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A Collective Household Labor Supply Model with Children and Non-Participation: Theory and Empirical Application

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Abstract

We extend the collective model of household behavior to consider public consumption (expenditures on children), together with non-participation in the labor market. Identification of individual preferences and the sharing rule from observing each individual's labor supply and the total expenditure on the public good rests on the existence of a distribution factor and the existence and uniqueness of individual reservation wages at which both members are indifferent whether a member participates or not. Using a sample of Mexican nuclear families, collective rationality is not rejected. No evidence is found that empowering mothers is more beneficial for children than empowering fathers.

Keywords: Collective models; Labor supply; Non-participation; Public goods; Children; Intrahousehold decision-making; Reservation wages; Sharing rule

JEL codes: D11, D12, D13, J12, J13, J22,

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1 Introduction

The goal of many conditional cash transfer (CCT) programs, in which a household receives a monetary compensation in exchange for the fulfillment of certain requirements that are positively related to household welfare, is to foster the human capital of children. Some programs give the cash transfer to a particular household member, often the mother, instead of the intended beneficiaries, the children.[1] Therefore, the impact of the cash transfer on the expenditures assigned to the children depends on how the intrahousehold allocation processes distribute this additional income. Using data from the Mexican Progresa/Oportunidades program, Bobonis (2009) found that the larger female non-labor income has increased the share of expenditures on children's goods. Similar results are obtained, also in a collective framework, by Martinelli and Parker (2008), who try to isolate the substitution, income, and bargaining effects Progresa/Oportunidades school subsidies have had on the share of expenditures devoted to children's clothing. The collective approach, unlike the unitary one, provides an adequate theoretical background for analyzing intrahousehold allocations and permits the recovery of individual preferences and the decision process household members' aggregate behavior. It draws upon the idea that an increase in the decision power of one household member changes household behavior in his or her favor, even though total household resources are kept constant.

In this paper we analyze the household's decisions regarding investments in children along with both parents' labor decisions. As in other developing countries, Mexico's female labor force participation is still at a very low level (Arceo and Campos 2010). However, the low participation rate does not imply that women's preferences are not taken into account in household resource allocations. If (potential) wages affect bargaining positions within a household, then any variation in the wage of a female household member will modify household behavior even if she does not work. It could be that female "empowerment" would lead to women's non-participation in the labor market, but also that women put so much emphasis on spending on children that they decide to work more hours.

The collective labor supply model of Chiappori (1988, 1992) has been used extensively for empirical applications, but generally considers the simplest possible case of household structure, childless households with two working members, making it difficult to apply to the broader definition of households typically found in developing countries: a two-adult household with a non-working female partner and at least one child. There is little literature on collective household labor supply behavior that considers the presence of children (Blundell, Chiappori, and Meghir 2005; Cherchye, de Rock, and Vermeulen 2012; Sarmiento 2012) or the decision to participate in the labor market (Donni 2003; Blundell, Chiappori, Magnac, and Meghir 2007; Bloemen 2010), and to the best of our knowledge there is no literature that considers the two issues simultaneously. To properly assess the collective framework as a useful tool for welfare evaluation and policy analysis on an intrahousehold level, it is necessary not to limit the analysis to childless households with members who participate in the labor market.

[1] Examples of CCT programs that gives the transfer to the mother are: Bono de Desarrollo Humano (Ecuador), Chile Solidario (Chile), Familias en Acción (Colombia), Progresa/Oportunidades, nowadays known as Prospera (Mexico).

The objective of this paper is to develop a theoretical collective framework that simultaneously takes into account the presence of children and the decision to participate in the labor market. We generalize Chiappori's (1992) model, employing the method of Donni (2003) to address the possibility of non-participation and introduce it in the scenario of Blundell, Chiappori, and Meghir (2005) that takes into account the presence of children in a household. The proposed model generalizes the identification results of Chiappori (1992); recovery of individual preferences and the sharing rule from observed behavior requires the knowledge of a distribution factor and the existence of a unique reservation wage for each adult household member at which both members are indifferent as to whether a member participates in the labor market or not.

The model relies on empirically testable restrictions on household labor supply to obtain information about aspects of the intrahousehold decision process that can be used for individual welfare analysis and policy evaluation. In an empirical application, the rule governing the sharing of household resources conditional on the level of expenditures on children is recovered from estimates of a system of equations comprising the woman's participation, the couple's labor supplies, and expenditures on children. We use data from the Mexican Family Life Survey (MxFLS); information on nuclear families in which the male partner works are used to estimate the model and test its implied restrictions. Despite rejection of the auxiliary assumption of continuity of both the male's labor supply and the sharing rule, the parameter restrictions that are imposed by the collective rationality are not rejected.

The paper is organized as follows. Section 2 briefly reviews the literature on collective household labor supply models, and especially those that either includes public consumption (such as expenses on children) or the possibility of non-participation in the labor market. Section 3 presents our theoretical model that integrates the participation decision and public goods into one framework. Section 4 proposes a parametric specification that will be used for an empirical implementation of the model. Section 5 reviews the data set that is used, and section 6 presents the empirical results. Section 7 concludes.

2 Intrahousehold Decisions, Labor Force Participation and Public Goods

The traditional unitary approach considers a household as a single decision-making unit, leaving unexplained how the household reaches its agreement to allocate resources. Its lack of a distinction between individual and household preferences is unsatisfactory from the perspective of welfare analysis, and implies that price changes are the only tool available for intrahousehold reallocations (Quisumbing and McClafferty 2006). Moreover, it lacks empirical support for its theoretical implications, such as the consideration of total income but not its source for household consumption decisions (the income pooling hypothesis),[2] and the assumption of symmetric

[2] Thomas (1990); Bourguignon, Browning, Chiappori, and Lechene (1993); Browning, Bourguignon, Chiappori, and Lechene (1994); Lundberg, Pollak, and Wales (1997); Fortin and Lacroix (1997), and for Mexico, Attanasio and Lechene (2002), among others.

Slutsky matrix of cross-price substitution effects (e.g., the compensated wage changes of spouses have the same effect on each other's labor supply).[3]

Alternative approaches, such as non-cooperative and cooperative (or collective) models, have tried to take into account the multiplicity and heterogeneity of decision makers in a household. On the one hand, in the absence of binding and enforceable agreements between household members, non-cooperative models have assumed that household members maximize their utility subject to an individual budget constraint, taking as given each other's behavior. The intrahousehold allocations under this framework are not necessarily Pareto efficient. In a household context this result is not very satisfactory, since possibilities for Pareto improvements may arise from daily interaction among their members.[4] On the other hand, the only assumption that household collective models have in common is that household decisions are Pareto efficient, so it is not necessary to specify the actual process that determines the intrahousehold allocation on the efficiency frontier, only to assume that it exists.

Efficiency means that no other consumption bundle could provide more utility for household members at the same cost. An equivalent interpretation of Pareto efficiency is that household members initially reach an agreement on the respective amount each is allowed to spend, a sharing rule. Then, all members independently choose their consumption, subject to their respective share. The approach does not impose a particular form on the rule; it only requires that it exists.

A continuum of different structural models can generate the same observable behavior (Chiappori and Ekeland 2009). Particular hypotheses over goods or preferences have been made within the collective framework to recover preferences and decision making from household aggregate demand. The main results have been obtained for the case where all goods consumed in a household are private (i.e., they are consumed non-jointly and exclusively by each member); where one member's consumption does not have a direct effect on another member's wellbeing; and at an interior solution for household demands. Intuitively, the quantities consumed by each member are a guide to the intrahousehold bargaining power distribution: the consumption of a good associated with a particular individual will be greater as his or her decision power increases.

Regarding the case of labor supply, the seminal collective model proposed by Chiappori (1988; 1992) allows, under certain assumptions, the recovery of some elements of the decision process from the observed labor supplies of household members. Since these results are derived for the simplest possible case, applications of this model are based on childless households composed of two adult members who participate in the labor market. Estimates obtained from this type of sample could be imprecise due to small sample size and may be subject to selection biases (Fortin and Lacroix 1997).

[3] Browning and Meghir (1991); Blundell, Pashardes, and Weber (1993); Fortin and Lacroix (1997); and Browning and Chiappori (1998), among others.

[4] The Pareto efficiency assumption can be justified if all household members are aware of the preferences and actions of the others, so they can decide to cooperate to make everyone better off by means of a binding agreement. Alternatively, this agreement can emerge if the relations between household members can be represented as a repeated game. For a more detailed discussion about assuming efficiency see Browning, Chiappori, and Weiss (2014).

When labor force participation and public goods are to be considered under a collective framework, there are certain aspects to take into account. First, the non-participation decision in the labor market may have an influence on outcomes even for individuals who are not directly affected by this decision. If a member's threat point involves participation in the labor market (e.g., because a woman's or man's participation involves credible outside options), (potential) wages could affect bargaining positions within a household. This result is the opposite of the one obtained within the unitary model, where only wages of working members matter, due to their effect on budget opportunities. Second, children are likely to be an important source of preference interdependence between parents, since it is reasonable to think that both parents could derive utility from their children's well-being (although not necessary to the same degree). Furthermore, the presence of children could generate non-separabilities in parents' commodity demand and labor supply (child care, say, may affect the tradeoff between consumption and labor force participation and hours of work at the individual level).

Advances have been made to include the possibilities of non-participation and of public consumption (i.e., goods from which both spouses derive utility, such as the amount spent on children, consumed jointly and not exclusively by each member) in the collective model, but along separate lines.

Donni (2003) and Blundell, Chiappori, Magnac, and Meghir (2007) have constructed theoretical frameworks to consider non-participation in the labor market. Donni's work, in which both members can freely choose their working hours, extends the results of Chiappori (1988; 1992) to take into account the case in which one of the two members does not work. An empirical application of Donni's framework has been made by Bloemen (2010) for the Netherlands. The work of Blundell et al. considers one discrete and another non-negative continuous labor supply. Donni (2007) develops a model similar to that of Blundell et al., fixing the male household member's labor supply at full-time instead of allowing a choice between working full-time or not at all. Structural elements of the decision process can be identified from Donni's model if the female household member's labor supply is observed together with at least one household commodity demand.

In the unitary model, the participation decision is included by means of a reservation wage at which an agent is indifferent between working and not working. Translating this concept to the collective framework, the central assumption of Pareto efficiency of the household decision process requires that if one member (say, the wife) is indifferent between working and not working, the other one (say, the husband) must be indifferent as well about the participation decision of the first member; Blundell, Chiappori, Magnac, and Meghir (2007) have called this condition the "double indifference" assumption.[5] Therefore, the participation decision in the

[5] Blundell et al. used the following example. Assume that at a wage infinitesimally below the reservation wage of a husband, he is indifferent between working and not working but that his wife experiences a strict loss if he is not working. Now suppose that at the reservation wage he decides to work and he receives ε more to spend on his private consumption than initially agreed. He is better off since he is indifferent between participating or not and his consumption increases (if the goods consumed are normal). If ε is small enough, the wife is better off too, since the participation of her spouse compensates her more than the reduction in her private consumption.

models of Donni (2003) and Blundell et al. (2007) relies on explicitly postulating a reservation wage; individual preferences and the sharing rule can be recovered for both models.

On the other hand, Blundell, Chiappori, and Meghir (2005, hereafter BCM) introduce children into Chiappori's (1988; 1992) model, assuming that both parents care about their children's welfare, or equivalently, considering that the expenditure on their children is a public good for them. In general, the decision process cannot be recovered; a continuum of different structural models can generate the reduced form of each individual's labor supply and expenditure on children. This result is due to the fact that the level of public consumption influences the analysis of labor supply not only through an income effect but also through its impact on the individual consumption/leisure trade-off. Under this approach, identifiability of the intrahousehold decision-making process can be obtained in two cases: first, when private consumption is separable from expenditures on children, so that the consumption/leisure trade-off effect disappears; or second, by introducing a distribution factor, that is, a variable that affects the decision process but not the individual preferences or the joint budget set. Empirical applications of this model are found in Cherchye, de Rock, and Vermeulen (2012) for the Netherlands and also in Sarmiento (2012) using Mexican data.

