuations.[6] We show that a belief-induced depreciation of the terms of trade shifts the upward-sloping aggregate labor demand schedules in each country in opposite directions, raising domestic output and consumption at the expense of foreign output and consumption. Consequently, in two-good indeterminacy models autonomous beliefs cannot on their own explain the data, since by causing a reallocation of output, they generate counterfactually negative cross-country correlations.

We next show that a number of the empirical puzzles can be resolved by allowing the forecast errors to be correlated with technology shocks. Now, the indeterminacy model can generate counter-cyclical behavior for the terms of trade and real net exports, while at the same time, increasing significantly the volatility of international relative prices and cross-country trade flows. This improvement in volatility over the business cycle is not at the cost of reduced volatility of the other aggregate variables, whose standard deviations relative to output are also increased.

The improved performance of the model is due to the transmission mechanism of technology shocks, which is fundamentally altered under indeterminacy. In our model, technology shocks induce a change in beliefs by causing agents in both countries to revise their expectations of the terms of trade. To explain the transmission mechanism, we show how to construct combined impulse responses that take into account the correlation of beliefs with fundamentals. We find that a very specific transmission of technology shocks, in which there is a negative response of employment to a positive technology shock and a delayed effect on output, best explains the data.[7]

[6] There is sizeable evidence to suggest that terms of trade shocks are an important source of business cycle fluctuations (see, e.g., Mendoza, 1995).

[7] This temporary contractionary transmission mechanism is not without empirical support. See, e.g., Galí (1999), Basu et al. (2006), Canova et al. (2010), Wang and Wen (2011), Giuli and Tancioni (2017).

In particular, a positive domestic technology shock causes a belief-induced depreciation (increase) of the terms of trade and the delayed expansion generates the desired negative correlation between the terms of trade and output. Moreover, real net exports are counter-cyclical, as in the data.[8] Finally, the depreciation of the terms of trade is sufficiently large relative to output that the model is able to generate volatile international relative prices. Overall, the positive correlation between self-fulfilling beliefs and productivity shocks can explain multiple features of international business cycles.

Our approach is similar to Schmitt-Grohé (2000) and Benhabib and Wang (2013) in that we select the properties of the forecast error and fundamental shocks which best match the key moments of the data, but we specifically focus on international fluctuations. In this way, we give the indeterminacy model the best chance at matching the international business cycle facts. However, two major discrepancies with the data remain. First, an international comovement puzzle arises, whereby the model counterfactually predicts a negative cross-country investment correlation. This happens because cross-country differences in the marginal productivity of capital induce investment flows to the most productive economy. Second, the model fails to resolve the so-called consumption-real exchange rate anomaly or Backus-Smith (1993) puzzle. The model predicts a positive correlation between the real exchange rate and relative consumption, whereas in the data this correlation is negative. This happens because a belief-induced depreciation of the terms of trade generates a relatively stronger reduction in employment abroad than in the domestic economy. This increases the ratio of consumption across the two countries, thereby counterfactually implying a positive correlation between international relative prices and relative consumption. We show that this transmission mechanism is at the heart of all indeterminacy models that have an upward-sloping aggregate labor demand schedule. Consequently, the results identified in this paper will hold for a wide-class of indeterminacy frameworks. We identify two ways to resolve the Backus-Smith puzzle. The first way requires a strong negative cross-country correlation for technology shocks.[9] Alternatively, we find that under indivisible labor the model can also give rise to a negative correlation between the real exchange rate and relative consumption.

In addition to the studies of Guo and Sturzenegger (1998) and Xiao (2004), the current paper builds upon several contributions within the indeterminacy literature. Recent studies have been

[8] As shown in the Appendix, the model also successfully predicts a S-curve cross-correlation relationship between the trade balance and the terms of trade.

[9] However, a negative cross-country correlation for technology shocks is not supported by other studies. See, e.g., Backus et al. (1992) and Heathcote and Perri (2004).

successful in quantitatively explaining closed-economy business cycles using indeterminacy. For example, Jaimovich (2007) and Pavlov and Weder (2017) using one-sector models, and Guo and Harrison (2010) and Dufourt et al. (2015) using two-sector models, can broadly reproduce several key features of U.S. business cycles. This paper contributes to this literature by examining whether indeterminacy can also successfully replicate some of the most well-known properties of international business cycles. Similar to Pintus et al. (2016) and Pavlov and Weder (2017), we solve the model under indeterminacy using the Farmer-Khramov-Nicolò (2015) method.[10] However, they estimate their closed-economy model using Bayesian techniques, whereas we use the method of moments approach to try and resolve some well-known empirical puzzles in international business cycle theory.

Finally, this paper is related to a number of recent studies who have also attempted to explain the international macro puzzles in directions different from ours. Beaudry et al. (2011) consider the role of news shocks and Levchenko and Pandalai-Nayar (2020) consider the role of sentiment shocks in driving international business cycles. Wen (2007) examines the role of demand shocks in explaining international comovements, while Dmitriev (2017) considers the role of time-nonseparable preferences. Heathcote and Perri (2013) show that taste shocks can successfully resolve the Backus-Smith puzzle. Karabarbounis (2014) considers the role of home production and labor wedges, whereas Raffo (2010) introduces an additional source of technological variation by including investment-specific technology shocks. This paper takes a different approach by examining how far indeterminacy and endogenous fluctuations can go in explaining international business cycles.

The remainder of the paper is organized as follows. Section 2 outlines the model economy. Section 3 discusses the calibration of the model and the solution method employed. Sections 4 and 5 presents the main results and Section 6 discusses the transmission mechanism. Finally, Section 7 briefly concludes. The Appendix reports additional results and robustness exercises.

2 Model

We develop a two-country extension of the imperfect competition model studied by Farmer and Guo (1994) and Schmitt-Grohé (1997) for the closed economy. Following Wen (1998), we assume

[10] A popular alternative to the Farmer-Khramov-Nicolò solution technique is the method of Lubik and Schorfheide (2003, 2004). As shown by Farmer et al. (2015), these two solution methods are equivalent. See also Bianchi and Nicolò (2020).

variable capacity utilization which significantly reduces the size of returns to scale needed to generate indeterminacy. Within each country there exists a representative agent, two final-good producers, and a continuum of intermediate-good producing firms. Intermediate firms operate under monopolistic competition and use domestic labor and capital as inputs to produce tradeable goods. The competitive final good producers use domestic and imported intermediate goods to produce non-tradeable consumption or investment goods, which are subsequently purchased by the domestic agent. However, final good producers are assumed to have a bias for domestically produced intermediate goods. While the law of one price is assumed to hold for all intermediate goods, with home bias, the real exchange rate deviates from purchasing power parity. The following presents the features of the model for the Home country on the understanding that the Foreign case can be analogously derived. All Foreign country variables are denoted by an asterisk.

2.1 Final good producers

In each country, there are two final goods, consumption and investment, which are produced with homogenous of degree one production functions using intermediate goods as the only inputs. The Home consumption final good C_t is produced by a competitive firm that uses $C_{H,t}$ and $C_{F,t}$ as inputs according to the following CES aggregation technology index:

$$C_t = \left[a^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where the constant elasticity of substitution between aggregate Home and Foreign intermediate goods is $\theta > 0$ and the relative share of domestic and imported intermediate inputs used in the production process is $0 < a < 1$. The Home investment final good I_t is produced according to the following CES aggregation technology index:

$$I_t = \left[b^{\frac{1}{\rho}} I_{H,t}^{\frac{\rho-1}{\rho}} + (1-b)^{\frac{1}{\rho}} I_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{2}$$

where $\rho > 0$ and $0 < b < 1$. The inputs $C_{H,t}$, $C_{F,t}$, $I_{H,t}$, and $I_{F,t}$ are defined as the quantity indices of domestic and imported intermediate goods respectively:

$$C_{H,t} = \left[\int_0^1 c_t(j)^{\frac{\kappa-1}{\kappa}} dj \right]^{\frac{\kappa}{\kappa-1}}, \qquad C_{F,t} = \left[\int_0^1 c_t(j^*)^{\frac{\kappa-1}{\kappa}} dj^* \right]^{\frac{\kappa}{\kappa-1}},$$
$$I_{H,t} = \left[\int_0^1 i_t(j)^{\frac{\kappa-1}{\kappa}} dj \right]^{\frac{\kappa}{\kappa-1}}, \qquad I_{F,t} = \left[\int_0^1 i_t(j^*)^{\frac{\kappa-1}{\kappa}} dj^* \right]^{\frac{\kappa}{\kappa-1}},$$

where the elasticity of substitution across domestic (imported) intermediate goods is $\kappa > 1$, and $c_t(j)$, $i_t(j)$, $c_t(j^*)$, $i_t(j^*)$ are the respective quantities of the domestic and imported type j and j^* intermediate goods. Intermediate firms sell their products to both consumption and investment final-good producers, where it is assumed that the law of one price holds. Cost minimization in final good production yields the demand conditions for Home and Foreign goods:

$$C_{H,t} = a \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t, \qquad C_{F,t} = (1-a) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t, \tag{3}$$

$$I_{H,t} = b \left(\frac{P^I_{H,t}}{P^I_t} \right)^{-\rho} I_t, \qquad I_{F,t} = (1-b) \left(\frac{P^I_{F,t}}{P^I_t} \right)^{-\rho} I_t, \tag{4}$$

and the corresponding aggregate price indices are given by:

$$P_t = \left[a P_{H,t}^{1-\theta} + (1-a) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad P^I_t = \left[b (P^I_{H,t})^{1-\rho} + (1-b)(P^I_{F,t})^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{5}$$

where P_t is the consumer price index, P^I_t is the price of investment goods, and $P_{H,t}$, $P^I_{H,t}$, $P_{F,t}$, $P^I_{F,t}$ are the respective price indices of Home and Foreign intermediate goods.

2.2 Intermediate goods producers

All intermediate firms have access to the same technology. A Home firm of type j has a production technology given by:

$$Y_t(j) = Z_t \left[u_t(j) K_t(j) \right]^{\alpha} L_t(j)^{\gamma} - \phi, \quad j \in [0,1] \tag{6}$$

where $K_t(j)$ and $L_t(j)$ represent the capital and labor usage of firm j, respectively, Z_t is the exogenous level of technology or productivity, and the input share is $\alpha + \gamma \geq 1$. Following Greenwood et al. (1988), the rate of capacity utilization $u_t \in (0,1)$ is endogenously determined. Similar to Mandelman et al. (2011), it is assumed that the depreciation rate of capital δ_t is higher if it is used more intensively:

$$\delta_t(j) = \widetilde{\delta} + \frac{h}{\mu} u_t(j)^{\mu}, \tag{7}$$

where $\mu > 1$ is the elasticity of the depreciation rate with respect to the utilization rate, and $h > 0$ and $\widetilde{\delta}$ are free parameters. A fixed cost of production $\phi > 0$ is also included in the production technology (6). Therefore, regardless of how much output Y_t is produced, a proportion ϕ of the intermediate good is used up in each period. As in Schmitt-Grohé (1997), allowing for a fixed production cost enables the model to generate zero profits without imposing any restrictions on the size of the steady-state markup.[11] Given competitive prices of labor and capital, cost-minimization yields:

$$w_t = \gamma mc_t(j) Z_t \left[u_t(j) K_t(j)\right]^{\alpha} L_t(j)^{\gamma - 1}, \tag{8}$$

$$rr_t + \delta_t(j) = \alpha mc_t(j) Z_t u_t(j)^{\alpha} K_t(j)^{\alpha - 1} L_t(j)^{\gamma}, \tag{9}$$

$$h u_t(j)^{\mu} = \alpha mc_t(j) Z_t u_t(j)^{\alpha} K_t(j)^{\alpha - 1} L_t(j)^{\gamma}, \tag{10}$$

where mc_t is real marginal cost, w_t is the real wage, and $rr_t + \delta_t(j)$ is the user cost of capital of firm j.

