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Abstract

This paper analyses the necessary and sufficient conditions to ensure that interest rate policy does not introduce real indeterminacy and thus self-fulfilling fluctuations into open economies. A key feature of the model is the incorporation of capital and investment spending into the analysis. The conditions for real determinacy are examined for two measures of inflation that central banks' can target in open economies: domestic vs. consumer price inflation. In stark contrast to previous studies, in the presence of investment activity monetary policy that targets domestic price inflation is more susceptible to self-fulfilling fluctuations than monetary policy rules that target consumer price inflation. However, the problem of indeterminacy identified under domestic price inflation can be ameliorated provided the policy rule also responds to either the exchange rate or to output.

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1 Introduction

There is a growing body of research that considers the issue of local equilibrium determinacy for the design of interest rate rules in sticky-price, open economy models.[1] A general conclusion to emerge from this literature is that the conditions for equilibrium determinacy crucially depend on the degree of openness to international trade, provided the policy rule is forward-looking. Consequently, the necessary and sufficient condition to ensure equilibrium determinacy for a closed economy, the so-called 'Taylor Principle' (i.e. an active policy stance), may no longer be appropriate for open economies.[2] In particular, one issue that has received increasing attention relates to whether the interest rate rule should target domestic or consumer price inflation.[3] The general consensus within the literature being that domestic price inflation targeting is preferable to consumer price inflation targeting under forward-looking interest rate rules, whereas under contemporaneous rules, the indicator of inflation targeted is irrelevant for equilibrium determinacy (e.g. Linnemann and Schabert, 2006; Llosa and Tuesta, 2008).

The above analyses are based on a framework where labour is the only factor of production. However, since Dupor (2001) a number of recent closed economy studies have shown that the conditions for equilibrium determinacy change significantly once the economic environment allows for capital and investment spending. For example, Carlstrom and Fuerst (2005) show that in the presence of investment spending equilibrium 'determinacy is essentially impossible' under a forward-looking interest rate rule.[4] For current-looking interest rate rules, Carlstom and Fuerst (2005), Sveen and Weinke (2005), and Benhabib and Eusepi (2005) all find that the Taylor Principle is not a sufficient condition for determinacy, although the range of indeterminacy generated is typically small.[5]

[1] For example, Zanna (2003), Batini *et al.* (2004), De Fiore and Liu (2005), Linnemann and Schabert (2006), Llosa and Tuesta (2008), Leith and Wren-Lewis (2009), and Bullard and Schaling (2009).

[2] The Taylor Principle is a policy that raises the nominal interest rate by proportionately more than the increase in inflation. For its derivation in sticky-price, closed economy models, see for example, Kerr and King (1996), Bernanke and Woodford (1997), and Clarida *et al.* (2000).

[3] Following Carlstrom *et al.* (2006) we interpret 'targeting' as 'reacting to'.

[4] Kurozumi and Van Zandweghe (2008) show that the range of determinacy can be significantly increased if the monetary authority implements an interest rate policy that responds to both expected inflation and current output. However, this outcome requires an infinite elasticity of labour supply. As shown by Huang *et al.* (2009), with a finite, empirically plausible labour supply elasticity, a policy that responds to both expected inflation and current output helps little in ensuring determinacy.

[5] Sveen and Weinke (2005) show that the range of indeterminacy is higher if firm-specific capital is assumed, relative to the more common assumption of a competitive rental market for capital. Benhabib and Eusepi

The current paper attempts to fill this gap by deriving the conditions for local equilibrium determinacy for economies that are open to international trade. Using a discrete-time, money-in-the-utility function framework, this paper develops a two country, sticky-price model that incorporates capital and investment spending.[6] Financial markets are assumed to be complete in the sense that agents in both countries have access to a complete set of contingent claims. Price stickiness is introduced following Calvo (1983). The degree of trade openness between the two countries is proxied by the degree of domestic bias in the consumption bundle for traded goods. Monetary policy is initially characterized by an interest rate rule that reacts solely to contemporaneous inflation, where there are two alternative price indexes the policy rule can target: domestic price inflation or consumer price inflation.[7] The Aoki (1981) decomposition approach is employed to analyse the determinacy properties of the model. The Aoki decomposition decomposes the open economy into two decoupled dynamic systems: the aggregate system that captures the properties of the closed world economy and the difference system that portrays the open economy dimension.