3 The Framework

Our model incorporates the decision to participate in the labor market into BCM (2005)'s framework of household labor supply with expenditures on children (considered as a public good) extending along the lines set out by Donni (2003), and simultaneously takes into account the possibility that (potential) wages affect the bargaining positions of household members, that the utility of each adult member depends on their children's wellbeing, and that individual consumption and labor supply decisions are not separable from the expenditure on children.

Subsection 3.1 presents the main assumptions of the model. Besides the assumptions of individualism and Pareto-efficiency common to the collective approach, the model assumes that both adult household members care about their own consumption (they have egoistic preferences), but also about their children. Subsection 3.2 shows that, as in the case considering only private consumption, the decision-making process can be represented as operating in two phases by the existence of a sharing rule conditional on the residual non-labor income after the expenditures on the public good. Subsection 3.3 shows how the model determines the level of expenditures on children. Here, the framework also addresses the effect of intrahousehold redistribution of power (e.g., a given policy that "empowers' a specific member of the household, such as the mother) regarding household expenditures on children. Subsection 3.4 introduces additional assumptions to guarantee the existence of a unique reservation wage for each partner that is consistent with the Pareto-efficiency assumption, employing the method used by Donni (2003). Finally, subsection 3.5 discusses the identification of the model and the corresponding restrictions on household labor supply. Given a set of (potential) wages, non-labor income, and a distribution factor, the framework can recover individual preferences and the conditional sharing rule if one or both partners work.

3.1 Commodities, Preferences, and the Decision Process

The model, following BCM, considers the case of an adult couple in a single time period. Labor supply of i, $i = m, f$, is denoted by h^i, with market wage equal to w^i. Total time endowment is normalized to one and domestic production is not considered.[6] A Hicksian composite good C is consumed by the household. This good is used for private (C^m, C^f) and public (K) consumption, with prices set to one. In a very general sense, the notion of public consumption should be understood as any expenditure that increases the utility of both partners, such as expenditures on heating, electricity, housecleaning, among others. A typical example of K is the amount spent on children by the household. Non-labor income is denoted by Y.

Each spouse's utility can be written as:

$$U^i = U^i(1-h^i, C^i, K), \quad i = m, f$$

where U^i is strongly quasi-concave, infinitely differentiable, and strictly increasing in all its arguments. Moreover, $\lim_{h^i \to 1} \partial U^i / \partial h^i = \lim_{C^i \to 0} \partial U^i / \partial C^i = \lim_{K \to 0} \partial U^i / \partial K = \infty$ for $i = m, f$, conditions that rule out cases where leisure, and individual and public consumption are equal to zero.

Household decisions are assumed to generate Pareto-efficient outcomes, whatever the mechanism used to reach this agreement. Therefore, there is a function λ such that the household allocation $(h^{m^*}, h^{f^*}, C^{m^*}, C^{f^*}, K^*)$ is the solution to the program:

$$\max_{h^{m^*}, h^{f^*}, C^{m^*}, C^{f^*}, K} \lambda U^m(1-h^m, C^m, K) + (1-\lambda) U^f(1-h^f, C^f, K) \tag{1}$$

$$s.t. \begin{cases} C^m + C^f + K = w^m h^m + w^f h^f + Y \\ 0 \le h^i \le 1, \quad i = m, f \end{cases}$$

The Pareto weight $\lambda \in [0,1]$ reflects the relative power of m in the household and $(1-\lambda)$ that of f; a larger λ corresponds to a larger weight of m's preferences in the household allocation problem, favoring the outcomes enjoyed by m (and likewise a smaller λ favors the outcomes of f). It is assumed that $\lambda = \lambda(w^m, w^f, Y, z)$ is a continuously differentiable function of wages, non-labor income, and at least one distribution factor z.

The bundle (w^f, w^m, Y, z) is assumed to vary within a compact subset $\mathcal{K}$ of $\mathbb{R}^3_+ \times \mathbb{R}$. Moreover, h^m, h^f, C, and K are observed (as functions of w^m, w^f, Y and z), whereas the individual consumptions C^m and C^f are unobserved. In general, household surveys do not collect information about intrahousehold allocation of expenditures but about aggregate consumption C.

[6] The model implicitly assumes that all non-market time corresponds to leisure; it does not consider the division of labor between household and market production. Extensions by Apps and Rees (1997), Chiappori (1997), Donni (2008), and Donni and Matteazzi (2016) with domestic production allow for non-participation, but do not consider children.

Third, it is assumed that both partners' wages are always observed, even when a partner does not participate in the labor market (we come back to that in subsection 4.3.1).

3.2 The Conditional Sharing Rule

The solution to the household program (1) can be thought of as a two-stage process in which the couple first agrees on the level of the public expenditure and how to distribute the resulting residual non-labor income between them. Next, conditional on the outcome of the first stage, each member decides, independently of each other, their individual consumption and labor supply.

Formally, let $h^{i^*}(w^m, w^f, Y, z)$, $C^{i^*}(w^m, w^f, Y, z)$, for $i = m, f$, and $K^*(w^m, w^f, Y, z)$ be the solution of program (1); then a function ϕ^i exists such that:

$$C^{i^*}(w^m, w^f, Y, z) = \phi^i(w^m, w^f, Y, z) + w^i h^{i^*}(w^m, w^f, Y, z), \quad i = m, f \ ,$$

where ϕ^m and ϕ^f characterize the *conditional sharing rule*, the portion of non-labor income allocated to each member once spending on the public good has been discounted:

$$\phi^m(w^m, w^f, Y, z) + \phi^f(w^m, w^f, Y, z) = Y - K^*(w^m, w^f, Y, z) \, .$$

Note that ϕ^i can be positive or negative; they could agree to spend beyond their non-labor income on the public good, and transfers between the two are also possible.

Fixing $\bar{K} = K^*(w^m, w^f, Y, z)$, the second stage of the household program (1), can be represented as:

$$max_{h^i, C^i} U^i(1-h^i, C^i, \bar{K}) \quad s.t. \quad C^i = w^i h^i + \phi^i, \quad i = m, f \tag{2}$$

with $h^{i^*}(w^m, w^f, Y, z)$ and $C^{i^*}(w^m, w^f, Y, z)$ as interior solutions to the individual problem. The structure of both partners' labor supplies can be described by:

$$h^{m^*}(w^m, w^f, Y, z) = H^m[w^m, \phi(w^m, w^f, Y, z)]$$

$$h^{f^*}(w^m, w^f, Y, z) = H^f[w^f, Y - K - \phi(w^m, w^f, Y, z)]$$

where $\phi = \phi^m$, and when ϕ is fixed, H^m and H^f are Marshallian labor supply functions. With the idea of expressing labor supplies in terms of public expenditures ($\bar{K}$) and maintaining the assumption that $\bar{K}$ is fixed, the following process is used. Let $\mathcal{O}$ be some open subset of $\mathcal{K}$ such that $\partial K / \partial z$ does not vanish on $\mathcal{O}$. The condition $K^*(w^m, w^f, Y, z) = \bar{K}$ is used to express z as a function ζ of $(w^m, w^f, Y, \bar{K})$ by the implicit function theorem. Following from this construction, the couple's labor supplies are:

$$\tilde{h}^f(w^m, w^f, Y, \bar{K}) = H^f[w^f, Y - K - \phi(w^m, w^f, Y, \zeta(w^m, w^f, Y, \bar{K}))] \tag{3}$$

$$\tilde{h}^m(w^m, w^f, Y, \bar{K}) = H^m[w^m, \rho(w^m, w^f, Y, \phi(w^m, w^f, Y, \bar{K}))] \tag{4}$$

In this way, i's labor supply is described as a function of wages, non-labor income, and a distribution factor z such that public expenditures are exactly $\bar{K}$. Hence, the values of w^m, w^f, and Y are not constrained to assure that $K^*(w^m, w^f, Y, z) = \bar{K}$; the key role of z is to guarantee that the level of public expenditure is exactly $\bar{K}$. This structure generates testable restrictions because the same function $\phi(w^m, w^f, Y, z)$ enters each member's labor supply (see footnote 8).

3.3 The Determination of Public Expenditures

The Bowen-Lindahl-Samuelson condition characterizes efficiency for public good expenditures. Formally, the first-order conditions for household program (1), with an interior solution for individual and public consumption, give:

$$\frac{\partial U^m / \partial K}{\partial U^m / \partial C} + \frac{\partial U^f / \partial K}{\partial U^f / \partial C} = 1 .$$

Equivalently, this condition can be expressed in terms of individual indirect utilities. First, let $V^i(w^i, \phi^i, K)$ denote the value of the second stage of the household program (2) for member i, that is, the maximum utility that i can achieve given his or her wage and conditional on the outcomes (ϕ^i, K) of the first stage decision. Next, returning to the first stage, efficiency leads to the following program:

$$\max_{\phi^m, \phi^f, K} \lambda V^m(w^m, \phi^m, K) + (1-\lambda) V^f(w^f, \phi^f, K) \quad s.t. \quad \phi^m + \phi^f + K = Y .$$

The first order conditions give:

$$\lambda \frac{\partial V^m}{\partial \phi^m} = (1-\lambda) \frac{\partial V^f}{\partial \phi^f} = \lambda \frac{\partial V^m}{\partial K} + (1-\lambda) \frac{\partial V^f}{\partial K} ,$$

and therefore:

$$\frac{\partial V^m / \partial K}{\partial V^m / \partial \phi^m} + \frac{\partial V^f / \partial K}{\partial V^f / \partial \phi^f} = 1 \qquad (5)$$

The ratio $\frac{\partial V^i / \partial K}{\partial V^i / \partial \phi^i}$ is i's marginal willingness to pay (MWP) for the public good. Thus, condition (5) states that individual MWPs must add up to the market price of expenditures on children. From BCM's Proposition 1 it follows that if i's preferences are such that both public and private consumption increase with non-labor income (i.e., K and ϕ^i are normal "goods", so i's MWP is decreasing in K and increasing in ϕ^i), a marginal increase in m's power will increase the household's expenditures on children if and only if m's MWP is more sensitive to changes in his income share than that of f, and vice versa. Because a positive transfer from one member to the other decreases the transferer's MWP for the public good and increases the MWP of the one who receives the transfer, this proposition establishes when the positive effect on the receiver is sufficient to compensate the reduction to the transferer. Hence, the key property for analyzing changes in the distribution of power within a household is not the magnitude of the MWPs (say, who cares more for children), but how the MWPs respond to changes in individual resources for private consumption.

Intuitively, empowering one household partner (say, the woman) comes with a higher fraction of household non-labor income for her. If both private and public goods are normal, she will consume more of all commodities, and, conversely, the male partner will see his share and consumption reduced. The reduction in household expenditures on the public good that comes from the male's share will be more than compensated by the increase of the female's share when the female partner is more sensitive to changes in her share than her partner, that is, when she is willing to spend a larger fraction than her partner on children of the additional monetary unit that comes via her empowerment.

3.4 The Participation Decision

Our aim is to extend the framework of BCM, which includes public goods but assumes interior solutions, with the labor force participation decision, thereby allowing for non-participation. The unitary framework's reservation wage is generalized to a collective model with two adult members such that at the reservation wage of one household member, not only that member is indifferent between working and not working, but also that the other member is indifferent (Blundell, Chiappori, Magnac, and Meghir 2007).

To characterize the participation decision of a household member, a procedure similar to the one used by Neary and Roberts (1980) is employed to model household behavior under rationing, which is characterized in terms of its unconstrained behavior when faced with shadow prices. Our logic follows the steps set out in a collective framework with non-participation and income taxes but without public goods by Donni (2003).

The reservation wage of i, ϖ^i, is defined by

$$\varpi^i = \frac{U^i_{h^i}(1, \phi^i, \bar{K})}{U^i_{C^i}(1, \phi^i, \bar{K})},$$

where U^i_x stands for the partial derivative of function U^i with respect to variable $x = h^i, C^i$. This equation is the marginal rate of substitution between leisure and private consumption computed along the axis $h^i = 0$ for a given sharing rule ϕ^i (and equal to C^i) and a level of public expenditures equal to $\bar{K}$.

