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NKPC-Based Inflation Forecasts with a Time-Varying Trend*

Stephen McKnight† Alexander Mihailov‡ Fabio Rumler§

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Abstract

Does theory aid inflation forecasting? This paper develops a forecasting procedure based upon a generalized New Keynesian Phillips Curve that incorporates time-varying trend inflation. Using quarterly data for the Euro Area and the United States over the period 1970-2015, we decompose inflation into trend and cyclical components and generate theory-implied predictions for both, which are recombined to obtain an overall inflation forecast. We find that our forecasting procedure outperforms in predictive accuracy the conventional random walk benchmark at all horizons considered (up to 20 quarters). Moreover, it also outperforms quantitatively the agnostic Atkeson-Ohanian (2001) benchmark that previous studies have found difficult to beat.

Keywords: time-varying trend, generalized New Keynesian Phillips Curve, inflation dynamics, inflation forecasts, predictive accuracy

JEL classification: C53, D43, E31, E37, F41, F47

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Contents

List of Figures

List of Tables

1 Introduction

> "Without resort to theory [...] conclusions relevant to the guidance of policies cannot be drawn." Koopmans (1947: p. 167)

Given its policy importance, there exists a substantial literature on inflation dynamics and the associated problem of inflation forecasting.[1] While one strand of the literature is broadly agnostic about the underlying macroeconomic model that generates the inflation path,[2] another strand has adopted a structural approach, with explicit links to macroeconomic models with microeconomic foundations. A key component of the microfounded approach to inflation dynamics is the estimation of the so-called New Keynesian Phillips Curve (NKPC), an aggregate inflation equation arising in dynamic stochastic general equilibrium (DSGE) models with price adjustment frictions.

Since Galí and Gertler (1999), the most popular approach to estimate the NKPC is via limited-information single-equation methods, which use the generalized method of moments (GMM) proposed by Hansen (1982) to operationalize the expectational terms under the rational expectations hypothesis.[3] An alternative approach is to employ a full-information system estimation using frequentist or Bayesian maximum likelihood techniques.[4] Whereas the latter approach has become widespread in conducting policy analysis, to date it has been less employed as a forecasting tool.[5]

Empirical findings from a number of studies have shown that Phillips curve-based inflation forecasts perform poorly in 'pseudo-out-of-sample' predictive eval-

[1] For comprehensive surveys, see Stock and Watson (1999, 2003, 2007, 2009); Edge and Gürkaynak (2010); Rossi and Sekhposyan (2010, 2014); Ball and Mazumder (2011); Faust and Wright (2013); Dotsey et al. (2018).

[2] This approach applies variants of popular time-series analysis techniques that impose minimal theoretical restrictions, e.g., Engle and Granger (1987), Lütkepohl (1987), Harvey (1989) and Johansen (1996). For a completely agnostic approach – known as Singular Spectrum Analysis – centred on the learning from pure data structures (without any a priori economic theory modeling), see, e.g., Golyandina et al. (2001), Hassani et al. (2009) and Patterson et al. (2010).

[3] See, e.g., Galí et al. (2001, 2003, 2005); Sbordone (2002, 2005); Leith and Malley (2007); Rumler (2007); Rumler and Valderrama (2010); Mihailov et al. (2011 a, b); King and Watson (2012); Kichian and Rumler (2014); Posch and Rumler (2015).

[4] See, e.g., Smets and Wouters (2003, 2007); Ireland (2004); Lindé (2005); An and Schorfheide (2007); Adolfson et al. (2007); Del Negro et al. (2015), Cai et al. (2018). These different econometric methods sometimes reach contradictory conclusions (e.g., the findings of Smets and Wouters, 2007, relative to Galí and Gertler, 1999). For further details, see King and Watson (2012).

[5] See, e.g., Adolfson et al., 2007; Faust and Wright, 2013; Gürkaynak et al., 2013; Wickens, 2014. However, recent work by Cai et al. (2018) based on the New York Fed DSGE model performs well at forecasting inflation since the Great Recession.

uation against a good univariate benchmark (e.g., Stock and Watson, 2009). Notably, Atkeson and Ohanian (2001), henceforth AO, found that since 1984 backward-looking Phillips curve forecasts for inflation in the United States (US) were inferior to a naïve forecast of 12-month inflation by its average rate over the previous 12 months. The AO finding has proven difficult to beat in inflation forecasting (e.g., Stock and Watson, 2007, 2009; Faust and Wright, 2013). Faust and Wright (2013) evaluate the predictive performance of a set of 16 commonly applied inflation forecasting methods and three judgemental forecasts of inflation based on private sector surveys and the Greenbook. They conclude that judgmental forecasts are remarkably hard to beat, and also find that very simple methods, which limit or avoid parameter estimation, tend to predict inflation relatively well. Again, the AO 'pseudo' or 'average' random walk (RW) forecast comes out among the best-performing forecasting methods in terms of predictive accuracy, not only for inflation forecasts for the US, but also for Canada, Germany, Japan, and the United Kingdom.

However, the numerous studies on NKPC derivation and estimation have typically ignored time-varying trend inflation, by assuming that net inflation is either zero or constant in the steady state.[6] The aim of this paper is to address this gap in the literature. As in Adolfson et al. (2007), but differently from Cogley and Sbordone (2008), we introduce drifting trend inflation by assuming price indexation to both last-period actual inflation and current-period trend inflation.[7] From a theoretical as well as empirical and policymaking perspective, such an indexation scheme by the private sector is important as it can capture not only inflation persistence but, notably, shifts in monetary policy preferences and inflation targets; since central bank mandates and policymaking frameworks evolve over time in response to changes in the macroeconomic and institutional environment, there should be potential gains in accounting for them, especially in medium- and longer-horizon inflation forecasting. Moreover, as shown in section 3, these assumptions result in inflation dynamics that are broadly consistent with the time-series properties of our data set. To the best of our knowledge, this is the first study to examine the predictive ability of the single-equation NKPC approach

[6] Constant trend inflation was introduced by Ascari (2004) and Ascari and Ropele (2007), while drifting trend inflation (with interpretation as a time-varying inflation target) appears to have been first modeled by Adolfson et al. (2007) in a small open-economy DSGE set-up and by Cogley and Sbordone (2008) within a single-equation estimation focus.

[7] Cogley and Sbordone (2008) incorporate drifting trend inflation by assuming price indexation to lagged inflation only. They show, using Bayesian time-varying coefficient VAR methods, that a purely forward-looking version of the NKPC fits US quarterly data well for the period 1960:1-2003:4.

with time-varying trend (TVT) inflation.

In this paper we propose a novel 'time-varying trend New Keynesian Phillips Curve (TVT-NKPC)' procedure to forecast inflation. It highlights the predictive value of allowing for the pervasive drifting trends in observed macrovariables when deriving from microfoundations the NKPC equation, and subsequently employing it in predicting inflation. It consists of first applying a one-sided Hodrick-Prescott (1997) filter to separate trend from cyclical components in each fixed-length rolling window or augmenting-length recursive window for re-estimation and prediction that resembles real-time forecasting in our pseudo-out-of-sample simulations.[8] Then, the TVT-NKPC equation for the cyclical component of inflation generates forecasts for this component via a corresponding theory-implied auxiliary vector autoregression (VAR).[9] Following Galí and Gertler (1999), unobservable real marginal cost (RMC) is proxied by real unit labor cost (RULC). Alternatively, we also construct a calibrated proxy for RMC from four observable time series and four parameters arising in the open-economy monetary model of McKnight and Mihailov (2015). In this case, the RMC proxy includes both terms-of-trade and real-balance effects, which complement the standard aggregate demand channel of monetary policy transmission. Our TVT-NKPC theory also generates predictions for trend inflation, which are employed in the forecasting of the trend component of inflation as a stochastic AR(1) process that incorporates a unit root. We build on the predictive accuracy comparisons in Stock and Watson (2007) and Faust and Wright (2013), by selecting the agnostic AO inflation forecast and driftless RW as the main benchmarks to evaluate the predictive performance of our TVT-NKPC inflation forecasting procedure.[10]

Predictive accuracy is assessed pseudo-out-of-sample via the commonly adopted criterion of root mean squared forecast error at the policy-relevant horizons of 1, 4 ('short run'), 8, 12 ('medium run'), 16 and 20 ('long run') quarters using quarterly time series for the Euro Area (EA) and the US that cover almost half a century,

[8] The rolling-window re-estimation, in particular, handles gradual (and possibly unknown) structural change, as argued in Bauwens et al. (2015), and is implemented in many inflation forecasting studies (see, e.g., Kascha and Ravazzolo, 2010).

[9] As far as the cyclical component of inflation is concerned, our approach is related to some extent to the so-called 'semi-structural' forecasting methods in the studies of Rumler and Valderrama (2010), Liu and Jansen (2011), Kichian and Rumler (2014) and Posch and Rumler (2015). However, this literature ignores any time-varying trend inflation and assumes agnostic time-series models for real unit labor cost as the single driving variable in the NKPC equation. The semi-structural approach has had only nonsystematic, partial success in specific countries and at specific horizons relative to inflation predictions exploiting the (driftless) random walk or a few simple AR benchmarks.

[10] The driftless RW has been the conventional forecasting benchmark since Meese and Rogoff (1983).

1970:1-2015:4 (184 quarters), and a forecasting evaluation period that constitutes, roughly, one-third of the sample, 2000:1-2015:4 (64 quarters). In contrast to the scepticism in the existing literature, the results from the inflation predictions generated from our theory-based method are positive, which points to the essential role played by modeling properly drifting trends in macroeconomic environments. We find that the TVT-NKPC forecasting procedure outperforms the conventional random walk benchmark at all horizons, up to a margin of 20-25 percentage points in the medium- and long-run inflation forecasts (significantly at 1 and 8 quarters in the EA and US data and also at 16 and 20 quarters in the EA case). Moreover, it also outperforms quantitatively, by 10-20 percentage points beyond the short run of 1 and 4 quarters, the AO benchmark that previous studies have found difficult to beat (significantly at the medium run of 8 and 12 quarters in the US case). Overall, therefore, our findings offer encouragement on the potential of the TVT-NKPC to generate reliable theory-based forecasts of inflation.

The paper is organized as follows. The next section derives the TVT-NKPC and justifies theoretically the concept of fundamental inflation and the law of motion of trend inflation employed in the forecasts. Section 3 discusses our data and empirical implementation, while section 4 reports and interprets our predictive evaluation results. The final section concludes. Supplementary material is available online that includes Appendices A–C.[11]

2 Theoretical Framework

In this section, we outline the key steps in the derivation of the TVT-NKPC, leaving the details in Appendix B.1. While typical NKPC equations are derived by approximating the equilibrium conditions that arise under Calvo (1983) price setting around a zero (net) inflation steady state, we deviate in two important dimensions. First, we derive the NKPC around a time-varying trend inflation assuming *full* indexation of non-optimized prices to *both* past actual inflation (as in Cogley and Sbordone, 2008) and current trend inflation (as in Adolfson et al., 2007). Secondly, to construct a proxy for real marginal costs we follow McKnight and Mihailov (2015) and include real money balances and openness to international trade.

[11] Appendix A contains a full description of our data set, with relevant sources and definitions; Appendix B provides additional derivations and technical detail; Appendix C presents and briefly discusses additional results (mostly illustrated in figures and tables to which we sometimes refer to later on).

2.1 Microfoundations of the NKPC with a Time-Varying Trend

Suppose that there is a continuum of monopolistically competitive firms each producing a differentiated product i. As standard in New Keynesian theory, the demand for good i is then

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t,$$

where ϵ denotes the elasticity of (intratemporal) substitution across the differentiated products. In any period t, a producer i is allowed to optimally change her price with probability $1-\alpha$ and with probability α this price is instead mechanically updated according to the rule

$$P_t(i) = \Pi_{t-1}^{\rho} \overline{\Pi}_t^{1-\rho} P_{t-1}(i). \tag{1}$$

Consequently, non-optimizing firms *fully* index their prices to a *mixture* of one-period lagged (gross) inflation with weight $0 \leq \rho \leq 1$, $\Pi_{t-1} \equiv P_{t-1}/P_{t-2}$, where P_t is a measure of the aggregate price level, and current-period time-varying trend (gross) inflation with weight $1-\rho$, $\overline{\Pi}_t \equiv \overline{P}_t/\overline{P}_{t-1}$.

Quite generally, and consistent with the time-series properties of our data set summarized in Table 2 of section 3, time-varying trend (gross) inflation is assumed to follow a law of motion that is known to the agents, and (re-)estimated by them recursively when forecasting trend inflation, which incorporates a *unit root*

$$\overline{\Pi}_t = \left(g_t^{\overline{\Pi}}\right) \overline{\Pi}_{t-1}, \tag{2}$$

where the (gross) *growth rate* of time-varying trend inflation $g_t^{\overline{\Pi}} \equiv \overline{\Pi}_t/\overline{\Pi}_{t-1}$ is an exogenous (again, re-estimated) *stationary* stochastic process. We model this process in the simplest possible way; that is, as a multiplicative AR(1) process,

$$g_t^{\overline{\Pi}} = \left(g_{t-1}^{\overline{\Pi}}\right)^{\theta} \varepsilon_{g^{\overline{\pi}},t}, \tag{3}$$

with $\varepsilon_{g^{\overline{\pi}},t} \rightsquigarrow i.i.d. \left(1, \sigma_{g^{\overline{\pi}}}^2\right)$ and $0 < \theta < 1$, the *deterministic* steady state value of which is 1.

In this set-up, if a producer gets stuck without being able to optimally change her price in $t+1$ as in $t-1$, the updating rule (1) becomes

$$P_{t+1}(i) = \left(\Pi_t^{\rho}\overline{\Pi}_{t+1}^{1-\rho}\right)\left(\Pi_{t-1}^{\rho}\overline{\Pi}_t^{1-\rho}\right)P_{t-1}(i).$$

The demand for intermediate good i at $t+s$ if producer i last reoptimized at t is thus

$$Y_{t,t+s}(i) = \left[\frac{\Psi_{t,t+s}P_t^*(i)}{P_{t+s}}\right]^{-\epsilon} Y_{t+s}, \tag{4}$$

where $P_t^*(i)$ is the optimal price for i chosen at t and

$$\Psi_{t,t+s} \equiv \prod_{j=0}^{s-1}\Pi_{t+j}^{\rho}\overline{\Pi}_{t+j+1}^{1-\rho}, \tag{5}$$

with the normalization $\Psi_{t,t} = 1$.

It follows that if a producer i is drawn to reset her price at time t, she will select $P_t^*(i)$ so as to maximize expected discounted future profits,[12]

$$\max_{P_t^*} E_t\sum_{s=0}^{\infty}(\alpha)^s M_{t,t+s}\left[\frac{\Psi_{t,t+s}P_t^*}{P_{t+j}}Y_{t,t+s} - TC_{t+s}\left(Y_{t,t+s}\right)\right], \tag{6}$$

where E_t is the expectation operator conditional on information available at time t,[13] $Y_{t,t+s}$ is firms' (expected) output at their optimally set price in t, subject to the sequence of demand constraints in (4), and the time-varying function $TC_{t+j,t}(\cdot)$ is the (expected) *real* total cost function. As is standard, the stochastic discount factor (or pricing kernel) is defined as

$$M_{t,t+s} \equiv \beta\frac{\Lambda_{t+s}}{\Lambda_t},$$

where β is the deterministic discount factor and Λ_t denotes the marginal utility of wealth of firm owners. The first-order necessary condition for P_t^* is

$$E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\frac{\Lambda_{t+s}}{\Lambda_t}\left[\frac{\Psi_{t,t+s}}{P_{t+s}}\left(\frac{\Psi_{t,t+s}P_t^*}{P_{t+s}}\right)^{-\epsilon}Y_{t+s} - \frac{\epsilon}{\epsilon-1}\frac{\Psi_{t,t+s}}{P_{t+s}}\left(\frac{\Psi_{t,t+s}P_t^*}{P_{t+s}}\right)^{-\epsilon-1}Y_{t+s}MC_{t,t+s}\right] = 0, \tag{7}$$

where $MC_{t,t+s}$ depends on terms that are specific to the firms resetting their price

[12] All firms that are given the opportunity to reset their price in period t behave in an identical manner. Hence, $P_t^*(i) = P_t^*$.

[13] Imposing rational expectations rather than assuming learning (e.g., through updating estimates of VAR coefficients as in Cogley and Sbordone, 2008) is inconsequential for the purposes of the forecasting exercise we are interested in here.

at t and not changing it through $t+s$,

$$MC_{t,t+s} = MC_{t+s}\left[\frac{\Psi_{t,t+s}P_t^*}{P_{t+s}}\right]^{-\epsilon} Y_{t+s}, \tag{8}$$

and MC_t denotes the *real* marginal cost function. Denoting the cumulative gross inflation rate between dates t and $t+s$ as

$$\Pi_{t,t+s} \equiv \frac{P_{t+s}}{P_t}$$

and the optimal relative price as

$$p_t^* \equiv \frac{P_t^*}{P_t}$$

allows us to express condition (7) more compactly:

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \frac{\Lambda_{t+s}}{\Lambda_t}\left(\frac{\Psi_{t,t+s}}{\Pi_{t,t+s}}p_t^*\right)^{-\epsilon} Y_{t+s}\left[\frac{\Psi_{t,t+s}}{\Pi_{t,t+s}}p_t^* - \frac{\epsilon}{\epsilon-1}MC_{t,t+s}\right] = 0. \tag{9}$$

With our assumptions on price-setting, the aggregate price level evolves according to

$$P_t = \left[(1-\alpha)\left(P_t^*\right)^{1-\epsilon} + \alpha\left(\Pi_{t-1}^{\rho}\overline{\Pi}_t^{1-\rho}P_{t-1}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \tag{10}$$

which can also be written in terms of the optimal relative price, p_t^*, as

$$1 = (1-\alpha)\left(p_t^*\right)^{1-\epsilon} + \alpha\left(\frac{\Pi_{t-1}^{\rho}\overline{\Pi}_t^{1-\rho}}{\Pi_t}\right)^{1-\epsilon}. \tag{11}$$

Let $\widehat{x}_t$ denote either *stationary* log-deviations of variables $\widetilde{x}_t$ from their drifting trend value $\overline{x}_t$ or *stationary* log gross growth rates. Log-linearizing equations (8), (9), and (11) around a time-varying trend (see Appendix B.1 for details) yields the following generalized NKPC:

$$\widehat{\Pi}_t - \rho\widehat{\Pi}_{t-1} + \rho\widehat{g}_t^{\overline{\Pi}} = \frac{(1-\alpha)(1-\alpha\beta g^{\Lambda}g^{Y})}{\alpha(1+\epsilon\omega)}\widehat{MC}_t + \beta g^{\Lambda}g^{Y}E_t\left[\widehat{\Pi}_{t+1} - \rho\widehat{\Pi}_t + \rho\widehat{g}_{t+1}^{\overline{\Pi}}\right], \tag{12}$$

where $\widehat{\Pi}_t \equiv \ln\widetilde{\Pi}_t = \ln\left(\Pi_t/\overline{\Pi}_t\right) = \ln\left(1+\pi_t\right) - \ln\left(1+\overline{\pi}_t\right) \approx \pi_t - \overline{\pi}_t \equiv \widehat{\pi}_t$ denotes the cyclical component of period t (net) inflation, $\widehat{MC}_t \equiv \ln\widetilde{MC}_t = \ln\left(MC_t/\overline{MC}_t\right) = \ln MC_t - \ln\overline{MC}_t = mc_t - \overline{mc}_t \equiv \widehat{mc}_t$ is the cyclical component of period t (log-) real marginal cost, and $\widehat{g}_t^{\overline{\Pi}} \equiv \ln\left(\overline{\Pi}_t/\overline{\Pi}_{t-1}\right) = \ln\left(1+\overline{\pi}_t\right) -$

$\ln(1+\overline{\pi}_{t-1}) \approx \overline{\pi}_t - \overline{\pi}_{t-1} \equiv \widehat{g}_t^{\overline{\pi}}$ is the (net) growth rate of the time-varying inflation trend in t relative to $t-1$. The parameters ω, β, $g^Y \equiv \overline{Y}_t/\overline{Y}_{t-1}$ and $g^\Lambda \equiv \overline{\Lambda}_t/\overline{\Lambda}_{t-1}$ capture, respectively, the degree of strategic complementarity, the deterministic discount factor, and the (gross) growth rates of output and of the marginal utility of wealth of firm owners, both evaluated at the drifting steady state.

2.2 Implications of Money and Openness for Real Marginal Costs

In addition to the standard RULC proxy for RMC, we also employ another RMC proxy based on the open-economy monetary model of McKnight and Mihailov (2015) with nonseparable utility preferences for consumption and real money balances. A log-linear approximation around the time-varying trend assumed here yields the following expression for the cyclical component of RMC (see Appendix B.2 for details),[14]

$$\widehat{mc}_t = \varpi\widehat{Y}_t + \sigma\widehat{C}_t - \chi\widehat{m}_t + (1-a)\,\widehat{S}_t, \tag{13}$$

where C_t, m_t, and S_t denote consumption, real money balances, and the (*Home* economy) terms of trade, respectively. For the parameters given in (13), $\varpi > 0$ is the output elasticity of real marginal cost, $\sigma > 0$ is the coefficient of relative risk aversion (CRRA), $\chi \geq 0$ is the degree of nonseparability of real money balances, and $0 < a \leq 1$ is the degree of home bias in production.[15] The structural parameters of (13) are calibrated using the values justified by McKnight and Mihailov (2015). Namely, we set $\varpi = 0.47$, $\sigma = 0.16$, $\chi = 0.02$ and $a = 0.85$. Consequently, given the four observable time series for Y_t, C_t, m_t and S_t, we can obtain a proxy for the unobservable (cyclical component of) real marginal cost implied by the equilibrium conditions of a microfounded open-economy monetary model.

[14] Under full price indexation, price dispersion has no real consequences up to the first order for the stationary distribution of the other endogenous variables. For further discussion, see, e.g., Schmitt-Grohé and Uribe (2007).

[15] As discussed by Woodford (2003), in popular *cashless* economies (i.e., $\chi = 0$), a policymaker does not face a trade-off between inflation and output stabilization. However, with $\chi > 0$ money enters (13) as a negative 'cost-push' shock.

2.3 Linking the TVT-NKPC Theory to Estimation and Inflation Forecasting

2.3.1 Re-estimation and Forecasting of Drifting Trend Inflation

The law of motion for drifting trend inflation embodied in (2) and (3) implies a straightforward one-step ahead forecast (for t-indexed variables assumed observed or estimated):

$$E_t\left[\overline{\Pi}_{t+1}\right] \equiv \overline{\Pi}_{t+1|t} = g_t^{\overline{\Pi}}\overline{\Pi}_t. \tag{14}$$

We then use this forecast in generating iterative forecasts for any horizon $h = 1, ..., 20$ quarters for the trend component of inflation after re-estimating (recursively) $g_t^{\overline{\Pi}}$ from the data. The h-step ahead forecast based on this stochastic AR(1) trend dynamics becomes

$$\overline{\Pi}_{t+h|t} = \left(g_t^{\overline{\Pi}}\right)^h \overline{\Pi}_t, \text{ for } h = 1, ... \tag{15}$$

This is the forecasting procedure for time-varying trend inflation we implement after re-estimation of $g_t^{\overline{\Pi}}$ at every prediction origin, and then add up to the corresponding forecast for the cyclical component of inflation generated according to the TVT-NKPC, which we describe next.

2.3.2 Re-estimation of the TVT-NKPC in Quasi-First Difference Form

We now transform the generalized NKPC equation (12) in a form that is more suitable for forecasting inflation. Following from (2) and (3), the evolution of the growth in trend inflation $\widehat{g}_t^{\overline{\pi}} \equiv \ln\left(\overline{\Pi}_t/\overline{\Pi}_{t-1}\right)$ obeys:

$$\widehat{g}_t^{\overline{\pi}} = \theta\widehat{g}_{t-1}^{\overline{\pi}} + \varepsilon_{\widehat{g}^{\overline{\pi}},t}, \tag{16}$$

where $0 < \theta < 1$ is the persistence parameter (to be re-estimated in recursive subsamples in our forecasting procedure) and $\varepsilon_{\widehat{g}^{\overline{\pi}},t} \equiv \ln \varepsilon_{g^{\overline{\pi}},t} \rightsquigarrow i.i.d.\left(0, \sigma^2_{\widehat{g}^{\overline{\pi}}}\right)$. Under this – theory and data consistent – AR(1)-trend-growth assumption, we can re-write (12) as

$$\widehat{\pi}_t - \rho\widehat{\pi}_{t-1} = \gamma\left[E_t\widehat{\pi}_{t+1} - \rho\widehat{\pi}_t\right] + \kappa\widehat{mc}_t + \rho\left(\theta\gamma - 1\right)\widehat{g}_t^{\overline{\pi}}, \tag{17}$$

where

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta g^{\Lambda}g^{Y})}{\alpha(1+\epsilon\omega)} \qquad \text{and} \qquad \gamma \equiv \beta g^{\Lambda}g^{Y}.$$

Equation (17) expresses the generalized NKPC in a quasi-first difference form, with $0 < \rho < 1$ being the weight of indexation of nonoptimized prices to last-period inflation. As discussed by Cogley and Sbordone (2008), estimates of ρ tend to cluster around the point estimate of 0.2.[16] Therefore, we calibrate $\rho = 0.2$ in (17) for the purposes of re-estimating the other two reduced-form parameters in the TVT-NKPC in quasi-first differences, κ and γ.