Given that the total demand for firm j's output can be expressed as:

$$Y_t(j) = \left(\frac{p_t(j)}{P_{H,t}}\right)^{-\kappa} \left[C_{H,t} + C^*_{H,t}\right] + \left(\frac{p_t(j)}{P^I_{H,t}}\right)^{-\kappa} \left[I_{H,t} + I^*_{H,t}\right],$$

it follows from the firm's profit maximization problem that the optimal price-setting rule is:

$$p_t(j) = \chi mc_t(j) P_t, \tag{11}$$

where $\chi \equiv \frac{\kappa}{\kappa - 1}$ is the markup.

2.3 Representative agent

The representative agent has an expected utility function of the form:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right),$$

[11] As discussed by Rotemberg and Woodford (1996), Schmitt-Grohé (1997), and Jaimovich (2007), positive profits are not observed in the U.S. economy despite the presence of market power.

where C_t and $L_t \equiv \int_0^1 L_t(j)dj$ are consumption and aggregate work effort, respectively, and the discount factor is $0 < \beta < 1$. Following Greenwood et al. (1988), we assume that the period utility function is given by:

$$U(C_t, L_t) = \frac{1}{1-\sigma}\left[\left(C_t - \frac{\psi}{1+\nu}L_t^{1+\nu}\right)^{1-\sigma} - 1\right],$$

where $\sigma > 0$ is the utility curvature parameter, $\nu \geq 0$ is the inverse of the Frisch elasticity of labor supply, and $\psi > 0$.

The representative agent during period t supplies labor and capital to intermediate-good producing firms, receiving real income from wages w_t, a rental return on capital rr_t, and nominal profits from the ownership of domestic intermediate firms Π_t. The agent then uses these resources to purchase the two final goods, dividing purchases between consumption C_t and investment I_t. The purchase of an investment good forms next period's capital according to the law of motion:

$$K_{t+1} = (1-\delta_t)K_t + I_t, \tag{12}$$

where $K_t \equiv \int_0^1 K_t(j)dj$ and $\delta_t \equiv \frac{\int_0^1 \delta_t(j)K_t(j)dj}{\int_0^1 K_t(j)dj}$.

The asset market structure is assumed to be incomplete. The Foreign agent is able to trade two non-state contingent bonds $B^*_{H,t}$ and $B^*_{F,t}$, whereas the Home agent can only purchase domestic bonds $B_{H,t}$.[12] All bonds are denominated in units of the domestic aggregate consumption index. For the Foreign agent, there is a transaction cost Ψ of adjusting the internationally traded bond $B^*_{H,t}$, where it is assumed that Ψ is a positive and differentiable function.[13] This transaction cost, which is paid to financial firms, captures the costs of adjusting bond holdings and is sufficient to ensure that bond holdings are stationary.[14] Consequently, the period budget constraints of the Home and Foreign agent can be expressed in real terms as:

$$\frac{B_{H,t}}{r_t} + C_t + \frac{P_t^I}{P_t}I_t \leq B_{H,t-1} + \int_0^1 w_t L_t(j)dj + \int_0^1 (rr_t + \delta_t(j))K_t(j)dj + \int_0^1 \Pi_t(j)dj + R_t, \tag{13}$$

[12] Similar to Benigno and Thoenissen (2008), asymmetry in the asset market structure is made for simplicity. Allowing the Home agent to purchase Foreign bonds introduces an additional optimality condition with no change in the results.

[13] Following Benigno (2009), we assume that $\Psi = 1$ when bond holdings are at their steady-state level and Ψ is positive, differentiable, and strictly decreasing in a neighborhood of the steady state.

[14] For a discussion of the stationary problem of incomplete market, open-economy models, see Schmitt-Grohé and Uribe (2003) and Ghironi (2006).

$$
\frac{B_{H,t}^{*}}{Q_t r_t}\frac{1}{\Psi(B_{H,t}^{*})}+\frac{B_{F,t}^{*}}{r_t^{*}}+C_t^{*}+\frac{P_t^{*I}}{P_t^{*}}I_t^{*}\leq\frac{B_{H,t-1}^{*}}{Q_t}+B_{F,t-1}^{*}+\int_0^1 w_t^{*}L_t^{*}(j^{*})dj^{*} \\
+\int_0^1 (rr_t^{*}+\delta_t^{*}(j^{*}))K_t^{*}(j^{*})dj^{*}+\int_0^1 \Pi_t^{*}(j^{*})dj^{*}+R_t^{*}, \tag{14}
$$

where R_t and R_t^* denote rebates from financial firms, r_t and r_t^* are the Home and Foreign (gross) real interest rates, and Q_t is the CPI-based real exchange rate.

The Home agent's maximization problem yields:

$$
U_c(C_t,L_t)=\left(C_t-\frac{\psi L_t^{1+\nu}}{1+\nu}\right)^{-\sigma}=\lambda_t, \tag{15}
$$

$$
-\frac{U_L(C_t,L_t)}{U_c(C_t,L_t)}=\psi L_t^{\nu}=w_t, \tag{16}
$$

$$
\lambda_t\frac{P_t^I}{P_t}=\beta E_t\left\{\lambda_{t+1}\left[rr_{t+1}+\delta_{t+1}+(1-\delta_{t+1})\frac{P_{t+1}^I}{P_{t+1}}\right]\right\}, \tag{17}
$$

$$
\beta r_t E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\right\}=1, \tag{18}
$$

where λ_t is the Lagrange multiplier on the agent's budget constraint (13). Analogous conditions to (15)-(18) apply for the Foreign agent, where the following interest-rate parity condition can be derived:

$$
\frac{r_t}{r_t^{*}}=\frac{E_t\left\{U_{c^*}\left(C_{t+1}^{*},L_{t+1}^{*}\right)\right\}}{E_t\left\{U_{c^*}\left(C_{t+1}^{*},L_{t+1}^{*}\right)\Psi\left(B_{H,t}^{*}\right)\frac{Q_t}{Q_{t+1}}\right\}}. \tag{19}
$$

Optimizing behavior implies that the budget constraints (13) and (14) hold with equality in each period and the appropriate transversality conditions are satisfied.

2.4 Market clearing and equilibrium

We now focus on a symmetric equilibrium where all firms in Home and Foreign set the same price in each period t, rent the same amount of capital, and employ the same amount of labor. Consequently, $p_t(j)=P_{H,t}=P_{H,t}^I$ and the index j can be dropped from all variables. Market clearing in the Home goods market requires:

$$
Y_t=C_{H,t}+C_{H,t}^{*}+I_{H,t}+I_{H,t}^{*}, \tag{20}
$$

and assuming that the Foreign non-state contingent bond is in zero net supply, bond market clearing requires:

$$B_{H,t} + B^*_{H,t} = 0, \qquad B^*_{F,t} = 0. \tag{21}$$

The aggregate resource constraint is given by:[15]

$$C_t + \frac{P^I_t}{P_t} I_t + \frac{B_{H,t}}{r_t} = B_{H,t-1} + \frac{P_{H,t}}{P_t} Y_t, \tag{22}$$

where

$$\frac{P^I_t}{P_t} = \frac{\left[b + (1-b) T_t^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[a + (1-a) T_t^{1-\theta}\right]^{\frac{1}{1-\theta}}}, \qquad \frac{P_{H,t}}{P_t} = \left[a + (1-a) T_t^{1-\theta}\right]^{\frac{1}{\theta-1}}, \tag{23}$$

follow from the aggregate price indices (5). The terms of trade T_t can be expressed as:

$$T_t \equiv \frac{P_{F,t}}{P_{H,t}} = \frac{\left[a + (1-a) T_t^{\theta-1}\right]^{\frac{1}{\theta-1}}}{\left[a + (1-a) T_t^{1-\theta}\right]^{\frac{1}{\theta-1}}} Q_t. \tag{24}$$

In what follows, we call an increase (decrease) in the terms of trade, or the real exchange rate, a depreciation (appreciation). Finally, we measure real net exports as the difference between exports and imports, divided by total output (all evaluated at steady state prices):[16]

$$RNX_t = \frac{C^*_{H,t} + I^*_{H,t} - \overline{T}\left(C_{F,t} + I_{F,t}\right)}{C_{H,t} + C^*_{H,t} + I_{H,t} + I^*_{H,t}}. \tag{25}$$

Equilibrium. An equilibrium for the world economy consists of a set of real prices r_t, r^*_t, w_t, w^*_t, rr_t, rr^*_t, δ_t, δ^*_t, mc_t, mc^*_t, λ_t, λ^*_t; a set of relative prices $\frac{P_{H,t}}{P_t}$, $\frac{P^*_{F,t}}{P^*_t}$, $\frac{P^I_t}{P_t}$, $\frac{P^{*I}_t}{P^*_t}$, Q_t, T_t; a collection of allocations for the Home and Foreign agent C_t, C^*_t, I_t, I^*_t, L_t, L^*_t, K_t, K^*_t, u_t, u^*_t, $B_{H,t}$, $B^*_{H,t}$, $B^*_{F,t}$; and a collection of allocations for Home and Foreign final and intermediate good producers Y_t, Y^*_t, $C_{H,t}$, $C_{F,t}$, $C^*_{H,t}$, $C^*_{F,t}$, $I_{H,t}$, $I_{F,t}$, $I^*_{H,t}$, $I^*_{F,t}$, RNX_t satisfying (i) the optimality conditions of each agent; (ii) the optimality conditions of final and intermediate good producing firms; (iii) all markets clear; and (iv) the aggregate resource constraints of both countries.

[15]By Walras' Law, the aggregate resource constraint of the Foreign country is redundant.
[16]Thus, our measure of net exports is unaffected by fluctuations in relative prices.