This paper shows that in the presence of investment spending, policies consistent with equilibrium determinacy for closed economies may not preclude indeterminacy in open economies. Moreover, the conditions for local determinacy under domestic and consumer price inflation are no longer equivalent in the presence of investment activity. Therefore it matters for equilibrium determinacy whether domestic or consumer price inflation is targeted. It is shown that targeting consumer price inflation is preferable to domestic price inflation targeting in order to avoid indeterminacy of equilibrium. Therefore, in contrast to the existing literature, this is one example where reacting to consumer price inflation might be important.

The intuition behind these results rests with how the degree of trade openness exacerbates the cost-channel of monetary policy which arises in sticky-price models with capital.

(2005) show that the range of parameter values that guarantee local determinacy do not necessarily guarantee global determinacy.

[6]The majority of the literature considers the issue of equilibrium determinacy using a small open economy framework, where the foreign sector is exogenously given. Similar to Batini *et al.* (2004) and Bullard and Schaling (2009), this paper instead utilizes a two country framework, where the optimizing decisions of one country can affect prices and allocations in the other country.

[7]Recall that in a closed economy, Carlstrom and Fuerst (2005) observe that determinacy is almost impossible under a forward-looking inflation rule. In McKnight (2007b), an extended working paper version of this paper, it is shown that for both inflation indicators determinacy is even less than almost impossible in the case of the open economy.

As discussed by Kurozumi and Van Zandweghe (2008) for a closed economy, under an active policy an increase in the real interest rate puts upward pressure on the expected future rental price of capital, which raises expected marginal cost. Consequently, indeterminacy can arise if the upward pressure on inflation generated through this cost channel outweighs the downward pressure on inflation generated through the standard aggregate demand channel of monetary policy. Allowing for trade openness exacerbates the upward pressure on expected marginal cost, since an increase in expected future inflation generates a current deterioration in the terms of trade and thus upward pressure on real marginal cost and current inflation. Consequently, in the open economy it is more likely that this cost channel can dominate the demand channel making a rise in domestic price inflation self-fulfilling. In contrast, since consumer price inflation depends on both domestic price inflation and the terms of trade, the upward pressure exerted on the latter by this cost channel of monetary policy can be more than offset by the downward pressure exerted on the terms of trade through the trade channel of monetary policy, thereby exerting downward pressure on the CPI inflation rate. This makes indeterminacy less likely if the policy rule reacts to consumer price inflation.

The contribution of this paper is to yield new insights into what indicator of inflation central banks should target in open economies. In this analysis we find that targeting consumer price inflation is preferable to domestic price inflation targeting in preventing self-fulfilling expectations. This is in stark contrast to the existing literature where the measure of inflation targeted is either deemed irrelevant for local determinacy under contemporaneous policy rules,[8] or domestic price inflation targeting is considered preferable to consumer price inflation under forward-looking policy rule specifications.[9] While this analysis gives one example where reacting to consumer price inflation can be deemed preferable to domestic price inflation, there are other factors other than the equilibrium determinacy criterion that influence the specification of the policy rule. For example, Clarida *et al.* (2002) show that for open economies the optimal monetary policy is to target domestic price inflation. Hence we outline alternative contemporaneous policy rules

[8] Linnemann and Schabert (2006) and Llosa and Tuesta (2008), both support Carlstrom *et al.* (2006) closed economy findings that the criteria for equilibrium determinacy does not imply a preference to any particular inflation indicator.

[9] For example, Batini *et al.* (2004), Linnemann and Schabert (2006), and Llosa and Tuesta (2008) all conclude that domestic price inflation targeting is preferable to consumer price inflation targeting.

specifications that can help overcome the indeterminacy problem with domestic price inflation. We show that a domestic price inflation policy that also responds either to the real or nominal exchange rate, or to output, helps mitigate the problem of indeterminacy. Indeed, we find that indeterminacy is easily preventable under domestic price inflation provided each central bank also targets current output.

The remainder of the paper is organized as follows. Section 2 develops the two country model. Section 3 presents the necessary and sufficient conditions for real indeterminacy under domestic and consumer price inflation targeting. Section 4 investigates a number of policy rules that can ameliorate the indeterminacy problem under domestic price inflation. Finally, Section 5 concludes.