The generalized TVT-NKPC given by (17) can be reduced to two common special cases when the weight of indexation of nonoptimized prices to lagged inflation takes its extreme values. If $\rho = 1$, indexation is *only* to last-period actual (gross) inflation, and this leads to the generalized TVT-NKPC estimated in Cogley and Sbordone (2008). For example, assuming a zero net inflation steady state (where trend inflation is constant and so $\widehat{g}_t^{\overline{\pi}} = 0$; and similarly $g^{\Lambda} = g^{Y} = 1$ so that $\gamma = \beta$), the NKPC given in (17) reduces to the well-known 'hybrid' NKPC, containing a backward-looking term, $\widehat{\pi}_{t-1}$, that arises from this indexation:

$$\widehat{\pi}_t = \frac{1}{1+\beta}\widehat{\pi}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1} + \kappa\widehat{mc}_t. \tag{18}$$

If $\rho = 0$, indexation is *only* to current trend inflation. For example, once again assuming a zero net inflation steady state, (17) reduces to the standard 'pure' NKPC:

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \kappa\widehat{mc}_t. \tag{19}$$

Comparing the hybrid NKPC (18) and pure NKPC (19) textbook versions with the generalized TVT-NKPC (17) shows that current inflation $\widehat{\pi}_t$ depends importantly on an additional driver, namely, current-period innovations to trend inflation, $\widehat{g}_t^{\overline{\pi}}$ (affecting the intercept of the TVT-NKPC). Furthermore, trend growth in output, g^Y, and in the marginal utility of wealth of firm owners, g^{Λ} – by entering the definitions of the reduced-form parameters of the TVT-NKPC, γ and κ – affects both its intercept and slope.

2.3.3 TVT-NKPC-Based Forecasting of Cyclical Inflation

Evaluating the implied 'fundamental inflation' measure that arises in equation (17) requires multiperiod forecasts of the forcing variables. These are generated in our case from a 3-variate VAR with 4 lags (denoted as 3VAR(4)) that includes the

[16] Most of these estimates relate to the *partial* indexation to past inflation only. However, Adolfson et al. (2007) estimate ρ with Bayesian methods assuming *full* indexation to *both* past actual inflation and current trend inflation, as we do, and report a posterior mode of 0.212 (with standard deviation of 0.066) and a posterior mean of 0.217.

theory-implied variables embodied in the key derived equation (17), expressing the generalized NKPC in a quasi-first difference form. More precisely, our 3VAR(4) includes our measures of real marginal cost, $\widehat{mc}_t$, trend inflation growth, $\widehat{g}_t^{\overline{\pi}}$, and cyclical inflation (in quasi-first difference form), $\widehat{\pi}_t - \rho\widehat{\pi}_{t-1}$.

In our case of the specific 3VAR(4) the relevant vector entering the forecasts is of dimension 12,

$$\underset{(1\times 12)}{\widehat{Z}_t'} = \left[\widehat{mc}_{t-j_p}, \widehat{g}_{t-j_p}^{\overline{\pi}}, \widehat{\pi}_{t-j_p} - \rho\widehat{\pi}_{t-(j_p+1)}\right]', \quad j_p = \{0, 1, 2, 3\},$$

where 'hats' denote, as in the preceding subsections, deviations from the respective time-varying trend values. Note that $0 < \kappa, \rho(\theta\gamma - 1) < 1$, as implied by the theory outlined earlier in the present section, and that $\widehat{Z}_{t+h|t} = \mathbf{A}^h \widehat{Z}_{t-1}$, where $\widehat{Z}_t$ denotes the vector entering the companion form matrix $\mathbf{A}$ of the VAR system and h the horizon of the forecasts with origin t. We can therefore apply the summation formula for infinite geometric sequences to the stationary forward-looking solution of the (re-)estimated TVT-NKPC equation (17) and obtain fundamental inflation in period t (in quasi-first difference form),

$$\widehat{\pi}_t - \rho\widehat{\pi}_{t-1} = \kappa \underset{(1\times 12)}{e_1'} \left(\mathbf{I}_{12} - \gamma \underset{(12\times 12)}{\mathbf{A}}\right)^{-1} \underset{(12\times 1)}{\widehat{Z}_t} + \rho(\theta\gamma - 1) \underset{(1\times 12)}{e_2'} \left(\mathbf{I}_{12} - \gamma \underset{(12\times 12)}{\mathbf{A}}\right)^{-1} \underset{(12\times 1)}{\widehat{Z}_t}. \tag{20}$$

In (20), the selection vector e_1' (with 1 as its first element and 0's elsewhere) extracts the forecast for $\widehat{mc}_{t+h}$ (first row of $\mathbf{A}$) while the selection vector e_2' (with 1 as its second element and 0's elsewhere) that for $\widehat{g}_{t+h}^{\overline{\pi}}$ (second row of $\mathbf{A}$).

In order to generate forecasts for the cyclical component of inflation, $\widehat{\pi}_{t+h|t}$, conditional on information at time t, lead expression (20) by one period and note that the 1-period-ahead forecast of $\widehat{Z}_t$ is $\widehat{Z}_{t+1|t} = \mathbf{A}\widehat{Z}_t$. Hence the 1-step-ahead forecast of the cyclical component of inflation, $\widehat{\pi}_{t+1|t}$, based on the model-consistent concept of fundamental inflation implied by our TVT-NKPC theory, is given in terms of current variables as follows:[17]

$$E_t[\widehat{\pi}_{t+1}] \equiv \widehat{\pi}_{t+1|t} = \rho\widehat{\pi}_t + \kappa e_1' (\mathbf{I} - \gamma\mathbf{A})^{-1} \mathbf{A}\widehat{Z}_t + \rho(\theta\gamma - 1) e_2' (\mathbf{I} - \gamma\mathbf{A})^{-1} \mathbf{A}\widehat{Z}_t. \tag{21}$$

Iterating this cyclical inflation forecast forward, we can construct an h-step-ahead forecast of cyclical inflation. The outcome is the following general forecasting equation,

[17] For simplicity, suppressing the explicit notation for the matrix dimensions.

$$\widehat{\pi}_{t+h|t} = \rho^h \widehat{\pi}_t + \kappa e_1' \left(\mathbf{I} - \gamma \mathbf{A}\right)^{-1} \sum_{i=1}^{h} \rho^{i-1} \mathbf{A}^i \widehat{Z}_t + \rho \left(\theta\gamma - 1\right) e_2' \left(\mathbf{I} - \gamma \mathbf{A}\right)^{-1} \sum_{i=1}^{h} \rho^{i-1} \mathbf{A}^i \widehat{Z}_t, \tag{22}$$

which is used to generate the forecasts for the cyclical component of inflation.

3 Empirical Implementation

In the present section we discuss our empirical implementation of the proposed theory of inflation dynamics to forecasting inflation in the EA and the US.

3.1 Data Set and Forecast Evaluation Design

We use quarterly data for the EA and the US over almost half a century, from 1970:1 to 2015:4 (184 quarters), and compare forecast accuracy in a pseudo-out-of-sample evaluation period of approximately the last third of our full sample, 2000:1-2015:4 (64 quarters), as is common practice. Appendix A (online) provides details of the data sources, definitions and mnemonics.

Our quarterly inflation measure is defined in a standard way (in annualized % terms),

$$\pi_t \equiv 400 \times \ln \frac{P_t}{P_{t-1}},$$

where P_t is the quarterly GDP-deflator price index. Our preference here for the GDP-deflator price index as a measure of the aggregate price level in the EA and the US follows the inflation forecasting literature, notably Stock and Watson (2007) and Faust and Wright (2013).[18]

[Figures 1 and 2 about here]

We follow Faust and Wright (2013) in performing iterated multistep forecasts, and not direct forecasts.[19] We also follow Faust and Wright (2013) in forecasting single-quarter inflation rates, and not cumulative inflation rates over a particular future horizon as done, e.g., in Stock and Watson (2007). The reason is that the

[18] As discussed in Clark and Doh (2014), this is a broader aggregate price measure than the alternative consumer price index (CPI) measure of inflation. While these two common inflation measures do not generally stray too far apart, they do not follow precisely the same dynamics. CPI inflation tends to be more volatile than GDP-deflator inflation, especially in the US data (see figures 1 and 2).

[19] See Appendix B.3 (online) for definitions and further detail on iterated vesrus direct forecasts.

former approach allows to judge in a more straightforward way how the forecast horizon may affect the predictability of inflation: if cumulating predicted inflation rates instead, the shorter- and longer-run forecast accuracy will be conflated over the duration of the respective forecast horizon.

[Table 1 about here]

Table 1 summarizes the models of inflation dynamics included in our predictive accuracy comparisons. As in Stock and Watson (2007), among others, we implement the forecasts in two variants, using recursively re-estimated (i) fixed-length rolling sample window and (ii) augmenting-length sample window. In addition, and also as a robustness check, the predictive performance of the TVT-NKPC forecasting procedure is examined in two other variants, depending on the proxy for unobservable real marginal cost (RMC), either (i) a monetary open economy (MOE) RMC proxy constructed from four observable variables as implied by equation (13) above, or (ii) the standard real unit labor cost (RULC) proxy for RMC. These TVT-NKPC forecasts are then compared against two typical univariate statistical specifications for inflation as (alternative and agnostic) benchmarks, (i) the conventional driftless random walk and (ii) the widely used AO pseudo random walk. Under the random walk without drift univariate benchmark, which predicts the same inflation for any quarter at any horizon h as that observed in the most recent quarter, the inflation forecast can be written as (assuming again that t-indexed variables are observed)

$$\pi_{t+h|t} = \pi_t. \tag{23}$$

For the second univariate benchmark we employ the AO pseudo random walk, which is essentially a random walk forecast designed for the horizon of 12 months, with Stock and Watson (2007) adapting it to quarterly data and extending the AO forecast to other horizons.[20] The iterated version of the AO forecast, adjusted to our quarterly frequency, can then be written as

$$\pi_{t+h|t} \equiv \frac{1}{4}\left(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right). \tag{24}$$

That is, the adjusted AO model forecasts the 4-quarter-ahead inflation and, by extension, the inflation in any earlier or future quarter, h $(= 1, \ldots, 20$ in our case),

[20] Atkeson and Ohanian (2001) proposed a naïve model of US inflation dynamics initially designed for the monthly frequency that has nevertheless performed very well at horizons of 1 and 2 years in the US data.

to be the same as the average of the latest four quarters of observed inflation.

It is important to emphasize as well that the congruence of the theoretical choices introduced earlier with the corresponding empirical implementation of our novel forecasting procedure. More specifically, the theory-based trend and cyclical inflation components implied by our derivation of the TVT-NKPC are also consistent with the stationarity properties of both the EA and US data, as evidenced by the formal test results reported in Table 2 and as illustrated visually in Figures 3–6, to which we return with some discussion further down. Accordingly – in a 'double', theory-informed and data-supported, justification – trend inflation is forecast using the univariate stochastic AR(1) trend model defined in (2) and (3), while cyclical inflation is forecast by the auxiliary 3VAR(4) model embodied in (22) as an empirical implementation of (17).[21]

In line with the literature, we report and compare accuracy for pseudo-out-of-sample predictions in terms of the Theil U-statistic, defined as the ratio of the root mean squared forecast error (MSFE) of the (theory-based) model of inflation dynamics (in the numerator of Theil's ratio) relative to that of the respective benchmark, RW or AO in our case, (in the denominator) for the policy-relevant horizons of 1, 4, 8, 12, 16 and 20 quarters. The modified Diebold-Mariano (1995) test (MDM), proposed by Harvey et al. (1997) to correct for small-sample bias, checks for the statistical significance of the difference in predictive accuracy between pairs of non-nested model forecasts.[22] We focus on the one-sided version of the p-value for the MDM test because we are interested here primarily in the improvement – not the deterioration – in predictive accuracy over the respective benchmark (i.e., Theil's U ratios lower than 1) that arises from the application of our TVT-NKPC forecasting procedure.

3.2 Stationarity Tests for Inflation, Its Components and Drivers

Figure 3 depicts the EA (blue) and US (red) quarterly GDP-deflator inflation at annualized % rate in our sample (1970:1–2015:4), where the shaded area corresponds to our forecasting evaluation period (2000:1–2015:4). The patterns of inflation dynamics in the EA and the US display some similarities as well as some

[21] The final instrument set and lags in the TVT-NKPC GMM re-estimation was selected with view to the Hansen (1982) J-test statistic for the validity of the overidentifying restrictions.

[22] For a detailed discussion of the rationale and advantages of using the MDM test in forecast accuracy comparisons, see Faust and Wright (2013), Clark and McCracken (2013), and Clark and Doh (2014).

differences. The high inflation of the 1970s, due to the global oil shocks, and the subsequent disinflation of the 1980s is a common feature, yet during these first two decades in our sample EA inflation was higher than US inflation (and, unsurprisingly, quite heterogeneous within the area). The 'great moderation' in the world economy, usually identified to have begun around the mid-1990s, after the European exchange-rate mechanism crisis of 1992:3–1993:2, and to have ended with the recent global financial crisis (GFC) of 2007:3–2009:4, is evident in inflation dynamics in both the EA and US data. Then, in the last 15 years of the sample, with the ECB now responsible for EA monetary policy, inflation has been low and relatively stable: lower and less volatile compared to the US, and almost reversing itself into deflation near the end of the GFC.

[Figure 3 around here]

We have kept the turbulent GFC quarters together with the preceding 'great moderation' low inflation subperiod and the subsequent 'deflation scare' subperiod, where a number of economies have been operating at the zero lower bound of nominal interest rates, in part with the goal to ensure a relatively long rolling window; in part we also wanted to subject our forecasting procedure to a more challenging empirical test. More precisely, the fixed-window length in the rolling estimation, which is equivalent to the initial estimation sample in the augmenting-window recursive estimation, is 113 effective quarters, with the first quarter of 2000 being the horizon of our first 1-step-ahead inflation prediction over the forecasting evaluation period.

[Table 2 about here]

The results from the stationarity tests we performed in our full sample are reported in Table 2. Inflation is conclusively found to be nonstationary in both the EA and US data; that is, by both the Augmented Dickey-Fuller (ADF) test with a null of unit root (with constant included as well as excluded)[23] at all conventional significance levels and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test with a null of stationarity (with constant included)[24] at the 5% and 10% significance levels (not at the 1% level). When a linear trend is also included, in addition to the constant in the ADF and KPSS specifications, this trend comes out statistically significant and negative for both the EA and the US, which is clearly

[23]See Dickey and Fuller (1979) and Said and Dickey (1984), as well as Schwarz (1978) and MacKinnon (1996).

[24]See Kwiatkowski et al. (1992), as well as Bartlett (1950) and Andrews (1991).

seen too by inspection of Figure 3. The formal statistical tests here regarding (non)stationarity are important in that their findings support the TVT-NKPC forecasting procedure. The methodology proposed here relies on first separating a time-varying inflation trend from the inflation cycle, before the prediction exercise for observed inflation is then undertaken by components (with potentially differing or diverging dynamic or stationarity properties).

[Figures 4 and 5 about here]

Figures 4 and 5 illustrate these *trend* and *cycle* components, respectively, of EA and US quarterly GDP-deflator inflation at annualized rate.[25] Again, checking Table 2, the ADF and KPSS tests are conclusive with regard to establishing the stationarity of the inflation cycle around the drifting inflation trend in both the EA and the US at all conventional levels of statistical significance. The KPSS test is also conclusive regarding the nonstationarity (in our sample) of trend inflation in both the EA and the US, whereas the ADF test confirms this finding at all conventional significance levels only for the US but not for the EA (at the 10% level).

[Figure 6 about here]

Figure 6 depicts the EA (blue) and US (red) change in quarterly (log-) GDP-deflator trend inflation at annualized % rate, as another major determinant of inflation dynamics around a time-varying trend according to the generalized NKPC equation (17). It is found stationary (in our sample) for both the EA and the US at all conventional significance levels by the KPSS test. In addition, for both the EA and the US, it is found stationary at the 5% and 10% levels by the ADF test too, but nonstationary at the 1% level.

[Figure 7 about here]

The (non)stationarity results from the ADF and KPSS tests we reported are, therefore, broadly consistent with the theoretical law of motion imposed in (3) and (16). As a further check of robustness of our particular – but typical in macroeconomic theory and empirics – detrending choice, Figure 7 presents *alternative estimates* for US trend inflation in the recent literature, such as Stock and Watson (2007) and Chan et al. (2013, 2016, 2017), all based on the CPI measure of

[25] In all figures illustrating trend or cycle of a time series, these two components have been separated for the whole sample by applying a two-sided Hodrick-Prescott (1997) filter.

the aggregate price level (for the EA we have not found a similar comparison). While trend inflation measures differ in time patterns and – especially volatility and smoothness – we can infer from this figure that our inflation trend measure based on Hodrick-Prescott (1997) filtering falls quite in the middle of these alternative estimates, in terms of dynamics and fluctuations. This is quite reassuring in the sense that our 'first pass' here based on one-sided Hodrick-Prescott detrending (to resemble real-time prediction) may be robust to alternative extraction of trend inflation, although we leave this issue (that could potentially improve our forecasting procedure) for further investigation.

[Figures 8 and 9 about here]

Finally, figures 8 and 9 depict the *cyclical* component of the EA (blue) and US (red) quarterly MOE and RULC proxies, respectively, for the (log-) real marginal cost at annualized % rate, as a major determinant of inflation dynamics according to the TVT-NKPC equation (17). The findings in Table 2 are convincing once again in establishing stationarity of the cyclical component of both these proxies for RMC around their drifting underlying trends in both the EA and the US data at all conventional levels of statistical significance. One could also notice that the MOE RMC measure and the RULC RMC alternative do not exhibit quite the same patterns of dynamics and volatility in our data set, and therefore may not be equally useful in the forecasting exercise at its various horizons.

The stationarity test results summarized above therefore support the TVT-NKPC forecasting procedure. The cyclical component of inflation, or rather its empirically constructed analogue in our sample, is stationary because all of its component drivers in equation (17) are stationary; hence the projection for the cyclical component for inflation extracted via the companion form of the theory-implied 3VAR(4), as in (22), is justified and in line with the required stationarity for the implemented forecasting iterations at any horizon ahead.

4 Results on Predictive Accuracy

In this section we present the key results from evaluating the predictive accuracy of our TVT-NKPC inflation forecasting procedure. Additional estimation and forecasting results, as further robustness checks, are presented in Appendix C (online). We begin by discussing our findings with regard to the EA data, before moving to the US case.

4.1 Forecasting EA Inflation

Our main results with regard to the EA, reported in Table 3, can be summarized as follows.

[Table 3 about here]

First, comparing the root mean square forecast errors in panel A of the table, it can be seen that the RW benchmark results in the worst predictive accuracy, that is the largest root MSFE, across all six horizons, relative to the AO benchmark forecast and all four variants of the TVT-NKPC procedure. This finding reveals that our theory-based forecasting procedure is able to extract predictive content from EA inflation data that helps to improve forecasting compared to the RW benchmark of nonpredictability. Moreover, at the medium (8 and 12 quarters) and longer (16 and 20 quarters) horizons, the TVT-NKPC procedure in all its four variants, that is, no matter the re-estimation of either a fixed- or augmenting-length recursive window or the choice of proxy used for RMC, achieves the most accurate prediction, beating in terms of root MSFE even the AO forecast. With bold fonts denoting the best forecast by horizon, one can see that for the EA data the RULC RMC rolling-window forecast dominates, winning at 8, 12 and 16 quarters, while the RULC RMC augmenting-window comes most accurate at the longest horizon of 20 quarters. However, at the two shorter-run horizons of 1 and 4 quarters the AO benchmark predicts EA inflation with the smallest root MSFE.[26]

We next consider the difference in predictive accuracy comparing the TVT-NKPC forecasts with the RW benchmark, reported in panel B of Table 3. The TVT-NKPC forecasts beat the RW forecast at all six horizons, with a gain ranging from 22.4% (statistically significant at horizon of 1 quarter, where the benefit of the MOE RMC proxy in the augmenting-window variant is illustrated) to 26.8% (statistically significant at horizon of 16 quarters, where the advantage of the RULC RMC proxy in the rolling-window variant becomes evident). More generally, our TVT-NKPC forecasts are significantly more accurate than the RW forecast at all horizons beyond the immediate short run of 1 quarter (where they are statistically worse) except 12 quarters (yet still numerically better by 19.1%) and 4 quarters (numerically better by 1.3%).

[26] Figures 9 (short run), 11 (medium run) and 13 (long run) in Appendix C of the supplementary online material provide a visual illustration for the best-performing variant of our TVT-NKPC forecasts against actual inflation in the EA.

Third, to further judge whether the best variant of the TVT-NKPC forecast is more accurate than the AO forecast in a statistically significant way, we compute Theil's U ratio using the root MSFE of the AO forecast as the denominator and the (one-sided) MDM statistics for this benchmark. Panel C of Table 3 reports that these two compared forecasts are not statistically distinguishable at all horizons. Of note, however, at the medium (8 and 12 quarters) and longer (16 and 20 quarters) horizons, the TVT-NKPC forecast (in all its four variants) outperforms the AO forecast, by a numerical margin of up to 9.9%.

Comparing the four variants of the TVT-NKPC forecasts, we see from Table 3 that re-estimating a fixed- versus augmenting-length recursive window of the data and using the RULC versus MOE RMC proxy tends to perform better, especially at horizons of 12 and 16 quarters. Yet the use of the MOE RMC proxy in its augmenting-window implementation brings a considerable advantage in coming very close to the best forecast at the immediate horizon of 1 quarter and at the longest horizon of 20 quarters.

Overall, for the EA we can conclude that the TVT-NKPC procedure significantly outperforms the RW benchmark at all horizons except 4 and 12 quarters ahead; however, even when the statistical significance is not confirmed, the Theil U ratio does indicate that there are important numerical gains to using the TVT-NKPC procedure. Moreover, while the TVT-NKPC procedure does not significantly outperform the AO benchmark at any of the six horizons, the numerical gains of using it are considerable, beating the AO forecast by almost 20% at the longer horizons of 12, 16 and 20 quarters.

4.2 Forecasting US Inflation

Turning to the US data, while our main results reported in Table 4 do not change the essence of the conclusions summarized in section 4.1 for the EA data; there are however some interesting differences and nuances worth mentioning, as follows.

[Table 4 about here]

First, now the MOE-RMC fixed rolling window variant comes out as the most accurate forecast (i.e., lowest root MSFE, in bold), winning at two horizons, 1 and 16 quarters. The AO forecast is best only at 4 quarters, whereas the RW forecast is not that much far off overall.[27]

[27] Figures 10 (short run), 12 (medium run) and 14 (long run) in Appendix C of the supplementary online material provide a visual illustration for the best-performing variant of our TVT-NKPC forecasts against actual inflation in the US.

Next, in panel B of Table 4 we see that in the US case the TVT-NKPC forecast in its MOE-RMC fixed rolling window version is statistically more accurate than the RW forecast at horizon of 1 quarter, whereas the RULC-RMC variant (in both windows, fixed and augmenting) is statistically dominating the RW at 8 quarters. At the remaining four prediction horizons the best TVT-NKPC variant is not statistically distinguishable from the RW forecast, but is numerically much more accurate, with gains from 7.6% at 4 quarters to 21% (MOE-RMC in augmenting window, again) at 20 quarters.

Third, panel C of Table 4 reveals that now, in the US data, the best variant of our TVT-NKPC forecast dominates the AO forecast in a statistically significant way at the medium run of 8 and 12 quarters. It also dominates numerically, by an important margin, the AO forecast at the long run of 16 (18.8%) and 20 (8.6%) quarters, remaining indistinguishable statistically at the short run of 1 and 4 quarters from the AO benchmark.