3 Numerical solution and calibration

3.1 The solution method under indeterminacy

To solve the indeterminacy model, we log-linearize the equilibrium conditions around a symmetric, deterministic steady state, where bond holdings are zero and the steady-state terms of trade is equal to 1.[17] Let $\boldsymbol{s}_t = \left[\widehat{K}_{t+1}, \widehat{K}^*_{t+1}, \tilde{B}_{H,t}, \widehat{T}_t, \widehat{C}_t, \widehat{C}^*_t, E_t\widehat{T}_{t+1}, E_t\widehat{C}_{t+1}, E_t\widehat{C}^*_{t+1}, \widehat{Z}_t, \widehat{Z}^*_t\right]'$ denote the vector of endogenous variables expressed in terms of percentage deviations from their steady state values.[18] The linearized system can be written as:

$$\boldsymbol{\Gamma}_0 \boldsymbol{s}_t = \boldsymbol{\Gamma}_1 \boldsymbol{s}_{t-1} + \boldsymbol{\Psi}\boldsymbol{\varepsilon}_t + \boldsymbol{\Pi}\boldsymbol{\eta}_t, \tag{26}$$

where $\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}$, and $\boldsymbol{\Pi}$ are matrices of structural parameters, $\boldsymbol{\varepsilon}_t = [\varepsilon_t, \varepsilon^*_t]'$ is the vector of fundamental or exogenous technology shocks, and $\boldsymbol{\eta}_t = \left[\eta^T_t, \eta^C_t, \eta^{C^*}_t\right]'$ is the vector of non-fundamental or endogenous shocks, which collects the one-step ahead forecast errors for the expectational variables of the system. The log of technology in both countries is assumed to follow an AR(1) process with zero mean. We assume that $E_{t-1}(\boldsymbol{\varepsilon}_t) = 0$ and $E_{t-1}(\boldsymbol{\eta}_t) = 0$.

If marginal costs are assumed to be decreasing in output (i.e., $\alpha + \gamma > 1$), then the system (26) may not have a unique solution. With our chosen value of increasing returns to scale (see Section 3.2 below), the number of non-predetermined variables exceeds the number of unstable roots by one, and thus we have one degree of indeterminacy.

The model is solved using the Farmer-Khramov-Nicolò (2015) solution method, whereby we redefine one expectational error $\eta_{f,t}$ of vector $\boldsymbol{\eta}_t$ as a new fundamental disturbance.[19] In this way the number of non-predetermined variables is decreased by one. This transformation enables us to treat the indeterminacy model as determinate and we use the popular algorithm of Uhlig (1999) to solve the model. Importantly, Farmer et al. (2015) show that the choice of which expectational error to redefine as a new fundamental shock is irrelevant.[20] Consequently, we choose the forecast error of the terms of trade as the new fundamental, $\eta_{f,t} = \eta^T_t \equiv \widehat{T}_t - E_{t-1}\widehat{T}_t$, and show in the

[17] In the steady state, the degree of increasing returns to scale can be expressed as the ratio between average and marginal costs, which is equal to the markup: i.e., $\frac{(\alpha+\gamma)(\overline{Y}+\phi)}{\overline{Y}} = \chi$. Consequently, for a steady state to exist, the steady-state markup cannot be lower than the degree of diminishing marginal cost i.e., $\chi \equiv \frac{\kappa}{\kappa-1} \geq \alpha + \gamma$.

[18] For bond holdings $\tilde{B}_{H,t}$, we take the linear deviation relative to steady-state Home consumption.

[19] Pintus et al. (2016) and Pavlov and Weder (2017) adopt a similar solution method.

[20] As demonstrated by Theorem 1 of Farmer et al. (2015, p.21), the same solution can be obtained under alternative specifications of the forecast error η^f_t, given a relatively mild regularity condition, which rules out linear dependence in the expectational errors.

Appendix that our results are robust to the choice of expectational error.[21] We refer to the forecast error η_t^T as a self-fulfilling expectation or belief.

An equilibrium is characterized by $\theta^* \in \boldsymbol{\Theta}$, where $\boldsymbol{\Theta}$ is a parameter space which includes the parameters of the structural equations, the variance covariance matrix of the original fundamental shocks, and the variance and covariances of the new fundamental shock with the original set of fundamentals:

$$\boldsymbol{\Theta} \equiv \left\{ \boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}, \boldsymbol{\Omega}_{\varepsilon\varepsilon}, \boldsymbol{\omega}_{\eta\varepsilon}, \sigma_\eta^2 \right\}, \tag{27}$$

where $\boldsymbol{\Omega}_{\varepsilon\varepsilon} \equiv E\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right)$, $\boldsymbol{\omega}_{\eta\varepsilon} \equiv \left[E\left(\varepsilon_t \eta_t^T\right), E\left(\varepsilon_t^* \eta_t^T\right)\right] = E\left(\eta_t^T \boldsymbol{\varepsilon}_t'\right)$, and $\sigma_\eta^2 \equiv E\left[\left(\eta_t^T\right)^2\right]$. By specifying a new fundamental shock together with $\boldsymbol{\omega}_{\eta\varepsilon}$ and σ_η^2 we select a unique rational expectations equilibrium. The covariance of η_t^T with $\boldsymbol{\varepsilon}_t$ represents the response of beliefs to the original set of fundamentals, which amplify or attenuate the effects of technological shocks in the economy (Dufourt et al., 2015).

Farmer et al. (2015) demonstrate that this representation of equilibrium under indeterminacy can be alternatively characterized in terms of a linear forecasting rule that expresses the forecast errors as a function of fundamentals and sunspot shocks. This alternative solution methodology has been proposed in the seminal contributions of Lubik and Schorfheide (2003, 2004). As shown by Farmer et al. (2015), the two representations of equilibrium indeterminacy are entirely equivalent, because for each indeterminate equilibrium $\theta^* \in \boldsymbol{\Theta}$ there exists a unique linear forecasting rule that implements equilibrium θ^*, and vice versa.

The equivalence between the two solution methods enables us to compute the parameters of a linear forecasting rule à la Lubik and Schorfheide, in order to illustrate the relationship between fundamental and sunspot disturbances. For our purposes, we specify the linear forecasting rule as follows:

$$\eta_t^T = [\beta_1, \quad \beta_2] \cdot \boldsymbol{\varepsilon}_t + \zeta_t = \boldsymbol{\beta} \cdot \boldsymbol{\varepsilon}_t + \zeta_t, \tag{28}$$

where the residual ζ_t can be interpreted as a "pure"sunspot shock uncorrelated with fundamentals: $E(\zeta_t) = 0$, $E\left(\zeta_t^2\right) \equiv \sigma_\zeta^2 > 0$, and $E(\zeta_t \boldsymbol{\varepsilon}_t) = \mathbf{0}$.

To aid our understanding of the indeterminacy model we consider two alternative assumptions. (i) *Autonomous beliefs*: shocks to the forecast error of the terms of trade η_t^T are the only source of business cycle fluctuations ($\boldsymbol{\Omega}_{\varepsilon\varepsilon} = \mathbf{0}$ and $\boldsymbol{\omega}_{\eta\varepsilon} = \mathbf{0}$); (ii) *Correlated beliefs*: the forecast error η_t^T is

[21] Table 5 of the Appendix summarizes the simulation results when the forecast error of Home consumption is selected as the new fundamental: $\eta_{f,t} = \widehat{C}_t - E_{t-1}\widehat{C}_t$.

correlated with fundamentals, thus both $\boldsymbol{\Omega}_{\varepsilon\varepsilon}$ and $\boldsymbol{\omega}_{\eta\varepsilon}$ are not restricted to be zero.[22] In this case, we can use the equivalence between the Farmer-Khramov-Nicolò and the Lubik and Schorfheide solution methods to recover $\boldsymbol{\beta}$ and σ_ζ^2 pertaining to equation (28).[23] Multiplying equation (28) by $\boldsymbol{\varepsilon}_t'$ and taking expectations yields:

$$\boldsymbol{\beta} = E\left(\eta_t^T \boldsymbol{\varepsilon}_t'\right) E\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right)^{-1} = \boldsymbol{\omega}_{\eta\varepsilon} \boldsymbol{\Omega}_{\varepsilon\varepsilon}^{-1}. \tag{29}$$

To compute the variance of the pure sunspot shock, first note:

$$\sigma_\zeta^2 = E\left(\zeta_t \zeta_t'\right) = E\left(\left[\eta_t^T - \boldsymbol{\beta} \cdot \boldsymbol{\varepsilon}_t\right] \left[\eta_t^T - \boldsymbol{\beta} \cdot \boldsymbol{\varepsilon}_t\right]'\right),$$

and since $E\left(\eta_t^T \boldsymbol{\varepsilon}_t'\right) = \boldsymbol{\beta} E\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right)$ it follows that:

$$\sigma_\zeta^2 = E\left[\left(\eta_t^T\right)^2\right] - \boldsymbol{\beta} E\left(\boldsymbol{\varepsilon}_t \eta_t^T\right) = \sigma_\eta^2 - \boldsymbol{\beta} \boldsymbol{\omega}_{\eta\varepsilon}'. \tag{30}$$

Next, we describe how we calibrate the structural parameters of matrices $\boldsymbol{\Gamma}_0$, $\boldsymbol{\Gamma}_1$, and $\boldsymbol{\Psi}$ in Section 3.2 below. Since the alternative assumptions of autonomous and correlated beliefs imply different strategies for the calibration of the stochastic processes, we discuss the calibration of σ_η^2, $\boldsymbol{\Omega}_{\varepsilon\varepsilon}$, and $\boldsymbol{\omega}_{\eta\varepsilon}$ separately in Sections 4 and 5 below.

3.2 Parameterization

The baseline parameter values used to compute the indeterminate equilibrium are summarized in Table 1. The U.S. is assumed to be the Home country and the rest of the world represents the Foreign country. The time interval is assumed to be a quarter. As is standard in the literature, we set the discount factor $\beta = 0.99$, which implies an annualized steady-state real interest rate of 4 percent. In the IRBC literature, the utility curvature parameter typically chosen lies between $1 \leq \sigma \leq 2$. Following Stockman and Tesar (1995), among others, we set $\sigma = 2$. The labor share

[22] In the indeterminacy literature (e.g., Dufourt et al., 2015), the forecast error is commonly assumed to be perfectly correlated with fundamentals by setting $\sigma_\zeta = 0$ in equation (28). Doing so would introduce an additional restriction. We choose to leave σ_ζ unrestricted so as not to lose any degree of freedom in our calibration strategy.

[23] Notice that under a linear forecasting rule the equilibrium is characterized by an alternative parameter space $\tilde{\boldsymbol{\Theta}}$ whereby $\boldsymbol{\beta}$ and σ_ζ^2 replace $\boldsymbol{\omega}_{\eta\varepsilon}$ and σ_η^2 in (27):

$$\tilde{\boldsymbol{\Theta}} \equiv \left\{\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}, \boldsymbol{\Omega}_{\varepsilon\varepsilon}, \boldsymbol{\beta}, \sigma_\zeta^2\right\}.$$

Alternatively, a researcher may want to consider a linear transformation of equation (28) and adjust the parameter space accordingly.