2 The Model

Consider a global economy that consists of two countries denoted home and foreign, where an asterisk denotes foreign variables. Within each country there exists a representative infinitely-lived agent, a representative final good producer, a continuum of intermediate good producing firms, and a monetary authority. The representative agent owns all domestic intermediate good producing firms, and supplies labour and capital to the production process. Intermediate firms operate under monopolistic competition and use domestic labour and capital as inputs to produce tradeable goods, which are sold to the home and foreign final good producers. The labour and rental capital markets are both assumed to be competitive. Each representative final good producer is a competitive firm that bundles domestic and imported intermediate goods into a non-tradeable final good, which is consumed and used for investment by the domestic agent. Preferences and technologies are symmetric across the two countries. The following presents the features of the model for the home country on the understanding that the foreign case can be analogously derived.

2.1 Final Good Producers

The home final good (Z) is produced by a competitive firm that uses Z_H and Z_F as inputs according to the following CES aggregation technology index:

$$Z_t = \left[a^{\frac{1}{\theta}} Z_{H,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} Z_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where the constant elasticity of substitution between aggregate home and foreign intermediate goods is $\theta > 0$ and the relative share of domestic and imported intermediate inputs used in the production process is $0.5 < a < 1$.[10] Thus $1-a$ captures the degree of trade openness. The inputs Z_H and Z_F are defined as the quantity indices of domestic and imported intermediate goods respectively:

$$Z_{H,t} = \left[\int_0^1 z_{H,t}(i)^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}}, \qquad Z_{F,t} = \left[\int_0^1 z_{F,t}(j)^{\frac{\lambda-1}{\lambda}} dj \right]^{\frac{\lambda}{\lambda-1}},$$

where the elasticity of substitution across domestic (foreign) intermediate goods is $\lambda > 1$, and $z_H(i)$ and $z_f(j)$ are the respective quantities of the domestic and imported type i and j intermediate goods. Let $p_H(i)$ and $p_F(j)$ represent the respective prices of these goods in home currency. Cost minimization in final good production yields the aggregate demand conditions for home and foreign goods:

$$Z_{H,t} = a \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} Z_t, \qquad Z_{F,t} = (1-a) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} Z_t, \tag{2}$$

where the demand for individual goods is given by

$$z_{H,t}(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\lambda} Z_{H,t}, \qquad z_{F,t}(j) = \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\lambda} Z_{F,t}. \tag{3}$$

Furthermore, since the final good producer is competitive, its price is set equal to marginal cost

$$P_t = \left[a P_{H,t}^{1-\theta} + (1-a) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{4}$$

[10] The analysis only considers this empirically relevant home bias case. See De Fiore and Liu (2005) and Wang (2008) for further details.

where P is the consumer price index and P_H and P_F are the respective price indices of home and foreign intermediate goods, all denominated in home currency:

$$P_{H,t} = \left[\int_0^1 p_{H,t}(i)^{1-\lambda} di \right]^{\frac{1}{1-\lambda}}, \qquad P_{F,t} = \left[\int_0^1 p_{F,t}(j)^{1-\lambda} dj \right]^{\frac{1}{1-\lambda}}.$$

We assume that there are no costs to trade between the two countries and the law of one price holds, which implies that

$$P_{Ht} = e_t P^*_{Ht}, \qquad P^*_{Ft} = \frac{P_{Ft}}{e_t}, \tag{5}$$

where e is the nominal exchange rate. Letting $Q = \frac{eP^*}{P}$ denote the real exchange rate, under the law of one price, the CPI index (4) and its foreign equivalent imply:

$$\left(\frac{1}{Q_t}\right)^{1-\theta} = \left(\frac{P_t}{e_t P^*_t}\right)^{1-\theta} = \frac{aP_{H,t}^{1-\theta} + (1-a)\left(e_t P^*_{F,t}\right)^{1-\theta}}{a\left(e_t P^*_{F,t}\right)^{1-\theta} + (1-a)P_{H,t}^{1-\theta}} \tag{6}$$

and hence the purchasing power parity condition is satisfied only in the absence of any bias between home and foreign intermediate goods (i.e. $a = 0.5$). The relative price T, the terms of trade, is defined as $T \equiv \frac{eP^*_F}{P_H}$.