Finally, looking across the four variants of implementing the TVT-NKPC forecasts, we see from Table 4, and somewhat consistent with the analogous conclusion in the EA data, that none of these variants really dominates the remaining ones across more than 1-2 of the 6 forecast horizons examined.

Overall, the findings for the US present more similarities than differences with respect to those for the EA. We can conclude that our TVT-NKPC forecasting procedure outperforms the conventional random walk benchmark at all horizons, significantly at 1 and 8 quarters in the EA and US data, and also at the longer run of 16 and 20 quarters in the EA case. Moreover, it also outperforms quantitatively, by about 10 to 20 percentage points in general, beyond the short run of 1 and 4 quarters, the agnostic AO benchmark that previous studies have found difficult to beat, significantly at the medium run of 8 and 12 quarters in the US case.

5 Concluding Comments

Previous results reported in the inflation forecasting literature have suggested some scepticism concerning the value added of theory-based models relative to the RW and AO forecasting benchmarks. Yet these models have typically exploited variants of the traditional and rather simple Phillips curve or of the New Keynesian Phillips Curve derived around a zero or constant trend inflation steady state in either single-equation estimation or DSGE system estimation, with King and Watson (2012) comparing the latter two alternative NKPC-based approaches and focusing on the choice of alternative real unit labor cost measures and some ensuing contradictory findings. This study has instead proposed a generalized New Keynesian Phillips Curve derived around a time-varying trend inflation with an additional proxy for unobservable marginal costs constructed from four observable time series arising from a monetary open-economy model that proved useful in the shortest and longest end of the forecast horizon we examined. On this basis, the corresponding model concept of fundamental inflation employed to predict the inflation cycle captured in the NKPC has been combined with a theory-informed but also data-supported prediction for the time-varying trend (TVT) to obtain a TVT-NKPC forecast for actual inflation. In effect, our limited information approach to forecasting inflation offers a simpler alternative to the recent advances in better predicting of inflation that is based on full-information DSGE systems (see, e.g., Cai et al., 2018). We believe that both these theory/NKPC approaches have their strengths and weaknesses and should be used as complementary to each other.

Our results on comparative forecasting accuracy, using two quarterly data sets, namely, for the EA and the US over 1970:1-2015:4, that include some quite variable inflation periods, suggest that there is a role for theory: in both data sets the TVT-NKPC forecasts were overall much more accurate numerically than the AO benchmark (by 10-20 percentage points in the medium to longer run), and sometimes in a statistically significant way too, as in the US data at the medium-run horizons of 8 and 12 quarters ahead, that has been found to be quite difficult to beat in previous comparative studies (e.g., Atkeson and Ohanian, 2001; Stock and Watson, 2007, 2009; Faust and Wright, 2013). These results go some way to re-establishing confidence in the merit of Koopmans's dictum that we should 'resort to theory' to guide policy – and, we would add, empirical work and forecasting.

This study is part of continuing research seeking to build better theory-based forecasting procedures, which in practice combine with econometric methods of implementation. Present lines of enquiry include searching for an optimal length of the re-estimation window, considering alternative methods of trend adjustment, and robustifying the forecasts to possible structural breaks (see, for example, Castle et al., 2016), all of which may improve the TVT-NKPC forecasts, and including metrics other than quadratic loss to evaluate the forecasts.

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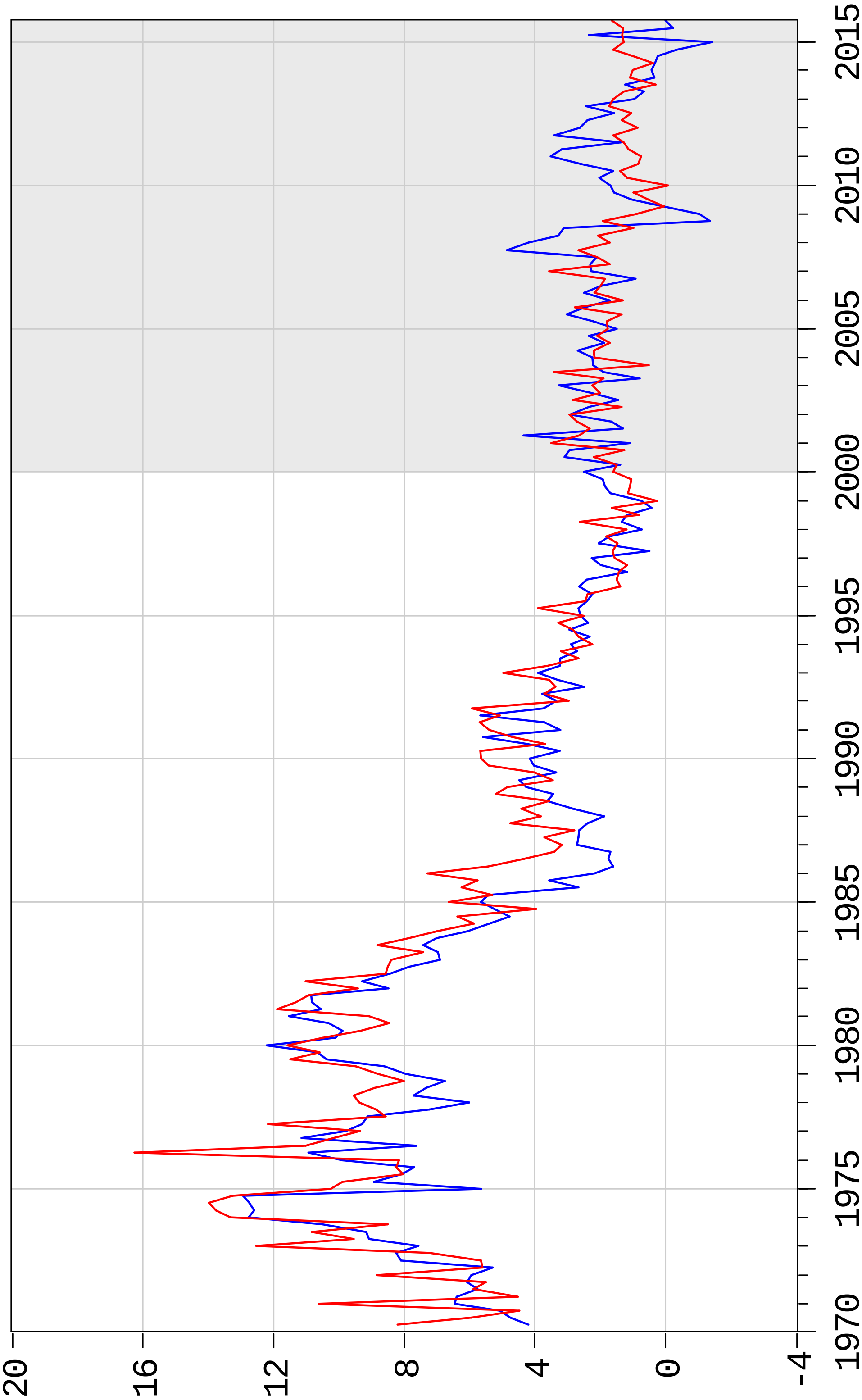

Figure 1: EA: GDP-deflator inflation (red) versus HICP inflation (blue) at annualized rate, % per annum; the shaded area corresponds to our forecasting evaluation period

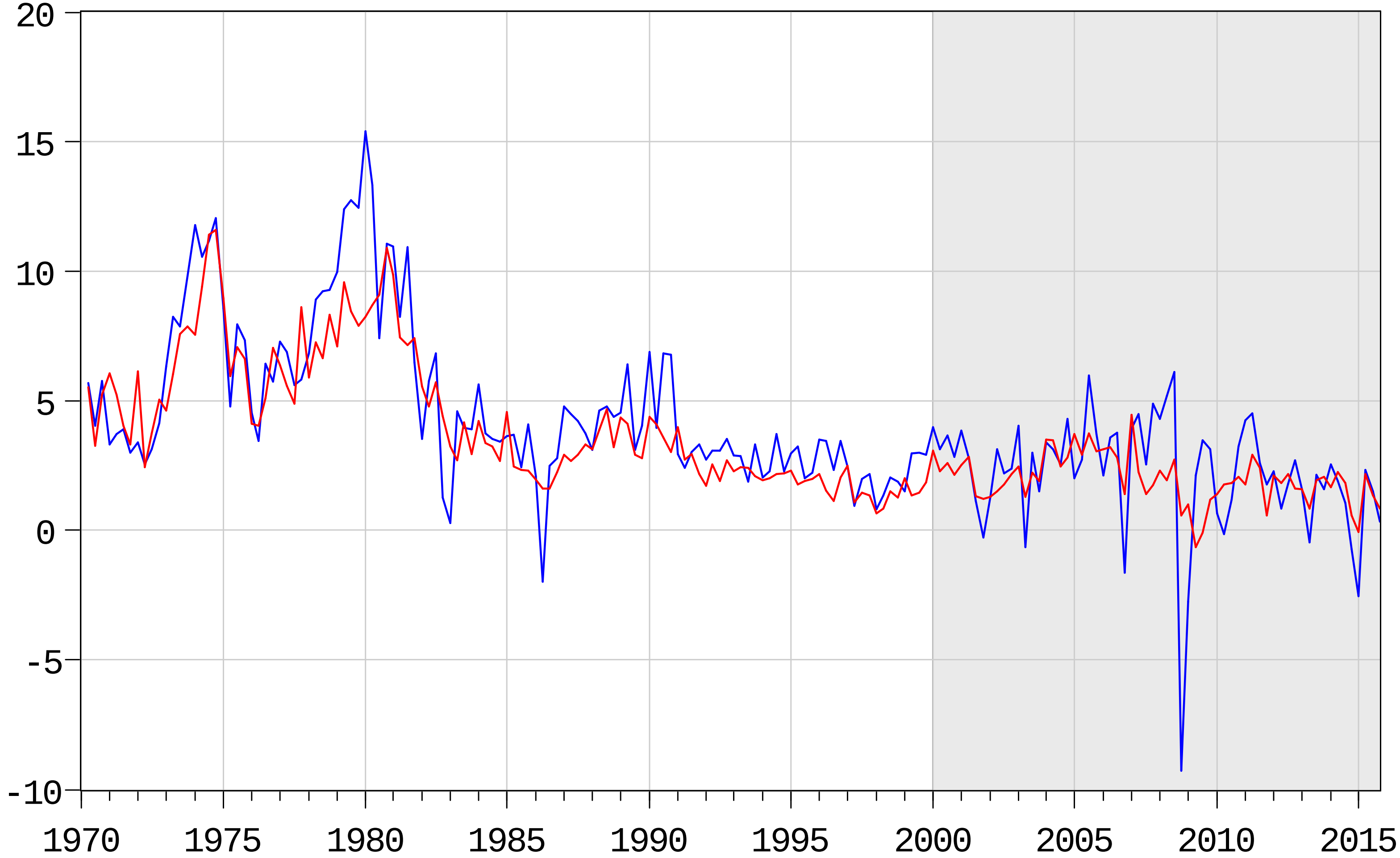

Figure 2: US: GDP-deflator inflation (red) versus CPI inflation (blue) at annualized rate, % per annum; the shaded area corresponds to our forecasting evaluation period

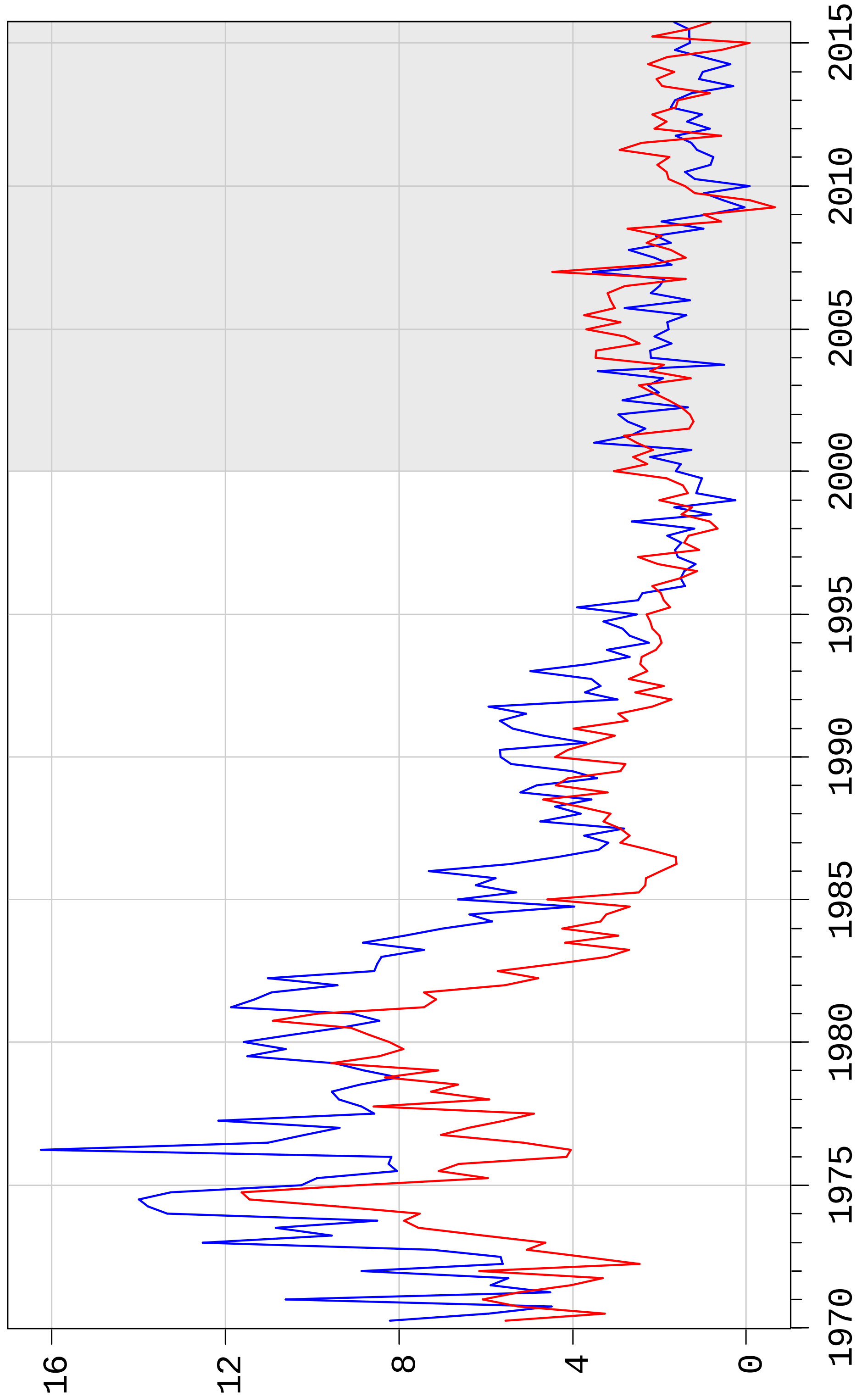

Figure 3: EA (blue) and US (red) quarterly GDP-deflator inflation at annualized rate, % per annum; the shaded area corresponds to our forecasting evaluation period

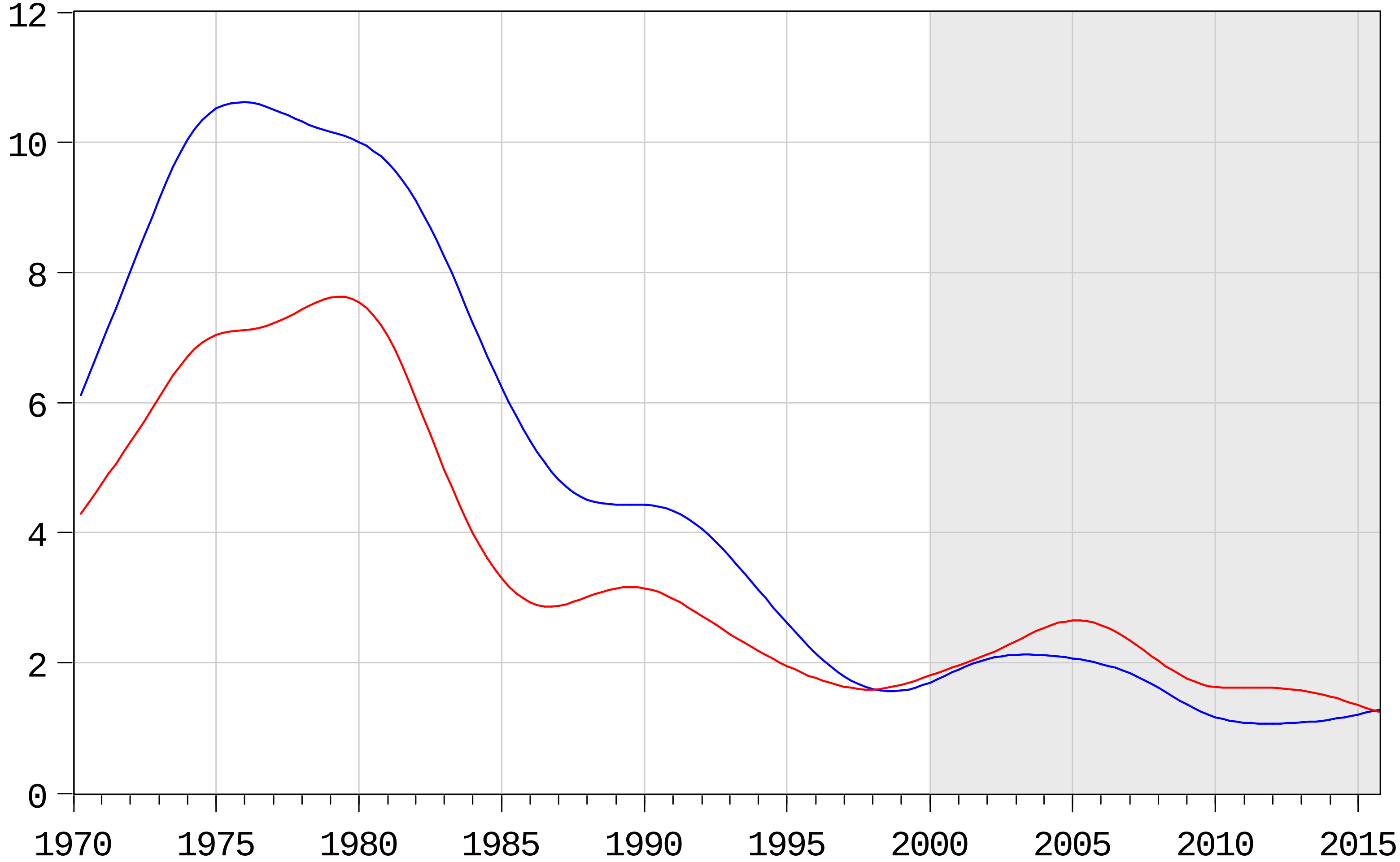

Figure 4: EA (blue) and US (red) quarterly GDP-deflator trend inflation component at annualized rate, % per annum; the trend is separated for the whole sample using a two-sided Hodrick-Prescott (1997) filter; the shaded area corresponds to our forecasting evaluation period

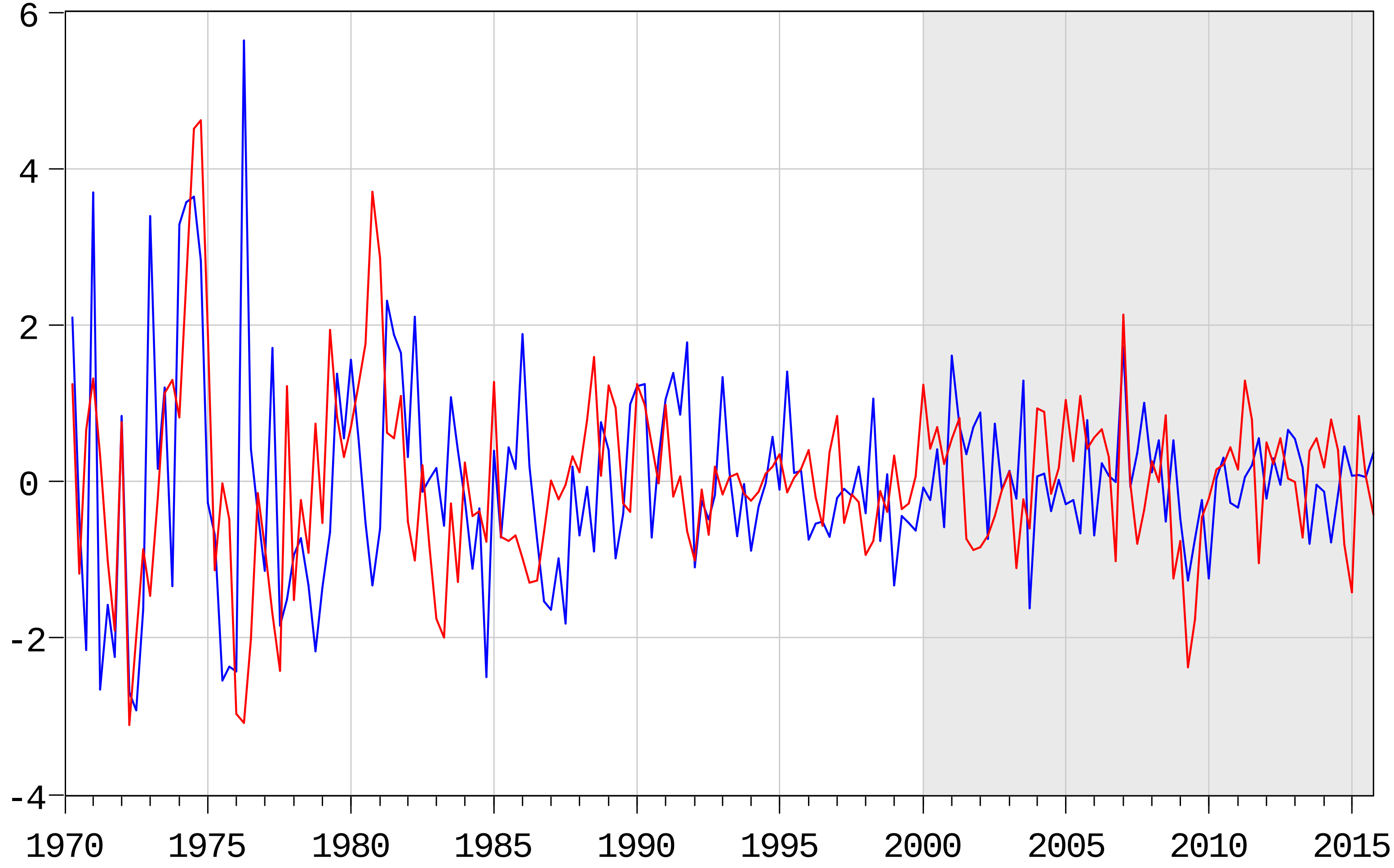

Figure 5: EA (blue) and US (red) quarterly GDP-deflator cyclical inflation component at annualized rate, % per annum; the cycle is separated for the whole sample using a two-sided Hodrick-Prescott (1997) filter; the shaded area corresponds to our forecasting evaluation period