Table 1: Baseline parameter values

β	0.99	Discount factor
$\overline{\delta}$	0.023	Steady-state depreciation rate of capital
$\overline{u}$	1	Steady-state capacity utilization rate
μ	1.1	Elasticity of the depreciation rate with respect to the utilization rate
$\overline{L}$	1/3	Steady-state hours worked
ν	0.33	Inverse elasticity of labor supply
σ	2	Utility curvature parameter
a	0.88	Home bias in consumption goods
b	0.88	Home bias in investment goods
θ	0.62	Elasticity of substitution between home & foreign consumption goods
ρ	0.62	Elasticity of substitution between home & foreign investment goods
ω	0.001	Bond adjustment cost
$\overline{T}$	1	Steady-state terms of trade
s_L	0.7	Labor share in production
χ	1.33	Steady-state markup
α	0.40	Elasticity of output with respect to capital
γ	0.93	Elasticity of output with respect to labor

in production is set equal to 0.7 and the preference parameter ψ is set so that in the steady state the agent in each country allocates one-third of their time to market activities.

In line with Benigno and Thoenissen (2008), we set the bond adjustment cost $\omega = 0.001$ and the steady-state terms of trade equal to 1. We set $a = b = 0.88$, which implies a steady-state imports to GDP ratio of 0.12, consistent with the U.S. economy. In terms of the trade price elasticities θ and ρ, recent estimates by Boehm et al. (2019) suggest a value for the elasticity of substitution in the range $0.42 \leq \theta \leq 0.62$. We initially set $\theta = \rho = 0.62$ at the upper range of these estimates, as in Mandelman et al. (2011), and check the robustness of the numerical results for variations in θ and ρ.

Using National Income and Product Accounts (NIPA) data from the U.S. Bureau of Economic Analysis (BEA) for the investment-output ratio ($\overline{I}/\overline{Y}$) and Penn World Table (PWT) version 9.1 data for the capital-output ratio ($\overline{K}/\overline{Y}$), the steady-state depreciation rate is calibrated to match the average $\overline{\delta} = \frac{\overline{I}/\overline{Y}}{\overline{K}/\overline{Y}} = 0.023$ for the period $1973 - 2007$, implying an annual rate of capital depreciation of 9 percent. In terms of the capacity utilization parameters, the parameters h and $\widetilde{\delta}$ are used to calibrate the steady-state capacity utilization rate $\overline{u}$ and the elasticity of the depreciation rate with respect to the utilization rate μ. Following Baxter and Farr (2005), we initially set $\mu = 1.1$, and similar to Mandelman et al. (2011) we normalize $\overline{u} = 1$.

Indeterminacy arises in our model provided that the aggregate labor demand schedule is upward sloping and steeper than the aggregate labor supply schedule (see Figure 1). The slope of the aggregate labor demand schedule is determined by the degree of returns to scale $\alpha + \gamma$, while the inverse elasticity of labor supply ν determines the slope of the aggregate labor supply schedule. We follow Dufourt et al. (2015) in setting $\alpha + \gamma = 1.33$, consistent with the point estimate of Basu and Fernald (1997) for the U.S. manufacturing industry. Given the degree of returns to scale, the value of the Frisch elasticity of labor supply $(1/\nu)$ cannot be too low, so as to avoid the aggregate labor supply schedule becoming steeper than the aggregate labor demand. Consequently, to help generate indeterminacy for empirically plausible values for the degree of returns to scale, a common assumption in the literature is to assume indivisible labor by setting $\nu = 0$. Instead, we follow Dufourt et al. (2015) and set $\nu = 1/3$, which implies a Frisch elasticity of 3. While a Frisch elasticity of 3 is on the high side of empirical estimates, it is consistent with the estimates of Peterman (2016) for the U.S. and is the value recommended by Rogerson and Wallenius (2009).[24]

Finally, since intermediate firms use only capital and labor in the production process (6), this implies that the markup is value added. As discussed by Jaimovich (2007), value-added markups for the U.S. are estimated to lie between 1.2 to 1.4. For simplicity, we follow Hornstein (1993) and equate the steady-state markup χ with the degree of returns to scale, thus $\chi = \alpha + \gamma = 1.33$.

4 Autonomous beliefs

4.1 The international business cycle facts

The estimated moments for the data, given in column 2 of Table 2, are for the period 1973(1) − 2007(4) and are taken from Gao et al. (2014), except for the moments for real net exports and the first-order autocorrelations, which we compute using data from the Quarterly National Accounts of the OECD.[25]

To understand the role of self-fulfilling beliefs, column 3 of Table 2 reports simulation results for the determinacy version of the model, where marginal costs are assumed to be constant (i.e., $\alpha + \gamma = 1$), expectational shocks do not exist, and technology shocks are assumed to follow an AR(1)

[24] For lower values of $1/\nu$, indeterminacy would require a larger degree of increasing returns to scale, which lie outside the upper range of empirical estimates.

[25] All series are logged, except real net exports, and Hodrick-Prescott (HP) filtered with a smoothing parameter of 1600. We adopt the HP filter to ensure comparability of our results with the existing literature. The statistics in Gao et al. (2014) are computed where the U.S. is taken as the Home country and the Foreign country is the aggregate of Canada, Japan, and 19 European countries.

process with zero mean.[26] The autocorrelation parameters are set equal to $\upsilon = \upsilon^* = 0.96$. The standard deviations and cross-correlation of Home and Foreign technology shocks are calibrated so as to match the standard deviation of U.S. output and the cross-correlation of U.S. and Foreign output.

To evaluate the ability of the indeterminacy model to explain international fluctuations, we compare its predictions with respect to a number of well-known stylized facts. In the data, the terms of trade and the real exchange rate are more volatile than output, whereas real net exports are significantly less volatile than output (*volatility anomalies*). Both the terms of trade and real net exports are counter-cyclical over the cycle (*output-correlation puzzles*). The data suggests that the cross-country correlation of output is greater than the cross-country correlation of consumption (*the cross-country correlation anomaly*). Finally, in the data the correlation between consumption and the real exchange rate is negative (*the Backus-Smith puzzle*). All these stylized facts have posed a challenge to international macro models (see, e.g., Thoenissen, 2010). By comparison of columns 2 and 3 of Table 2, the determinacy version of the model fails to generate any of these key features of the data except a negative output correlation for real net exports (-0.44).[27]

4.2 Results

Column 4 of Table 2 reports the results under autonomous beliefs. Here, the forecast error is assumed to be the only source of business cycle fluctuations, and the standard deviations and correlations with technology shocks are set equal to zero: $\boldsymbol{\Omega}_{\varepsilon\varepsilon} = \mathbf{0}$ and $\boldsymbol{\omega}_{\eta\varepsilon} = \mathbf{0}$. Since we choose $\eta_{f,t} = \eta_t^T$, under autonomous beliefs equation (28) is simply:

$$\widehat{T}_t - E_{t-1}\widehat{T}_t = \zeta_t.$$

In this scenario we treat the standard deviation σ_ζ as a free parameter and we calibrate it so as to match the standard deviation of U.S. output in all our experiments.[28]

By comparing columns 2 and 4 of Table 2, one observes that the model is able to generate significantly more volatility for real net exports (0.35) nearly matching the data (0.38). However,

[26] The parameter values used in the simulations are the same as in Table 1 of Section 3.2 above with the exception that $\alpha + \gamma = 1$.

[27] If the capacity utilization rate is assumed to be constant, as is the case in traditional IRBC models, the determinacy model generates a counterfactual positive output correlation of 0.45 for real net exports.

[28] For example, Table 2 shows that in the baseline parametrization we set $\sigma_\zeta = 0.55$ in order to produce a standard deviation of output of 1.49.

Table 2: Main results: Second moments of alternative model versions

	Data	Determinacy	Indeterminacy: Autonomous Beliefs	Indeterminacy: Correlated Beliefs
Standard deviations				
Consumption	0.62	0.61	0.63	0.74
Investment	2.92	2.70	3.52	3.53
Employment	0.68	0.55	0.71	0.81
Terms of Trade	1.77	0.51	0.49	1.46
Real Exchange Rate	2.38	0.39	0.37	1.11
Real Net Exports	0.38*	0.09	0.35	0.73
First-order autocorrelations				
Output	0.87*	0.71	0.73	0.76
Real Exchange Rate	0.82*	0.73	0.74	0.72
Real Net Exports	0.85*	0.73	0.73	0.74
Correlations with output				
Consumption	0.82	0.99	1.00	0.98
Investment	0.94	0.98	1.00	0.65
Employment	0.85	1.00	1.00	0.99
Terms of Trade	-0.16	0.45	0.99	-0.35
Real Net Exports	-0.47*	-0.44	-1.00	-0.10
Cross-country correlations				
Output	0.58	0.58	-1.00	0.30
Consumption	0.43	0.77	-1.00	0.43
Investment	0.41	0.20	-1.00	-0.58
Employment	0.45	0.69	-1.00	0.39
Correlation with the terms of trade				
Real Net Exports	0.47*	-0.86	-0.99	-0.33
Correlation with the real exchange rate				
Relative Consumption	-0.17	0.97	0.99	0.61
Shock processes				
s.d. of ε_t (σ_ε)		0.20	-	0.25
s.d. of ε_t^* (σ_{ε^*})		0.20	-	0.20
s.d. (σ_η)		-	0.55	1.68
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$		0.14	-	0.001
cross-correlation $\rho_{\eta,\varepsilon}$		-	-	0.98
cross-correlation $\rho_{\eta,\varepsilon^*}$		-	-	-0.03

Notes: The estimated sample moments for the data are taken from Gao et al. (2014), except for values denoted by $*$ which are from the authors' own calculations. With the exception of real net exports, all standard deviations are relative to the standard deviation of U.S. GDP (1.49).

the model cannot resolve any other major empirical irregularity of the data in relation to international relative prices or quantities. The terms of trade and the real exchange rate are less volatile than output and the terms of trade and output are predicted to move in the same direction leading to a counterfactual positive correlation. The model generates cross-country correlations which are equal to -1, and the correlation between the real exchange rate and relative consumption is positive and very close to 1, such that the Backus-Smith puzzle arises. The model generates a correlation between the terms of trade and real net exports of -0.99, implying that a deterioration in the terms of trade counterfactually results in a real net trade deficit. While the model can generate counter-cyclical real net exports, the negative correlation generated between real net exports and output is -1, which is much stronger than the data (-0.47).

An important element in understanding how self-fulfilling beliefs are transmitted relates to the labor market. The log-linearized Home and Foreign aggregate labor demands can be expressed as:

$$\widehat{w}_t = \left[\frac{\alpha(\eta-1)}{\eta-\alpha}\right]\widehat{K}_t + \left[\frac{\eta\gamma}{\eta-\alpha}-1\right]\widehat{L}_t - \left[\frac{(1-a)\eta}{\eta-\alpha}\right]\widehat{T}_t + \left[\frac{\eta}{\eta-\alpha}\right]\widehat{Z}_t, \tag{31}$$

$$\widehat{w}_t^* = \left[\frac{\alpha(\eta-1)}{\eta-\alpha}\right]\widehat{K}_t^* + \left[\frac{\eta\gamma}{\eta-\alpha}-1\right]\widehat{L}_t^* + \left[\frac{(1-a)\eta}{\eta-\alpha}\right]\widehat{T}_t + \left[\frac{\eta}{\eta-\alpha}\right]\widehat{Z}_t^*, \tag{32}$$

where in our parameterization $\eta-\alpha>0$ and $\frac{\eta\gamma}{\eta-\alpha}-1>0$. With decreasing marginal costs, the source of indeterminacy arises from an upward-sloping aggregate labor demand schedule, which is steeper than the aggregate labor supply schedule. The labor market of each country is depicted in Figure 1. Under autonomous beliefs, the demand and supply schedules only shift after changes in $\widehat{T}_t$, since $\widehat{Z}_t=\widehat{Z}_t^*=0$.