2.2 Intermediate Goods Producers

Intermediate firms hire labour and rent capital to produce output given a (real) wage rate w_t and capital rental cost rr_t. A firm of type i has a production technology:

$$y_t(i) = K_t(i)^\alpha L_t(i)^{1-\alpha}, \tag{7}$$

where K and L represent capital and labour usage respectively, and the input share is $0 < \alpha < 1$. Given competitive prices of labour and capital, cost-minimization yields:

$$w_t = mc_t(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)\left(\frac{K_t(i)}{L_t(i)}\right)^\alpha, \tag{8}$$

$$rr_t = mc_t\alpha\left(\frac{P_{H,t}}{P_t}\right)\left(\frac{L_t(i)}{K_t(i)}\right)^{1-\alpha}, \tag{9}$$

where $mc_t \equiv \frac{MC_t}{P_{H,t}}$ is real marginal cost.

Firms set prices according to Calvo (1983), where in each period there is a constant probability $1-\psi$ that a firm will be randomly selected to adjust its price, which is drawn independently of past history. A domestic firm i, faced with changing its price at time t, has to choose $p_{H,t}(i)$ to maximize its expected discounted value of profits, taking as given the indexes P, P_H, P_F, Z and Z^*:[11]

$$\max_{p_{H,t}(i)} \sum_{s=0}^{\infty} (\beta\psi)^s X_{t,t+s} \left\{ \left[p_{H,t}(i) - MC_{t+s}(i)\right] \left[z_{H,t+s}(i) + z^*_{H,t+s}(i)\right] \right\}, \tag{10}$$

where

$$z_{H,t+s}(i) + z^*_{H,t+s}(i) \equiv \left(\frac{p_{H,t}(i)}{P_{H,t+s}}\right)^{-\lambda} \left[Z_{H,t+s} + Z^*_{H,t+s}\right]$$

and the firm's discount factor is $\beta^s X_{t,t+s} = \beta^s [U_c(C_{t+s})/U_c(C_t)][P_t/P_{t+s}]$.[12] Firms that are given the opportunity to change their price, at a particular time, all behave in an identical manner. The first-order condition to the firm's maximization problem yields

$$\widetilde{P}_{H,t} = \frac{\lambda}{\lambda-1} \sum_{s=0}^{\infty} q_{t,t+s} MC_{t+s}. \tag{11}$$

The optimal price set by a domestic home firm $\widetilde{P}_{H,t}$ is a mark-up $\frac{\lambda}{\lambda-1}$ over a weighted average of future nominal marginal costs, where the weight $q_{t,t+s}$ is given by

$$q_{t,t+s} = \frac{(\beta\psi)^s X_{t,t+s} P^{\lambda}_{H,t+s} \left(Z_{H,t+s} + Z^*_{H,t+s}\right)}{\sum_{s=0}^{\infty} (\beta\psi)^s X_{t,t+s} P^{\lambda}_{H,t+s} \left(Z_{H,t+s} + Z^*_{H,t+s}\right)}.$$

Since all prices have the same probability of being changed, with a large number of firms, the evolution of the price sub-indexes is given by

$$P^{1-\lambda}_{H,t} = \psi P^{1-\lambda}_{H,t-1} + (1-\psi)\widetilde{P}^{1-\lambda}_{H,t}, \tag{12}$$

since the law of large numbers implies that $1-\psi$ is also the proportion of firms that adjust their price each period.

[11] While the demand for a firm's good is affected by its pricing decision $p_{H,t}(i)$, each producer is small with respect to the overall market.

[12] Under the assumption that all firms are owned by the representative agent, this implies that the firm's discount factor is equivalent to the agent's intertemporal marginal rate of substitution.

2.3 Representative Agent

The representative agent is infinitely lived with preferences over consumption C, domestic real money balances M/P, and leisure $1-L$ given by:[13]

$$\sum_{t=0}^{\infty} \beta^t U\left[C_t, \frac{M_t}{P_t}, 1-L_t\right], \tag{13}$$

where the discount factor is $0<\beta<1$. For analytical simplicity we assume that the period utility function is separable among its three arguments and the labour supply elasticity is infinite.