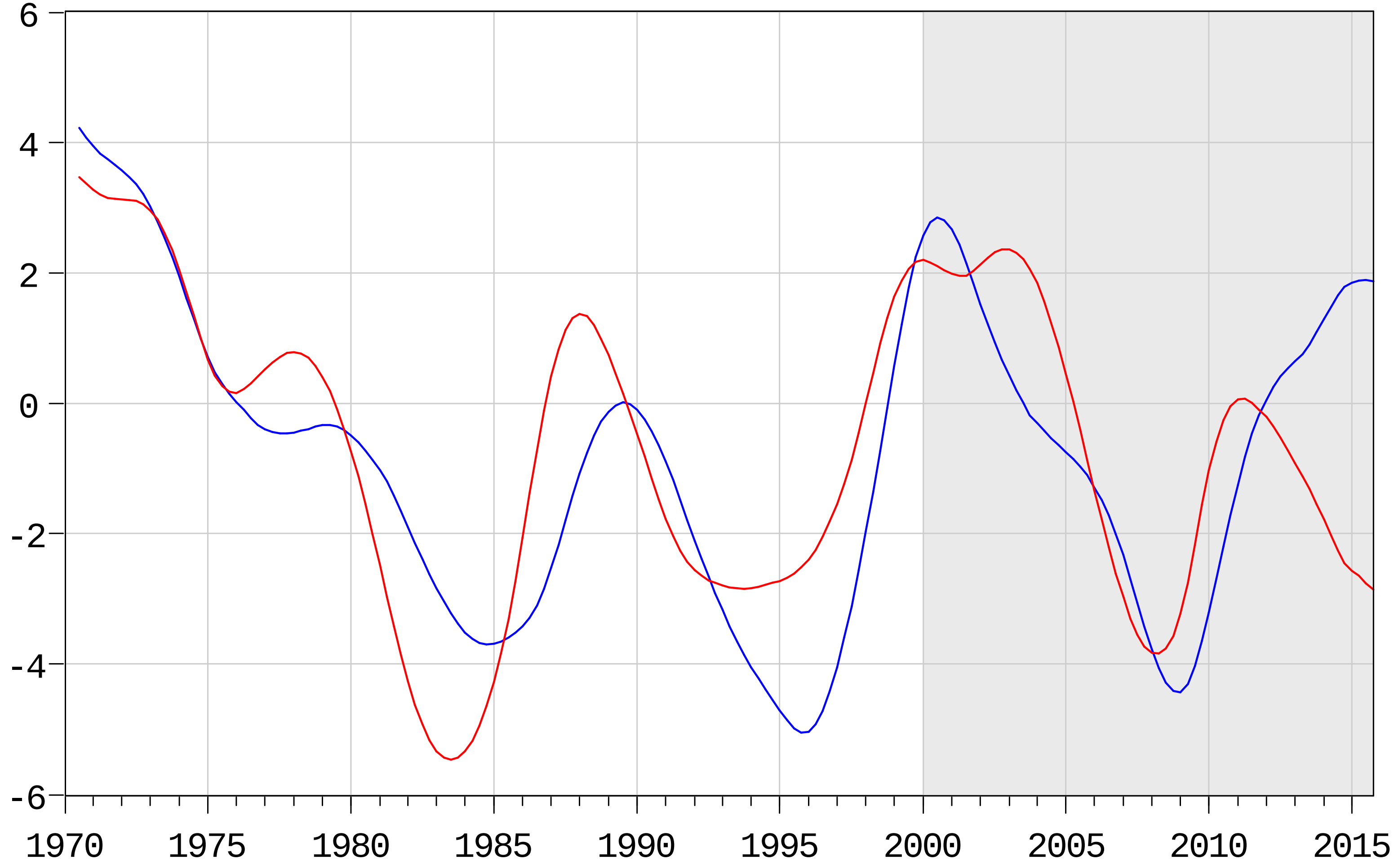

Figure 6: EA (blue) and US (red) change in quarterly GDP-deflator trend inflation at annualized rate, % per annum; the trend is separated for the whole sample using a two-sided Hodrick-Prescott (1997) filter; the shaded area corresponds to our forecasting evaluation period

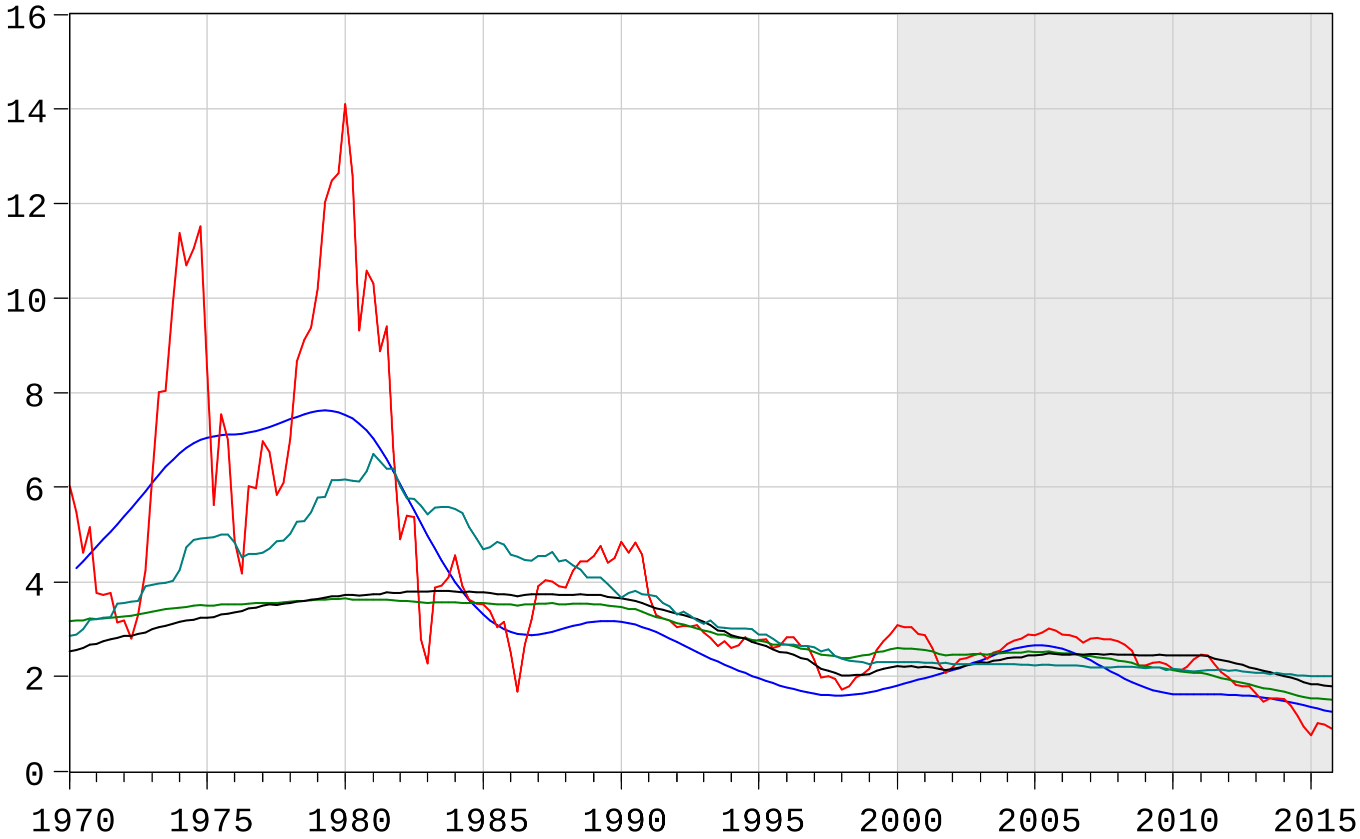

Figure 7: US: Our measure, using the two-sided Hodrick-Prescott (1997) filter, of quarterly GDP-deflator trend inflation (blue) at annualized rate, % per annum against alternative estimates in the literature, all based on CPI inflation data, as follows: (i) univariate unobserved component stochastic volatility (UCSV) model (red), as in Stock and Watson (2007); (ii) univariate bounded inflation trend model ('trend-bounded', green), as in Chan et al. (2013); (iii) bivariate model with bounded inflation trend and NAIRU ('trend-bounded2', black), as in Chan et al. (2016); (iv) UC model augmented with long-run inflation expectations (UC-E, grey), as in Chan et al. (2017). The underlying data for the alternative trend measures we use in this graph are available in Excel format on Chen's website.

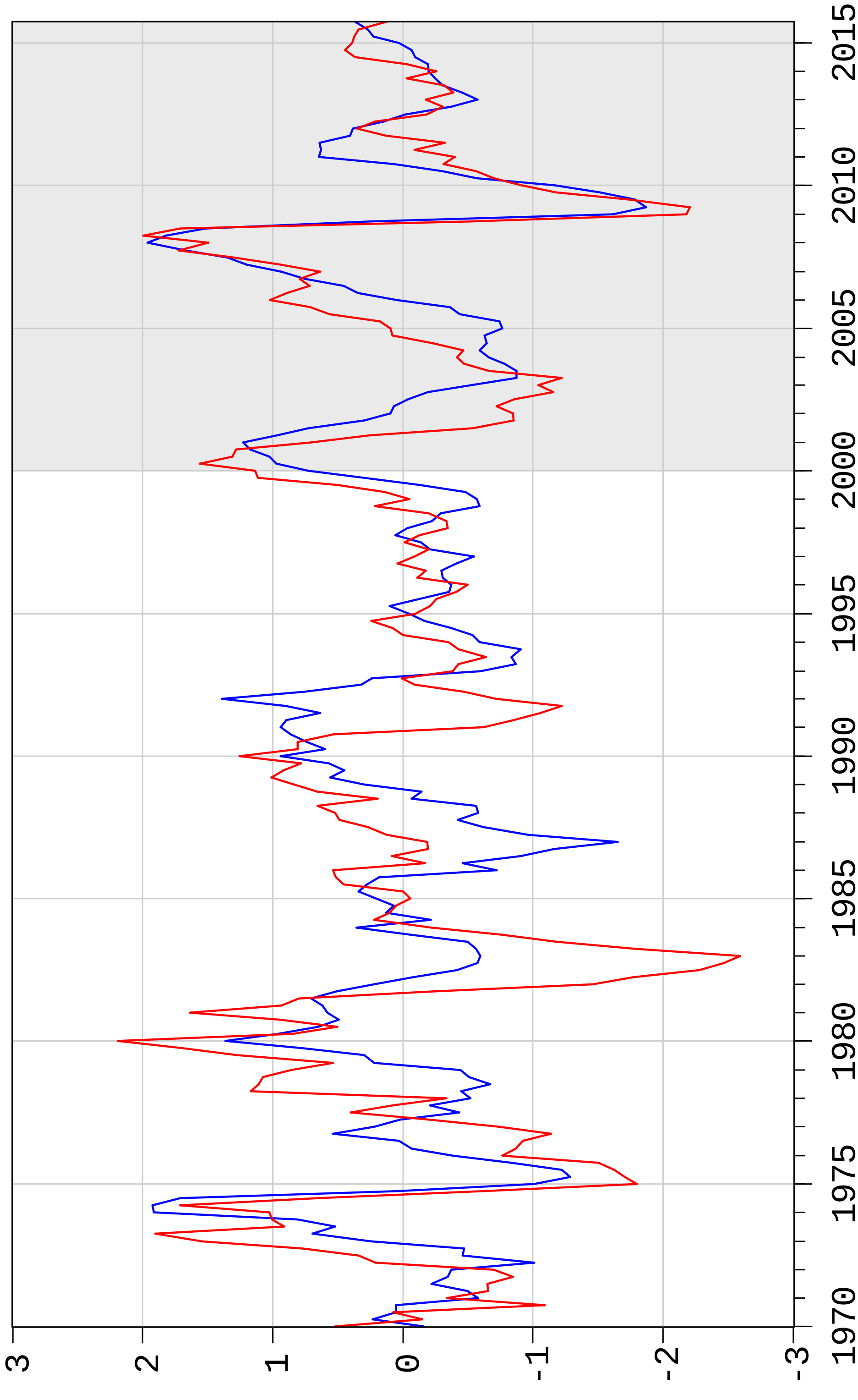

Figure 8: EA (blue) and US (red) cyclical component of the quarterly monetary open-economy measure of real marginal cost at annualized rate, % per annum; the cycle is separated for the whole sample using a two-sided Hodrick-Prescott (1997) filter; the shaded area corresponds to our forecasting evaluation period

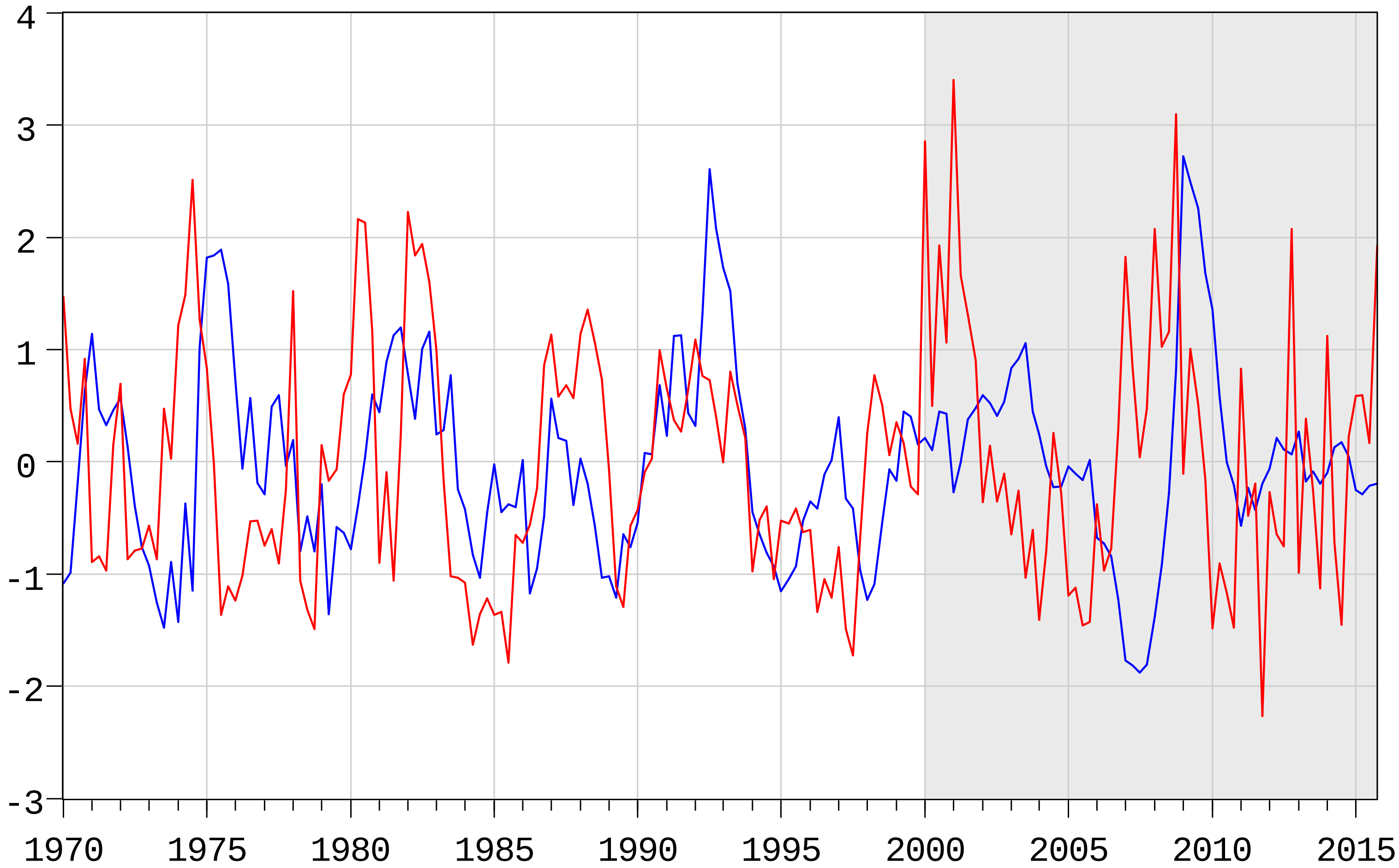

Figure 9: EA (blue) and US (red) cyclical component of the quarterly real unit labor cost measure of real marginal cost at annualized rate, % per annum; the cycle is separated for the whole sample using a two-sided Hodrick-Prescott (1997) filter; the shaded area corresponds to our forecasting evaluation period

Shorthand Notation	Models of Inflation Dynamics
I. TVT-NKPC Forecasting Procedures	
TVT-NKPC1	re-estimated MOE RMC proxied cycle and AR(1) trend
TVT-NKPC2	re-estimated RULC RMC proxied cycle and AR(1) trend
II. Univariate Forecasting Benchmarks	
U1	RW (1st benchmark): driftless random walk
U2	AO (2nd benchmark): Atkeson-Ohanian (2001) pseudo RW

Table 1: Models of Inflation Dynamics Compared in Predictive Accuaracy

Note: 'TVT-NKPC' denotes the 'time-varying trend New Keynesian Phillips Curve' theory-based forecasting procedure we propose in this paper, in two implementations: (i) 'MOE RMC' is our monetary open-economy real marginal cost proxy, and its structural parameters are calibrated according to McKnight and Mihailov (2015); (ii) 'RULC RMC' denotes the usual proxy for RMC, real unit labor cost (RULC), as employed originally in Galí and Gertler (1999). 'U' denotes the univariate forecasting benchmarks, in two alternatives: (i) a random walk ('RW') without drift, typical in the forecasting litearture since Meese and Rogoff (1983); (ii) the Atkeson-Ohanian (2001) pseudo RW ('AO') taht has been among the most successful agnostic models to forecast inflation – see, e.g., Stock and Watson (2007) and Faust and Wright (2013).

Generalized NKPC component	ADF test t-stat pv	KPSS test LM stat
		1% cv = 0.7390
		5% cv = 0.4630
		10% cv = 0.3470
EA: full (adjusted) sample	179 to 182 observations	183 to 184 observations
EA inflation	0.6647	0.5998
Cycle of EA inflation	0.0000	0.0191
Cycle of EA MOE real marginal cost	0.0000	0.0201
Cycle of EA real unit labor cost	0.0000	0.0214
EA (log-)trend inflation	0.0558	1.4018
Growth rate of EA trend inflation	0.0402	0.2157
US: full (adjusted) sample	181 to 182 observations	183 to 184 observations
US inflation	0.1956	0.5639
Cycle of US inflation	0.0000	0.0181
Cycle of US real marginal cost	0.0001	0.0204
Cycle of US real unit labor cost	0.0000	0.0214
US (log-)trend inflation	0.2040	1.2243
Growth rate of US trend inflation	0.0474	0.1661

Table 2: Stationarity Tests for the Components and Drivers of the TVT-NKPC

Note: Separation of the cyclical component from the trend component of the respective time series has been obtained in the whole (adjusted) sample by applying a two-sided Hodrick-Prescott (1997) filter. The null of the Augmented Dickey-Fuller (ADF) test is nonstationarity (unit root): MacKinnon (1996) one-sided probability values (pv) for the ADF test t-statistic are provided in the table; the lag length is automatic, based on the Schwarz (1978) Information Criterion (SIC), with a maximum lag set at 13 quarters; the reported results are for the ADF specification that includes a constant. The null of the Kwiatkowski-Phillips-Schmidt-Shin (KPSS, 1992) test is stationarity: asymptotic critical values (cv) for the KPSS test LM-statistic at conventional levels are provided in the table; the reported results are for the KPSS specification that includes a constant; the bandwidth is selected automatically, based on Andrews (1991) and using Bartlett (1950) kernel.

Forecasting evaluation period	2000:1–2015:4					
Forecast horizon, quarters	1	4	8	12	16	20
Panel A: Root MSFE						
Theory-Based TVT-NKPC Procedures of Inflation Forecasting						
MOE RMC (fix)	0.729	0.884	0.912	0.829	0.745	0.883
MOE RMC (aug)	0.706	0.879	0.914	0.842	0.759	0.838
RULC RMC (fix)	0.751	0.873	**0.892**	**0.816**	**0.739**	0.880
RULC RMC (aug)	0.765	0.879	**0.892**	0.822	0.747	**0.833**
Agnostic Univariate Benchmarks of Inflation Forecasting						
RW Forecast	0.909	0.885	1.038	1.001	0.996	1.092
AO (Pseudo-RW) Forecast	**0.669**	**0.815**	0.913	0.894	0.819	0.921
Panel B: Theil U-stat to RW ≡ root MSFE of TVT-NKPC w.r.t. RW forecast						
MOE RMC (fix)	0.802***	1.000	0.879	0.821	0.748*	0.808
MOE RMC (aug)	0.776***	0.993	0.881	0.834	0.762*	0.767*
RULC RMC (fix)	0.826**	0.987	0.859*	0.809	0.742*	0.805
RULC RMC (aug)	0.842**	0.993	0.860*	0.814	0.751*	0.763*
Panel C: Theil U-stat to AO ≡ root MSFE of TVT-NKPC w.r.t. AO forecast						
MOE RMC (fix)	1.090**	1.086	0.999	0.927	0.909	0.958
MOE RMC (aug)	1.053	1.078	1.001	0.942	0.926	0.910
RULC RMC (fix)	1.121***	1.072	0.977	0.914	0.901	0.955
RULC RMC (aug)	1.143**	1.079*	0.977	0.920	0.912	0.905

Table 3: Predictive Performance of TVT-NKPC Forecasts in the EA Data - MDM Test

Note: Bold font indicates the best forecast by horizon. All forecasts are iterated (see Appendix B.3), and implemented in two versions; (i) 'fix' denotes fixed-length rolling window; (ii) 'aug' denotes augmenting-length recursive window. 'TVT-NKPC' is eq. (17), derived in Appendix B.1; its reduced-form parameters γ and κ – after calibrating $\rho = 0.2$, see Adolfson et al. (2007) and Cogley and Sborodone (2008) – are re-estimated in the pseudo-out-of-sample forecasting simulation via GMM using 5 lags of the dependent variable and 6 lags of the other variables in eq. (17) as instruments. As discussed in section 2, we employ two proxies for real marginal cost (RMC) in eq. (17): (i) 'MOE RMC' denotes a monetary open-economy RMC proxy; (ii) 'RULC RMC' denotes a real unit labor cost proxy. The modified Diebold-Mariano (1995) t-statistic (see Harvey et al., 1997) with p-values using Newey-West (1987) HAC standard errors, tests the null of no significant difference in the forecast accuracy of two compared (non-nested) models; statistical significance of the (one-sided) test is shown at conventional levels: *** 1%, ** 5%, and * 10%. These results are robust to applying instead the original Diebold-Mariano (one-sided) test – see Table 1 in Appendix C.

Forecasting evaluation period	2000:1–2015:4					
Forecast horizon, quarters	1	4	8	12	16	20
Panel A: Root MSFE						
Theory-Based TVT-NKPC Procedures of Inflation Forecasting						
MOE RMC (fix)	**0.854**	1.156	1.165	1.249	**1.255**	1.662
MOE RMC (aug)	0.986	1.125	1.438	1.370	1.291	**1.144**
RULC RMC (fix)	0.914	1.041	**1.100**	1.229	1.286	1.655
RULC RMC (aug)	0.922	1.036	1.127	**1.227**	1.261	1.179
Agnostic Univariate Benchmarks of Inflation Forecasting						
RW Forecast	0.970	1.127	1.333	1.360	1.470	1.447
AO (Pseudo-RW) Forecast	0.868	**1.027**	1.222	1.344	1.376	1.251
Panel B: Theil U-stat to RW ≡ root MSFE of TVT-NKPC w.r.t. RW forecast						
MOE RMC (fix)	0.881*	1.034	0.874	0.918	0.854	1.149
MOE RMC (aug)	1.016	0.998	1.079	1.007	0.878	0.790
RULC RMC (fix)	0.942	0.924	0.826*	0.903	0.875	1.114
RULC RMC (aug)	0.950	0.919	0.845*	0.902	0.858	0.812
Panel C: Theil U-stat to AO ≡ root MSFE of TVT-NKPC w.r.t. AO forecast						
MOE RMC (fix)	0.984	1.134**	0.953	0.929	0.912	1.329
MOE RMC (aug)	1.135***	1.096	1.177*	1.020	0.938	0.914
RULC RMC (fix)	1.053	1.014	0.901**	0.914*	0.935	1.323
RULC RMC (aug)	1.061	1.001	0.922**	0.913*	0.916	0.943

Table 4: Predictive Performance of TVT-NKPC Forecasts in the US Data - MDM Test

Note: See the note below Table 3 and Table 2 in Appendix C.

For online publication: Supplementary Appendix to "NKPC-Based Inflation Forecasts with a Time-Varying Trend"

Stephen McKnight* Alexander Mihailov† Fabio Rumler‡

July 2018

Abstract

This supplementary online appendix provides the sources and definitions of our data (in section A) as well as further technical derivations and details (in section B). It also presents and briefly discusses additional results in appendix tables and figures (in section C) that are sometimes also refrerred to in the main text. All references used in this supplementary online material are listed in the main text only, and not hereafter. For replication purposes, a zip file archive is available upon request that contains our data set, codes and the respective input and output files (using the econometric software package EViews, version 9.5 – but also works with the earlier versions 9.0, 8.1 and 7.2; the one-sided Hodrick-Prescott filter add-in needs to be downloaded from the EViews website).

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Contents

List of Tables

List of Figures

A Data Sources and Definitions

This Appendix A provides details on our data sources and definitions.