By fixing public expenditures at some arbitrary level $\bar{K}$, problem (1) is basically reduced to that considered by Donni (2003), in which the participation decision is analyzed in a framework with only private goods. As above, let $\mathcal{O}$ be some open subset of $\mathcal{K}$ such that $\partial K / \partial z$ does not vanish on $\mathcal{O}$, and impose the condition $K^*(w^m, w^f, Y, z) = \bar{K}$, where z, by the implicit function theorem, is equivalent to $z = \zeta(w^m, w^f, Y, \bar{K})$. Let $y = Y - \bar{K}$ denote the portion of non-labor income not devoted to public expenditures, which could be positive or negative (labor income can also be used for public consumption). Therefore, i's reservation wage ϖ^i is implicitly defined as a function of (w^m, w^f, y):

$$w^i = \varpi^i(w^m, w^f, Y, \zeta(w^m, w^f, Y, \bar{K})) = \varpi^i(w^m, w^f, Y, \bar{K}) = \varpi^i(w^m, w^f, y). \quad (6)$$

Without additional assumptions, equation (6) could have several solutions, i.e., the uniqueness of a reservation wage for member i has to be explicitly postulated. Intuitively, there are two reasons that explain why there can be many wage rates for which i is indifferent between working and not working. The first comes from the assumption that the sharing rule ϕ^i depends on i's wage, so there could be more than one combination of w^i and ϕ^i at which i is indifferent. The second is related to the possibility that the sharing rule itself may depend on the non-participation of household members.

A sufficient condition to obtain a unique reservation wage (fixed point) for each member is to define that the function ϖ^i is a contraction mapping (cf. Donni, 2003):

Assumption R. *For any* (w^{m^*}, w^{f^*}, y) *and* $(w^{m^\circ}, w^{f^\circ}, y) \in \mathbb{R}^2_+ \times \mathbb{R}$, *preferences and the sharing rule are such that there is some non-negative real number* $r < 1$ *for which the following condition is satisfied:*

$$\max_{i=m,f}[|\varpi^i(w^{m^*}, w^{f^*}, y) - \varpi^i(w^{m^\circ}, w^{f^\circ}, y)|] \leq r \max_{i=m,f}(|w^{i^*} - w^{i^\circ}|).$$

This condition does not affect the level of public expenditure; z varies to guarantee that public expenditure is exactly $\bar{K}$. Consequently, the distribution factor allows that w^m, w^f, and Y — and thus also ϖ^i — can vary freely, whereas K is kept constant. Moreover, the assumption only applies in the neighborhood of the participation frontier; in the interior of other household participation sets the allocation of additional income stemming from the participation of one member could be more complex.

In essence, Assumption R restricts the impact on both individual shares (and hence individual consumption) of a change in one household member's wage. This amounts to assuming that the Pareto weights are smooth functions of both wages and non-labor income, and therefore that the smoothness of the individual utilities is preserved at the participation frontier of each individual.[7] Assumption R is not expected to be very restrictive and it simplifies the analysis by not having to use more restrictive fixed point theorems to ensure the existence of a well-behaved participation frontier.

Under this assumption, the system of equations ϖ^m and ϖ^f is a contraction with respect to w^m and w^f for any y. Using the Banach contraction principle (Green and Heller 1981), two corollaries are:

[7] To understand better the intuition behind Assumption R, we analyze the effect on *m*'s private consumption at *m*'s participation frontier for infinitesimal increases in each one of the wages. First, when *m*'s wage increases, the increase in *m*'s private consumption depends on his participation. When *m* does not participate, the wage increase has a positive impact on his bargaining power, and his reservation wage and consumption share increase. When *m* participates, the increase in his wage also has a positive effect on household income, and *m*'s consumption share increases more. Second, when *f*'s wage increases, the effect on *m*'s private consumption depends also on *f*'s participation. When *f* does not participate, the increase in her wage reduces *m*'s bargaining power, reducing his share. If leisure is a normal good, the decrease of *m*'s share is associated with a reduction in *m*'s reservation wage. When *f* does participate, an increase in her wage also has a positive effect on household income, which may compensate *m*'s share for the increase in *f*'s bargaining power. Then, the condition that the difference in *m*'s reservation wage cannot be greater in absolute value than the initial increase in *m* (*f*)'s wage is satisfied when *m*'s consumption share responds less, in absolute value, to changes in *m* (*f*)'s wage when *m* (*f*) is not participating than when *m* (*f*) is participating.

1. For any y, the functions ϖ^m and ϖ^f have a unique fixed point. Then, there exists a unique pair of wages, $\hat{w}^m(y)$ and $\hat{w}^f(y)$, such that both adult members are indifferent between working and not working.
2. For any $w^j (j \neq i)$ and y, each ϖ^i has a unique fixed point with respect to w^i. Then, there exists a function $\gamma^i(w^j, y)$ such that member i participates in the labor market if and only if $w^i > \gamma^i(w^j, y)$, $i = m, f$.

3.5 Identification

This section discusses the empirical restrictions on each household member's labor supply implied by the collective setting with children and non-participation, and shows that it is possible to recover the structural model (preferences and the sharing rule) by observing the labor supplies and the household expenditure on children.

Considering the possible combinations of household members' participation decisions, four sets can be defined. First, the set of (w^m, w^f, y) for which both household members choose to work defines the Participation set P. Second, f's non-participation set, N^f, is formed by the combinations of (w^m, w^f, y) for which f chooses not to work and m chooses to work. Similarly, third, the combinations for which m chooses not to work and f chooses to work define m's non-participation set, N^m. Finally, the non-participation set N, consists of (w^m, w^f, y) such that both household members choose not to work; this set is not taken into account in identifying individual utilities and the decision process given the lack of information for this purpose – if the hours of work for both partners are zero, the sharing rule within the household cannot be deduced, so individual utilities cannot be recovered.

Therefore, it is assumed that at least one of the partners' labor supplies is an interior solution to (1). The following theorem establishes the identification and testability results.

Theorem 1. *Let $(\tilde{h}^m, \tilde{h}^f)$ be a pair of labor supplies, satisfying the regularity conditions listed in Lemmas 1-3 (below). Under Assumption R:*

1. *Both labor supplies have to satisfy some testable restrictions in the form of partial equations on the participation set P.*
2. *Individual preferences and the sharing rule are identified up to some additive constant $D(\bar{K})$ when at least one of the partners works. Moreover, for each choice of $D(\bar{K})$, preferences are exactly identified.*

The proof of this theorem is developed in the next subsections. First, subsection 3.5.1 identifies the sharing rule in the participation set in which both household members choose to work (P). Next, subsection 3.5.2 identifies ϕ in the set in which one of the couple does not work (N^f and N^m).

3.5.1 Identification When Both Partners Participate

This case considers only a positive labor supply for both adults. This is the only situation implicitly considered by BCM (2005). The knowledge of the two labor supplies in the set P allows recovery of ϕ by applying a theorem from Chiappori (1992). For any $(w^m, w^f, y) \in P$ such that $\tilde{h}_y^m \bullet \tilde{h}_y^f = 0$, the following definitions are introduced: $A(w^m, w^f, y) = \dfrac{\tilde{h}_{w^f}^m (w^m, w^f, y)}{\tilde{h}_y^m (w^m, w^f, y)}$, and $B(w^m, w^f, y) = \dfrac{\tilde{h}_{w^m}^f (w^m, w^f, y)}{\tilde{h}_y^f (w^m, w^f, y)}$. Note that A and B are the marginal rates of substitution of the sharing rule $\left(\dfrac{\phi_{w^f}}{\phi_y} = \dfrac{\tilde{h}_{w^f}^m (w^m, w^f, y)}{\tilde{h}_y^m (w^m, w^f, y)} \text{ and } \dfrac{\phi_{w^m}}{\phi_y} = \dfrac{\tilde{h}_{w^m}^f (w^m, w^f, y)}{\tilde{h}_y^f (w^m, w^f, y)} \right)$, which can be identified in terms of the observable labor supplies of m and f.

Lemma 1. *Assume that* $\tilde{h}_y^m \bullet \tilde{h}_y^f \neq 0$, *and* $AB_y - B_{w^f} \neq BA_y - A_{w^m}$ *for any* $(w^m, w^f, y) \in P$. *Then for any given* $\bar{K}$, *the individual preferences and the sharing rule are identified on* P *up to an increasing function of* $\bar{K}$.

Proof. See Lemma 1 in BCM (2005) and proposition 4 in Chiappori (1992). □

The sketch of the proof is as follows. Under a collective framework the labor supply of spouse i is affected by changes either in the non-labor income or in j's wage by means of their effects on the sharing rule. Therefore, from (3) and (4) it is possible to obtain a system of two partial differential equations in ϕ, $\phi_{w^f} - A\phi_y = 0$, and $\phi_{w^m} - B\phi_y = -B$.

The indifference surfaces of i's share can be derived in the space (w^j, y) from noting that if there is a simultaneous change in non-labor income and in j's wage that maintain i's labor supply at the same level, then i's share also remains constant. In addition, j's share can be derived from the fact that both shares must add up to the non-labor income devoted to non-public consumption. The system of partial differential equations can be solved if it is differentiated again and if the symmetry of cross-partial derivatives is taken into account.[8]

[8] The solution consists of partial derivatives of the sharing rule that can be deduced from observed labor supplies. Assuming that $AB_y - B_{w^f} \neq BA_y - A_{w^m}$, let $\alpha = \left(1 - \dfrac{BA_y - A_{w^m}}{AB_y - B_{w^f}} \right)^{-1}$ and $\beta = 1 - \alpha$. The partial derivatives are given by $\phi_y = \alpha$, $\phi_{w^f} = A\alpha$, and $\phi_{w^m} = B(\alpha - 1) = -B\beta$. In words, $\alpha(\beta)$ is the share of marginal non-labor income not devoted to public expenditures received by $m(f)$.

The sharing rule and couples' preferences have to be adjusted to consider the presence of public expenditures. For the sharing rule ϕ and the pair of utilities U^m and U^f there exists a constant $D(\bar{K})$ such that, for all $(w^m, w^f, y) \in P$,

$$\tilde{\phi}(w^m, w^f, y) = \phi(w^m, w^f, y) + D(\bar{K})$$
$$\tilde{U}^m(h^m, C^m, \bar{K}) = g^m[U^m(h^m, C^m - D(\bar{K}), \bar{K}), \bar{K}]$$
$$\tilde{U}^f(h^f, C^f, \bar{K}) = g^f[U^f(h^f, C^f + D(\bar{K}), \bar{K}), \bar{K}]$$

where g^m and g^f are twice continuously differentiable mappings, increasing in their first argument. The functions $\tilde{U}^i$ and U^i are different, although impossible to distinguish solely from observation of labor supplies,[9] but once $D(\bar{K})$ has been chosen, $\tilde{U}^i$ and g^i coincide up to an increasing function of $\bar{K}$.

3.5.2 Identification When One Member of the Couple Does Not Participate

In the case where only one of the adult household members, say i,works, the observation of i's labor supply characterizes the sharing rule on the set N^j. In addition, the values of the partial derivatives of the sharing rule are identified on j's frontier by Lemma 1, providing boundary conditions for the identification of the sharing rule on N^j. By continuity of $\tilde{h}^i$ and ϕ,[10] the recovery of the sharing rule on P can be extended to the frontier between P and N^j if w^j approaches the participation frontier $\gamma^j(w^i, v)$.

In particular, consider the participation set N^f in which member m works and f does not (i.e., $w^m > \gamma^m(w^f, y)$ and $w^f \leq \gamma^f(w^m, y)$). For any $(w^m, w^f, y) \in \text{int}(N^f)$ such that $\tilde{h}^m_y \neq 0$, define:

$$A(w^m, w^f, y) = \frac{\tilde{h}^m_{w^f}(w^m, w^f, y)}{\tilde{h}^m_y(w^m, w^f, y)}$$

Along f's participation frontier, for any set I^f of (w^m, y) such that $w^m \geq \hat{w}^m(y)$, the following definition is made by a continuity argument, if $\lim_{w^f \uparrow \gamma^f} \tilde{h}^m_y \neq 0$:

[9] The intuition in the case of member m is the following. Switching from ϕ and U^m to $\tilde{\phi}$ and $\tilde{U}^m$ affects, first,: the budget constraint of m, with a vertical translation of magnitude $D(K)$, and second, all of m's indifference curves shift downward by $D(\bar{K})$, so m's labor supply does not change. Because m's consumption C^m cannot be observed, (ϕ, U^m) is empirically indistinguishable from $(\tilde{\phi}, \tilde{U}^m)$.