The representative agent owns the capital stock K and makes all investment decisions I according to the law of motion

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{14}$$

where $0<\delta<1$ is the depreciation rate of capital. The representative agent carries M_{t-1} units of money and B_t nominal bonds into period t. Before proceeding to the goods market, the agent visits the financial market where a state contingent nominal bond B_{t+1} can be purchased that pays one unit of domestic currency in period $t+1$ when a specific state is realized at a period t price $\Gamma_{t,t+1}$. Letting R_t denote the gross nominal yield on a one-period discount bond, then in the absence of uncertainty, $R_t^{-1} \equiv \Gamma_{t,t+1}$. During period t the agent supplies labour and capital to the intermediate good producing firms, receiving real income from wages w_t, a rental return on capital rr_t, nominal profits from the ownership of domestic intermediate firms Π_t and a lump-sum nominal transfer Υ_t from the monetary authority. Hence the period budget constraint is given by

$$\Gamma_{t,t+1}B_{t+1} + M_t + P_t\left(C_t + I_t\right) \leq B_t + M_{t-1} + P_t w_t L_t + P_t rr_t K_t + \int_0^1 \Pi_t d(h) - \Upsilon_t. \tag{15}$$

[13] To facilitate comparison with the existing literature, this paper adopts the traditional convention that end-of-period money balances enter the utility function. This is equivalent to a cashless economy framework provided the utility function is separable between consumption and real money balances. Assuming an alternative timing-assumption on money would have important consequences for equilibrium determinacy, as discussed by Carlstrom and Fuerst (2001), Kurozumi (2006) and McKnight (2007a).

The first-order conditions from the home agent's maximization problem yield:

$$\beta R_t \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{P_t}{P_{t+1}} = 1 \tag{16}$$

$$\frac{U_L(L_t)}{U_c(C_t)} = w_t \tag{17}$$

$$U_c(C_t) = \beta U_c(C_{t+1}) \left[rr_{t+1} + (1 - \delta) \right] \tag{18}$$

$$\frac{U_m(m_t)}{U_c(C_t)} = \frac{R_t - 1}{R_t}. \tag{19}$$

Equation (16) is the consumption Euler equation for the holdings of domestic bonds, which must hold for each possible state and the money demand equation is given by (19). Equations (17) and (18) are the respective labour supply and optimal investment conditions. Optimizing behavior implies that the budget constraint (15) holds with equality in each period and the appropriate transversality condition is satisfied. Analogous conditions apply to the foreign agent.

From the first-order conditions for the home and foreign agent, the following risk-sharing conditions can be derived:

$$R_t = R_t^* \frac{e_{t+1}}{e_t} \tag{20}$$

$$Q_t = q_0 \frac{U_c(C_t^*)}{U_c(C_t)} \tag{21}$$

where the constant $q_0 = Q_0 \left[\frac{U_c(C_0)}{U_c(C_0^*)} \right]$. Equation (20) is the standard uncovered interest rate parity condition,[14] and equation (21) is the risk sharing condition associated with complete asset markets, which equates the real exchange rate Q with the marginal utilities of consumption.

2.4 Monetary Authority

Monetary policy is specified as a Taylor rule in which the nominal interest rate is a function of current inflation. The monetary authority can adjust the nominal interest

[14] With the exception of Zanna (2003) the existing literature has analysed the issue of equilibrium determinacy in open economy models where the uncovered interest parity condition (20) holds. While this condition is rejected by empirical studies (e.g. see Lewis, 1995 and the references within), for analytical tractability and comparative purposes we follow the existing literature.

rate in response to changes in domestic price inflation π_t^h or to changes in consumer price inflation π_t, according to the rules:

$$R_t = \mu\left(\pi_t^h\right) = \overline{R}\left(\frac{\pi_t^h}{\overline{\pi}^h}\right)^{\mu}, \tag{22}$$

$$R_t = \mu\left(\pi_t\right) = \overline{R}\left(\frac{\pi_t}{\overline{\pi}}\right)^{\mu}, \tag{23}$$

where $\overline{R} > 1$ and $\mu \geq 0$.