A.1 Euro Area

- *Source:* 16th update to the database underlying the Area Wide Model (AWM), see Fagan, Henry and Mestre (2001) at http://www.ecb.europa.eu/; the units of the series follow Eurostat or ECB conventions, that is:
  - real GDP and its components are in millions of ECU/euro corrected with reference year 1995;
  - nominal series are (typically) in millions of ECU/euro corrected;
  - deflators are (generally) set to 1 in 1995.
- *Mnemonics and Definitions – all variables are released at quarterly frequency and as seasonally adjusted at the data source:*
  - EAMTD ⇔ MTD in the AWM: Imports of Goods and Services Deflator;
  - EAPCR ⇔ PCR in the AWM: Private Consumption (Real);
  - EATOT ≡ EAMTD / EAXTD;
  - EARULC ⇔ ULC in the AWM: Unit Labour Costs, calculated as the ratio of compensation of employees and real GDP (ULC = WIN / YER);
  - EAXTD ⇔ XTD in the AWM: Exports of Goods and Services Deflator;
  - EAYED ⇔ YED in the AWM: GDP Deflator;
  - EAYER ⇔ YER in the AWM: GDP (Real).
- *Source:* FRED (Federal Reserve Economic Data), Economic Research Division, Federal Reserve Bank of St. Louis; http://research.stlouisfed.org/fred2/ (accessed on 2 September 2017)
- *Mnemonics and Definitions:*
  - EAM1 ⇔ MYAGM1EZQ196N at FRED via the International Monetary Fund / *International Financial Statistics* (IMF/IFS): M1 for Euro Area, Euros; Frequency: Quarterly, Not Seasonally Adjusted (subsequently seasonally adjusted, for consistency with the remaining raw data employed in the present study, using the Census-X12 procedure);
  - EAM1R ≡ EAM1 / EAYED.

A.2 United States

- *Source:* FRED (Federal Reserve Economic Data), Economic Research Division, Federal Reserve Bank of St. Louis; http://research.stlouisfed.org/fred2/ (accessed on 2 September 2017)

- *Mnemonics and Definitions – all variables are released at quarterly frequency and as seasonally adjusted at the respective original data source:*

  - USGDPD ⇔ GDPDEF at FRED via the Bureau of Economic Analysis (BEA): Gross Domestic Product – Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted;
  - USM1 ⇔ MANMM101USQ189S at FRED via the Organization for Economic Co-operation and Development / *Main Economic Indicators* (OECD/MEI): M1 for the United States, National Currency, Quarterly, Seasonally Adjusted
  - USM1R ≡ USM1 / USGDPD;
  - USMD ⇔ A021RD3Q086SBEA at FRED via BEA: Imports of Goods and Services – Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted;
  - USRGDP⇔ GDPC1 at FRED via BEA: Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate;
  - USRPCE ⇔ PCECC96 at FRED via BEA: Real Personal Consumption Expenditures, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate;
  - USULC: ULCNFB_20120606 at FRED via BEA: Nonfarm Business Sector, Unit Labor Cost, Index 2009=100, Quarterly, Seasonally Adjusted;
  - USRULC ≡ USULC / USGDPD;
  - USTOT ≡ USMD / USXD;
  - USXD ⇔ A020RD3Q086SBEA at FRED via BEA: Exports of Goods and Services – Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted.

B Technical Appendix

This Appendix B provides further technical details.

B.1 Derivation of a Generalized NKPC with Drifting Trend Inflation

In the present section, we provide the steps in the derivation of our generalized TVT-NKPC, eqs. (12) and (17) in the main text, whose theoretical framework was outlined in section 2 of the paper.

B.1.1 Log-Linear Optimality Condition of Price-Setting Firms

In line with the New Keynesian literature, the *individual* real marginal cost at $t+s$ of a firm i drawn to set optimally its price at t, denoted as $MC_{t,t+s}$, is assumed to be related to the *average* (or *aggregate*) real marginal cost in the model economy, MC_{t+s}, according to

$$MC_{t,t+s} = MC_{t+s}\left[\left(\frac{\Psi_{t,t+s}P_t^*}{P_{t+s}}\right)^{-\epsilon}\right]^{\omega} = MC_{t+s}\left(\frac{\Psi_{t,t+s}}{\Pi_{t,t+s}}p_t^*\right)^{-\epsilon\omega}, \quad (1)$$

where ω denotes the elasticity of firm i's real marginal cost to its own real output. Substituting the above expression in the firm's price-setting FONC (9) in the main text yields

$$E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\,\frac{\Lambda_{t+s}}{\Lambda_t}\left(\frac{\Psi_{t,t+s}}{\Pi_{t,t+s}}p_t^*\right)^{-\epsilon}Y_{t+s}\left[\frac{\Psi_{t,t+s}}{\Pi_{t,t+s}}p_t^* - \frac{\epsilon}{\epsilon-1}MC_{t+s}\left(\frac{\Psi_{t,t+s}}{\Pi_{t,t+s}}p_t^*\right)^{-\epsilon\omega}\right] = 0.$$

collecting the P_{t+j} and Ψ_{tj} terms, and using standard definitions in NKPC derivations for the numerator and denominator, we obtain

$$(p_t^*)^{1+\epsilon\omega} = \frac{\frac{\epsilon}{\epsilon-1}E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\,\frac{\Lambda_{t+s}}{\Lambda_t}\left(\frac{\Psi_{t,t+s}}{\Pi_{t,t+s}}\right)^{-(1+\omega)\epsilon}Y_{t+s}MC_{t+s}}{E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\,\frac{\Lambda_{t+s}}{\Lambda_t}\left(\frac{\Psi_{t,t+s}}{\Pi_{t,t+s}}\right)^{1-\epsilon}Y_{t+s}} \equiv \frac{C_t}{D_t}. \quad (2)$$

Next, using the fact that $\Psi_{t,t} = \Pi_{t,t} = 1$, we re-write C_t as

$$C_t = \frac{\epsilon}{\epsilon-1}MC_tY_t + \alpha\beta\frac{\epsilon}{\epsilon-1}E_t\sum_{s=1}^{\infty}(\alpha\beta)^s\,\frac{\Lambda_{t+s}}{\Lambda_t}\left(\frac{\Psi_{t,t+s}}{\Pi_{t,t+s}}\right)^{-(1+\omega)\epsilon}MC_{t+s}Y_{t+s}$$

In the next steps, we express the sum term as an expression involving C_{t+1}. To do so, one needs to replace all t-indexed terms by $t+1$-indexed terms. First, the above expression can be re-written as

$$\begin{aligned} C_t &= \frac{\epsilon}{\epsilon-1} MC_t Y_t + \alpha\beta \frac{\epsilon}{\epsilon-1} \times \\ & E_t \sum_{s=0}^{\infty} (\alpha\beta)^{s-1} \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\Lambda_{t+1+(s-1)}}{\Lambda_{t+1}} \left(\frac{\Psi_{t,t+1+(s-1)}}{\Pi_{t,t+1+(s-1)}} \right)^{-(1+\omega)\epsilon} MC_{t+1+(s-1)} Y_{t+1+(s-1)} \end{aligned}$$

Then, terms like $\Psi_{t,t+1+(s-1)}$ above are expressed as corresponding terms in $\Psi_{t+1,t+1+(s-1)}$ below, as follows:

$$\begin{aligned} \frac{\Psi_{t,t+1+(s-1)}}{\Pi_{t,t+1+(s-1)}} &= \frac{\Psi_{t,t+1+(s-1)}}{\Pi_{t,t+1+(s-1)}} \frac{\Psi_{t+1,t+1+(s-1)}}{\Psi_{t+1,t+1+(s-1)}} \frac{\Pi_{t+1,t+1+(s-1)}}{\Pi_{t+1,t+1+(s-1)}} \\ &= \frac{\Psi_{t+1,t+1+(s-1)}}{\Pi_{t+1,t+1+(s-1)}} \frac{\Psi_{t,t+1+(s-1)}}{\Psi_{t+1,t+1+(s-1)}} \frac{\Pi_{t+1,t+1+(s-1)}}{\Pi_{t,t+1+(s-1)}}. \end{aligned}$$

Now, using the definitions of $\Pi_{t,t+s}$ and $\Psi_{t,t+s}$ we obtain

$$\frac{\Pi_{t+1,t+1+(s-1)}}{\Pi_{t,t+1+(s-1)}} = \frac{\frac{P_{t+1+(s-1)}}{P_{t+1}}}{\frac{P_{t+1+(s-1)}}{P_t}} = \frac{\frac{1}{P_{t+1}}}{\frac{1}{P_t}} = \frac{P_t}{P_{t+1}} = \frac{1}{\Pi_{t+1}}$$

and

$$\frac{\Psi_{t,t+1+(s-1)}}{\Psi_{t+1,t+1+(s-1)}} = \frac{\Psi_{t,t+s}}{\Psi_{t+1,t+1+(s-1)}} = \frac{\prod_{j=0}^{s-1} \left(\Pi_{t+j}^{\rho} \overline{\Pi}_{t+j+1}^{1-\rho} \right)}{\prod_{j=0}^{s-2} \left(\Pi_{t+1+j}^{\rho} \overline{\Pi}_{t+1+j+1}^{1-\rho} \right)}.$$

Notice that

$$\prod_{j=0}^{s-2} \left(\Pi_{t+(1+j)}^{\rho} \overline{\Pi}_{t+1+(j+1)}^{1-\rho} \right) = \prod_{j=1}^{s-1} \left(\Pi_{t+j}^{\rho} \overline{\Pi}_{t+1+j}^{1-\rho} \right)$$

so that

$$\frac{\Psi_{t,t+1+(s-1)}}{\Psi_{t+1,t+1+(s-1)}} = \frac{\prod_{j=0}^{s-1} \left(\Pi_{t+j}^{\rho} \overline{\Pi}_{t+j+1}^{1-\rho} \right)}{\prod_{j=1}^{s-2} \left(\Pi_{t+j}^{\rho} \overline{\Pi}_{t+1+j}^{1-\rho} \right)} = \Pi_t^{\rho} \overline{\Pi}_{t+1}^{1-\rho},$$

and, finally,

$$\frac{\Psi_{t,t+1+(s-1)}}{\Pi_{t,t+1+(s-1)}} = \frac{\Psi_{t+1,t+1+(s-1)}}{\Pi_{t+1,t+1+(s-1)}} = \frac{\Pi_t^{\rho} \overline{\Pi}_{t+1}^{1-\rho}}{\Pi_{t+1}}.$$

Hence, it follows that

$$\begin{aligned} C_t &= \frac{\epsilon}{\epsilon-1}MC_tY_t + \alpha\beta\frac{\epsilon}{\epsilon-1} \times \\ & E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Pi_t^{\rho}\overline{\Pi}_{t+1}^{1-\rho}}{\Pi_{t+1}}\right)^{-(1+\omega)\epsilon}\sum_{s=0}^{\infty}(\alpha\beta)^s\frac{\Lambda_{t+1+s}}{\Lambda_{t+1}}\left(\frac{\Psi_{t+1,t+1+s}}{\Pi_{t+1,t+1+s}}\right)^{-(1+\omega)\epsilon}MC_{t+1+s}Y_{t+1+s}\right] \end{aligned}$$

which can be re-written recursively as

$$C_t = \frac{\epsilon}{\epsilon-1}MC_tY_t + \alpha\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Pi_t^{\rho}\overline{\Pi}_{t+1}^{1-\rho}}{\Pi_{t+1}}\right)^{-(1+\omega)\epsilon}C_{t+1}\right]. \quad (3)$$

Proceeding in a similar fashion with D_t, we obtain its analogous recursive representation:

$$D_t = Y_t + \alpha\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Pi_t^{\rho}\overline{\Pi}_{t+1}^{1-\rho}}{\Pi_{t+1}}\right)^{1-\epsilon}D_{t+1}\right]. \quad (4)$$

To induce stationarity, we next define

$$\widetilde{C}_t \equiv \frac{C_t}{Y_t}, \qquad \widetilde{D}_t \equiv \frac{D_t}{Y_t},$$

and obtain further

$$\widetilde{C}_t \equiv \frac{\epsilon}{\epsilon-1}MC_t + \alpha\beta E_t\left[g_{t+1}^{\Lambda}g_{t+1}^{Y}\left(\frac{\Pi_t^{\rho}\overline{\Pi}_{t+1}^{1-\rho}}{\Pi_{t+1}}\right)^{-(1+\omega)\epsilon}\widetilde{C}_{t+1}\right] \quad (5)$$

and

$$\widetilde{D}_t \equiv 1 + \alpha\beta E_t\left[g_{t+1}^{\Lambda}g_{t+1}^{Y}\left(\frac{\Pi_t^{\rho}\overline{\Pi}_{t+1}^{1-\rho}}{\Pi_{t+1}}\right)^{1-\epsilon}\widetilde{D}_{t+1}\right], \quad (6)$$

where

$$g_{t+1}^{\Lambda} \equiv \frac{\Lambda_{t+1}}{\Lambda_t}, \qquad g_{t+1}^{Y} \equiv \frac{Y_{t+1}}{Y_t}.$$

Since $\overline{\Pi}_t$ in incorporates a random walk, we define

$$\widetilde{\Pi}_t \equiv \frac{\Pi_t}{\overline{\Pi}_t}.$$

Using this, we get

$$\widetilde{C}_t = \frac{\epsilon}{\epsilon-1}MC_t + \alpha\beta E_t\left[g_{t+1}^{\Lambda}g_{t+1}^{Y}\left(\frac{\widetilde{\Pi}_t^{\rho}}{\widetilde{\Pi}_{t+1}}\left(g_{t+1}^{\overline{\Pi}}\right)^{-\rho}\right)^{-(1+\omega)\epsilon}\widetilde{C}_{t+1}\right],$$

$$\widetilde{D}_t = 1 + \alpha\beta E_t \left[g^{\Lambda}_{t+1} g^{Y}_{t+1} \left(\frac{\widetilde{\Pi}^{\rho}_t}{\widetilde{\Pi}_{t+1}} \left(g^{\overline{\Pi}}_{t+1} \right)^{-\rho} \right)^{1-\epsilon} \widetilde{D}_{t+1} \right].$$

and their ratio,

$$\frac{\widetilde{C}_t}{\widetilde{D}_t} = \left(\frac{P^*_t}{P_t} \right)^{1+\epsilon\omega} = (p^*_t)^{1+\epsilon\omega}. \tag{7}$$

Defining 'hat' variables to denote log-deviations of stationary variables around the drifting steady state (as in the main text), i.e., $\widehat{C}_t \equiv \ln \widetilde{C}_t = \ln \frac{C_t}{\overline{C}_t}$, we further obtain

$$(1 + \epsilon\omega)\, p^*_t = \widehat{C}_t - \widehat{D}_t,$$

and using the steady state restriction

$$\widetilde{C}_t = \frac{1}{1 - \alpha\beta g^{\Lambda} g^{Y}} \frac{\epsilon}{\epsilon - 1} \widetilde{MC}_t$$

we arrive at

$$\widehat{C}_t = \left(1 - \alpha\beta g^{\Lambda} g^{Y}\right) \widehat{MC}_t + \alpha\beta g^{\Lambda} g^{Y} E_t \left[g^{\Lambda}_{t+1} + g^{Y}_{t+1} + (1+\omega)\,\epsilon \left(\widehat{\Pi}_{t+1} - \rho\widehat{\Pi}_t + \rho\widehat{g}^{\overline{\pi}}_{t+1} \right) + \widehat{C}_{t+1} \right].$$

Proceeding in an analogous way for D, we begin from the steady state restriction

$$\widetilde{D}_t = \frac{1}{1 - \alpha\beta g^{\Lambda} g^{Y}}$$

and arrive at

$$\widehat{D}_t = \alpha\beta g^{\Lambda} g^{Y} E_t \left[g^{\Lambda}_{t+1} + g^{Y}_{t+1} - (1-\epsilon) \left(\widehat{\Pi}_{t+1} - \rho\widehat{\Pi}_t + \rho\widehat{g}^{\overline{\pi}}_{t+1} \right) + \widehat{D}_{t+1} \right].$$

Forming the difference,

$$\widehat{C}_t - \widehat{D}_t = \left(1 - \alpha\beta g^{\Lambda} g^{Y}\right) \widehat{MC}_t + \alpha\beta g^{\Lambda} g^{Y} E_t \left[(1+\omega\epsilon) \left(\widehat{\Pi}_{t+1} - \rho\widehat{\Pi}_t + \rho\widehat{g}^{\overline{\pi}}_{t+1} \right) + \widehat{C}_{t+1} - \widehat{D}_{t+1} \right]$$

or equivalently

$$\widehat{p}^*_t = \frac{1 - \alpha\beta g^{\Lambda} g^{Y}}{1 + \epsilon\omega} \widehat{MC}_t + \alpha\beta g^{\Lambda} g^{Y} E_t \left[\left(\widehat{\Pi}_{t+1} - \rho\widehat{\Pi}_t + \rho\widehat{g}^{\overline{\pi}}_{t+1} \right) + \widehat{p}^*_{t+1} \right]. \tag{8}$$

B.1.2 Log-Linear Aggregate Price Level and TVT-NKPC

Following Cogley and Sbordone (2008), we next appropriately transform condition (10) in the main text into a log-linear approximation around the drifting steady state. The aggregate price level obeys

$$P^{1-\epsilon}_t = \int_0^1 P_t(j)^{1-\epsilon}\, dj.$$

Using backward induction, one can obtain

$$P_t^{1-\epsilon} = (1-\alpha)\left(P_t^*\right)^{1-\epsilon} + \alpha\left(\Pi_{t-1}^{\rho}\overline{\Pi}_t^{1-\rho}P_{t-1}\right)^{1-\epsilon}$$

Dividing by P_t,

$$1 = (1-\alpha)\left(p_t^*\right)^{1-\epsilon} + \alpha\left(\frac{\Pi_{t-1}^{\rho}\overline{\Pi}_t^{1-\rho}}{\Pi_t}\right)^{1-\epsilon},$$

$$1 = (1-\alpha)\left(p_t^*\right)^{1-\epsilon} + \alpha\left(\frac{\Pi_{t-1}^{\rho}}{\overline{\Pi}_t^{\rho}}\frac{\overline{\Pi}_t}{\Pi_t}\right)^{1-\epsilon}.$$

Using the *gross steady-state growth rate*,

$$g_t^{\overline{\pi}} \equiv \frac{\overline{\Pi}_t}{\overline{\Pi}_{t-1}},$$

we further obtain

$$1 = (1-\alpha)\left(p_t^*\right)^{1-\epsilon} + \alpha\left(\frac{\Pi_{t-1}^{\rho}}{\left(g_t^{\overline{\pi}}\overline{\Pi}_{t-1}\right)^{\rho}}\frac{1}{\widetilde{\Pi}_t}\right)^{1-\epsilon},$$

$$1 = (1-\alpha)\left(p_t^*\right)^{1-\epsilon} + \alpha\left(\frac{1}{\left(g_t^{\overline{\pi}}\right)^{\rho}}\frac{\Pi_{t-1}^{\rho}}{\overline{\Pi}_{t-1}^{\rho}}\frac{1}{\widetilde{\Pi}_t}\right)^{1-\epsilon},$$

$$1 = (1-\alpha)\left(p_t^*\right)^{1-\epsilon} + \alpha\left(\frac{\widetilde{\Pi}_{t-1}^{\rho}}{\widetilde{\Pi}_t}\left(g_t^{\overline{\pi}}\right)^{-\rho}\right)^{1-\epsilon}. \quad (9)$$

Transforming (9) to express it in terms of the stationary variables, we get:[1]

$$1 = (1-\alpha)\left(\overline{p}_t^*\right)^{1-\epsilon}\left(\widetilde{p}_t^*\right)^{1-\epsilon} + \alpha\left(g_t^{\overline{\pi}}\right)^{-\rho(1-\epsilon)}\widetilde{\Pi}_{t-1}^{\rho(1-\epsilon)}\widetilde{\Pi}_t^{-(1-\epsilon)}. \quad (10)$$

Thus, in a *drifting* steady state at t, equation (10) yields:

$$1 = (1-\alpha)\,\overline{p}_t^* + \alpha\left(g_t^{\overline{\pi}}\right)^{-\rho(1-\epsilon)}\left(g_t^{\overline{\pi}}\right)^{\rho(1-\epsilon)},$$

$$1 = 1 - \alpha\overline{p}_t^* + \alpha,$$

$$\Rightarrow \overline{p}_t^* = 1. \quad (11)$$

Using the 'hat' variables,

$$\widehat{\Pi}_t \equiv \ln\widetilde{\Pi}_t = \ln\left(\Pi_t/\overline{\Pi}_t\right) = \ln\Pi_t - \ln\overline{\Pi}_t,$$

$$\widehat{g}_t^{\overline{\pi}} \equiv \ln g_t^{\overline{\pi}},$$

[1] Which is a generalization of eq. (26) in Appendix A of Cogley and Sbordone (2008) arising from the indexation (a fraction $1-\rho$) to *current trend inflation* we consider here in addition to the indexation (a fraction ρ) to *past actual inflation* as assumed by the latter authors.

the log-linear approximation of (10) around the (time-varying) steady state $\overline{p}_t^* = 1$ is:

$$\widehat{p}_t^* = \frac{\alpha}{1-\alpha}\left(\widehat{\Pi}_t - \rho\widehat{\Pi}_{t-1} + \rho\widehat{g}_t^{\overline{\pi}}\right). \tag{12}$$

Employing relation (8), we finally arrive at

$$\widehat{\Pi}_t - \rho\widehat{\Pi}_{t-1} + \rho\widehat{g}_t^{\overline{\Pi}} = \frac{(1-\alpha)(1-\alpha\beta g^{\Lambda} g^{Y})}{\alpha(1+\epsilon\omega)}\widehat{MC}_t + \beta g^{\Lambda} g^{Y} E_t\left[\widehat{\Pi}_{t+1} - \rho\widehat{\Pi}_t + \rho\widehat{g}_{t+1}^{\overline{\Pi}}\right],$$

which is equation (12) in the main text.

B.2 Log-linear Marginal Cost Condition of McKnight and Mihailov (2015)

Linearizing equation (8) of McKnight and Mihailov (2015):

$$\begin{aligned} mc_t &= w_t\left(\frac{P_t}{P_{H,t}}\right), \\ \Rightarrow \widehat{mc}_t &= \widehat{w}_t + \left(\widehat{P}_t - \widehat{P}_{H,t}\right). \end{aligned} \tag{13}$$

Linearizing the first-order condition (13) of McKnight and Mihailov (2015) but using hereafter our notation for $\overline{\omega}$ and σ in the present paper,

$$\begin{aligned} w_t &= \frac{v_h(h_t)}{u_c(C_t, m_t)}, \\ \Rightarrow \widehat{w}_t &= \overline{\omega}\widehat{h}_t + \sigma\widehat{C}_t - \chi\widehat{m}_t, \end{aligned} \tag{14}$$

where the coefficients are defined as

$$\overline{\omega} \equiv \frac{\overline{h}_t v_{hh}}{v_h} > 0, \qquad \chi \equiv \frac{\overline{m}_t u_{cm}}{u_c},$$

and the coefficient of relative risk aversion (CRRA) is given by

$$\sigma \equiv -\frac{\overline{C}_t u_{cc}}{u_c} > 0.$$

The aggregate version of the production function (7) of McKnight and Mihailov (2015) is

$$d_t Y_t = h_t,$$

where d_t is the measure of price dispersion:

$$d_t \equiv \int_0^1 \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varphi} di.$$

Combining equations (13) and (14) above yields:

$$\widehat{mc}_t = \overline{\omega}\widehat{d_t Y_t} + \sigma \widehat{C}_t - \chi \widehat{m}_t + \left(\widehat{P}_t - \widehat{P}_{H,t}\right). \tag{15}$$

Rewriting the price index, equation (2) of McKnight and Mihailov (2015):

$$\begin{aligned} P_t^{1-\epsilon} &= a P_{H,t}^{1-\epsilon} + (1-a) P_{F,t}^{1-\epsilon}, \\ \Rightarrow \left(\frac{P_t}{P_{H,t}}\right)^{1-\epsilon} &= a + (1-a)\left(S_t\right)^{1-\epsilon}, \end{aligned}$$

after using the definition of the terms of trade S_t. Linearizing the above yields:

$$\left(\widehat{P}_t - \widehat{P}_{H,t}\right) = (1-a)\left[\frac{\left(\overline{S}_t\right)^{1-\epsilon}}{a + (1-a)\left(\overline{S}_t\right)^{1-\epsilon}}\right]\widehat{S}_t. \tag{16}$$

Combining equations (15) and (16) above gives us the definition of mc in a monetary open economy:

$$\widehat{mc}_t = \overline{\omega}\widehat{d_t Y_t} + \sigma \widehat{C}_t - \chi \widehat{m}_t + (1-a)\left[\frac{\left(\overline{S}_t\right)^{1-\epsilon}}{a + (1-a)\left(\overline{S}_t\right)^{1-\epsilon}}\right]\widehat{S}_t. \tag{17}$$

Assuming that in the steady state $\overline{S}_t = 1$, equation (17) simplifies to:

$$\widehat{mc}_t = \overline{\omega}\widehat{d_t Y_t} + \sigma \widehat{C}_t - \chi \widehat{m}_t + (1-a)\widehat{S}_t. \tag{18}$$

Ignoring $\widehat{d_t Y_t}$ for the moment, nothing has changed in terms of our definition of mc provided the linearization is performed assuming $\overline{S}_t = 1$, which we assume as most of the open economy literature does.