[10] Although $\tilde{h}^m$, $\tilde{h}^f$, and ϕ are generally nondifferentiable along the participation frontiers, it can be shown that couples' labor supplies and the sharing rule are infinitely differentiable in all their arguments on P, $int(N^f)$, and $int(N^m)$ (for a proof, see Theorem A.3 of Magnus and Neudecker 2007: 163).

$$a(w^m, y) = A(w^m, \gamma^f(w^m, y), y)$$

Lemma 2. *Assume that* $\lim_{w^f \uparrow \gamma^f} \tilde{h}_y^m \neq 0$ *and* $1 + a \bullet \gamma_y^f \neq 0$ *for any* $(w^m, y) \in I^f$ *and* $\tilde{h}_y^m \neq 0$ *for any* $(w^m, w^f, y) \in int(N^f)$. *Then the sharing rule is identified on* N^f *up to some additive constant* $D(\bar{K})$.

Proof. The same technique used by Donni (2003) can be applied; the only adjustment that must be made is that the additive constant is indexed by the level of public expenditures. From Lemma 1, it is known that ϕ must satisfy the partial differential equation

$$\phi_{w^f} - A\phi_y = 0 \tag{7}$$

that characterizes the sharing rule on N^f. Additionally, the sharing rule along the participation frontier $(w^f - \gamma^f(w^m, y) = 0)$ gives a boundary condition for the partial differential equation. From standard theorems in partial differential equations theory, the identification of the sharing rule (up to an additive constant) is achieved if the following condition is fulfilled. First, (7) can be written as $\nabla\phi\mathbf{u} = 0$, where $\nabla\phi$ denotes the gradient of ϕ and $\mathbf{u}$ is the vector $(0,1,-A)$. Now, the condition is that $\mathbf{u}$ is not tangent to f's participation frontier. The intuition behind this condition is the following: (7) defines the indifference surfaces of the sharing rule (the values of w^f, w^m, and y that keep constant the sharing rule at some level) that pass through f's participation frontier. Since $\nabla\phi$ is a vector normal to surfaces of constant ϕ, and $\mathbf{u}$ indicates the direction in which the sharing rule is constant, (7) states that $\mathbf{u}$ is perpendicular to $\nabla\phi$ everywhere. Therefore, $\mathbf{u}$ is a vector that is tangent to the surfaces of constant ϕ at every point and, in particular, is a tangent vector to the surface in the participation frontier of f. Given that, on the frontier, A coincides with a, this condition states that, for all $(w^m, y) \in I^f$:

$$1 + a \bullet \gamma_y^f \neq 0$$

If this condition is fulfilled on the frontier, then the partial differential equation (7) together with the boundary condition defines ϕ up to an additive constant, $D(\bar{K})$, in the context analyzed. □

For the participation set N^m in which only member f works, *mutatis mutandis*, the reasoning is identical, giving rise to the following:

Lemma 3. *Assume that* $\lim_{w^m \uparrow \gamma^m} \tilde{h}_y^f \neq 0$ *and* $1 + b \bullet \gamma_y^m \neq 0$ *for any* $(w^f, y) \in I^m$ *and* $\tilde{h}_y^f \neq 0$ *for any* $(w^m, w^f, y) \in \mathrm{int}(N^m)$. *Then the sharing rule is identified on* N^m *up to some additive constant* $D(\bar{K})$.

Proof. As above, using the partial differential equation $\phi_{w^m} - B\phi_y = -B$, with $B(w^m, w^f, y) = \dfrac{\tilde{h}_{w^m}^f(w^m, w^f, y)}{\tilde{h}_y^f(w^m, w^f, y)}$, and the boundary condition $w^m - \gamma^m(w^f, y) = 0$. □

4 Parametric Specification and Empirical Implementation

For a simple but realistic empirical illustration of the collective model with expenditures on children and non-participation proposed in the previous section, subsection 4.1 discusses the specific functional forms and simplifying assumptions that have been chosen, while subsection 4.2 addresses the restrictions implied by the identifiability assumptions. Subsection 4.3 discusses the stochastic specification and the likelihood function used for the estimations.

4.1 Preferences, Labor Supply, Expenditures on Children, and the Sharing Rule

For the empirical implementation it is important to have a relatively simple parametric specification in mind. Following the semi-log specification popular in empirical work in general (Blundell, MaCurdy, and Meghir 2007), and used in the empirical literature of collective models as well (Chiappori, Fortin, and Lacroix 2002), when both partners work, their individual structural labor supply functions are be specified as:

$$h^m = \psi_0 + \psi_1 \phi^m + \psi_2 \ln w^m + \psi_3 K \tag{8}$$

$$h^f = \gamma_0 + \gamma_1 \phi^f + \gamma_2 \ln w^f + \gamma_3 K \tag{9}$$

Equations (8) and (9) are linear in parameters, what eases the estimation process. Applying Roy's identity to the underlying indirect utility functions of the Stern (1986) type,

$$V^m(w^m, \phi^m) = \left(\frac{\exp\left(\psi_1 w^m\right)}{\psi_1} \right) \left(\psi_0 + \psi_1 \phi^m + \psi_2 \ln w^m + \psi_3 K \right) - \frac{\psi_2}{\psi_1} \int_{\infty}^{\psi_1 w^m} \frac{\exp(t)}{t} dt$$

for men (m), and similar for women (f) but with γs instead of ψs., yields the individual labor supply system (8) and (9). In this specification, K appears non-separable in the utility function of both members. Note that the efficiency condition (5) for public-good expenditures implies the following restriction in parameters:

$$\frac{\gamma_3 - \gamma_1}{\gamma_1} = -\frac{\psi_3}{\psi_1} \tag{10}$$

As in Chiappori, Fortin, and Lacroix (2002), the sharing rule is specified as:[11]

$$\begin{aligned} \phi = \phi^m &= \alpha_0 + \alpha_1 Y + \alpha_2 \ln w^m + \alpha_3 \ln w^f + \alpha_4 \ln w^m \ln w^f + \alpha_5 z \\ &= \boldsymbol{\alpha}'\mathbf{W} \end{aligned} \tag{11}$$

From the definition of the sharing rule, the expenditure on children has to satisfy the identity $K = Y - (\phi^m + \phi^f)$, so that the reduced form is specified as:

$$\begin{aligned} K &= c_0 + c_1 Y + c_2 \ln w^m + c_3 \ln w^f + c_4 \ln w^m \ln w^f + c_5 z \\ &= \mathbf{c}'\mathbf{W} \end{aligned} \tag{12}$$

Inserting the sharing rule (11) in the structural labor supply functions (8) and (9), the reduced-form functions are:

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[11] The interaction between log wage rates is included because the identifiability of the sharing rule depends on the first and second derivatives of both partners' labor supply functions; the second-order cross-partial derivatives with respect to wages do not vanish.

$$h^m = a_0 + a_1 Y + a_2 \ln w^m + a_3 \ln w^f + a_4 \ln w^m \ln w^f + a_5 z$$
$$= \mathbf{a}'\mathbf{W} \tag{13}$$
$$h^f = b_0 + b_1 Y + b_2 \ln w^m + b_3 \ln w^f + b_4 \ln w^m \ln w^f + b_5 z$$
$$= \mathbf{b}'\mathbf{W} \tag{14}$$

4.2 Restrictions of the Model

With the intention to focus on labor supplies, the level of public expenditures is fixed to $K(w^m, w^f; Y, z) = \bar{K}$. Hence, using the change in variable $y = Y - \bar{K}$ rearranging equation (12), the distribution factor can be expressed as:

$$z = \frac{1}{c_5}[(1-c_1)\bar{K} - c_0 - c_1 y - c_2 \ln w^m - c_3 \ln w^f - c_4 \ln w^m \ln w^f] \tag{15}$$

Using (15), the reduced-form labor supply functions (13) and (14) can be written as:

$$h^m = A_0 + A_1 y + A_2 \ln w^m + A_3 \ln w^f + A_4 \ln w^m \ln w^f + A_5 \bar{K} \tag{16}$$
$$h^f = B_0 + B_1 y + B_2 \ln w^m + B_3 \ln w^f + B_4 \ln w^m \ln w^f + B_5 \bar{K} \tag{17}$$

The relation between the parameters of the equations (13)-(14) and the parameters of equations (16)-(17) is shown in Table 1.

Table 1. Relation between parameters of the reduced labor supply functions

Member m, eq. (16)	Member f, eq. (17)
$A_0 = a_0 - \frac{a_5 c_0}{c_5}$	$B_0 = b_0 - \frac{b_5 c_0}{c_5}$
$A_1 = a_1 - \frac{a_5 c_1}{c_5}$	$B_1 = b_1 - \frac{b_5 c_1}{c_5}$
$A_2 = a_2 - \frac{a_5 c_2}{c_5}$	$B_2 = b_2 - \frac{b_5 c_2}{c_5}$
$A_3 = a_3 - \frac{a_5 c_3}{c_5}$	$B_3 = b_3 - \frac{b_5 c_3}{c_5}$
$A_4 = a_4 - \frac{a_5 c_4}{c_5}$	$B_4 = b_4 - \frac{b_5 c_4}{c_5}$
$A_5 = a_1 + \frac{a_5 (1-c_1)}{c_5}$	$B_5 = b_1 + \frac{b_5 (1-c_1)}{c_5}$

Using equations (16) and (17), the conditional sharing rule when both partners work, in terms of the household non-labor income devoted to private expenditures and wages, is characterized by the partial derivatives (see footnote 8):

$$\phi_y = \frac{A_1 B_4}{A_1 B_4 - B_1 A_4}$$

$$\phi_{w^m} = \frac{A_4 B_2 + A_4 B_4 \ln w^f}{A_1 B_4 w^m - B_1 A_4 w^m}$$

$$\phi_{w^f} = \frac{A_3 B_4 + A_4 B_4 \ln w^m}{A_1 B_4 w^f - B_1 A_4 w^f}$$

Solving this system of differential equations, the conditional sharing rule recovered is:

$$\phi = \tilde{\alpha}_0 + \tilde{\alpha}_1 y + \tilde{\alpha}_2 ln w^m + \tilde{\alpha}_3 ln w^f + \tilde{\alpha}_4 \ln w^m \ln w^f \tag{18}$$

Table 2 shows the parameters of the sharing rule (11) and its conditional version (18) in terms of the parameters of the reduced-form labor supply functions,

Table 2. Parameters of the sharing rule in terms of the parameters of the reduced-form labor supply functions

Parameter	Reduced-form labor supply functions † eqs. (13)-(14)	eqs. (16)-(17)
Sharing rule (eq. (11))		
α_0	$\tilde{\alpha}_0 - \frac{c_0 (a_1 c_5 - a_5 c_1)(b_4 c_5 - b_5 c_4)}{\Delta}$	$\tilde{\alpha}_0 - \frac{c_0 A_1 B_4}{A_1 B_4 - B_1 A_4}$
α_1	$\frac{(1 - c_1)(a_1 c_5 - a_5 c_1)(b_4 c_5 - b_5 c_4)}{\Delta}$	$\frac{(1 - c_1) A_1 B_4}{A_1 B_4 - B_1 A_4}$
α_2	$\frac{(a_4 c_5 - a_5 c_4)(b_2 c_5 - c_2 b_5) - c_2 (a_1 c_5 - a_5 c_1)(b_4 c_5 - b_5 c_4)}{\Delta}$	$\frac{A_4 B_2 - c_2 A_1 B_4}{A_1 B_4 - B_1 A_4}$
α_3	$\frac{(a_3 c_5 - a_5 c_3)(b_4 c_5 - b_5 c_4) - c_3 (a_1 c_5 - a_5 c_1)(b_4 c_5 - b_5 c_4)}{\Delta}$	$\frac{A_3 B_4 - c_3 A_1 B_4}{A_1 B_4 - B_1 A_4}$
α_4	$\frac{(a_4 c_5 - a_5 c_4)(b_4 c_5 - b_5 c_4) - c_4 (a_1 c_5 - a_5 c_1)(b_4 c_5 - b_5 c_4)}{\Delta}$	$\frac{A_4 B_4 - c_4 A_1 B_4}{A_1 B_4 - B_1 A_4}$
α_5	$\frac{-c_5 (a_1 c_5 - a_5 c_1)(b_4 c_5 - b_5 c_4)}{\Delta}$	$\frac{-c_5 A_1 B_4}{A_1 B_4 - B_1 A_4}$
Conditional sharing rule (eq. (18))		
$\tilde{\alpha}_1$	$\frac{(a_1 c_5 - a_5 c_1)(b_4 c_5 - b_5 c_4)}{\Delta}$	$\frac{A_1 B_4}{A_1 B_4 - B_1 A_4}$
$\tilde{\alpha}_2$	$\frac{(a_4 c_5 - a_5 c_4)(b_2 c_5 - b_5 c_2)}{\Delta}$	$\frac{A_4 B_2}{A_1 B_4 - B_1 A_4}$
$\tilde{\alpha}_3$	$\frac{(a_3 c_5 - a_5 c_3)(b_4 c_5 - b_5 c_4)}{\Delta}$	$\frac{A_3 B_4}{A_1 B_4 - B_1 A_4}$
$\tilde{\alpha}_4$	$\frac{(a_4 c_5 - a_5 c_4)(b_4 c_5 - b_5 c_4)}{\Delta}$	$\frac{A_4 B_4}{A_1 B_4 - B_1 A_4}$

† With $\Delta = (a_1 c_5 - a_5 c_1)(b_4 c_5 - b_5 c_4) - (a_4 c_5 - a_5 c_4)(b_1 c_5 - b_5 c_1)$ and $\tilde{\alpha}_0$ an unknown constant.