To understand the poor performance of the indeterminacy model under autonomous beliefs, consider a positive revision to the terms of trade forecast, which results in a depreciation (increase) of the terms of trade $\widehat{T}_t$. The impulse response functions are depicted in Figure 2 and the underlying transmission mechanism is illustrated in Figure 1. After a terms of trade depreciation, the upward-sloping Home aggregate labor demand schedule shifts down (Equation 31 and Figure 1) increasing Home employment, which raises Home output and consumption. Consequently, belief-induced fluctuations counterfactually generate a positive correlation between the terms of trade and output. As the demand for imports increases in the Home country due to higher consumption, real net exports decrease implying a counterfactual negative correlation between the terms of trade and real net exports. For the Foreign country, the Foreign aggregate labor demand schedule (Equation

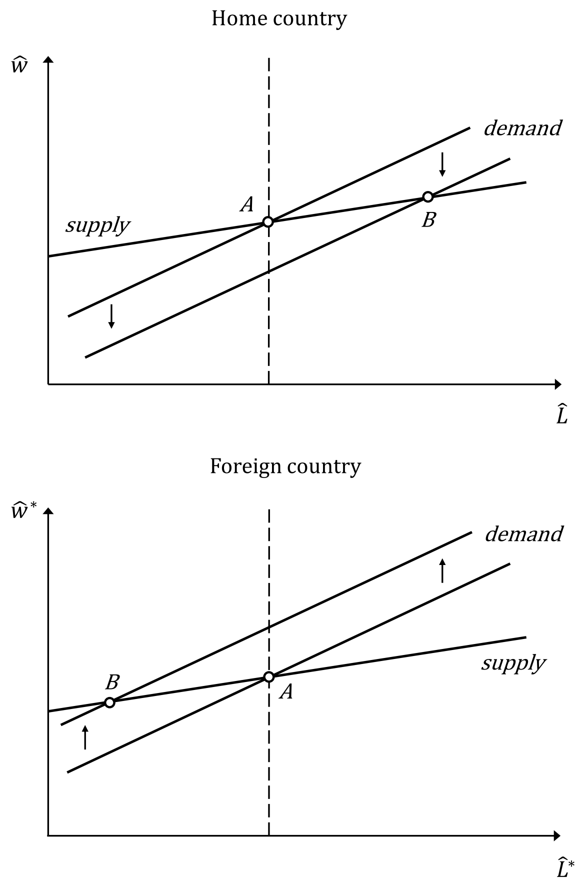

Figure 1: The transmission of a pure sunspot shock in the two country model

32) shifts up in Figure 1, and as a result, Foreign employment decreases, reducing Foreign output and consumption. This explains the perfect negative cross-country correlations generated under autonomous beliefs. Furthermore, while the data suggests that relative consumption decreases in response to a depreciation of international relative prices, self-fulfilling expectations induce a counterfactual positive correlation between relative consumption and the terms of trade. Overall, the indeterminacy model under autonomous beliefs cannot replicate the observed behavior for international relative prices and quantities nor solve the Backus-Smith puzzle.

The above mechanism is in stark contrast to the two-country, one-good indeterminacy models of Guo and Sturzenegger (1998) and Xiao (2004), where self-fulfilling expectations result in positive cross-country correlations for consumption and output. Due to the absence of international relative prices in these models, belief-induced fluctuations stimulate consumption and output in both counties. In our two-good indeterminacy model, self-fulfilling beliefs are global extrinsic

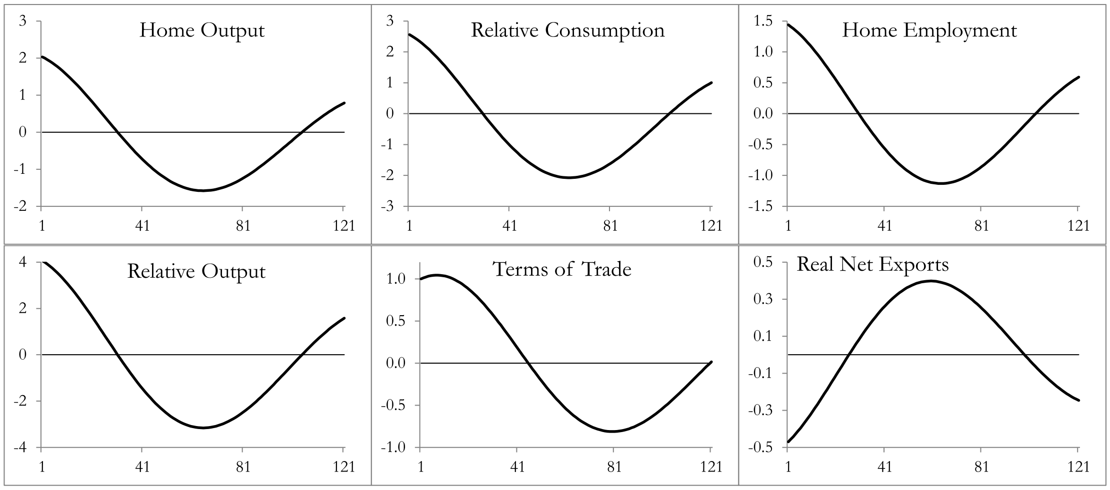

Figure 2: Impulse responses under a positive 1% shock to the terms of trade forecast. Vertical axes: % deviation from the steady state; Horizontal axes: quarters. Autonomous beliefs.

shocks that affect the terms of trade, inducing an output reallocation between the two countries. Consequently, cross-country correlations for consumption and output are negative.

The above analysis shows that the inability of the autonomous beliefs model to replicate the stylized facts stems from the labor market. The transmission of pure (uncorrelated) sunspot shocks is at odds with the data because the upward-sloping Home and Foreign labor demands, which are steeper than supply, move in opposite directions (Figure 1). However, since an upward-sloping aggregate labor demand is at the core of many indeterminacy models, our results will extend to all two-good, open-economy indeterminacy models with sunspot shocks as the only source of fluctuations.

5 Correlated beliefs

The quantitative results from the previous section showed that self-fulfilling beliefs alone cannot replicate the basic international business cycle facts. However, when the forecast error of the terms of trade is correlated with productivity shocks the analysis differs significantly. Inspection of the aggregate labor demand equations (31) and (32) suggest that the indeterminacy model should perform better under positively correlated shocks. In this case, a Home technology shock $\widehat{Z}_t$ causes a revision of expectations (Equation 28) and therefore a belief-induced change in the terms of trade $\widehat{T}_t$. Since both a positive $\widehat{Z}_t$ and a positive $\widehat{T}_t$ affect the aggregate labor demand schedule (Figure 1) in opposite directions, the response of domestic employment and output will not be as strong as under autonomous beliefs. Indeed, both domestic employment and output could actually fall provided the shocks to Home technology and the expectational error are sufficiently positively correlated to generate an upward shift of the Home labor demand schedule (31), and consequently, the correlation between the terms of trade and output would become negative, as in the data. Furthermore, if a temporary domestic contraction results in low Home imports then the model would also generate countercyclical real net exports. Finally, if the model can induce a large enough adjustment in the terms of trade relative to output then the model could potentially generate sizeable volatility improvements for international relative prices and quantities, helping to resolve the volatility anomalies.

5.1 Shock processes

To test the above conjecture we introduce technology shocks and leave the covariances between the fundamental shocks and the forecast error η_t^T unrestricted, and therefore, the matrix $\boldsymbol{\Omega}_{\varepsilon\varepsilon}$ and the vector $\boldsymbol{\omega}_{\eta\varepsilon}$ are not assumed to be zero. As a result, we have six free parameters: the standard deviations of the technology shocks and forecast error (σ_ε, σ_{ε^*}, and σ_η), and the cross correlations between the shocks ($\rho_{\varepsilon,\varepsilon^*}$, $\rho_{\eta,\varepsilon}$, and $\rho_{\eta,\varepsilon^*}$). The vector $\boldsymbol{\omega}_{\eta\varepsilon}$ of the covariances between η_t^T and the technology shocks can be interpreted as a coordination mechanism for revising expectations, which amplify (or attenuate) the effects of technological shocks on the economy.

In line with the IRBC literature, we assume that the stochastic processes for productivity are quite persistent and we set the Home and Foreign autocorrelation parameters equal to $\upsilon = \upsilon^* = 0.96$. Similar to Schmitt-Grohé (2000) and Benhabib and Wang (2013), the standard deviations and cross-correlations of the stochastic processes are calibrated using a method of moments approach, where we include all the moments that define the main stylized facts of international business cycle fluctuations in the objective function. Thus, we explicitly look for the shock properties that maximize the model's ability to match the data, as we want to give the indeterminacy model the best chance at matching the international business cycles facts. Specifically, we calibrate the volatility and cross-correlations of the shocks so as to minimize the distance between selected model moments and data moments.[29] Consistent with the empirical evidence of Backus et al. (1992) and Heathcote and Perri (2004), the cross-country correlation of technology shocks is restricted to be non-negative. We check that the covariance matrix of the shocks that minimizes the objective

[29]Model moments are computed using frequency domain techniques as described in Uhlig (1999).

function is positive semi-definite.[30]

The objective function is computed as the sum of the squared differences between HP-filtered model moments and data moments, and we initially set the weighting matrix equal to the identity matrix. The following eight moments are included in the objective function: the standard deviations of output, the terms of trade, and net exports; the correlations with output of the terms of trade and net exports; the cross-country correlations of output and consumption; and the correlation of the real exchange rate with relative consumption. Therefore, the number of moment conditions exceeds the number of parameters to be estimated by two. In all our estimations we ensure that the standard deviation of output is closely matched, adjusting the weighting matrix if necessary.

5.2 Results

For the parameter values given in Table 1, the final column of Table 2 summarizes the simulation results when self-fulfilling expectations are correlated with technology shocks. Under correlated beliefs, the quantitative performance of the indeterminacy model improves significantly in terms of replicating the data. Now, both the terms of trade and the real exchange rate are more volatile than output generating 83% of the observed standard deviation of the terms of trade. The volatilities of international relative prices have increased by a factor of over 2.8 relative to the determinacy baseline model and by a factor of 3 in comparison to autonomous beliefs.[31] Furthermore, the model can also simultaneously generate sufficient volatility for real net exports. Remarkably, under correlated beliefs the indeterminacy model results in a standard deviation for real net exports over twice as large than under autonomous beliefs and over 8 times larger than the determinacy baseline.