2.5 Market Clearing and Equilibrium

Market clearing for the home goods market requires

$$Z_{H,t} + Z_{H,t}^{*} = Y_t. \tag{24}$$

Total home demand must equal the supply of the final good,

$$Z_t = C_t + I_t, \tag{25}$$

and the labour, capital, money and bond markets all clear:

$$\Upsilon_t = M_t - M_{t-1} \qquad B_t + B_t^{*} = 0. \tag{26}$$

Definition 1 **(Perfect Foresight Equilibrium)**: Given an initial allocation of B_{t_0}, $B_{t_0}^{*}$, K_{t_0}, $K_{t_0}^{*}$, and M_{t_0-1}, $M_{t_0-1}^{*}$, a perfect foresight equilibrium is a set of sequences $\{C_t$, C_t^{*}, M_t, M_t^{*}, L_t, L_t^{*}, K_t, K_t^{*}, I_t, I_t^{*}, B_t, B_t^{*}, R_t, R_t^{*}, MC_t, MC_t^{*}, w_t, w_t^{*}, rr_t, rr_t^{*}, Y_t, Y_t^{*}, e_t, Q_t, P_t, P_t^{*}, $P_{H,t}$, $\widetilde{P_{H,t}}$, $\widetilde{P_{F,t}^{*}}$, $P_{H,t}^{*}$, $P_{F,t}$, $P_{F,t}^{*}$, Z_t, Z_t^{*}, $Z_{H,t}$, $Z_{F,t}$, $Z_{H,t}^{*}$, $Z_{F,t}^{*}\}$ for all $t \geq t_0$ characterized by: (i) the optimality conditions of the representative agent, (16) to (19), and the capital accumulation equation (14); (ii) the intermediate firms' first-order conditions (8) and (9), price-setting rules, (11) and (12), and the aggregate version of the production function (7); (iii) the final good producer's optimality conditions, (2), and (4); (iv) all markets clear, (24) to (26); (v) the representative agent's budget constraint (15) is satisfied and the transversality conditions hold; (vi) the monetary policy rule is satisfied,

(22) or (23); along with the foreign counterparts for (i)-(vi) and conditions (5), (6), (20) and (21).

2.6 Local Equilibrium Dynamics

In order to analyse the equilibrium dynamics of the model, a first-order Taylor approximation is taken around the steady state. To be precise the model is linearized around a symmetric steady state in which inflation is zero ($\overline{\pi} = \overline{\pi}^* = 1$) and prices in the two countries are equal ($\overline{P}_H = \overline{P}_F = \overline{P} = \overline{P}^* = \overline{P}^*_H = \overline{P}^*_F$). Then by definition the steady state terms of trade and nominal and real exchange rates are $\overline{T} = \overline{e} = \overline{Q} = 1$. In what

Table 1: Linearized system of equations

Cross-Country Differences	
$\widehat{C}^R_{t+1} = \widehat{C}^R_t + \sigma \widehat{R}^R_t - \sigma \widehat{\pi}^R_{t+1}$	IS^R
$\widehat{R}^R_t = \Delta \widehat{e}_{t+1}$	UIP
$\widehat{\pi}^{R(h-f^*)}_t = \kappa \widehat{mc}^R_t + \beta \widehat{\pi}^{R(h-f^*)}_{t+1}$	AS^R
$\widehat{mc}^R_t = \frac{\widehat{C}^R_t}{\sigma} + \alpha \widehat{x}^R_t + 2(1-a)\widehat{T}_t$	Marginal cost
$\widehat{C}^R_{t+1} = \widehat{C}^R_t + \sigma \Lambda_1 \left[\widehat{mc}^R_{t+1} + (1-\alpha)\widehat{x}^R_{t+1} - 2(1-a)\widehat{T}_{t+1} \right]$	Investment
$\widehat{K}^R_{t+1} = \frac{(1-\alpha)}{(2a-1)} \left(\frac{\overline{Z}}{\overline{K}} \right) \widehat{x}^R_t - \left(\frac{\overline{C}}{\overline{K}} \right) \widehat{C}^R_t - \frac{4\theta a(1-a)}{(2a-1)} \left(\frac{\overline{Z}}{\overline{K}} \right) \widehat{T}_t + \Lambda_2 \widehat{K}^R_t$	Resource constraint
$\widehat{R}^R_t = \mu \widehat{\pi}^R_t$ *or* $\widehat{R}^R_t = \mu \widehat{\pi}^{R(h-f^*)}_t$	Taylor rule
$\widehat{\pi}^R_t = (2a-1)\widehat{\pi}^{R(h-f^*)}_t + 2(1-a)\Delta \widehat{e}_t$	Inflation
$\widehat{Q}_t = \frac{1}{\sigma}\widehat{C}^R_t = (2a-1)\widehat{T}_t$	RER
World Aggregates	
$\widehat{C}^W_{t+1} = \widehat{C}^W_t + \sigma \widehat{R}^W_t - \sigma \widehat{\pi}^W_{t+1}$	IS^W
$\widehat{\pi}^W_t = \kappa \widehat{mc}^W_t + \beta \widehat{\pi}^W_{t+1}$	AS^W
$\widehat{mc}^W_t = \frac{\widehat{C}^W_t}{\sigma} + \alpha \widehat{x}^W_t$	Marginal cost
$\widehat{C}^W_{t+1} = \widehat{C}^W_t + \sigma \Lambda_1 \left[\widehat{mc}^W_{t+1} + (1-\alpha)\widehat{x}^W_{t+1} \right]$	Investment
$\widehat{K}^W_{t+1} = (1-\alpha) \left(\frac{\overline{Z}}{\overline{K}} \right) \widehat{x}^W_t - \left(\frac{\overline{C}}{\overline{K}} \right) \widehat{C}^W_t + \left[\frac{\overline{Z}}{\overline{K}} + 1 - \delta \right] \widehat{K}^W_t$	Resource constraint
$\widehat{R}^W_t = \mu \widehat{\pi}^W_t$	Taylor rule