Besides the parameter constraints from the efficiency condition for public-good expenditures (10), under a collective approach and with the chosen functional form of the labor supply functions, it is required that the ratio of the marginal effects of the interaction between the log wage rates has to be equal to the corresponding ratio of the marginal effects of the distribution factor on labor supplies:

$$\frac{a_4}{b_4} = \frac{a_5}{b_5} \tag{19}$$

This restriction stems from the fact that the cross term and the distribution factor enter the labor supply functions only through the sharing rule.

Moreover, collective rationality has implications for the ratio of the marginal effects of the expenditures on children (K) on each partner's labor supply functions:

$$\frac{a_1 + \frac{a_5}{c_5}(1\text{-}c_1)}{b_1 + \frac{b_5}{c_5}(1\text{-}c_1)} = 1, \tag{20}$$

where the marginal effect of K is the sum of two terms. The first is the marginal effect that corresponds to the individual preferences via a change in the household's non-labor income (a_1 and b_1), and the second term is the marginal change of K on the sharing rule via the distribution factor ($a_5 / c_5 (1 - c_1)$ and $b_5 / c_5 (1 - c_1)$). Therefore, changes in the expenditures on children only impact individual labor supply functions through income effects, the impact for both partners being equal. Equations (19) and (20) impose testable cross-equation restrictions in the couple's labor supply functions.

Finally, the parameters of the structural labor supplies (8) and (9) can be expressed in terms of the parameters of their reduced form (Table 3).

Table 3: Parameters of the structural labor supply functions in terms of the reduced-form parameters

Member m, eq. (8)	Member f, eq. (9)
$\psi_0 = A_0 - c_0 A_1 + \frac{A_1 B_5 - A_5 B_1}{B_1 - B_5}\left(\tilde{\alpha}_0 - \frac{c_0 A_1 B_4}{A_1 B_4 - A_4 B_1}\right)$	$\gamma_0 = B_0 - c_0 B_1 + \frac{A_1 B_5 - A_5 B_1}{A_1 - A_5}\left(\tilde{\alpha}_0 - \frac{c_0 A_4 B_1}{\left(A_1 B_4 - A_4 B_1\right)}\right)$
$\psi_1 = \frac{B_1 A_5 - A_1 B_5}{B_1 - B_5}$	$\gamma_1 = \frac{A_1 B_5 - B_1 A_5}{A_1 - A_5}$
$\psi_2 = A_2 + B_2 \frac{A_5 - A_1}{B_1 - B_5}$	$\gamma_2 = B_3 + A_3 \frac{B_5 - B_1}{A_1 - A_5}$
$\psi_3 = A_5$	$\gamma_3 = B_5$

If the female partner does not work, there is a regime switch in the male partner's labor supply and the sharing rule, and the parameters change:

$$\begin{aligned} h^m &= \breve{a}_0 + \breve{a}_1 Y + \breve{a}_2 \ln w^m + \breve{a}_3 \ln w^f + \breve{a}_4 \ln w^m \ln w^f + \breve{a}_5 z \\ &= \breve{\mathbf{a}}'\mathbf{W} \end{aligned} \tag{21}$$

$$\begin{aligned} \phi &= \breve{\alpha}_0 + \breve{\alpha}_1 Y + \breve{\alpha}_2 \ln w^m + \breve{\alpha}_3 \ln w^f + \breve{\alpha}_4 \ln w^m \ln w^f + \breve{\alpha}_5 z \\ &= \breve{\boldsymbol{\alpha}}'\mathbf{W} \end{aligned} \tag{22}$$

To identify the decision process, the model imposes the restrictions that both the male's labor supply function and the sharing rule have to be continuous along the female's participation frontier:

$$\breve{\mathbf{a}}'\mathbf{W} = \mathbf{a}'\mathbf{W} + s \cdot (\mathbf{b}'\mathbf{W}) \tag{23}$$

$$\breve{\boldsymbol{\alpha}}'\mathbf{W} = \boldsymbol{\alpha}'\mathbf{W} + r \cdot (\mathbf{b}'\mathbf{W}) \tag{24}$$

Using the partial differential equation of the male's labor supply in ϕ, a relation between s and r is obtained when the female partner does not work:

$$\frac{\tilde{\alpha}_3 + rB_3 + (\tilde{\alpha}_4 + rB_4)\ln w_m}{(\tilde{\alpha}_1 + rB_1) w_f} = \frac{A_3 + sB_3 + (A_4 + sB_4)\ln w_m}{(A_1 + sB_1) w_f}$$

Using the equalities of the parameters of the sharing rule (18) shown in Table 2, the relation $r = \frac{sB_4}{\Delta}$ is obtained.

4.3 Stochastic Specification and the Likelihood Function

For household t, starting from equations (12)-(14) and (21), the complete system of equations to be estimated, is:

$$\begin{aligned} K_t &= \mathbf{c}'\mathbf{W_t} + \mathbf{\Gamma_K X_{tK}} + \varepsilon_{tK} \\ h_t^f &= \begin{cases} h_t^{f*} = \mathbf{b}'\mathbf{W_t} + \mathbf{\Gamma_f X_{tf}} + \varepsilon_{tf} & \text{if } h_t^{f*} > 0 \\ 0 & \text{if } h_t^{f*} \leq 0 \end{cases} \\ h_t^m &= \begin{cases} h_{tp}^m = \mathbf{a}'\mathbf{W_t} + \mathbf{\Gamma_m X_{tm}} + \varepsilon_{tp} & \text{if } h_t^{f*} > 0 \\ h_{tnp}^m = \mathbf{a}'\mathbf{W_t} + \mathbf{\Gamma_m X_{tm}} + s \cdot (\mathbf{b}'\mathbf{W_t} + \mathbf{\Gamma_f X_{tf}}) + \varepsilon_{tnp} & \text{if } h_t^{f*} \leq 0 \end{cases} \end{aligned} \tag{25}$$

where $\mathbf{X_{tl}}$ is a vector of exogenous variables. A stochastic model is obtained through the inclusion of the error terms on the right-hand side of each equation, where the vector of errors $(\varepsilon_{tp}, \varepsilon_{tnp}, \varepsilon_{tf}, \varepsilon_{tK})$ follows a joint normal distribution with a covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma^2{}_p & \sigma_p \sigma_{np} \rho_{p,np} & \sigma_p \sigma_f \rho_{p,f} & \sigma_p \sigma_K \rho_{p,K} \\ \sigma_p \sigma_{np} \rho_{p,np} & \sigma^2{}_{np} & \sigma_{np} \sigma_f \rho_{np,f} & \sigma_{np} \sigma_K \rho_{np,K} \\ \sigma_p \sigma_f \rho_{p,f} & \sigma_{np} \sigma_f \rho_{np,f} & \sigma^2{}_f & \sigma_f \sigma_K \rho_{f,K} \\ \sigma_p \sigma_K \rho_{p,K} & \sigma_{np} \sigma_K \rho_{np,K} & \sigma_f \sigma_K \rho_{f,K} & \sigma^2{}_K \end{bmatrix} \tag{26}$$

The stochastic model is a type 4 Tobit model (Amemiya 1985) or switching regression model (Maddala 1983), with simultaneity. The log-likelihood function of the econometric model is:[12]

[12] With $\phi(\bullet)$ and $\Phi(\bullet)$ the density and distribution function, respectively, of the standard normal distribution.

$$\ln L = \sum_{t=1}^{T} \left\{ \begin{array}{l} \ln\left(\dfrac{1}{\sigma_K}\phi\left(S_{tK}\right)\right) + I_t\left[\ln\left(\dfrac{1}{\sigma_{z_p}}\phi\left(S_{tp}\right)\right) + \ln\left(\dfrac{1}{\sigma_{z_f}}\phi\left(S_{tf}\right)\right)\right] \\ +\left(1-I_t\right)\left[\ln\left(\dfrac{1}{\sigma_{z_{np}}}\phi\left(S_{tnp}\right)\right) + \ln\left(1-\Phi\left(\eta_{tf}\right)\right)\right] \end{array} \right\}$$

where

$$I_t = \begin{cases} 1 \text{ if } h_t^{f*} > 0 \\ 0 \text{ if } h_t^{f*} \le 0 \end{cases},$$

$$S_{tK} = \frac{K_t - \mathbf{c}'\mathbf{W}_t}{\sigma_K},$$

$$S_{tp} = \frac{\left[h_{tp}^m - \mathbf{a}'\mathbf{W}_t\right] - \sigma_p\left[\left(\dfrac{\rho_{p,f} - \rho_{p,K}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\left(\dfrac{h_t^f - \mathbf{b}'\mathbf{W}_t}{\sigma_f}\right) + \left(\dfrac{\rho_{p,K} - \rho_{p,f}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\left(\dfrac{K_t - \mathbf{c}'\mathbf{W}_t}{\sigma_K}\right)\right]}{\sigma_p\sqrt{1-\left(\dfrac{\rho_{p,f} - \rho_{p,K}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\rho_{p,f} - \left(\dfrac{\rho_{p,K} - \rho_{p,f}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\rho_{p,K}}},$$

$$\sigma_{z_p} = \sigma_p\sqrt{1-\left(\frac{\rho_{p,f} - \rho_{p,K}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\rho_{12} - \left(\frac{\rho_{p,K} - \rho_{p,f}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\rho_{p,K}},$$

$$S_{tf} = \frac{\left[h_t^f - \mathbf{b}'\mathbf{W}_t\right] - \sigma_f\rho_{f,K}\left(\dfrac{K_t - \mathbf{c}'\mathbf{W}_t}{\sigma_K}\right)}{\sigma_f\sqrt{1-\rho_{f,K}^2}},$$

$$\sigma_{z_f} = \sigma_f\sqrt{1-\rho_{f,K}^2},$$

$$S_{tnp} = \frac{\left[\dfrac{h_{tnp}^m - \left(\mathbf{a}'\mathbf{W}_t + s\cdot\left(\mathbf{b}'\mathbf{W}_t\right)\right)}{\sigma_{np}}\right] - \rho_{np,K}\left(\dfrac{K_t - \mathbf{c}'\mathbf{W}_t}{\sigma_K}\right)}{\sqrt{1-\rho_{np,K}^2}},$$

$$\sigma_{z_{np}} = \sigma_{np}\sqrt{1-\rho_{np,K}^2},$$

$$\eta_{tf} = \frac{\left(\dfrac{\mathbf{b}'\mathbf{W}_t}{\sigma_f}\right) + \rho_{np,f}\left(\dfrac{h_{tnp}^m - \left(\mathbf{a}'\mathbf{W}_t + s\cdot\left(\mathbf{b}'\mathbf{W}_t\right)\right)}{\sigma_{np}}\right) + \left(\rho_{f,K} - \rho_{np,f}\rho_{np,K}\right)\left(\dfrac{K_t - \mathbf{c}'\mathbf{W}_t}{\sigma_K}\right)}{\sqrt{\left(1-\rho_{f,K}^2\right)\left(1-\dfrac{\left(\rho_{np,f} - \rho_{np,K}\rho_{f,K}\right)^2}{\left(1-\rho_{np,K}^2\right)\left(1-\rho_{f,K}^2\right)}\right)}}.$$