In terms of output correlations, the indeterminacy model with correlated beliefs correctly predicts that both real net exports and the terms of trade are counter-cyclical. In stark contrast to autonomous beliefs, the perfect negative correlation between net exports and output no longer arises. However, the counterfactual negative cross-country correlation of investment remains. The indeterminacy model also fails to generate cross-country output correlations higher than cross-country consumption correlations and the model predicts a counterfactual negative correlation

[30] In a small number of cases the estimated covariance matrix is not positive semi-definite. In these cases, we replace the estimated covariance matrix with its closest positive semi-definite matrix.

[31] Under correlated beliefs, the model still generates less than half the volatility for the real exchange rate relative to the data. This is unsurprising since the real exchange rate in our model is a linear transformation of the terms of trade (due to the assumption of the law of one price and the absence of non-traded goods). See Corsetti et al. (2008) for further discussion.

between the terms of trade and real net exports.[32] Another important discrepancy between the model and the data relates to the correlation between the real exchange rate and relative consumption. Although the model generates a significantly lower positive correlation between the real exchange rate and relative consumption than under autonomous beliefs (0.61 vs. 0.99) and the determinacy baseline model (0.61 vs. 0.97), this correlation remains counterfactual with the data where a negative correlation is observed (−0.17).

In terms of the calibrated standard deviations and shock cross-correlations given in the bottom panel of Table 2, we find that to best match the data revisions to the terms of trade forecasts must be positively correlated with Home productivity shocks, the correlation of beliefs with Home productivity shocks should be close to one, and the standard deviation of beliefs are relatively high.[33] The high values of $\rho_{\eta,\varepsilon}$ and σ_η drive the coefficient β_1 in equation (28) above unity, and the relative low value of $\rho_{\eta,\varepsilon^*}$ drives the coefficient β_2 near zero. In fact, the implied belief vector $\boldsymbol{\beta} = [6.54,\ -0.22]$, which controls how expectations are affected by technology shocks, indicate that domestic productivity shocks, amplified by self-fulfilling beliefs (revisions to the terms of trade forecasts), have a much stronger effect on the business cycle than Foreign productivity shocks.

6 Inspecting the mechanism

6.1 Indeterminacy and the propagation of technology shocks

With the notable exception of the Backus-Smith puzzle, our results show that when self-fulfilling beliefs are correlated with Home productivity shocks the indeterminacy model can solve several international relative price and quantity puzzles. Since this correlation is key, conventional impulse responses cannot completely represent the transmission of productivity shocks. Thus, we now illustrate how to derive combined impulse responses that are applicable when expectational errors are correlated with fundamentals.

Letting $\overline{\Phi}_{\varepsilon}^{X}$ denote the impulse response of variable X to an uncorrelated Home technology shock and $\overline{\Phi}_{\eta_T}^{X}$ denote the impulse response of variable X to a shock to the terms of trade forecast

[32] As shown in the Appendix, the model is successful at predicting a S-curve cross-correlation relationship between the trade balance and the terms of trade.

[33] Under the baseline calibration, we find that a one percent shock to the forecast error has a relatively modest impact on the variables compared to technology shocks. Consequently, the estimation procedure selects a relatively higher standard deviation for the forecast error in order to match the selected moments.

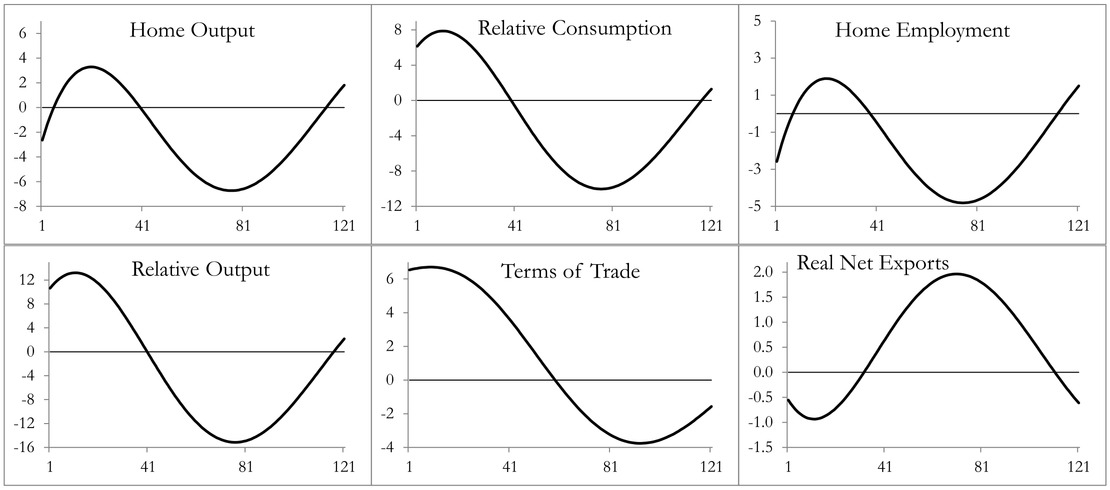

Figure 3: Combined impulse responses under a positive 1% productivity shock in the Home country. Vertical axes: % deviation from the steady state; Horizontal axes: quarters. Correlated beliefs.

η^T, then the combined impulse response Φ_{ε}^{X} to a positive productivity shock is given by:[34]

$$\Phi_{\varepsilon}^{X} = \overline{\Phi}_{\varepsilon}^{X} + E_t\left(\eta_t^T|\varepsilon_t = 1\right)\overline{\Phi}_{\eta_T}^{X} = \overline{\Phi}_{\varepsilon}^{X} + \beta_1\overline{\Phi}_{\eta_T}^{X}, \tag{33}$$

where β_1 is the first element of vector $\boldsymbol{\beta}$ given in (29). For simplicity we have abstracted from Foreign technology shocks.[35] Figure 3 depicts selected impulse response functions, which combine the effect of Home technology shocks with the revision of expectations.

When shocks to the terms of trade are positively correlated with Home technology shocks, a positive Home technology shock results in a belief-induced increase (depreciation) in the terms of trade $\widehat{T}_t$. From inspection of (31), the rise in $\widehat{Z}_t$ and $\widehat{T}_t$ shift the Home aggregate labor demand schedule in opposite directions. The top-right panel of Figure 3 suggests that the increase in employment caused by an increase in $\widehat{T}_t$ is more than offset by the fall in employment caused by the rise in $\widehat{Z}_t$. Consequently, the Home aggregate labor demand schedule in Figure 1 shifts upwards causing an overall fall in Home employment and thus Home output. However, as shown in Figure 3, the negative effect on these variables is small and temporary.

Are the contractionary effects of technology shocks empirically plausible? Since the seminal work of Galí (1999), there is now a large literature that has investigated the labor market effects of

[34]$\overline{\Phi}_{\varepsilon}^{X}$ and $\overline{\Phi}_{\eta_T}^{X}$ are obtained using the Farmer-Khramov-Nicolò (2015) solution method under the assumption that all shocks are uncorrelated.

[35]We ignore the cross-country correlation of Home and Foreign technology shocks, since in our simulated results $\rho_{\varepsilon,\varepsilon^*} = 0.001$.

technology shocks.[36] Several papers have found that positive technology shocks induce a temporary decline in employment and hours (see, for example, Galí, 1999; Basu et al., 2006; Canova et al., 2010, among others), which is at odds with standard RBC theory.[37] On the other hand, due to the upward-sloping aggregate labor demand of Figure 1, indeterminacy models can more easily replicate the temporary contractionary effect on employment in response to a positive technology shock.

The contractionary effect of technology shocks on output is more controversial, but not without empirical support (Galí, 1999; Basu et al., 2006; Canova et al., 2010; Wang and Wen, 2011; Giuli and Tancioni, 2017).[38] A negative output transmission (i.e., a rise in $\widehat{Z}_t$ and $\widehat{T}_t$ accompanied by a fall in output) is crucial for the model to match the negative correlation between output and the terms of trade.[39] However, as shown by Figure 3, the effect on output is small and temporary, delaying the increase in output by a few quarters.

The combined transmission of technology and beliefs is key for understanding the model generated second-order moments given in Table 2. As highlighted by Figure 3, the change in the terms of trade is large relative to domestic output. Therefore, international relative prices are now more volatile than output, as in the data. This is in stark contrast to the model with autonomous beliefs, which also predicts a perfect negative correlation between real net exports and output. Recall that when the international business cycle is driven only by sunspot shocks, domestic output and aggregate demand are stimulated. Imports rise more than exports such that real net exports fall. Under correlated beliefs, the correlation between real net exports and output stays negative but its size is reduced.[40]

The perfect negative cross-country correlations generated under autonomous beliefs no longer arises with correlated beliefs. Under autonomous beliefs, a belief-induced increase in $\widehat{T}_t$ stimulates Home output. In the Foreign country, the rise in $\widehat{T}_t$ causes the Foreign aggregate labor demand curve in Figure 1 to shift up, and the resulting fall in Foreign employment, investment, and

[36] For a recent summary of the literature, see Ramey (2016).

[37] See, for example, Lindé (2009) and Wang and Wen (2011).

[38] Giuli and Tancioni (2017) find a small fall in output in the short-run after a technology improvement. Galí (1999), Basu et al. (2006), Canova et al. (2010), and Wang and Wen (2011) all find little to no effect on output of positive technology shocks in the short-run. However, their estimated confidence intervals imply that a unambiguous short-run reduction in output is possible.

[39] Note that the contractionary effects of technology shocks in our model hinges on the size and direction of the shift in the aggregate labor demand schedule, which depend on the estimated $\boldsymbol{\omega}_{\eta\varepsilon}$. In all our robustness exercises we find a negative output transmission because the correlation between output and the terms of trade is included in the objective function.

[40] As shown in the Appendix, the sensitivity analysis finds that the correlation between real net exports and output is sensitive to the choice of trade price elasticities. The lower the trade elasticities chosen, the less countercyclical is the behaviour of real net exports.

output generates counterfactual negative cross-country correlations. However, when self-fulfilling expectations and Home technology shocks are sufficiently positively correlated, the increase in $\widehat{Z}_t$ more than offsets the belief-induced rise in $\widehat{T}_t$. Now the aggregate labor demand schedules in both countries shift upwards, resulting in positive cross-country correlations for employment and output. However, the cross-country investment correlation remains negative as investment flows to the Home country and falls in the Foreign country. This is because (due to the relatively higher Home employment and the increase in $\widehat{Z}_t$) the marginal productivity of capital is relatively higher in the Home country.

While a negative investment cross-correlation also arises in standard IRBC models, it can be reduced by introducing investment adjustment costs to curb the movement of investment towards the country with higher marginal returns to capital. Alternatively, Baxter and Farr (2005) show that low values of the depreciation rate elasticity μ also help to induce a positive comovement for investment.[41] Both these strategies, however, are not effective in indeterminacy models, since under high investment adjustment costs or a low μ indeterminacy cannot arise.[42] The investment cross-country correlation can be further improved under indivisible labor (as shown in Table 3), although it remains negative.