The relative price dispersion measure is:

$$d_t = \int_0^1 \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} di.$$

With Calvo (1983) price-setting, the above can be written as:

$$\begin{aligned} d_t \quad = \quad & (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \alpha(1-\alpha)\left[\frac{P_{t-1}^* \Pi_{t-1}^{\rho} \overline{\Pi}_t^{1-\rho}}{P_t}\right]^{-\epsilon} \\ & + \alpha^2(1-\alpha)\left[\frac{P_{t-2}^*\left(\Pi_{t-1}\Pi_{t-2}\right)^{\rho}\left(\overline{\Pi}_t\overline{\Pi}_{t-1}\right)^{1-\rho}}{P_t}\right]^{-\epsilon} + \ldots \end{aligned}$$

Recalling that $\Pi_t \equiv P_t/P_{t-1}$, collecting terms gives:

$$\begin{aligned} d_t &= (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \alpha\left(\Pi_{t-1}^{-\epsilon}\right)^{\rho}\left(\overline{\Pi}_t^{-\epsilon}\right)^{1-\rho} \\ &\quad \times \Pi_t^{\epsilon} \underbrace{\left[(1-\alpha)\left(\frac{P_{t-1}^*}{P_{t-1}}\right)^{-\epsilon} + \alpha(1-\alpha)\left(\frac{P_{t-2}^*\Pi_{t-2}^{\rho}\overline{\Pi}_{t-1}^{1-\rho}}{P_{t-1}}\right)^{-\epsilon} + \ldots\right]}_{\equiv d_{t-1}}. \end{aligned}$$

Recalling that $p_t^* \equiv P_t^*/P_t$, the above expression simplifies to:

$$d_t = (1-\alpha)\left(p_t^*\right)^{-\epsilon} + \alpha\left(\Pi_{t-1}^{-\epsilon}\right)^{\rho}\left(\overline{\Pi}_t^{-\epsilon}\right)^{1-\rho}\Pi_t^{\epsilon} d_{t-1}. \tag{19}$$

Transforming (19) to express it in terms of stationary variables:

$$\overline{d}_t \cdot \widetilde{d}_t = (1-\alpha)\left(\overline{p}_t^*\right)^{-\epsilon} \cdot \left(\widetilde{p}_t^*\right)^{-\epsilon} + \alpha\overline{d}_{t-1} \cdot \widetilde{d}_{t-1}\left(\overline{\Pi}_{t-1}^{-\epsilon} \cdot \widetilde{\Pi}_{t-1}^{-\epsilon}\right)^{\rho}\left(\overline{\Pi}_t^{-\epsilon}\right)^{-\rho} \cdot \widetilde{\Pi}_t^{\epsilon},$$

$$\overline{d}_t \cdot \widetilde{d}_t = (1-\alpha)\left(\overline{p}_t^*\right)^{-\epsilon} \cdot \left(\widetilde{p}_t^*\right)^{-\epsilon} + \alpha\overline{d}_{t-1} \cdot \widetilde{d}_{t-1}\left(\widetilde{\Pi}_{t-1}^{-\epsilon}\right)^{\rho}\left[\left(g_t^{\overline{\pi}}\right)^{-\epsilon}\right]^{-\rho} \cdot \widetilde{\Pi}_t^{\epsilon}, \tag{20}$$

since $g_t^{\overline{\pi}} \equiv \overline{\Pi}_t/\overline{\Pi}_{t-1}$.

Now recall, that in a time-varying steady state at t,

$$\widetilde{\Pi}_t \equiv \Pi_t/\overline{\Pi}_t = \overline{\Pi}_t/\overline{\Pi}_t = 1,$$

$$\widetilde{p}_t^* \equiv p_t^*/\overline{p}_t^* = \overline{p}_t^*/\overline{p}_t^* = 1,$$

$$\widetilde{d}_t \equiv d_t/\overline{d}_t = \overline{d}_t/\overline{d}_t = 1,$$

Also recall that in the steady state $\overline{p}_t^* = 1$. Thus, equation (20) becomes

$$\overline{d}_t = (1-\alpha) + \alpha\overline{d}_{t-1}g_t^{\overline{d}}, \tag{21}$$

and after substituting in (21) by the definition $\overline{d}_t \equiv \overline{d}_{t-1}g_t^{\overline{d}}$, we obtain $\overline{d}_t = 1$. Linearizing (20) around the steady state yields:

$$\widehat{d}_t = -\epsilon(1-\alpha)\widehat{p}_t^* + \alpha\widehat{d}_{t-1} + \alpha\epsilon\left(\widehat{\Pi}_t - \rho\widehat{\Pi}_{t-1} + \rho\widehat{g}_t^{\overline{\pi}}\right). \tag{22}$$

Recalling that

$$\widehat{p}_t^* = \frac{\alpha}{1-\alpha}\left(\widehat{\Pi}_t - \rho\widehat{\Pi}_{t-1} + \rho\widehat{g}_t^{\overline{\pi}}\right), \tag{23}$$

equations (22) and (23) imply that $\widehat{d}_t = \alpha\widehat{d}_{t-1}$. As discussed by Schmitt-Grohé and Uribe (2007), d_t has no real consequences up to first order in the stationary distribution of other endogenous variables. This means that our linear approximations to the equilibrium conditions around the steady state are justified in ignoring the variable d_t. Consequently equation (18) above simplifies to

$$\widehat{mc}_t = \overline{\omega}\widehat{Y}_t + \sigma\widehat{C}_t - \chi\widehat{m}_t + (1-a)\,\widehat{S}_t,$$

which is eq. (13) in the main text.

B.3 Alternative Types of Multistep Univariate Forecasts

Let $y_t \equiv \Delta^d X_t$ denote the stationary transformation of a time series in levels or log-levels X_t, where Δ^d is the d-th difference for X_t being integrated of order d, that is, X_t is $I(d)$ and $d = \{0, 1, 2\}$. To forecast X_t at horizon h, one has to forecast first the appropriate *stationary* transformation of X_t, y_t, at the same horizon. When $h > 1$, multistep univariate forecasts can be generated in two ways, namely, by *iterated* autoregressive (AR) forecasts or by *direct* AR forecasts.

B.3.1 Iterated Multistep Univariate Forecasts

The *iterated* forecast always begins with the 1-step-ahead forecasting AR model for y_t, which can be written as (see Marcellino et al., 2006, pp. 502–503, whose notation we follow here)

$$y_{t+1} = \alpha + \sum_{i=1}^{p} \phi_i y_{t+1-i} + \varepsilon_{t+1}. \tag{24}$$

After estimating recursively by OLS the parameters in (24), the multistep forecasts of y_{t+h} are obtained iteratively from

$$\widehat{y}^I_{t+h|t} = \widehat{\alpha} + \sum_{i=1}^{p} \widehat{\phi}_i \widehat{y}^I_{t+1-i|t}, \text{ where } \widehat{y}^I_{j|t} = y_j \text{ for } j \leq t. \tag{25}$$

Iterated forecasts of X_{t+h} are then computed by accumulating the values of $\widehat{y}^I_{t+h|t}$ depending on the order of integration of X_t, as follows:

$$\widehat{X}^I_{t+h|t} = \begin{cases} \widehat{y}^I_{t+h|t} & \text{if } X_t \text{ is } I(0), \\ X_t + \sum_{i=1}^{h} \widehat{y}^I_{t+i|t} & \text{if } X_t \text{ is } I(1), \\ X_t + h\Delta X_t + \sum_{i=1}^{h}\sum_{j=1}^{i} \widehat{y}^I_{t+j|t} & \text{if } X_t \text{ is } I(2). \end{cases} \tag{26}$$

B.3.2 Direct Multistep Univariate Forecasts

The *direct* estimates of the parameters are the recursive minimizers of the h-step-ahead criterion function. Accordingly, the parameters are estimated by OLS from the following direct forecasting AR model (again, we follow Marcellino et al., 2006, p. 503):

$$y^h_{t+h} = \beta + \sum_{i=1}^{p} \rho_i y_{t+1-i} + \varepsilon_{t+h}, \tag{27}$$

where now

$$y_{t+h}^{h} = \begin{cases} X_{t+h} & \text{if } X_t \text{ is } I(0), \\ X_{t+h} - X_t & \text{if } X_t \text{ is } I(1), \\ \sum_{i=1}^{h}\sum_{j=1}^{i}\Delta^2 X_{t+j} = X_{t+h} - X_t - h\Delta X_t & \text{if } X_t \text{ is } I(2). \end{cases}$$

The direct estimator of the parameters is obtained by recursive OLS estimation of (27) where data through period t are used, so that the last observation includes y_t^h on the left-hand side of the regression. Then, the direct multistep forecasts of y_{t+h}^h are given by

$$\widehat{y}_{t+h|t}^{D,h} = \widehat{\beta} + \sum_{i=1}^{p}\widehat{\rho}_i y_{t+1-i}. \tag{28}$$

By analogy with the iterated multistep AR forecasts in (26), direct multistep AR forecasts of X_{t+h} are recovered from $\widehat{y}_{t+h|t}^{D,h}$ depending on the order of integration of X_t:

$$\widehat{X}_{t+h|t}^{D} = \begin{cases} \widehat{y}_{t+h|t}^{D,h} & \text{if } X_t \text{ is } I(0), \\ X_t + \widehat{y}_{t+h|t}^{D,h} & \text{if } X_t \text{ is } I(1), \\ X_t + h\Delta X_t + \widehat{y}_{t+h|t}^{D,h} & \text{if } X_t \text{ is } I(2). \end{cases} \tag{29}$$

C Additional Results and Robustness Checks

This Appendix C provides additional results and robustness checks, some of which are referred to in the main text, presented in the tables and figures that follow.

Tables 3 and 4 in the main text reported the key results in evaluating the predictive performance of our TVT-NKPC forecasting procedure, where the statistical significance in the Diebold-Mariano (1995) test based on the null of equal predictive accuracy (EPA) was indicated by the *modified* DM statistic, to correct for small sample bias, as proposed by Harvey et al. (1997). Since our forecasting evaluation period is not that small, in effect, ranging from 44 (if the horizon is 20 quarters ahead) to 64 (if the horizon is 1 quarter ahead) quarters, we here also present a version of the same results in appendix tables 1 and 2 in this section, but now applying the *original* DM statistic in judging about statistical significance.

It can be seen, comparing the respective significance levels in tables 1 and 2 in this Appendix C with tables 3 and 4, respectively, already reported in the main text that such an alteration, which ignores the small sample size correction, results in even more favorable outcomes regarding the predictive accuracy of our method. More precisely, the final conclusions in the main text and its abstract can now be slightly modified, in the following sense. Based on the *original* DM statistic, we conclude that our TVT-NKPC forecasting procedure significantly outperforms the conventional random walk benchmark at all horizons (remaining statistically indistinguishable only at 4 quarters). Moreover, it also outperforms quantitatively, by about 10 percentage points beyond the short run of 1 and 4 quarters, the agnostic Atkeson-Ohanian (2001) benchmark that previous studies have found difficult to beat, and in a statistically significant fashion in the US at the medium run of 8 and 12 quarters.

Forecasting evaluation period	2000:1–2015:4					
Forecast horizon, quarters	1	4	8	12	16	20
Panel A: Root MSFE						
Theory-Based TVT-NKPC Procedures of Inflation Forecasting						
MOE RMC (fix)	0.729	0.884	0.912	0.829	0.745	0.883
MOE RMC (aug)	0.706	0.879	0.914	0.842	0.759	0.838
RULC RMC (fix)	0.751	0.873	**0.892**	**0.816**	**0.739**	0.880
RULC RMC (aug)	0.765	0.879	**0.892**	0.822	0.747	**0.833**
Agnostic Univariate Benchmarks of Inflation Forecasting						
RW Forecast	0.909	0.885	1.038	1.001	0.996	1.092
AO (Pseudo-RW) Forecast	**0.669**	**0.815**	0.913	0.894	0.819	0.921
Panel B: Theil U-stat to RW ≡ root MSFE of TVT-NKPC w.r.t. RW forecast						
MOE RMC (fix)	0.802***	1.000	0.879	0.821	0.748**	0.808
MOE RMC (aug)	0.776***	0.993	0.881	0.834	0.762**	0.767**
RULC RMC (fix)	0.826**	0.987	0.859*	0.809*	0.742**	0.805
RULC RMC (aug)	0.842**	0.993	0.860*	0.814*	0.751**	0.763**
Panel C: Theil U-stat to AO ≡ root MSFE of TVT-NKPC w.r.t. AO forecast						
MOE RMC (fix)	1.090**	1.086	0.999	0.927	0.909	0.958
MOE RMC (aug)	1.053	1.078	1.001	0.942	0.926	0.910
RULC RMC (fix)	1.121***	1.072*	0.977	0.914	0.901	0.955
RULC RMC (aug)	1.143***	1.079**	0.977	0.920	0.912	0.905

Table 1: Predictive Performance of TVT-NKPC Forecasts in the EA Data - DM Test

Note: See the note below Table 3 in the main text. Differently here, the original Diebold-Mariano (1995) t-statistic is used, and is again computed with p-values using Newey-West (1987) HAC standard errors; it tests the null of no significant difference in the forecast accuracy of two compared (non-nested) models; statistical significance of the (one-sided) test is shown at conventional levels: *** 1%, ** 5%, and * 10%. These results are robust to applying instead the modified DM t-statistic test of Harvey et al. (1997) that corrects for small-sample bias – see Table 3 in the main text.

Forecasting evaluation period	2000:1–2015:4					
Forecast horizon, quarters	1	4	8	12	16	20
Panel A: Root MSFE						
Theory-Based TVT-NKPC Procedures of Inflation Forecasting						
MOE RMC (fix)	**0.854**	1.156	1.165	1.249	**1.255**	1.662
MOE RMC (aug)	0.986	1.125	1.438	1.370	1.291	**1.144**
RULC RMC (fix)	0.914	1.041	**1.100**	1.229	1.286	1.655
RULC RMC (aug)	0.922	1.036	1.127	**1.227**	1.261	1.179
Agnostic Univariate Benchmarks of Inflation Forecasting						
RW Forecast	0.970	1.127	1.333	1.360	1.470	1.447
AO (Pseudo-RW) Forecast	0.868	**1.027**	1.222	1.344	1.376	1.251
Panel B: Theil U-stat to RW ≡ root MSFE of TVT-NKPC w.r.t. RW forecast						
MOE RMC (fix)	0.881*	1.034	0.874	0.918	0.854	1.149
MOE RMC (aug)	1.016	0.998	1.079	1.007	0.878	$0.790^{\diamond}$
RULC RMC (fix)	0.942	0.924	0.826*	0.903*	0.875	1.114
RULC RMC (aug)	0.950	0.919	0.845*	0.902*	0.858*	0.812
Panel C: Theil U-stat to AO ≡ root MSFE of TVT-NKPC w.r.t. AO forecast						
MOE RMC (fix)	0.984	1.134**	0.953	0.929	0.912	1.329*
MOE RMC (aug)	1.135***	1.096	1.177*	1.020	0.938	0.914
RULC RMC (fix)	1.053	1.014	0.901**	0.914**	0.935	1.323*
RULC RMC (aug)	1.061	1.001	0.922**	0.913*	0.916	0.943

Table 2: Predictive Performance of TVT-NKPC Forecasts in the US Data - DM Test

Note: See the note below Table 1 in this Appendix B, as well as Table 4 in the main text for the corresponding US results when applying the modified DM t-statistic instead of the original DM t-statistic here. $\diamond$-superscript denotes that the p-value of the DM t-statisitic in the respective cell of the table above is just marginal at the 10% sigificance level here, 0.1092.

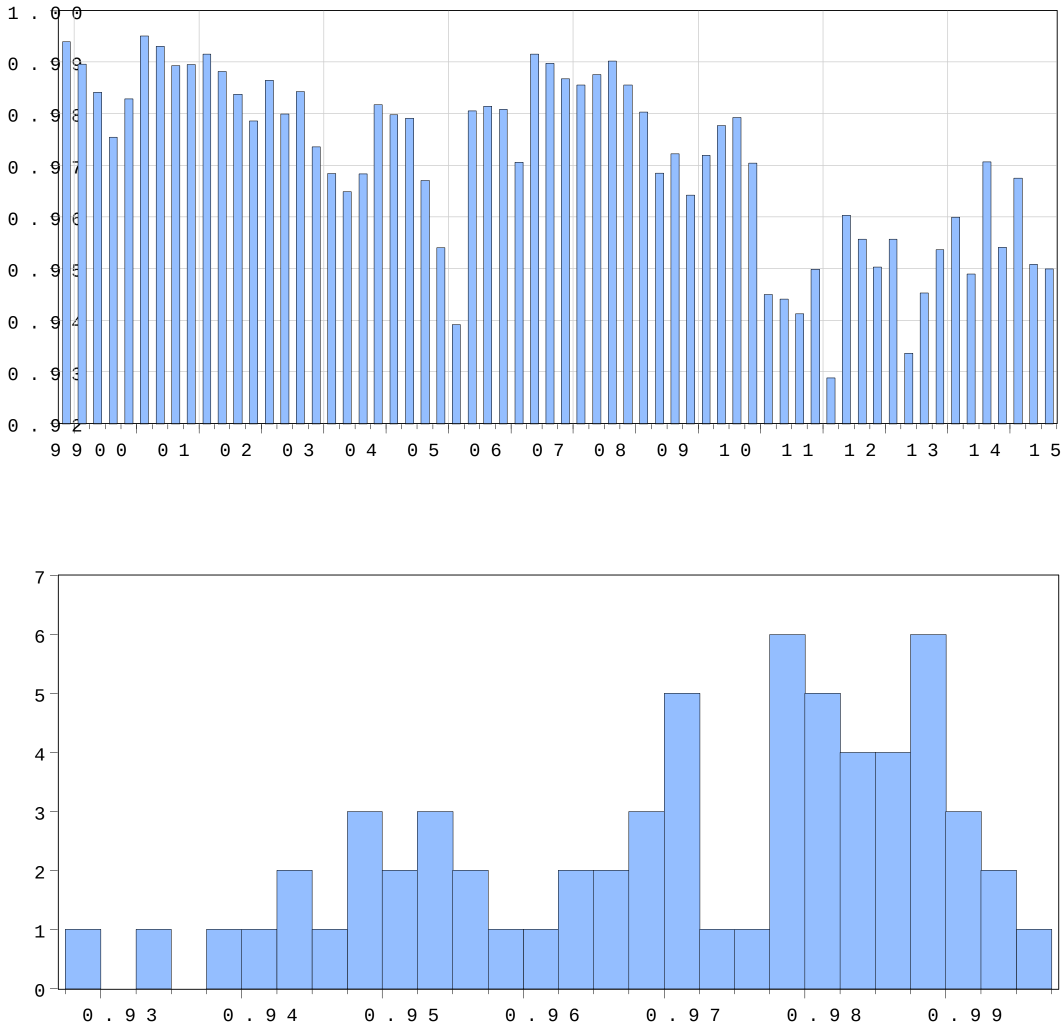


Figure 1: EA – Recursively Re-estimated Persistence ($g_t^{\overline{\Pi}}$) of Trend Inflation (fixed rolling window of 113 quarters), plot (top graph) and histogram (bottom graph); 1999Q4–2015Q3, 64 observations; mean 0.971, median 0.975, max 0.995, min 0.929, SD 0.017

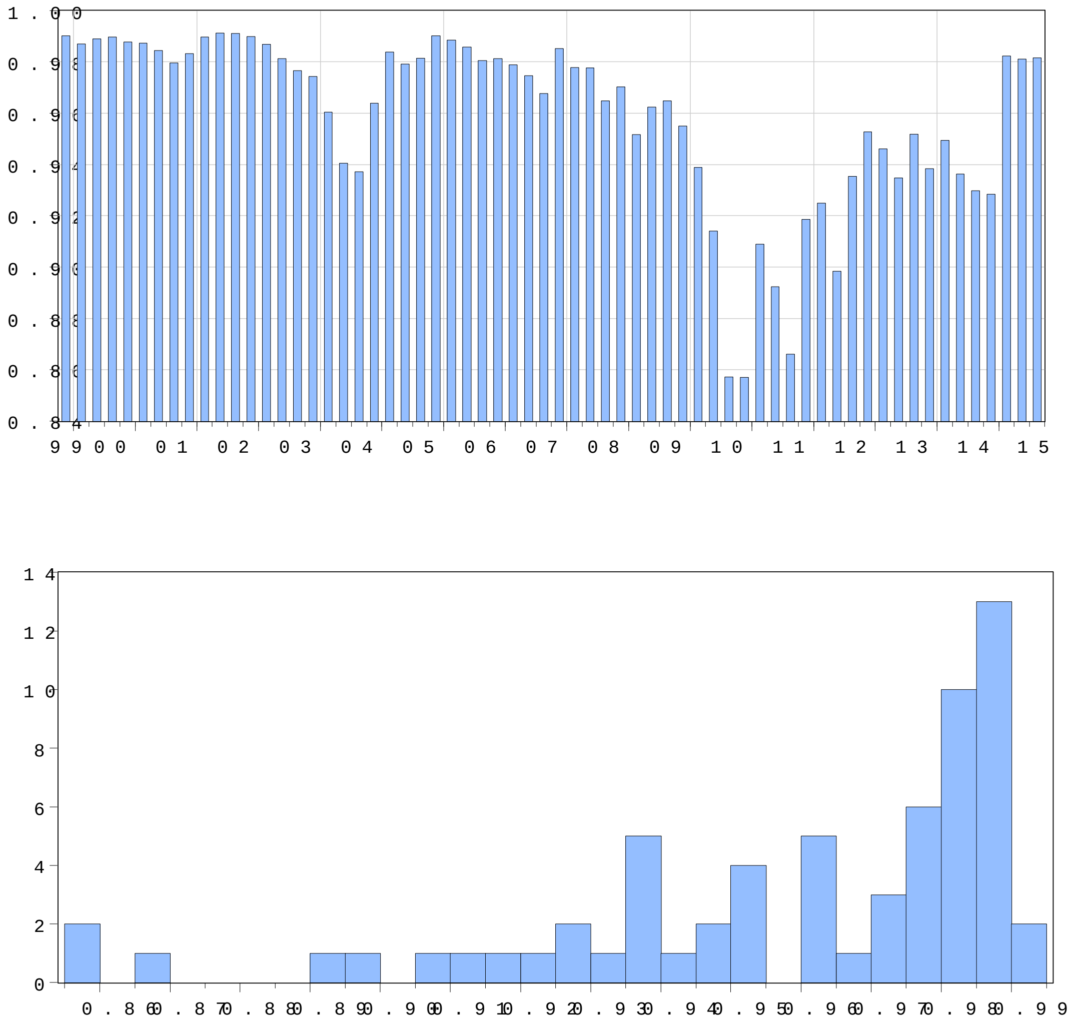
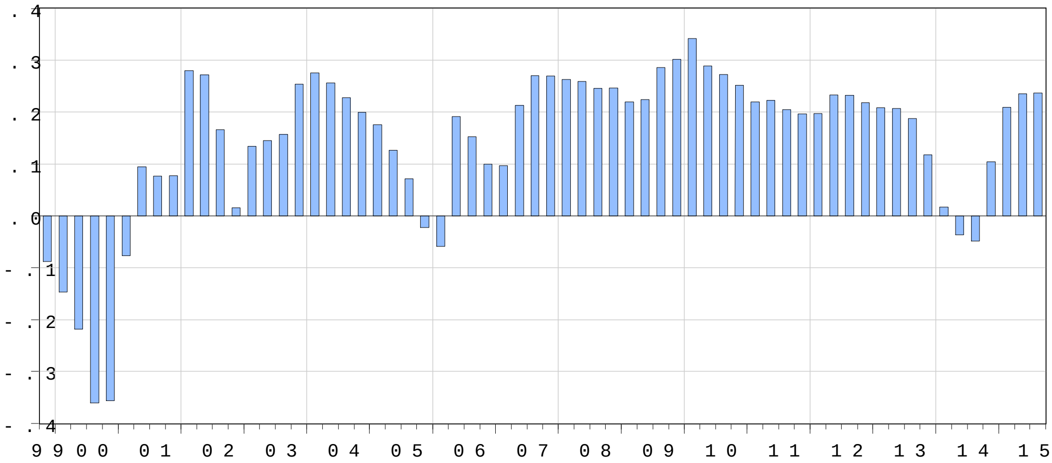