4.3.1 Imputation of potential wages

Up to this point it has been assumed that both partners' wages are always observed, even if someone is not working. Based on Wooldridge (2010), for non-working women the empirical analysis uses a Tobit selection procedure to impute both a wage rate and the interaction between the couple's wage rates, taking into account the simultaneity between expenditures on children and the couple's labor decisions. First, using the full sample, a standard Tobit of h_t^f on all the exogenous variables is estimated:

$$h_t^f = b_0 + b_1 Y + b_2 \ln w^m + \boldsymbol{\pi}'\mathbf{X}_{tw^f} + \boldsymbol{\psi}'\mathbf{X}_{tw^f w^m} + b_5 z + \boldsymbol{\Gamma}_f \mathbf{X}_{tf} + \boldsymbol{\Gamma}_k \mathbf{X}_{tK} + v_t \ .$$

Then, using observations for which $h_t^f > 0$, the following equations for female log wage and for the interaction between the couples' wages are estimated, including the residuals $\hat{v}_t$ from the previous step as a covariate:

$$\ln w_t^f = \boldsymbol{\pi}'\mathbf{X}_{tw^f} + \delta_1 \hat{v}_t + u_{tw^f} \tag{27}$$

$$\ln w_t^m \ln w_t^f = \boldsymbol{\psi}'\mathbf{X}_{tw^f w^m} + \delta_2 \hat{v}_t + u_{tw^f w^m} \tag{28}$$

Equations (27) and (28) are identified from the exclusion of household non-labor income, the distribution factor, the male partner's age and education, a second-order polynomial in the number of children in the household under 15, and a dummy variable for the number of children under five. To identify the effect of the woman's log wage rate and the cross product of log wages on the woman's labor supply, it is necessary that $\mathbf{X}_{tw^f}$ and $\mathbf{X}_{tw^f w^m}$ each contain at least one variable not in $\mathbf{X}_{tf}$ and $\mathbf{X}_{tK}$. The chosen variables for $\mathbf{X}_{tf}$ are the cross product of the woman's age and education (see, e.g., Mroz 1987), and the unemployment rate by state and by year-quarter of the first survey visit to the household as a means of accounting for local labor market conditions. For $\mathbf{X}_{tw^f w^m}$, the male partner's log wage, the same variables considered for $\mathbf{X}_{tf}$, and the interaction between them are chosen. The choice of instruments is based on Wooldridge (2010)'s discussion of identification in simultaneous equations models that are nonlinear in endogenous variables, particularly models with interactions between exogenous variables (here, $\ln w_t^m$) and endogenous variables (here, $\ln w_t^f$).

Finally, the fitted values of $\ln w^f$ and $\ln w^m \ln w^f$ are calculated, correcting for selection bias $\left(\widehat{\ln w_t^f} = \hat{\boldsymbol{\pi}}'\mathbf{X}_{tw^f}, \widehat{\ln w^m \ln w^f} = \hat{\boldsymbol{\psi}}'\mathbf{X}_{tw^f w^m}\right)$.

5 Data

A survey that satisfies the data requirements is the Mexican Family Life Survey (MxFLS). From the original sample of 8,328 households, a subsample is extracted from the second wave (held in 2005-2006) that consists of nuclear families that only have children under 15 years of age (1,921 households, 48.15% of nuclear families). By using these nuclear families, the focus is on households where the decision process is centralized in the parents, reducing the possibility of interaction with other kin within the household. The selection of children under 15 is because a child of this age is less likely to have bargaining power in household decisions.

Furthermore, the sample is restricted to couples living together where both partners are less than 60 years old. We exclude households where a member is unemployed (the choice between working or not has to be freely made, to avoid misinterpretation of the findings), and selfemployed or working without remuneration (to avoid problems in measuring labor income). Also households where the male partner is not employed, a negligible number, are dropped. These criteria and the exclusion of households with missing and outlier data leave us with a total of 1,002 households. The information on wage rates and working hours of both partners is used, as well as information on women with missing wage rates. Expenditures on children include education (enrollment fees, exams, school supplies, uniforms, and transportation), clothes and shoes, toys, and clothes and items for babies. Non-labor income is the annual household current income minus the couple's labor incomes.

Table 4 reports descriptive statistics of the final sample. The low female participation rate, only 18 percent of the women (180 of 1,002) participate in the labor market, represents a challenge to the model estimation since the procedure for imputing potential wages to all women in the sample is based on the information from working women. The mean annual number of working hours is 308 for all women in the sample and 2,408 for men. However, working women have on average a higher hourly wage rate than men (MXN $43 versus $29). Using the procedure described in subsection 4.3.1, the female's log wage rate and the interaction between the couple's log wage rates are replaced – for all observations – by their fitted values (see Appendix Table A.1). There is no significant difference in years of education (approximately eight years), while women are on average two-and-a-half years younger than their husbands.

Table 4: Descriptive statistics

	Mean	Std. Dev.
Woman		
Employed (percentage)	17.96	
Working hours per year	307.71	762.41
Wage rate (MXN per hour)	42.81	88.98
Age	30.15	6.46
Years of education	8.50	3.74
Man		
Working hours per year	2,407.66	880.61
Wage rate (MXN per hour)	28.30	43.68
Age	32.70	7.02
Years of education	8.69	3.93
Expenditures on children (MXN per year)	4,105.25	6,362.91
Non-labor income (MXN per year)	9,822.41	15,686.06
Number of children under 15 years	2.15	1.02
Children under 5 years (percentage)	62.77	
Sex ratio:		
Age-to-age	0.90	0.07
2-year-band	0.88	0.07
Number of observations	1,002	

In the collective framework, the intrahousehold decision process depends on distribution factors, variables that leave the individual preferences and the joint budget set unchanged and only shift the distribution of power. The sex ratio is a frequently used distribution factor that proxies the situation in the marriage market, reflecting the couple's outside opportunities (Angrist (2002), Chiappori, Fortin, and Lacroix (2002), Grossbard-Shechtman and Neuman (2003), Park (2007)). A higher sex ratio – a smaller percentage of women on the marriage market – improves the female's bargaining position; if the relationship dissolves, she has a higher probability to find a new partner than he does, so he is willing to concede to her a larger share of the gains of living in a couple in order to avoid an end to the relationship. Following Park (2007), two kinds of sex ratio variables at the state level are constructed using the *Conteo de Poblacion y Vivienda* of 2005. The age-to-age sex ratio is the number of men of the same age as the male partner of each household over the corresponding number of women. A 2-year-band sex ratio is also calculated; this ratio uses the weighted sum of women who are at most two years younger than the male partner of the household, based on the assumption that a man and a woman aged 15 years or older can form a couple with an equal chance if the man is between zero and two years older than the woman, which reflects the age difference observed in the sample.

6 Estimation Results

Tables 5-7 and Appendix Table A.2 show the parameter estimates of the unrestricted model (25)-(26), which assumes that the male's labor supply function is continuous along the female's participation frontier, and its associated collective version, which imposes the restrictions (19)-(20) in the estimation process. Two versions are estimated, one using the age-to-age sex ratio variable as a distribution factor (columns labeled (age)), and the other using the 2-year-band sex ratio variable (columns labeled (2yr)). Using the log-likelihood values for each model it is possible to construct likelihood-ratio statistics to test the collective restrictions (19)-(20). In the version employing the age-to-age (2-year-band) sex ratio the test statistic of 1.79 (4.55) is to be compared with the critical value of $\chi^2_{0.05}(2)$ = 5.99. Hence, for both sex ratios, the collective model cannot be rejected, a finding that is consistent with the hypothesis that the presence of children in a household generates non-separabilities in individual consumption. Others that have not explicitly considered this aspect have usually rejected the collective rationality when analyzing a household with children (see Fortin and Lacroix 1997; Donni 2007).

Table 5 presents the estimates of the parameters of expenditures on children. The magnitudes of the coefficients are very similar in the unrestricted and the collective versions. The marginal effect of a change in the male's wage rate on the expenditures on children is $(c_2 + c_4 \ln w^f)/w^m$, so for all specifications and everything else being equal, an increase in the male's wage rate implies an increase in the money spent on children if the female's wage is more than MXN \$8 (that is if $w^f > \exp(-c_2/c_4)$), which is the case for the large majority of the sample. In both versions of the unrestricted model, at the mean wage rate of both parents, a MXN \$1 increase in the male's wage rate (equivalent to an annual increase of MXN \$2,408 in labor income at the mean hours worked by men) increases the annual expenditure on children by approximately MXN \$61. The marginal effect of the female's wage rate is determined by $(c_3 + c_4 \ln w^m)/w^f$, and is positive if the male's wage is larger than MXN \$23 using the age-to-age sex ratio as distribution factor, and \$26 with the 2-year-band, which is the case for just over half of the sample. In the unrestricted model with the age-to-age sex ratio as distribution factor and at the mean wage rate of both parents, a MXN \$1 increase in the mother's wage (equivalent to an annual increase of MXN \$308 in her labor income, at the mean hours worked by women) increases the annual expenditure on children by approximately MXN $5 (approximately $2 with the 2-year-band). The non-labor income is not statistically significant at conventional levels.

Table 5: Parameter Estimates. Expenditures on Children †

	Unrestricted Model		Collective Model	
	(age)	(2yr)	(age)	(2yr)
$\ln w^m$	-2,110.244***	-2,052.694***	-2,107.689***	-2,052.502***
	(755.282)	(761.974)	(755.251)	(761.966)
$\ln w^f$	-3,218.229**	-3,282.515**	-3,227.715**	-3,303.747**
	(1,570.866)	(1,586.456)	(1,570.793)	(1,586.447)
$\ln w^m \ln w^f$	1,023.365***	1,012.113***	1,023.439***	1,013.046***
	(278.762)	(281.249)	(278.765)	(281.280)
Non-labor income (††)	0.020	0.019	0.020	0.019
	(0.013)	(0.013)	(0.013)	(0.013)
Sex ratio (††):				
Age-to-age	-9,224.334**		-8,948.663**	
	(3,831.198)		(3,805.978)	
2-year-band		-5,914.022		-5,343.813
		(4,218.697)		(4,203.801)
Female's education	303.542***	300.054***	303.477***	300.087***
	(68.441)	(68.209)	(68.439)	(68.209)
Female's age	102.993*	100.598*	102.779*	100.317*
	(54.265)	(54.496)	(54.262)	(54.492)
Male's education	214.400***	217.284***	214.513***	217.274***
	(58.844)	(58.949)	(58.842)	(58.950)
Male's age	-36.331	-48.868	-36.631	-48.690
	(43.559)	(43.455)	(43.555)	(43.452)
No. of children < 15	1,584.207**	1,623.509**	1,587.529**	1,625.201**
	(681.113)	(682.118)	(681.137)	(682.180)
No. of children < 15 squared	-177.249	-181.418	-177.746	-181.634
	(126.852)	(127.079)	(126.863)	(127.091)
Children < 5	-1,292.749***	-1,300.210***	-1,293.948***	-1,300.883***
	(448.442)	(449.293)	(448.448)	(449.314)
Intercept	11,112.689*	8,774.182	10,912.489*	8,349.874
	(6,521.690)	(6,728.217)	(6,514.149)	(6,725.028)
Region dummies	Yes	Yes	Yes	Yes

Note. * p<0.10, ** p<0.05, *** p<0.01. Standard errors in parentheses. The regions are: North, Capital, Gulf, Pacific, South, Central-North, and Central.
† Estimation of equations (25)-(26), with restrictions (19)-(20) imposed in the collective model
†† Parameter constrained in the estimation process of the collective model by imposing the restrictions (19)-(20).