Another weakness of the correlated-beliefs model is its inability to resolve the Backus-Smith puzzle. To understand this, we focus again on the transmission of Home technology shocks, which have a more marked effect on the revision to the terms of trade forecasts than Foreign technology shocks.[43] Recall that a positive Home technology shock causes a belief-induced increase (deterioration) in the terms of trade, and therefore an increase (depreciation) in the real exchange rate. Consequently, in order to solve the Backus-Smith puzzle the response of Foreign consumption must be above the response of Home consumption for relative consumption to fall, thereby generating a negative correlation with the real exchange rate. However, this cannot happen in the baseline calibration. From Figure 1, the upward shift of the Foreign aggregate labor demand schedule caused by the belief-induced increase in $\widehat{T}_t$ is greater than that of the Home country, since the rise in $\widehat{T}_t$ partially offsets the upward shift of the Home aggregate labor demand schedule due to $\widehat{Z}_t$. Because Foreign employment is relatively lower than Home employment, the response of For-

[41] This is confirmed in Table 2, where the determinacy version of the model generates a positive investment cross-correlation with $\mu = 1.1$.

[42] Table 6 of the Appendix presents results for the maximum level of investment adjustment costs that permit indeterminacy. Although the cross-country investment correlation improves, it remains negative.

[43] As discussed in Section 3, with correlated beliefs the vector $\boldsymbol{\beta}$ (Equation 28) controls how terms-of-trade expectations are affected by technology shocks. In the baseline calibration, β_2 is close to zero.

eign consumption is below Home consumption, generating the counterfactual positive correlation between relative consumption and $\widehat{T}_t$.

The failure of the model to generate a negative correlation between relative consumption and the real exchange rate results in the model counterfactually predicting a negative correlation between real net exports and the terms of trade. Since the response of Foreign consumption is below Home consumption, it follows that Home exports are below imports in the short run, implying a negative correlation between real net exports and international relative prices, as illustrated by Figure 3. In the Appendix we show that this result is robust to the choice of trade elasticities θ and ρ.

6.2 Indeterminacy and the Backus-Smith puzzle

To test whether the performance of the model can be improved, we conduct a number of sensitivity exercises and search for alternative parameterizations to the baseline calibration given in Table 1 that may resolve some of the outstanding puzzles. We find that there are two ways of resolving the Backus-Smith puzzle. The first approach involves setting very low values for the inverse labor supply elasticity υ. The second approach is to remove the restriction that the cross-country correlation of productivity shocks must be nonnegative. The cross-correlation of Foreign productivity shocks is the key to the success of both approaches.

We find that the positive correlation between the real exchange rate and relative consumption is a robust feature of the model, although it can switch sign for values of the inverse labor supply elasticity υ close to zero. The fourth column of Table 3 reports the second moments setting $\upsilon = 0$.[44] By inspection, the correlation between the real exchange rate and relative consumption is negative (-0.06) in the case of indivisible labor. The key mechanism is the estimated negative correlation between the belief and Foreign technology shocks ($\rho_{\eta,\varepsilon^*} = -0.34$), which alters the transmission mechanism of the latter.[45] In this case, a positive Foreign technology shock, which shifts the Foreign labor demand schedule upwards (leading to a fall in Foreign output and demand, thereby increasing relative consumption), is now accompanied by a fall (appreciation) in $\widehat{T}_t$. Figure 4 depicts selected impulse response functions combining the effect of Foreign technology shocks with beliefs.[46] By inspection, the terms of trade and relative consumption now move in opposite directions after a positive Foreign technology shock.

[44] The values of the other parameters are set as in Table 1 with the exception of $\mu = 1.3$.

[45] The estimated standard deviations and shock cross-correlations under indivisible labor imply a belief vector $\boldsymbol{\beta} = [3.62,\ -1.80]$, where β_1 is lower and β_2 is higher in absolute value compared to the baseline calibration.

[46] We ignore the cross-country correlation of Home and Foreign technology shocks, since in our simulated results

Table 3: Simulated results under correlated beliefs with unrestricted cross-country correlations for the productivity shocks ($\rho_{\varepsilon,\varepsilon^*}$) or indivisible labor ($\upsilon = 0$)

	Data	Unrestricted $\rho_{\varepsilon,\varepsilon^*}$	Indivisible labor
Standard deviations			
Consumption	0.62	0.71	0.95
Investment	2.92	3.07	2.01
Employment	0.68	0.79	1.07
Terms of Trade	1.77	1.69	1.70
Real Exchange Rate	2.38	1.28	1.29
Real Net Exports	0.38	0.47	0.45
First-order autocorrelations			
Output	0.87	0.80	0.72
Real Exchange Rate	0.82	0.72	0.72
Real Net Exports	0.85	0.75	0.71
Correlations with output			
Consumption	0.82	0.98	0.98
Investment	0.94	0.95	0.89
Employment	0.85	0.98	0.98
Terms of Trade	-0.16	-0.17	-0.26
Real Net Exports	-0.47	-0.51	-0.38
Cross-country correlations			
Output	0.58	0.30	0.53
Consumption	0.43	0.34	0.55
Investment	0.41	-0.25	-0.15
Employment	0.45	0.30	0.55
Correlation with the terms of trade			
Real Net Exports	0.47	0.82	0.78
Correlation with the real exchange rate			
Relative Consumption	-0.17	-0.14	-0.06
Shock processes			
s.d. of ε_t (σ_ε)		0.27	0.44
s.d. of ε_t^* (σ_{ε^*})		0.23	0.37
s.d. (σ_η)		1.97	1.97
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$		-0.91	0.003
cross-correlation $\rho_{\eta,\varepsilon}$		0.98	0.82
cross-correlation $\rho_{\eta,\varepsilon^*}$		-0.98	-0.34

Notes: See Table 2.

$\rho_{\varepsilon,\varepsilon^*} = 0.003$.

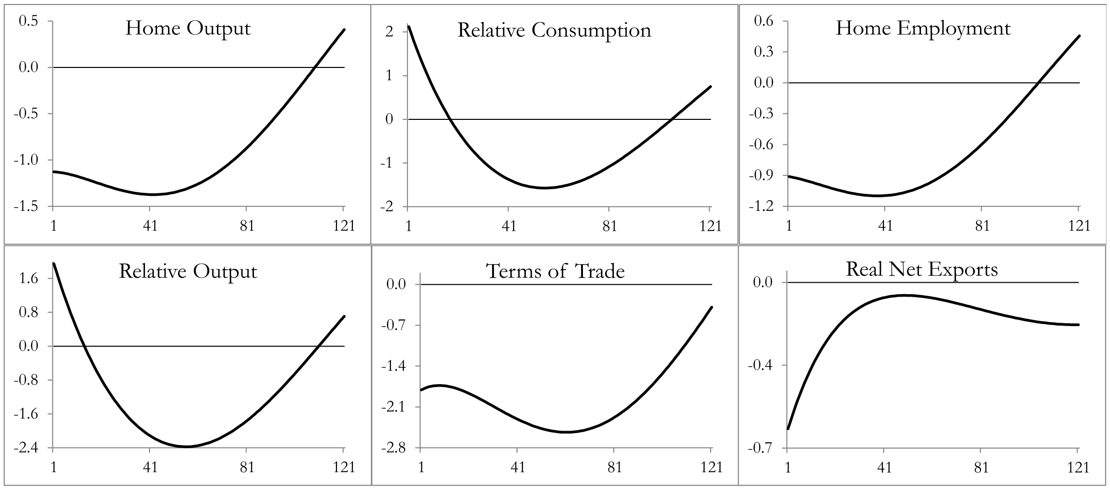

Figure 4: Combined impulse responses under a positive 1% productivity shock in the Foreign country. Vertical axes: % deviation from the steady state; Horizontal axes: quarters. Correlated beliefs.

Table 4 reports the correlation between relative consumption and the real exchange rate for three alternative values of the degree of increasing returns to scale $\alpha + \gamma$. As in the baseline calibration, setting $\alpha + \gamma = 1.33$ can generate a negative correlation only if the inverse labor supply elasticity υ is set equal to zero. While the model can solve the Backus-Smith puzzle under finite labor supply elasticities for a higher degree of increasing returns $\alpha + \gamma = 1.40$, the value of υ required remains close to zero.[47] Moreover, when the returns to scale are set equal to 1.2, the Backus-Smith puzzle cannot be resolved regardless of the value of υ. Therefore, our sensitivity exercises suggest that for empirically plausible returns of scale, the indeterminacy model can only generate a negative correlation between the relative consumption and the real exchange rate for values of the inverse labor supply elasticity close to zero. While the existing literature has highlighted the role of lower levels of aggregate increasing returns for a lower υ in generating indeterminacy, our results show that also the quantitative performance of indeterminacy models is sensitive to the slopes of the aggregate labor demand and supply schedules.

As a second approach, we allow the cross-country correlation of productivity shocks to take any value. The third column of Table 3 shows that the removal of the constraint $\rho_{\varepsilon,\varepsilon^*} \geq 0$ enables the indeterminacy model to generate a negative correlation between the real exchange rate and relative consumption (-0.14) almost matching the data (-0.17). The estimated negative

[47] Using the baseline value of $\upsilon = 0.33$, a value of $\alpha + \gamma = 1.65$ is required to induce a negative correlation between the real exchange rate and relative consumption (-0.05), which is well outside the range supported by the empirical literature.

Table 4: Correlation between relative consumption and the real exchange rate for different values of υ and $\alpha+\gamma$

	Degree of increasing returns		
	1.20	1.33	1.40
$\upsilon = 0$	0.32	-0.06	-0.15
$\upsilon = 0.025$	0.42	-0.02	-0.12
$\upsilon = 0.05$	0.52	0.04	-0.09
$\upsilon = 0.075$	0.64	0.10	-0.05
$\upsilon = 0.10$	0.77	0.17	0.00

Notes: The values of the remaining parameters are set as in Table 1, with the exception of $\mu = 1.3$ and $\chi = \alpha + \gamma$.

correlation between Home and Foreign technology shocks ($\rho_{\varepsilon,\varepsilon^*} = -0.91$) is central in solving the Backus-Smith puzzle. When the correlation between technology shocks cannot be negative, the minimization algorithm selects a very high positive correlation between Home technology shocks and the belief shock, but (almost) no correlation between Foreign technology shocks and the other two shocks (see Table 2). As a result, relative consumption increases after a positive Home technology shock, since the upward shift of the aggregate labor demand schedule is greater in the Foreign country than the Home country. However, if $\rho_{\varepsilon,\varepsilon^*} < 0$, the upward shift of the Foreign aggregate labor demand schedule in Figure 1 caused by the belief-induced increase in $\widehat{T}_t$ is now offset by a fall in $\widehat{Z}_t^*$. Consequently, the response of $\widehat{C}_t^*$ is now above $\widehat{C}_t$, which generates the desired negative correlation between relative consumption and international relative prices.