Notes: The index R refers to the difference between home and foreign variables e.g. $\widehat{C}^R_t \equiv \left(\widehat{C}_t - \widehat{C}^*_t \right)$, $\widehat{\pi}^{R(h-f^*)}_t \equiv \left(\widehat{\pi}^h_t - \widehat{\pi}^{*f}_t \right)$. The index W refers to world aggregates where $\pi^W = \frac{\pi + \pi^*}{2} = \frac{\pi^h + \pi^{*f}}{2}$ and $\Delta \widehat{e}_t \equiv \widehat{e}_t - \widehat{e}_{t-1}$. The parameters are defined as: $\kappa \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi}$, $\Lambda_1 = 1 - \beta(1-\delta)$ and $\Lambda_2 = \left[\frac{\overline{Z}}{\overline{K}} \frac{1}{(2a-1)} + 1 - \delta \right]$ where the steady state levels are given by $\frac{\overline{Z}}{\overline{K}} = \frac{\overline{C}}{\overline{K}} + \delta$ and $\frac{\overline{C}}{\overline{K}} = \frac{1}{\alpha} \left[\frac{1}{\beta} - (1-\delta) \right] \frac{\lambda}{\lambda - 1} - \delta$.

follows, a variable $\widehat{X}_t$ denotes the percentage deviation of X_t with respect to its steady state value $\overline{X}$ $\left(\text{i.e. } \widehat{X}_t = \frac{X_t - \overline{X}}{\overline{X}}\right)$.

In order to obtain analytical conditions for local determinacy we employ the Aoki (1981) decomposition,[15] which decomposes the model into two decoupled dynamic systems: the aggregate system that captures the properties of the closed world economy and the difference system that portrays the open economy dimension. Thus, we solve both for cross-country differences $X^R \equiv \widehat{X} - \widehat{X}^*$ and worldwide aggregates $X^W \equiv \frac{\widehat{X}}{2} + \frac{\widehat{X}^*}{2}$.[16] Determinacy of the aggregate and difference systems implies determinacy at the individual country level since $\widehat{X} = X^W + \frac{X^R}{2}$ and $\widehat{X}^* = X^W - \frac{X^R}{2}$. The complete linearized system of equations is summarized in Table 1.[17] In what follows below it will also be convenient to define the labour-capital ratio $x_t \equiv \frac{L_t}{K_t}$.

Table 2: Baseline parameter values

β	Discount factor	0.99
α	Cost share of capital	0.36
δ	Depreciation rate of capital	0.025
ψ	Degree of price rigidity	0.75
κ	Real marginal cost elasticity of inflation	0.08
λ	Degree of monopolistic competition	7.66

In order to illustrate the conditions for determinacy, the ensuing analysis using the following baseline parameter values summarized in Table 2. The parameters β, α and δ are taken from Sveen and Weinke (2005) and λ from Rotemberg and Woodford (1998).[18] As noted by Benhabib and Eusepi (2005) empirical estimates of nominal rigidity find ψ to be between 0.66 and 0.83. Following Taylor (1999) we set $\psi = 0.75$ which constitutes an

[15] The Aoki decomposition requires that the steady state is symmetric and both countries follow identical policies.