The age-to-age sex ratio has a negative and statistically significant effect on expenditures on children; for example, a one-standard deviation increase in the age-to-age sex ratio (0.07 points) reduces the annual amount spent on children by approximately MXN $646 in the unrestricted model. Because an increase in the sex ratio is related to an increase in the bargaining power of the female partner (and a corresponding decrease in that of the male partner), this result suggests that fathers care more about their children than mothers (although, under the proposed specification, the adequate indicator of parents' preferences regarding children is their marginal willingness to pay, whose estimated values are shown later). These results reject the implication of the unitary approach that no distribution factor is associated with intrahousehold allocations.

Most other control variables are statistically significant at conventional levels. As expected, the presence of a larger number of children under 15 increases the expenditure on them. However, if a child under five is present, all else equal, the expenditures are reduced. Children under five contribute to higher expenditures through the total number of children, but an autonomous correction is made since there are no school expenditures for them. Parental education has a positive effect on the expenditures on children, especially the female's; while an additional year in the male's education increases the annual amount spent on children by approximately MXN $215, that same factor in the female's education increases the expenditure by MXN $300.

The estimates of the reduced-form female household member's labor supply function are shown in Table 6. The own-wage effect of female labor supply is determined by $(b_3 + b_4 \ln w^m) / w^f$, and is positive at male hourly wage rates inferior to MXN $9 but the negative backward bending effect dominates for higher male wage rates. Therefore, if the husband earns more than MXN $9 per hour, a higher potential wage for the woman does not result in a greater labor supply for her; only if the man earns less than MXN $9 the wife is inclined to work more hours. The cross-wage effect of female labor supply, $(b_2 + b_4 \ln w^f) / w^m$, is positive for female wage rates less than MXN $62 in the model with the age-to-age sex ratio as a distribution factor (and less than MXN $59 using the 2-year-band). Thus, for the most relevant female wage range, all other factors being equal, women who participate work more if the husband has a higher wage, while for those women who do not work the probability of starting to participate increases with the wage of their partner. In sum, the own-wage income effect tends to dominate the substitution effect for very small values of the male wage rate, while a woman tends to increase her working hours upon a wage increase of her partner within a wide range of her own wage rate.

The parameter of the sex ratio variable in the couple's reduced labor supply functions is the result of two effects, one of the sharing rule and the other of the expenditures on children (see the structural labor supply functions (8) and (9)). Interestingly, the effect of both sex ratios on the female's labor supply is positive, but imprecisely determined, in both the unrestricted and collective model. In the collective version, the magnitude of both sex ratios is smaller and better determined: the age-to-age sex ratio parameter passes from a *p*-value of 60% in the unrestricted model to 21% in the collective one, while the corresponding value for the 2-year-band falls from 53% to 33%.

With respect to the control variables, the female household member's age and education have a significantly positive effect on her labor supply. As expected, an increase in the number of children, other factors being equal, is accompanied by a decrease in her number of hours worked; the presence of a pre-school child also reduces the number of hours worked.

Table 6: Parameter Estimates. Female Labor Supply †

	Unrestricted Model		Collective Model	
	(age)	(2yr)	(age)	(2yr)
$\ln w^m$	1,089.380**	1,105.518**	1,096.719**	1,105.610**
	(450.590)	(455.006)	(436.139)	(441.196)
$\ln w^f$	574.838	589.798	617.201	630.653
	(875.231)	(881.747)	(859.036)	(865.680)
$\ln w^m \ln w^f$ (††)	-264.210*	-270.585*	-267.814*	-270.757*
	(151.476)	(152.873)	(146.559)	(148.305)
Non-labor income (††)	0.008	0.007	0.008	0.008
	(0.006)	(0.006)	(0.006)	(0.006)
Sex ratio (††):				
Age-to-age	1,187.142		121.586	
	(2,261.005)		(97.847)	
2-year-band		1,585.192		72.249
		(2,518.236)		(74.347)
Female's education	210.729***	209.617***	209.857***	209.338***
	(39.431)	(39.240)	(39.300)	(39.106)
Female's age	95.593***	95.413***	96.436***	96.679***
	(33.292)	(33.419)	(33.249)	(33.326)
Male's education	4.616	4.477	5.157	5.322
	(34.418)	(34.452)	(34.394)	(34.390)
Male's age	-24.030	-21.364	-23.447	-23.349
	(26.560)	(26.694)	(26.525)	(26.533)
No. of children < 15	-355.098***	-357.618***	-357.286***	-357.594***
	(129.531)	(129.528)	(129.411)	(129.376)
Children < 5	-578.354**	-574.808**	-581.987**	-582.713**
	(267.455)	(267.615)	(267.518)	(267.409)
Intercept	-8,138.800**	-8,559.303**	-7,371.339**	-7,373.822**
	(3,929.801)	(4,075.067)	(3,438.628)	(3,480.545)
Region dummies	Yes	Yes	Yes	Yes

Note. * p<0.10, ** p<0.05, *** p<0.01. Standard errors in parentheses. The regions are: North, Capital, Gulf, Pacific, South, Central-North, and Central.

† Estimation of equations (25)-(26), with restrictions (19)-(20) imposed in the collective model.

†† Parameter constrained in the estimation process of the collective model by imposing the restrictions (19)-(20).

Table 7 reports the estimates of the parameters of the reduced-form male labor supply function. In a working couple, the own-wage effect of the labor supply, $(a_2 + a_4 \ln w^f)/w^m$, is always negative and the cross-wage effect, $(a_3 + a_4 \ln w^m)/w^f$, is positive for a wide range of male wage rates. The former indicates a backward bending of the male labor supply, and the latter suggests that men tend to increase working hours upon a wage increase of their partner. Evidence of similar male labor supply behavior has been found for the Netherlands by Bloemen (2010) and Kapteyn, Kooreman, and van Soest (1990) when male and female labor supply is estimated simultaneously.

Comparing the unrestricted with the collective model, there is a change of sign in the effect of both sex ratios on the male labor supply; it passes from a negative effect to a positive. The constraints (19) and (20) imposed by the collective model seem to be restrictive regarding the influence of distribution factors on the male's hours worked. Nevertheless, only in the unrestricted model with the 2-year-band sex ratio the distribution factor is statistically significant at the 5% level. Regarding the control variables, only the male's education is significant, with a positive sign, in the male's labor supply.

Table 7: Parameter Estimates. Male Labor Supply †

	Unrestricted Model		Collective Model	
	(age)	(2yr)	(age)	(2yr)
$\ln w^m$	-182.662	-174.075	-185.539	-185.434
	(142.057)	(143.793)	(141.054)	(142.222)
$\ln w^f$	492.879**	517.573**	466.796**	467.926**
	(221.783)	(223.809)	(220.338)	(222.300)
$\ln w^m \ln w^f$ (††)	-65.346	-68.001	-63.438	-63.478
	(45.570)	(46.151)	(45.206)	(45.613)
Non-labor income (††)	-0.002	-0.002	-0.003	-0.003
	(0.002)	(0.002)	(0.002)	(0.002)
Sex ratio (††):				
Age-to-age	-631.514		28.800	
	(530.699)		(39.170)	
2-year-band		-1,164.464**		16.938
		(586.188)		(25.351)
Female's education	15.906	16.632	15.410	15.690
	(17.084)	(17.090)	(16.984)	(16.929)
Female's age	12.218	12.831	11.666	11.755
	(10.046)	(10.087)	(10.063)	(10.094)
Male's education	19.426**	19.736**	19.503**	19.486**
	(8.080)	(8.069)	(8.090)	(8.091)
Male's age	-7.494	-8.715	-8.159	-8.124
	(6.242)	(6.160)	(6.209)	(6.206)
No. of children < 15	-12.051	-12.624	-9.483	-9.406
	(37.455)	(37.684)	(37.466)	(37.492)
Children < 5	-67.505	-70.096	-65.831	-65.657
	(76.937)	(76.875)	(77.083)	(77.120)
Intercept	1,948.241*	2,321.786**	1,490.660	1,485.768
	(1,112.535)	(1,152.092)	(1,007.380)	(1,011.289)
s	-0.043	-0.046	-0.041	-0.040
	(0.083)	(0.084)	(0.083)	(0.083)
Region dummies	Yes	Yes	Yes	Yes

Note. * p<0.10, ** p<0.05, *** p<0.01. Standard errors in parentheses. The regions are: North, Capital, Gulf, Pacific, South, Central-North, and Central.

† Estimation of equations (25)-(26), with restrictions (19)-(20) imposed in the collective model

†† Parameter constrained in the estimation process of the collective model by imposing the restrictions (19)-(20).

The parameter estimate of s, associated with (23), the assumption of a regime switch in the male's labor supply and its continuity along the female participation frontier, is negative but not estimated precisely. Bloemen (2010), under a similar logic of the parametric specification for a sample with all possible combinations of working and non-working partners in the Netherlands, has found that the corresponding parameter for a working husband and a non-working wife is statistically significant, whereas the parameter associated with a working wife and a non-working husband is not significantly different from zero. This unsatisfactory result does not constitute a rejection of the collective approach but instead a rejection of the auxiliary assumptions of a continuous regime switch of the male labor supply function due to a change in the female's participation decision. Female non-participation in the labor market affects the working hours of her partner via her potential wage and the correlation between them ($\rho_{npf} \approx -0.53$, see Table A.2), but a non-working female partner does not involve a continuous shift in the male labor supply. The reason for the rejection of a regime switch may be that the female reservation wage tends to show little variation and is only captured by the correlation coefficient.

With respect to the nuisance parameters (Table A.2), all the standard deviations of the dependent variables are estimated precisely. Additionally, the only correlations that are statistically significant at the 10% level are those between the female's participation equation and the male's labor supply when she does not work (negative), and the female's participation equation and the expenditures on children (positive). These findings suggest that unobserved variables that influence women's decision to participate in the labor market are negatively correlated with those that influence men's hours worked, and positively with the expenditures on children.

Although the effect of some important variables is quite precisely measured, the limited number of significant parameters can be explained, at least partially, by the small size of the sample.

6.1 Structural model parameters

The estimates presented in Tables 5-7, by use of the expressions in Table 2, enable the recovery of the parameters of the (conditional) sharing rule (11) and (18) when both partners work, as well as the parameter r in (24) that allows a regime switch in the sharing rule if the female partner does not work. The parameters, presented in Table 8, turn out to be not very precisely estimated; the most significant parameter is the one related to non-labor income (both the total in specification (11) and the one that discounts the expenditures on children in specification (18)), with a *p*-value of approximately 10.3%. The parameter of non-labor income is around 0.57, indicating that couples seem to share their non-labor income such that 57% goes to man and the remaining 43% to the woman.

The marginal effect of the male and female wage rate on the sharing rule (11) is $(\alpha_2 + \alpha_4 \ln w^f)/w^m$ and $(\alpha_3 + \alpha_4 \ln w^m)/w^f$, respectively, and similarly for specification (18) using the $\tilde{\alpha}_i$ instead of α_i. The estimated parameters of the sharing rule using the age-to-age sex ratio imply that, as long as the female's hourly wage is less than approximately MXN $67, all other factors being equal, the female partner benefits, in terms of a non-labor income transfer, from an increase in the male's wage (and for rates less than approximately MXN $74 with the 2-year-band). The female's share also benefits from increases in her wage within a wide range of the male's wage rate. By way of illustration, the parameter estimates of the conditional sharing rule equation (18), with the level of expenditures on children fixed, indicate that in the collective model with the age-to-age sex ratio variable as distribution factor and at the mean wage rate of both parents, a MXN $1 increase in the male's hourly wage (MXN $2,408 annually at the mean) induces him to transfer an additional MXN $214 to the female partner. Also, an extra MXN $1,367 will be transferred to the female partner when her wage increases MXN $1 (MXN $308 annually at the mean hours worked). Hence, at the mean wage rate of both parents, part of the male's gain in labor income is transferred to his partner, whereas the female's wage increase dramatically improves her bargaining position; she is able to keep the direct gains and in addition extract a larger portion of household non-labor income devoted to private expenditures.