Under both approaches, the improved performance with respect to the Backus-Smith puzzle is also accompanied by a positive correlation between real net exports and the terms of trade, as in the data. This is because changes in relative consumption affect the demand for exports and imports. When relative consumption falls in response to a rise in international relative prices, real net exports are more likely to increase, thereby moving in the same direction as the terms of trade. Moreover, although the model still struggles to generate cross-country output correlations higher than cross-country consumption correlations, it does much better compared to the determinacy baseline.

In summary, in order to solve the Backus-Smith puzzle, the indeterminacy model requires either a negative cross-country correlation for technology shocks, which is not supported by the empirical evidence, or an infinite (or very large) labor-supply elasticity. Moreover, our analysis suggests that simulation results obtained using indivisible labor may be a special case that cannot be replicated even with small deviations from $\upsilon = 0$. Since the above transmission mechanism originates from the labor market, and is not a specific feature of our model, the findings of this paper will extend to all indeterminacy models with an upward-sloping aggregate labor-demand schedule wishing to explain the key features of the open-economy data.

7 Conclusion

We have analyzed whether equilibrium indeterminacy and self-fulfilling belief-driven fluctuations can explain the major features of international business cycles. We have found that when beliefs are correlated with technology shocks, the indeterminacy model can solve the volatility and output-correlation puzzles for the terms of trade and real net exports. In order to explain some other important features of the open-economy data, the model is shown to require a perfectly elastic labor supply. Under indivisible labor, the indeterminacy model generates significantly improved statistics for the cross-correlation anomaly than the determinate benchmark and a positive contemporaneous correlation between real net exports and the terms of trade. Moreover, the model now correctly predicts a negative correlation between international relative prices and relative consumption, overcoming the Backus-Smith puzzle. However, the model suffers from an international comovement puzzle for investment, where cross-country differences in the marginal productivity of capital counterfactually induce investment flows to the most productive country.

The mechanism we use to generate indeterminacy requires an upward-sloping aggregate labor demand schedule, a common feature of many indeterminacy models. While previous studies in this literature have highlighted the role played by the Frisch elasticity and the degree of returns to scale in generating indeterminacy, we showed that the slopes of the aggregate labor demand and supply schedules also have important quantitative effects. We found that our model performs quantitatively better under a perfectly elastic labor supply curve, which many indeterminacy models rely on. Our analysis suggests that caution is required when using indivisible labor in indeterminacy models, since the results may not be robust with seemingly minor changes in the inverse labor supply elasticity.

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Appendix

AI: Sensitivity analysis

This appendix provides a summary of the results from additional sensitivity exercises that are discussed in the main text. Table 5 presents simulation results for both the autonomous and correlated beliefs cases when the forecast error of Home consumption is selected (instead of the terms of trade as in the main text) as the new fundamental: $\eta_{f,t} = \widehat{C}_t - E_{t-1}\widehat{C}_t$. Tables 2 and 5 show that the estimated moments are the same, regardless of the choice of forecast error, as explained by Farmer et al. (2015).[48]

Table 6 considers the sensitivity of our results to variations in the trade elasticity parameters θ and ρ. Most moments remain unaffected to the choice of trade elasticities, except for real net exports, which become more counter-cyclical the higher the values of θ and ρ and less negatively correlated with the terms of trade. However, the Backus-Smith puzzle remains.

Additionally, we introduce investment adjustment costs in the following way:

$$K_{t+1} = (1 - \delta_t)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t,$$

where in the steady state $S(1) = S'(1) = 0$ and the investment adjustment cost parameter is defined as $\Phi \equiv S''(1) > 0$. With adjustment costs, Home and Foreign investment become additional state variables, making indeterminacy harder to achieve. For the baseline calibration, the maximum level of the investment adjustment costs parameter that permits indeterminacy is a very low value, $\Phi = 0.013$. The final column of Table 6 shows that there is little improvement in the cross-country correlation of investment and the first-order autocorrelations become significantly lower than the data.

[48] Note that the restriction $\rho_{\varepsilon,\varepsilon^*} \geq 0$ was imposed on both sets of estimates.

Table 5: Simulated results under an alternative choice of forecast error (Home consumption): $\eta_{f,t} = \widehat{C}_t - E_{t-1}\widehat{C}_t$

	Data	Autonomous Beliefs	Correlated Beliefs
Standard deviations			
Consumption	0.62	0.63	0.74
Investment	2.92	3.52	3.53
Employment	0.68	0.71	0.81
Terms of Trade	1.77	0.49	1.46
Real Exchange Rate	2.38	0.37	1.11
Real Net Exports	0.38	0.35	0.73
First-order autocorrelations			
Output	0.87	0.73	0.76
Real Exchange Rate	0.82	0.74	0.72
Real Net Exports	0.85	0.73	0.74
Correlations with output			
Consumption	0.82	1.00	0.98
Investment	0.94	1.00	0.65
Employment	0.85	1.00	0.99
Terms of Trade	-0.16	0.99	-0.35
Real Net Exports	-0.47	-1.00	-0.10
Cross-country correlations			
Output	0.58	-1.00	0.30
Consumption	0.43	-1.00	0.43
Investment	0.41	-1.00	-0.58
Employment	0.45	-1.00	0.39
Correlation with the terms of trade			
Real Net Exports	0.47	-0.99	-0.33
Correlation with the real exchange rate			
Relative Consumption	-0.17	0.99	0.61
Shock processes			
s.d. of ε_t (σ_ε)		-	0.25
s.d. of ε_t^* (σ_{ε^*})		-	0.20
s.d. (σ_η)		0.70	0.76
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$		-	0.001
cross-correlation $\rho_{\eta,\varepsilon}$		-	-0.80
cross-correlation $\rho_{\eta,\varepsilon^*}$		-	-0.12

Notes: See Table 2.

Table 6: Simulated results under correlated beliefs: Sensitivity analysis

	Data	Trade price elasticities $\theta = \rho = 0.23$	Trade price elasticities $\theta = \rho = 1.0$	Investment adj. costs
Standard deviations				
Consumption	0.62	0.73	0.72	0.73
Investment	2.92	3.63	3.53	3.13
Employment	0.68	0.79	0.82	0.79
Terms of Trade	1.77	1.35	1.53	1.26
Real Exchange Rate	2.38	1.02	1.16	0.95
Real Net Exports	0.38	0.81	0.70	0.53
First-order autocorrelations				
Output	0.87	0.75	0.78	0.13
Real Exchange Rate	0.82	0.73	0.72	0.52
Real Net Exports	0.85	0.74	0.73	0.22
Correlations with output				
Consumption	0.82	0.98	0.99	0.99
Investment	0.94	0.57	0.73	0.78
Employment	0.85	0.99	0.99	0.99
Terms of Trade	-0.16	-0.24	-0.40	-0.29
Real Net Exports	-0.47	0.01	-0.21	-0.23
Cross-country correlations				
Output	0.58	0.30	0.34	0.27
Consumption	0.43	0.43	0.46	0.39
Investment	0.41	-0.67	-0.43	-0.45
Employment	0.45	0.38	0.43	0.37
Correlation with the terms of trade				
Real Net Exports	0.47	-0.53	0.00	-0.41
Correlation with the real exchange rate				
Relative Consumption	-0.17	0.57	0.65	0.70
Shock processes				
s.d. of ε_t (σ_ε)		0.23	0.26	0.22
s.d. of ε_t^* (σ_{ε^*})		0.21	0.21	0.14
s.d. (σ_η)		1.54	1.76	1.62
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$		0.001	0.001	0.013
cross-correlation $\rho_{\eta,\varepsilon}$		0.96	0.98	0.99
cross-correlation $\rho_{\eta,\varepsilon^*}$		-0.07	-0.03	0.15

Notes: See Table 2.

AII: The S-curve

This appendix investigates whether the correlated-beliefs version of the indeterminacy model can generate a S-shaped cross-correlation between the trade balance and the terms of trade. For this purpose, we follow Backus et al. (1994) and define the trade balance NX_t as the ratio of nominal net exports to nominal output, which in our model:

$$NX_t = \frac{P_{H,t}\left(C^*_{H,t} + I^*_{H,t}\right) - P_{F,t}\left(C_{F,t} + I_{F,t}\right)}{P_{H,t}Y_t},$$

where Y_t is defined from equation (20) of the main text.

Figure 5 depicts the cross-correlations between the trade balance (time $t+j$) and the terms of trade (time t) for leads and lags of j up to ten quarters for two versions of the indeterminacy model: the baseline model of Section 5 and the model under indivisible labor. As in Dimitriev (2017), we plot the cross-correlations for two sample periods covering 1973Q1–2007Q4 and the Great Moderation 1984Q1–2007Q4. By inspection of Figure 5, the indeterminacy model generates a (horizontal) S-curve relationship for cross-correlations between the trade balance and the terms of trade. Similar to the predictions of the workhorse two-good IRBC model of Backus et al. (1994), our model generates a negative contemporaneous cross-correlation between NX_{t+j} and T_t at $j = 0$.

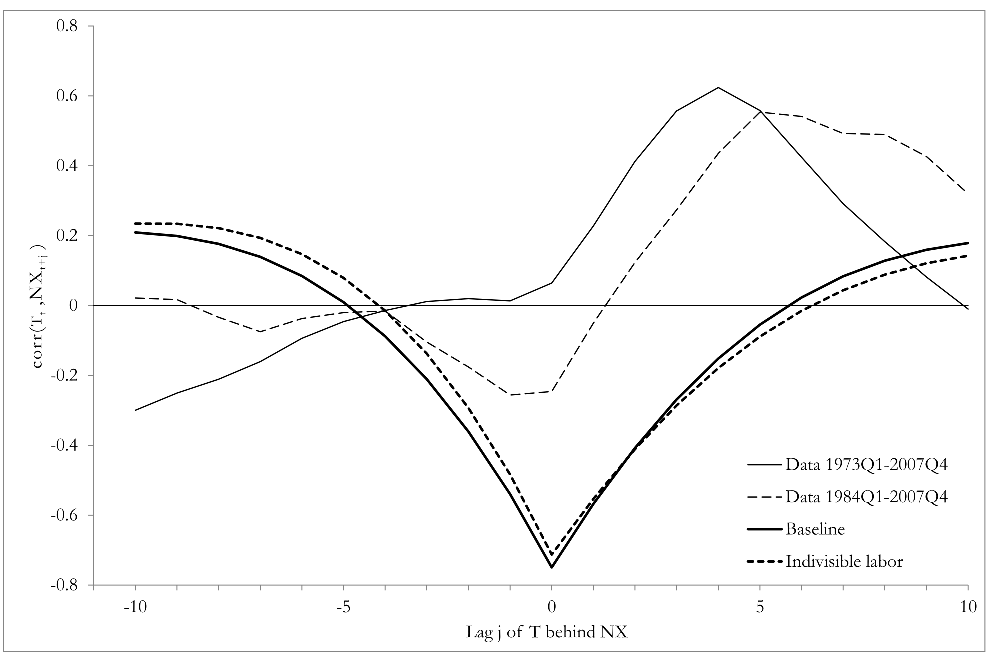

Figure 5: The S-curve: Cross-correlations between the trade balance and the terms of trade

Consequently, the model fits the data best for the Great Moderation period.