Figure 2: US – Recursively Re-estimated Persistence ($g_t^{\overline{\Pi}}$) of Trend Inflation (fixed rolling window of 113 quarters), plot (top graph) and histogram (bottom graph); 1999Q4–2015Q3, 64 observations; mean 0.959, median 0.974, max 0.991, min 0.857, SD 0.034



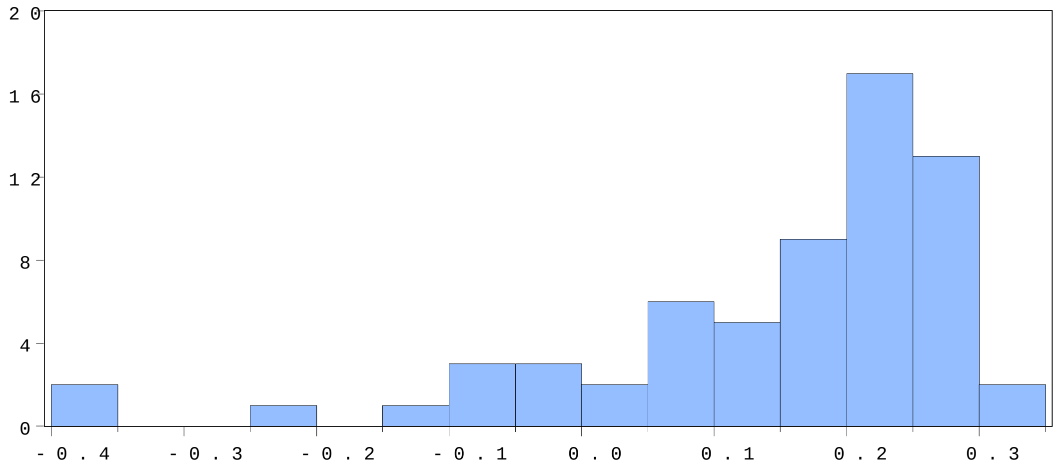


Figure 3: EA – Recursively Re-estimated Persistence (θ) of Trend Inflation Growth (fixed rolling window of 113 quarters), plot (top graph) and histogram (bottom graph); 1999Q4–2015Q3, 64 observations; mean 0.146, median 0.202, max 0.341, min –0.361, SD 0.150

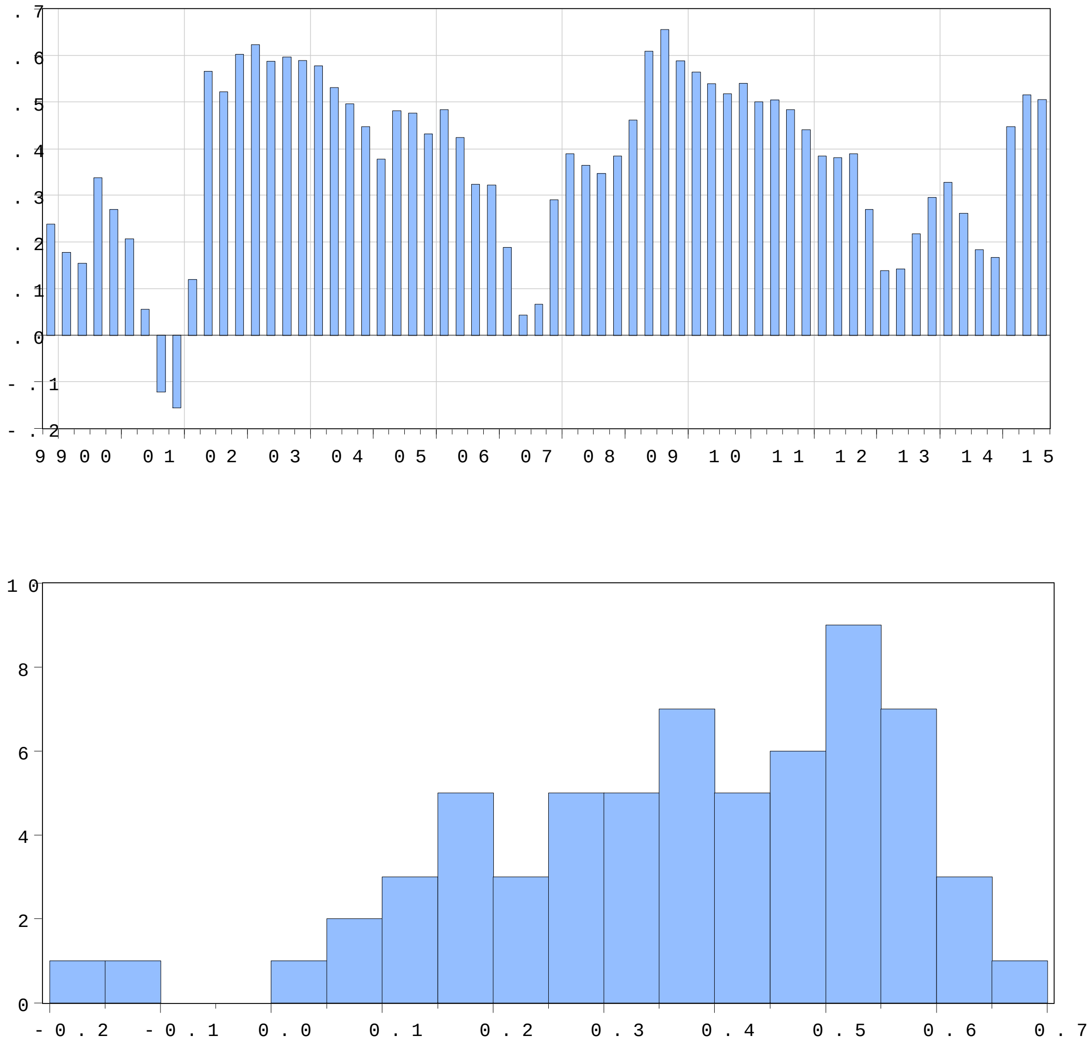
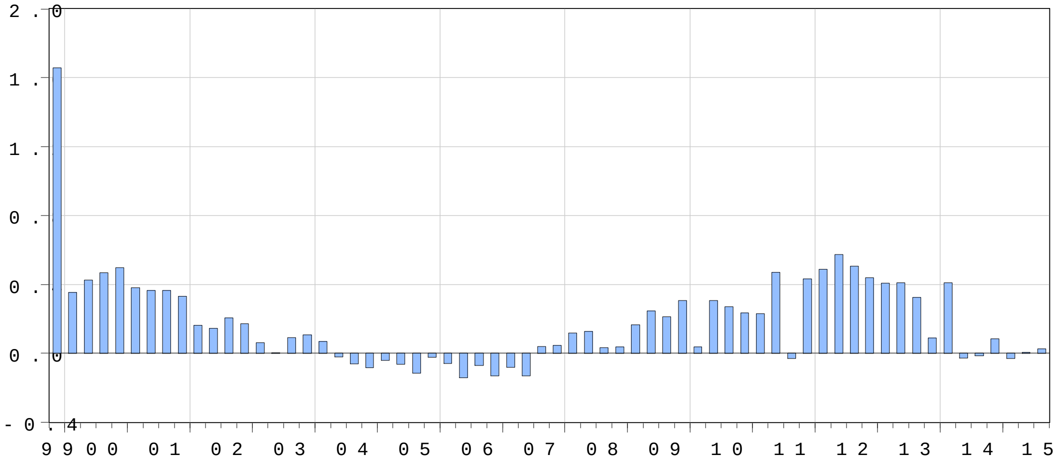


Figure 4: US – Recursively Re-estimated Persistence (θ) of Trend Inflation Growth (fixed rolling window of 113 quarters), plot (top graph) and histogram (bottom graph); 1999Q4–2015Q3, 64 observations; mean 0.373, median 0.389, max 0.655, min –0.156, SD 0.186



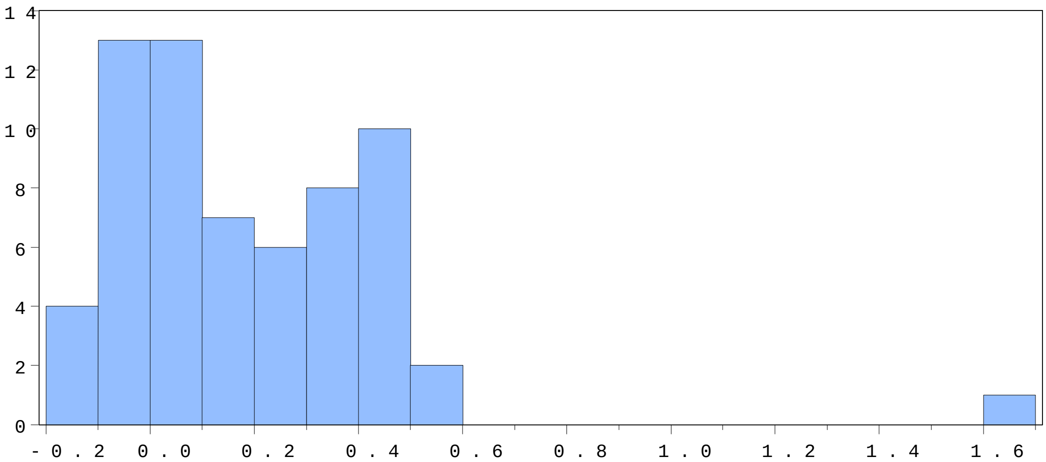
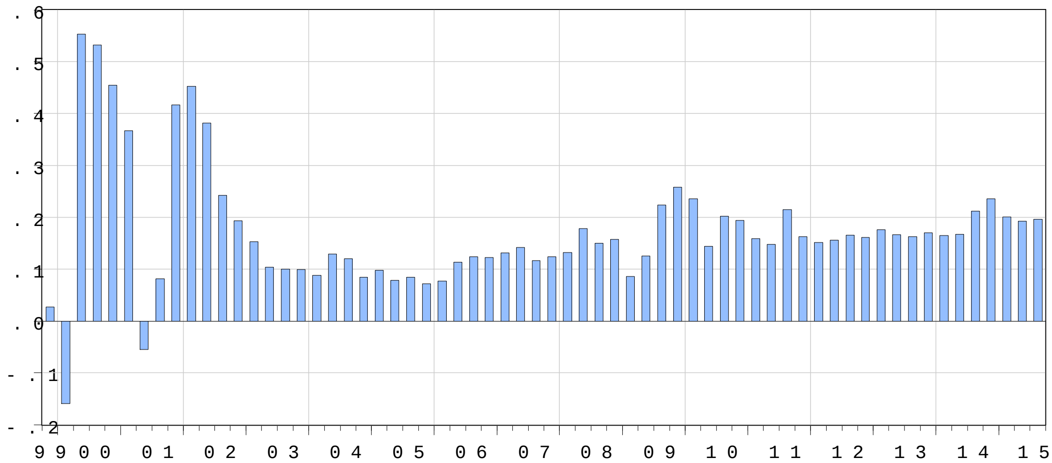


Figure 5: EA – Recursively Re-estimated Slope (κ) of the TVT-NKPC at Horizon of 1 Quarter (MOE RMC proxy, fixed rolling window of 113 quarters), plot (top graph) and histogram (bottom graph); 1999Q4–2015Q3, 64 observations; mean 0.185, median 0.124, max 1.656, min –0.141, SD 0.273



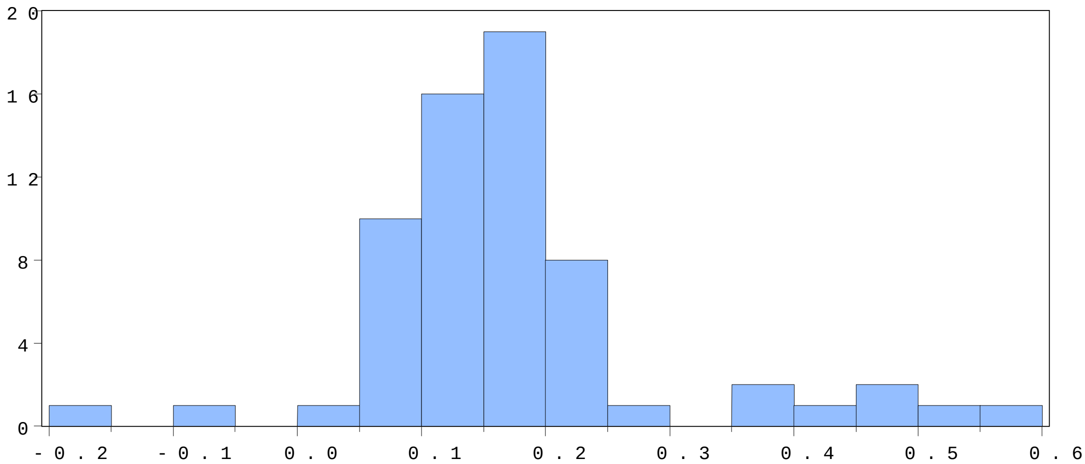


Figure 6: US – Recursively Re-estimated Slope (κ) of the TVT-NKPC at Horizon of 1 Quarter (MOE RMC proxy, fixed rolling window of 113 quarters), plot (top graph) and histogram (bottom graph); 1999Q4–2015Q3, 64 observations; mean 0.173, median 0.157, max 0.553, min –0.159, SD 0.120

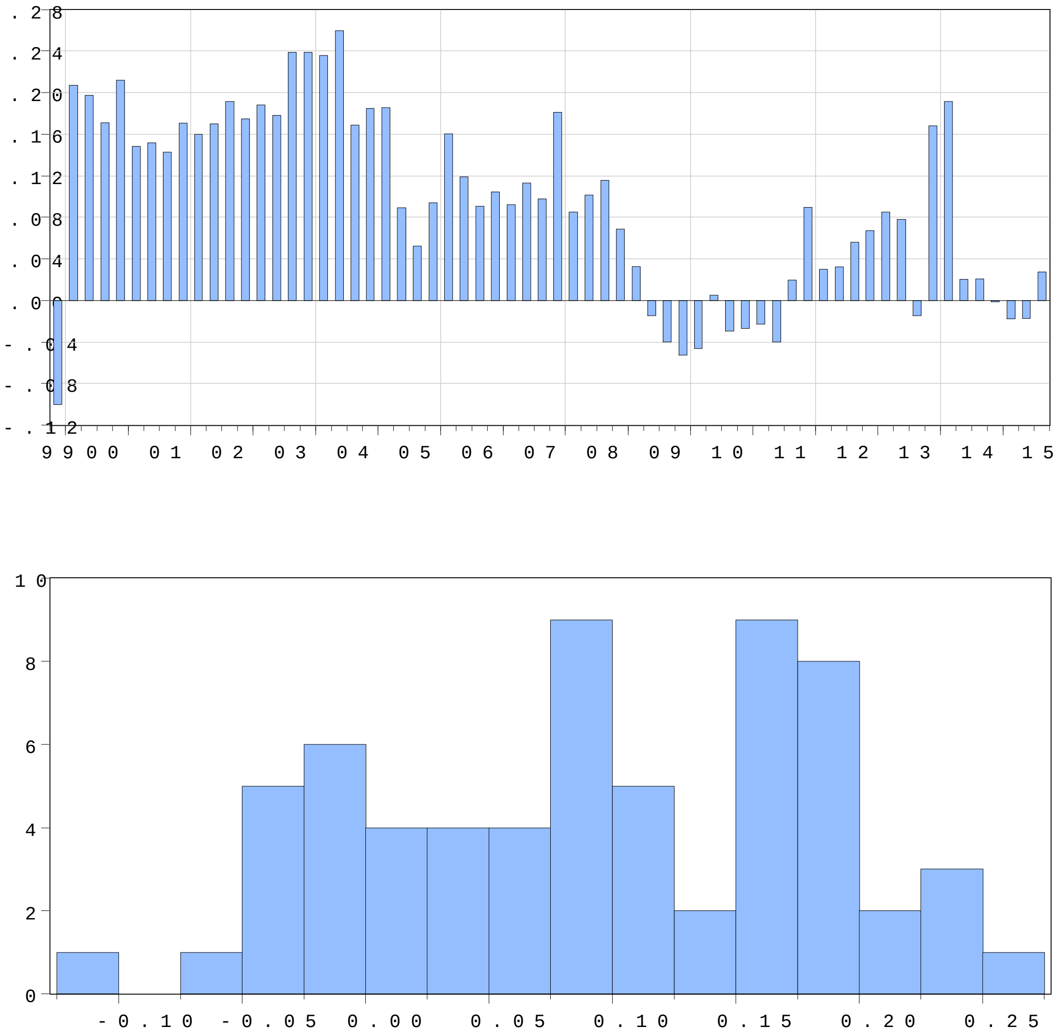


Figure 7: EA – Recursively Re-estimated Slope (κ) of the TVT-NKPC at Horizon of 1 Quarter (RULC RMC proxy, fixed rolling window of 113 quarters), plot (top graph) and histogram (bottom graph); 1999Q4–2015Q3, 64 observations; mean 0.094, median 0.093, max 0.260, min –0.100, SD 0.089

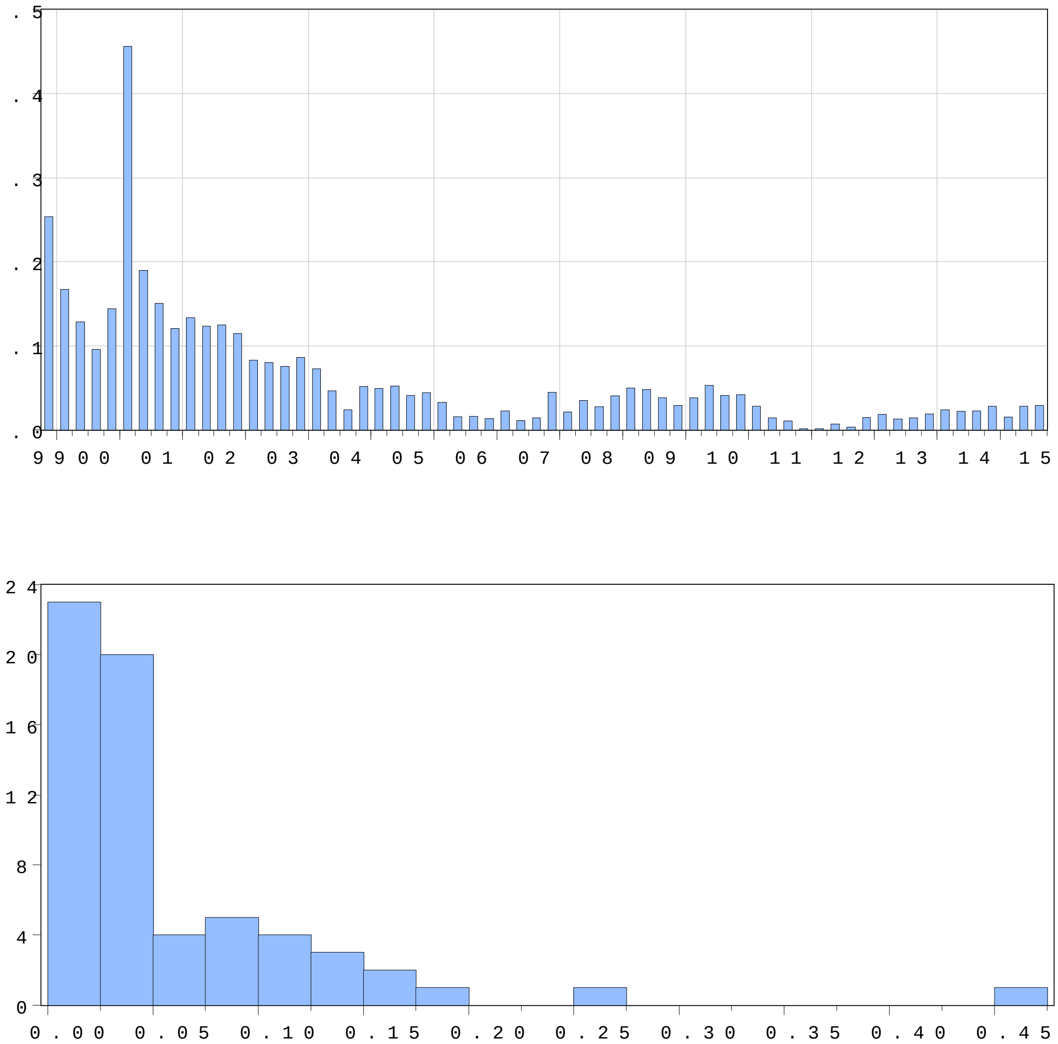
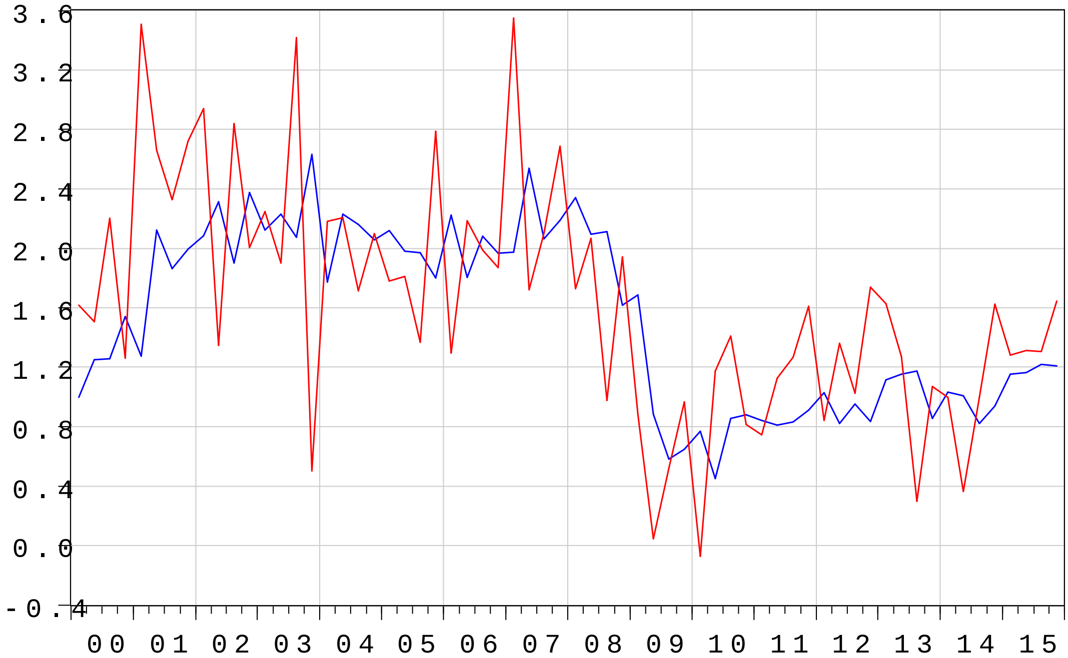


Figure 8: US – Recursively Re-estimated Slope (κ) of the TVT-NKPC at Horizon of 1 Quarter (RULC RMC proxy, fixed rolling window of 113 quarters), plot (top graph) and histogram (bottom graph); 1999Q4–2015Q3, 64 observations; mean 0.060, median 0.038, max 0.456, min 0.001, SD 0.072