Table 8: Parameter Estimates of the Sharing Rule †

	Collective Model	
	(age)	(2yr)
Sharing rule (eq. (11))		
$\alpha_1(Y)$	0.561	0.565
	(0.345)	(0.346)
$\alpha_2(\ln w^m)$	-56,771.396	-57,038.836
	(50,343.536)	(50,809.781)
$\alpha_3(\ln w^f)$	-102,744.125	-103,492.947
	(85,950.793)	(86,834.318)
$\alpha_4(\ln w^m \ln w^f)$	13,196.633	13,301.720
	(10,110.434)	(10,246.393)
$\alpha_5(z)$	5,126.260	3,077.755
	(3,861.181)	(3,113.048)
Conditional Sharing Rule (eq. (18))		
$\tilde{\alpha}_1(y)$	0.573	0.576
	(0.352)	(0.353)
$\tilde{\alpha}_2(\ln w^m)$	-57,978.791	-58,220.969
	(50,030.373)	(50,502.335)
$\tilde{\alpha}_3(\ln w^f)$	-104,593.129	-105,395.732
	(86,306.474)	(87,197.639)
$\tilde{\alpha}_4(\ln w^m \ln w^f)$	13,782.912	13,885.181
	(10,041.201)	(10,176.163)
r	1.161e-07	3.253e-07
	(2.376e-07)	(6.699e-07)

Note. * p<0.10, ** p<0.05, *** p<0.01. Standard errors in parentheses.
† Estimation of equations (11) and (18) using the results from Tables 5-7 and the expressions in Table 2.

The estimate of r, associated with the assumption of a regime switch in the sharing rule and its continuity along the female's participation frontier (eq. (24)), is not significantly different from zero; the estimated values of the sharing rule's parameters are maintained when the female partner does not work. Also in Bloemen (2010) the corresponding parameter for a working woman with a non-working husband is not significantly different from zero. Although the non-participation of a female partner would have reduced overall household resources, it does not imply a shift in the resources toward her; the female's bargaining power does not seem to be affected by her non-participation in the labor market. Nevertheless, the male partner's share decreases if the wage rate of his partner increases, regardless of her labor status: the wage rate of a non-working woman may still function as a threat point.

The reason that the male labor supply and the sharing rule of a working man and his non-working female partner is not significantly different from those of a working couple may be that reservation wages of women tend to be very low and show little variation in the sample used. In this scenario, there is a negligible reduction in overall resources for the household when the woman is not working, so there is no visible response in the male partner's hours worked or in the distribution of household non-labor income.

Table 9: Parameter Estimates of the Structural Labor Supply Functions †

	Collective Model	
	(age)	(2yr)
Male labor supply function (eq. (8))		
$\psi_1(\phi^m)$	-0.004	-0.004
	(0.004)	(0.004)
$\psi_2(\ln w^m)$	-445.322	-444.640
	(446.214)	(502.561)
$\psi_3(K)$	-0.006	-0.006
	(0.005)	(0.006)
Female labor supply function (eq. (9))		
$\gamma_1(\phi^f)$	0.018	0.019
	(0.020)	(0.022)
$\gamma_2(\ln w^f)$	-1,353.463	-1,365.233
	(3,361.797)	(3,845.730)
$\gamma_3(K)$	-0.006	-0.006
	(0.014)	(0.018)
Marginal Willingness to Pay		
Male	1.310*	1.306
	(0.756)	(0.958)
Female	-0.310	-0.306
	(0.756)	(0.958)

Note. * p<0.10, ** p<0.05, *** p<0.01. Standard errors in parentheses.

† Estimation of equations (8) and (9) using the results from Tables 5-7 and the expressions in Table 3.

The parameters of the structural individual labor supply functions (8) and (9) can be computed using the expressions in Table 3. In general terms, the parameters in Table 9 are not estimated precisely. The small sample size, together with the low variation in the potential wage, can explain part of this result. Nevertheless, if the marginal willingness to pay for expenditures on children is calculated for each member ($MWP^m = \psi_3 / \psi_1$ and $MWP^f = \gamma_3 / \gamma_1$), the male partner seems to care more about the children than the female: an increase of MXN $1 in the male's share, ϕ^m , is associated with an increase of MXN $1.3 in the money spent on children; a corresponding increase in the female's share is associated with a reduction of MXN $0.3. Using the same database but considering only working couples and including home production, Sarmiento (2012) also found that when time and expenditure on children's education is evaluated, fathers care more than mothers.

7 Conclusions

The richness of collective models comes from the opportunities the framework provides for the theoretical foundations of how individuals share resources within an intragroup decision-making process such as a household. In this sense, the approach could serve as an empirical tool for understanding intrahousehold allocations, particularly when evaluating policies with a targeting purpose. However, the literature on the identification of the structural elements of household behavior in a more general case than private consumption with interior solutions is relatively recent.

This paper extends Chiappori's (1992) model of collective labor supply to bring together the decision to participate in the labor market and expenditures on public goods, such as expenditures on children. The paper unites in a single framework the work of Blundell, Chiappori, and Meghir (2005) for children and Donni (2003) for non-participation. The model generates testable restrictions on household labor supply behavior. In particular, labor supply functions have to satisfy certain structural conditions in the form of partial differential equations. Moreover, the model can recover individual preferences and the sharing rule from the observation of adult members' labor supply and expenditure on children. Identifiability when at least one of the partners works requires i) the knowledge of a distribution factor to control for the effect of public consumption on the optimal individual choice of consumption and labor supply; and ii) the explicit postulation of a unique reservation wage to identify the structure in the non-participation sets of each household member.

In an empirical application the model is estimated using Mexican nuclear households that have only children under 15 years from the MxFLS 2005-2006 wave. Specifying each partner's labor supply function, based on individual preferences, as a linear function of their own log wage rate, the sharing rule, and expenditures on children, and specifying the sharing rule and expenditures on children as linear functions of individuals' and the cross product of the couple's log wage rates, household non-labor income, and a distribution factor, the paper provides evidence on the relevance of factors that influence the couple's bargaining positions, such as the female's potential wage rate and the state-level sex ratio, and through these factors the household resource allocations. Unconstrained and constrained versions of the model are estimated.

The estimated parameters satisfy the conditions imposed by the proposed collective labor supply model. Previous studies that included a household with the presence of more than one child or pre-school children have generally rejected the restrictions implied by the collective rationality

(Fortin and Lacroix 1997; Donni 2007). As in Sarmiento (2012), we do not find evidence that empowering mothers is more beneficial to the children than empowering fathers; indeed, there is a larger increase in expenditure on children if their fathers, rather than mothers, are empowered. Cherchye, de Rock, and Vermeulen (2012) have found this unanticipated behavior in a sample of Dutch couples. Martinelli and Parker (2008) and Rubalcava et al. (2009), using different modeling and identification strategies in a sample of the poorest Mexican households, found indications that female empowerment is beneficial for children. Our results suggest that this does not generally hold, something found also by Handa et al. (2009) for the poorest households. Our results confirm that the rejection of the unitary model as found by Attanasio and Lechene (2002) for the poorest households in Mexico does hold more generally.

Another important finding is that expenditures on children and male labor supply vary significantly with the female wage even when the woman is not working. Nevertheless, the auxiliary assumptions of a continuous regime switch on the male labor supply and sharing rule functions to a change on the female participation decision are rejected; the difference between the labor supply and sharing rule functions of a working man and his non-working partner and the corresponding functions of a working couple are not statistically significant. The reservation wages of non-working female partners may be relatively low and without sufficient fluctuation.

Future research should consider household production; welfare comparisons at the individual level can be biased if household production is not taken into account. For example, the specialization of a woman in domestic activities is interpreted as an increase in her individual leisure consumption; her share of the non-labor income is interpreted as a lump-sum transfer from her partner instead of the exchange of her domestic production for market goods. Another line for future research consists of the use of a closed form for the female's shadow wage rate that accounts for rationing in the woman's hours worked. Introducing this wage into the male's labor supply function, the latter is continuous everywhere. Additionally, one can assume that the sharing rule is the same without considering the female's labor participation change. The lack of precision of the sharing rule actually indicates avenues for further empirical exploration. For instance, although the sample of households of working couple without offspring was enlarged by including households with a non-working female partner and children under 15 years of age, the imprecision of some parameters may still be due to the small sample size. In particular, the female's potential wage rate has been estimated using information from the 18% of households that have working women. Also, because extended families are common in developing countries, it would be desirable to extend the model to include the possibility of a household with more than two persons with bargaining power. In that case, a private good that was consumed by each member with power and a distribution factor that affected the distribution of power for each of those members would be needed.

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Appendix. Further Empirical Results

Table A.1 shows the parameters estimates of the female log wage rate (eq. (27)) and the cross product of the couple's log wage rates (eq. (28)) used to overcome the unobservalibility of the wages of the non-participating women in our sample. The fitted (predicted) values for these two variables are used as the potential wages in the estimation of the model formed by equations (25)-(26).

Table A.1: Parameter Estimates of female's log wage rate and the cross product of couple's log wage rates

	$\ln w^f$		$\ln w^m \ln w^f$	
	(age)	(2yr)	(age)	(2yr)
Residuals female's participation equation	-0.000***	-0.000***	-0.001***	-0.001***
	(0.000)	(0.000)	(0.000)	(0.000)
Female's education	-0.024	-0.024	-0.174	-0.173
	(0.107)	(0.106)	(0.330)	(0.329)
Female's age	-0.036	-0.036	-0.142	-0.142
	(0.035)	(0.035)	(0.108)	(0.108)
Female's education x age	0.001	0.001	-0.010	-0.010
	(0.003)	(0.003)	(0.011)	(0.011)
Unemployment rate by state	0.236**	0.236**	1.312	1.323
	(0.104)	(0.104)	(0.804)	(0.803)
$\ln w^m$	—	—	1.745*	1.767*
			(0.942)	(0.941)
$\ln w^m \times$ Female's education $\times$ age	—	—	0.005***	0.005***
			(0.002)	(0.002)
$\ln w^m \times$ Unemployment rate by state	—	—	-0.080	-0.083
			(0.180)	(0.179)
Intercept	3.883***	3.907***	7.523	7.479
	(1.413)	(1.410)	(5.377)	(5.363)
Region dummies	Yes	Yes	Yes	Yes

Note. * p<0.10, ** p<0.05, *** p<0.01. Standard errors in parentheses. The regions are: North, Capital, Gulf, Pacific, South, Central-North, and Central.

Table A.2 presents the estimates of the variance-covariance matrix Σ of the model formed by equations (25)-(26). The other parameters are presented in Tables (5)-(7).

Table A.2: Parameter Estimates. Standard deviations and correlation coefficients †

	Unrestricted Model		Collective Model	
	(age)	(2yr)	(age)	(2yr)
σ_p	798.817***	801.662***	796.530***	796.604***
	(44.682)	(44.994)	(44.418)	(44.426)
σ_{np}	848.225***	845.188***	849.784***	850.002***
	(27.838)	(27.752)	(27.913)	(27.923)
σ_f	2,423.854***	2,424.039***	2,424.987***	2,424.993***
	(152.166)	(152.208)	(152.252)	(152.254)
σ_K	5,892.423***	5,903.727***	5,892.357***	5,903.810***
	(131.837)	(132.085)	(131.833)	(132.094)
$\rho_{p,f}$	0.139	0.141	0.135	0.136
	(0.147)	(0.147)	(0.147)	(0.147)
$\rho_{p,K}$	0.081	0.082	0.083	0.082
	(0.052)	(0.052)	(0.052)	(0.052)
$\rho_{np,f}$	-0.527***	-0.521***	-0.529***	-0.530***
	(0.101)	(0.102)	(0.100)	(0.100)
$\rho_{np,K}$	0.050	0.050	0.049	0.049
	(0.041)	(0.041)	(0.041)	(0.041)
$\rho_{f,K}$	0.104***	0.104***	0.104***	0.103***
	(0.035)	(0.035)	(0.035)	(0.035)
Log-likelihood function	-20,138.027	-20,138.624	-20,138.922	-20,140.897

Note. * p<0.10, ** p<0.05, *** p<0.01. Standard errors in parentheses.

† Estimation of equations (25)-(26), with restrictions (19)-(20) imposed in the collective model