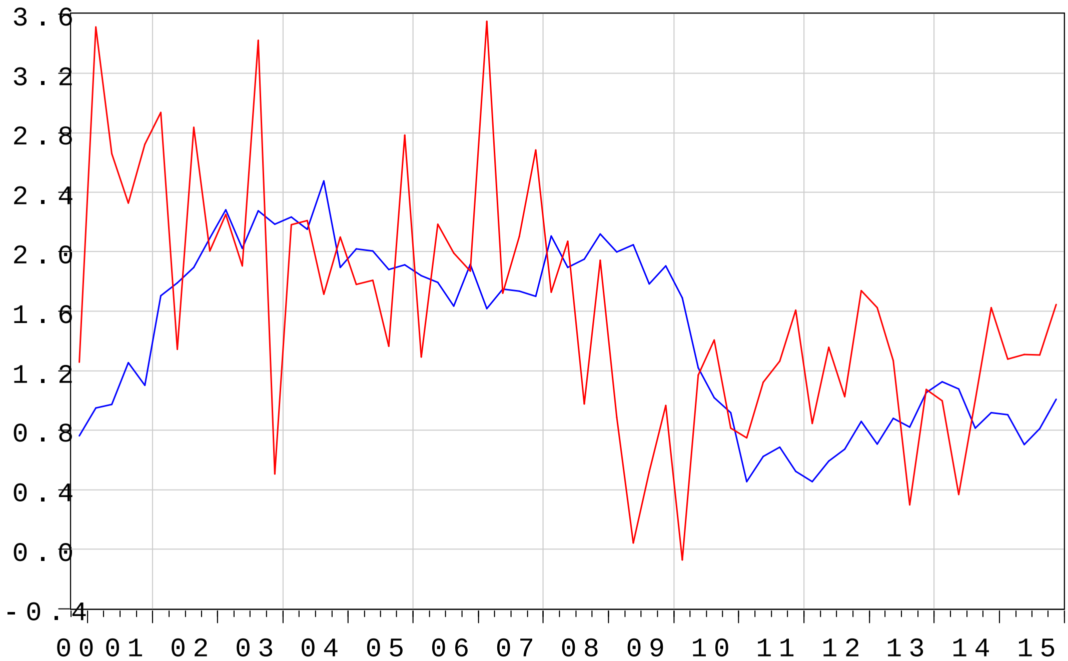
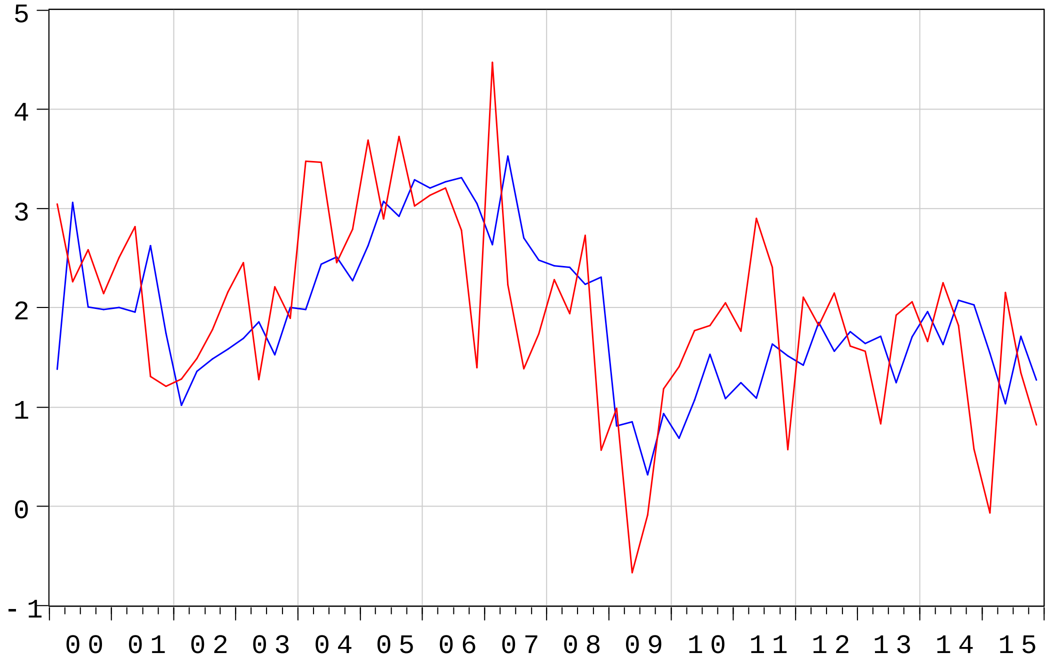


Figure 9: EA – Best TVT-NKPC forecasts (blue) against actual inflation (red) in the short run (as indicated by the bold fonts in Table 3 in the main text and Table 1 in this Appendix C): top graph – 1 quarter ahead; bottom graph – 4 quarters ahead; % per annum at annualized rate



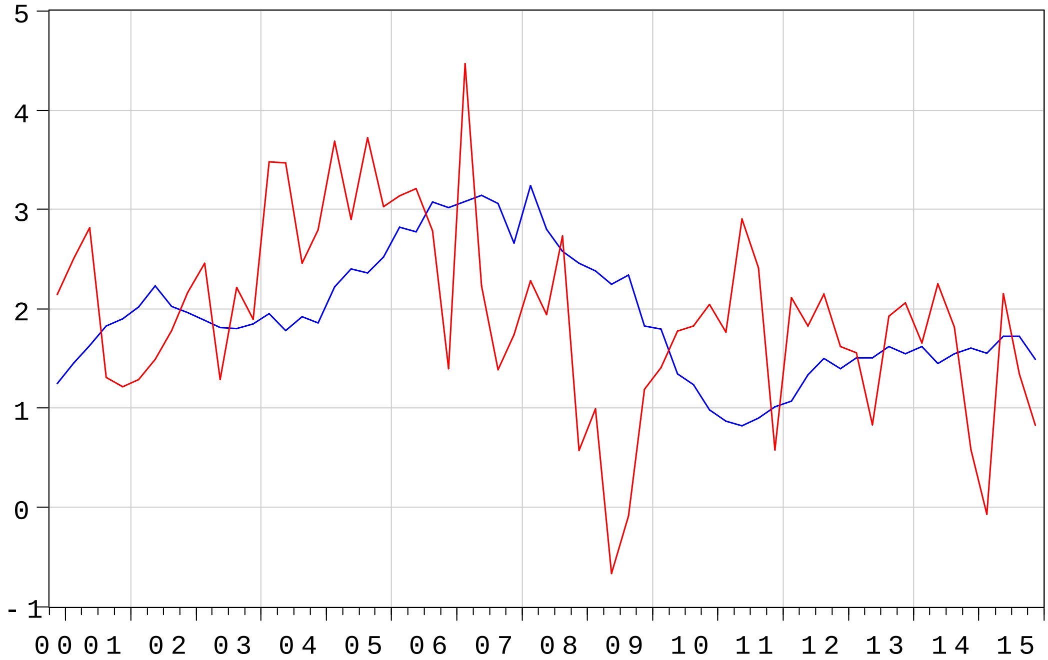
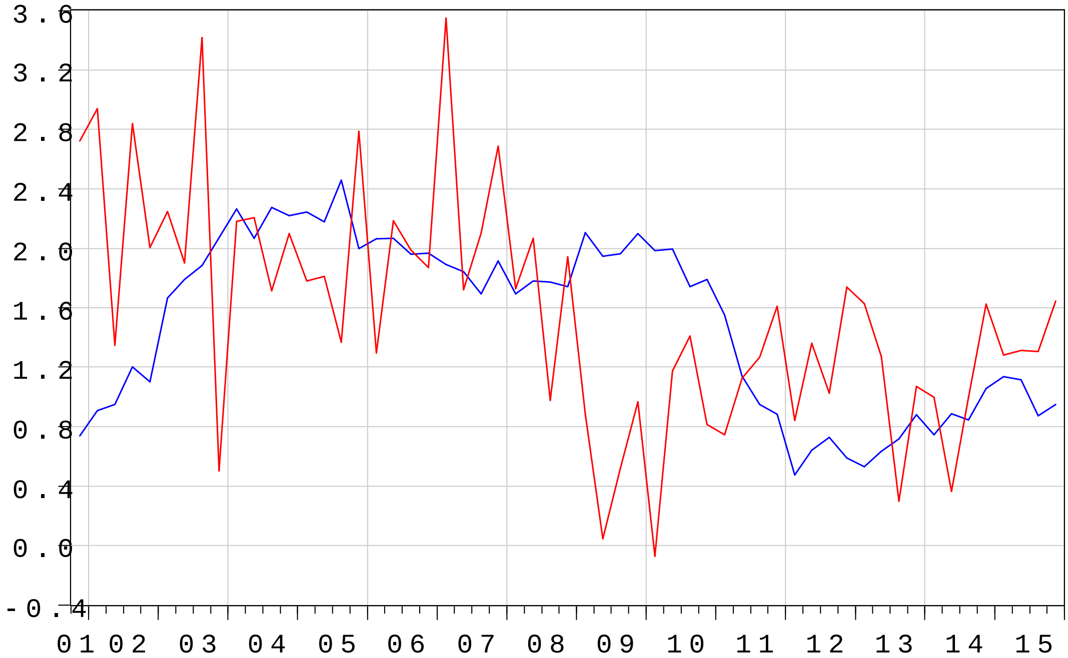


Figure 10: US – Best TVT-NKPC forecasts (blue) against actual inflation (red) in the short run (as indicated by the bold fonts in Table 4 in the main text and Table 2 in this Appendix C): top graph – 1 quarter ahead; bottom graph – 4 quarters ahead; % per annum at annualized rate



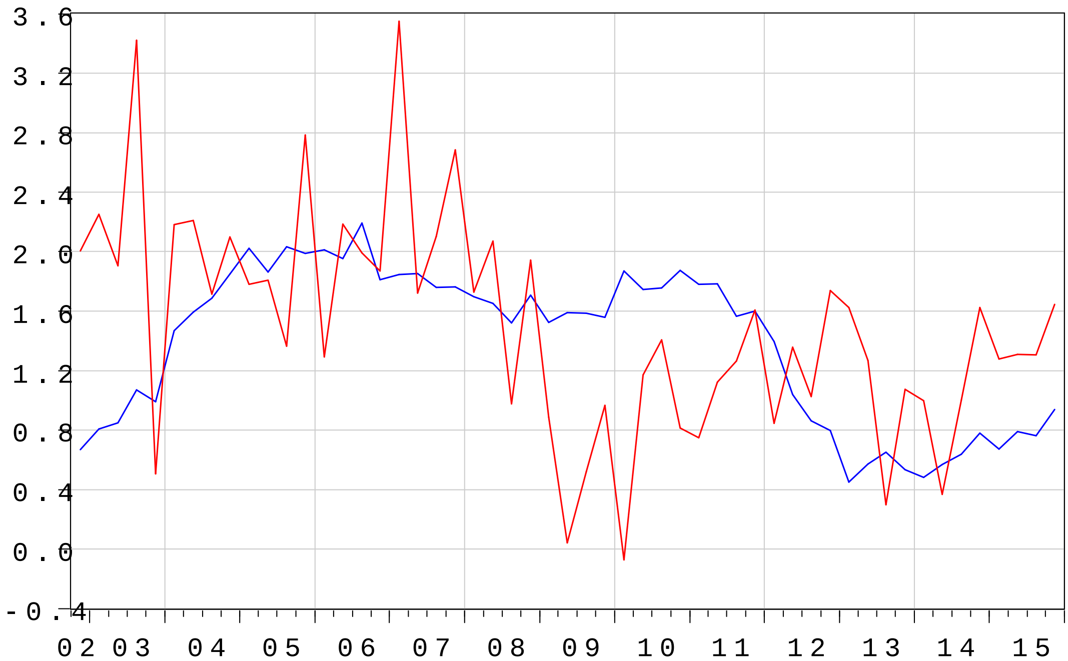


Figure 11: EA – Best TVT-NKPC forecasts (blue) against actual inflation (red) in the medium run (as indicated by the bold fonts in Table 3 in the main text and Table 1 in this Appendix C): top graph – 8 quarters ahead; bottom graph – 12 quarters ahead; % per annum at annualized rate

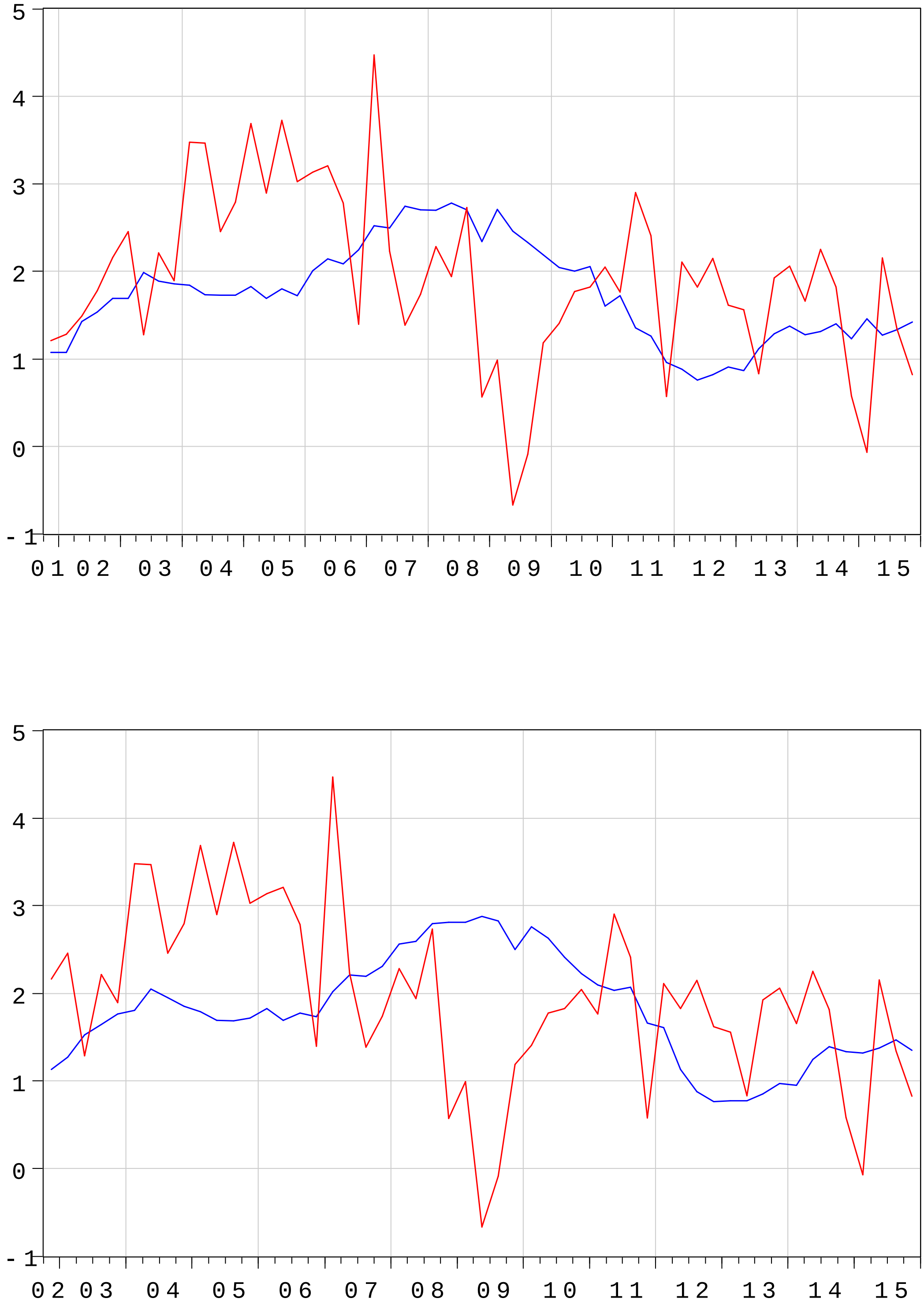
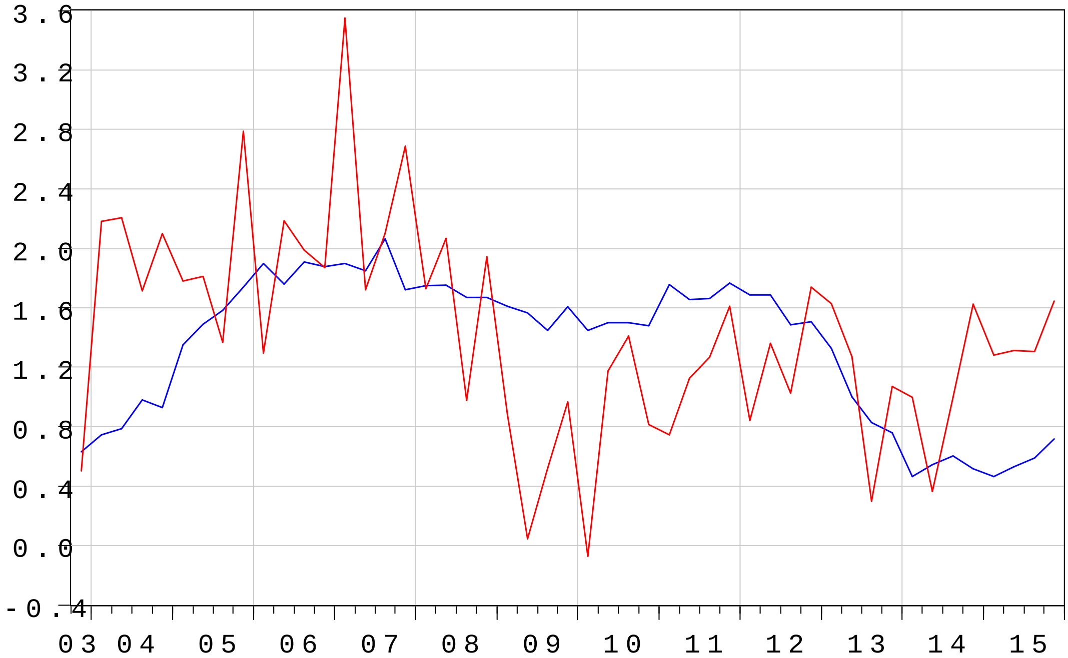


Figure 12: US – Best TVT-NKPC forecasts (blue) against actual inflation (red) in the medium run (as indicated by the bold fonts in Table 4 in the main text and Table 2 in this Appendix C): top graph – 8 quarters ahead; bottom graph – 12 quarters ahead; % per annum at annualized rate



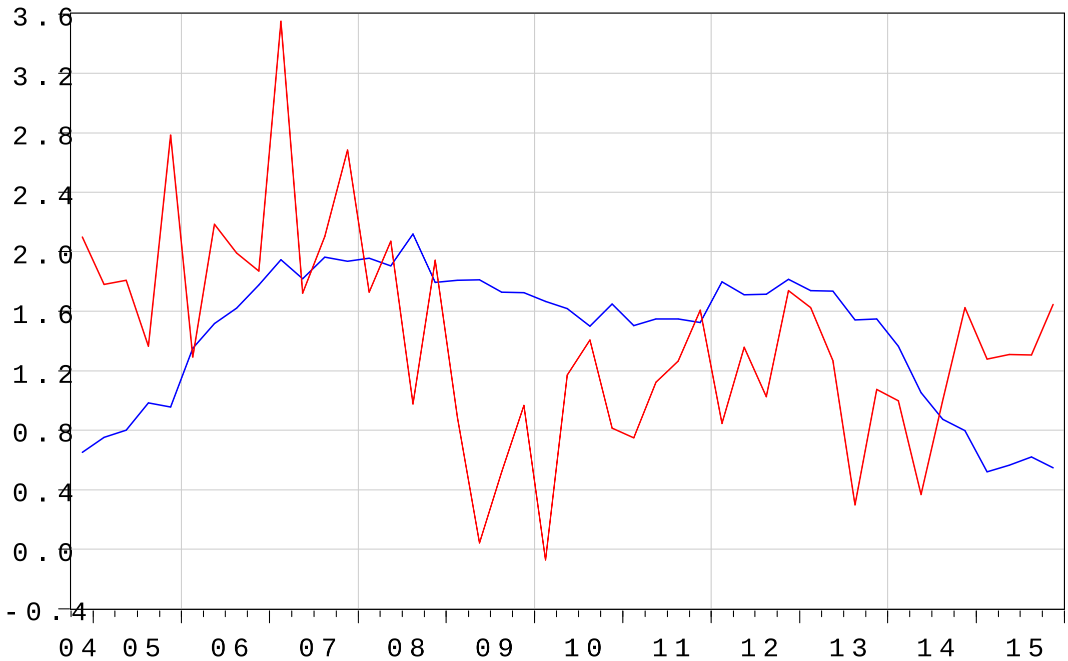
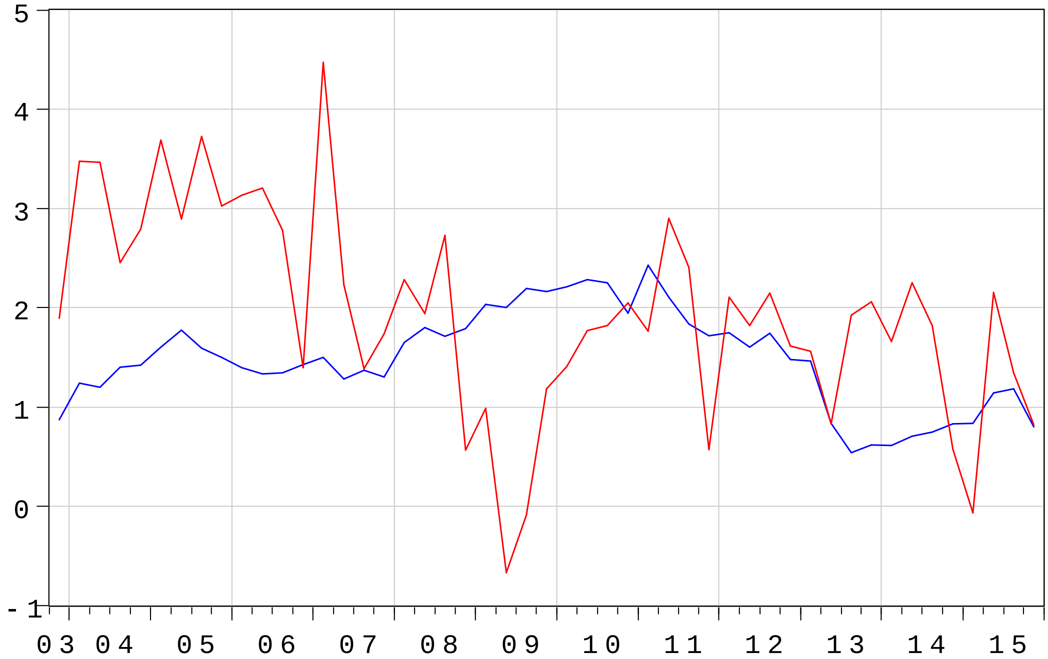


Figure 13: EA – Best TVT-NKPC forecasts (blue) against actual inflation (red) in the long run (as indicated by the bold fonts in Table 3 in the main text and Table 1 in this Appendix C): top graph – 16 quarters ahead; bottom graph – 20 quarters ahead; % per annum at annualized rate



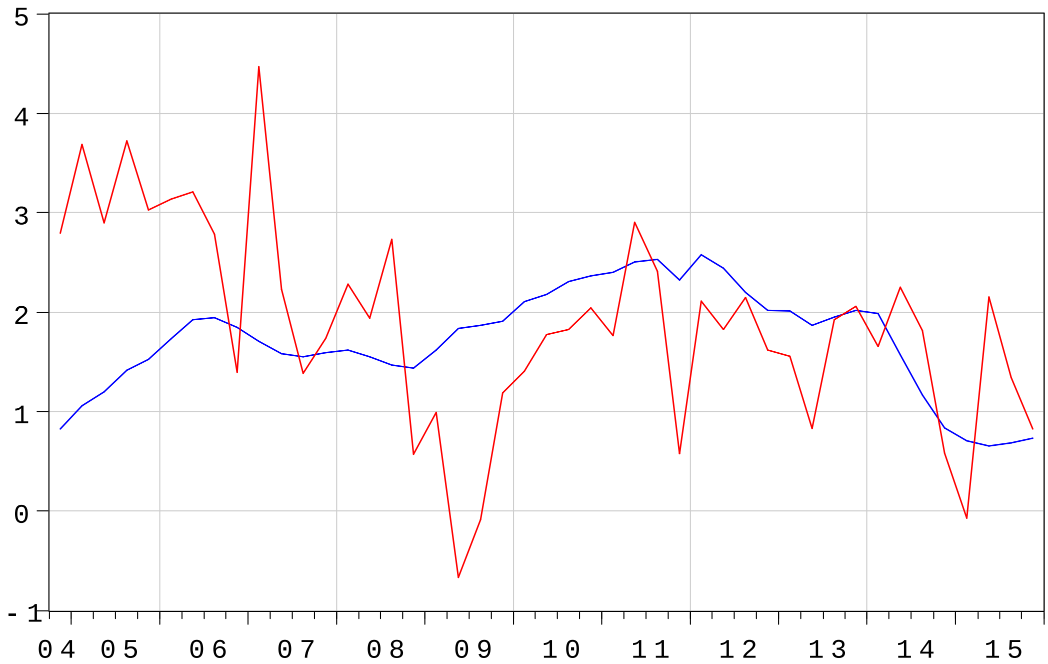


Figure 14: US – Best TVT-NKPC forecasts (blue) against actual inflation (red) in the long run (as indicated by the bold fonts in Table 4 in the main text and Table 2 in this Appendix C): top graph – 16 quarters ahead; bottom graph – 20 quarters ahead; % per annum at annualized rate