[16] The determinacy conditions for the aggregate system are identical to comparable closed-economy New Keynesian models (e.g. Carlstrom and Fuerst (2005)). The measure of inflation targeted is irrelevant in the aggregate system since domestic and consumer price inflation are the same concept; i.e.

$$\pi^W = \frac{\pi + \pi^*}{2} = \frac{\pi^h + \pi^{*f}}{2}.$$

[17] The money demand equations are omitted since the remaining conditions determine local equilibrium determinacy in the absence of real balance effects.

[18] As will be shown, the analytical results for determinacy do not depend on the parameters σ, which measures the intertemporal substitution elasticity of consumption, or θ, which measures the elasticity of substitution between aggregate home and foreign goods.

average price duration of one year and this implies that the real marginal cost elasticity of inflation $\kappa = 0.08$. However, the robustness of the numerical results are examined for variations in ψ.

3 Policy Response to Current Inflation

This section considers the issue of local determinacy for monetary policy rules that react to contemporaneous inflation. A key conclusion to arise from the analysis is that the conditions for local determinacy under domestic and consumer price inflation are no longer equivalent in the presence of investment activity. Indeed, the criteria for equilibrium determinacy suggest that targeting consumer price inflation is preferable to domestic price inflation targeting. The analysis proceeds in three steps. First, the conditions for determinacy of the aggregate system (or closed economy) is examined. Second, the determinacy conditions of the difference system are examined when monetary policy reacts to domestic price inflation. Finally, we consider the determinacy conditions of the difference system when consumer price inflation is the price-index targeted.

3.1 Aggregate System

The set of linearized equations for the world aggregates, given in Table 1, can be reduced to the following four-dimensional system:

$$\mathbf{b}_{t+1}^W = \mathbf{A}^W \mathbf{b}_t^W, \qquad \mathbf{b}_t^W = \left[\widehat{mc}_t^W \ \widehat{x}_t^W \ \widehat{\pi}_t^W \ \widehat{K}_t^W\right]',$$

$$\mathbf{A}^W \equiv \begin{bmatrix} (1-\alpha) + \frac{\kappa}{\beta}\left[1 + \frac{\alpha(1-\Lambda_1)}{\Lambda_1}\right] & -\alpha(1-\alpha) & \left(\mu - \frac{1}{\beta}\right)\left[1 + \frac{\alpha(1-\Lambda_1)}{\Lambda_1}\right] & 0 \\ \frac{\kappa(1-\Lambda_1)}{\Lambda_1\beta} - 1 & \alpha & \left(\mu - \frac{1}{\beta}\right)\left[\frac{1-\Lambda_1}{\Lambda_1}\right] & 0 \\ -\frac{\kappa}{\beta} & 0 & \frac{1}{\beta} & 0 \\ -\left(\frac{\overline{C}}{\overline{K}}\right)\sigma & \frac{(1-\alpha)\overline{Z}}{\overline{K}} + \sigma\alpha\frac{\overline{C}}{\overline{K}} & 0 & 1 + \frac{\overline{C}}{\overline{K}} \end{bmatrix}.$$

Since the dynamics of mc, x, and π are independent of the capital stock dynamics, one eigenvalue of the system is $1 + \frac{\overline{C}}{\overline{K}} > 1$. Consequently, given that capital is the only predetermined variable in the column vector $\mathbf{b}^W$, equilibrium determinacy requires that two of the remaining eigenvalues of $\mathbf{A}^W$ are outside the unit circle and one eigenvalue is inside the unit circle. The Appendix proves the following:

Proposition 1 *If the policy rule reacts to contemporaneous inflation, then the necessary and sufficient conditions for local equilibrium determinacy of the aggregate system is* $\mu > 1$ *and either*

$$(2\beta - 1)\Lambda_1 < \kappa\left[1 - \beta(1-\delta)(1-\alpha)\right] \tag{27}$$

$$\text{or} \quad \frac{\mu\kappa\alpha}{\Lambda_1\beta}\left[\frac{\kappa\alpha(\mu-1)}{\Lambda_1} - 1 - \kappa(1-\alpha)\right] + 1 - \beta + \kappa\mu(1-\alpha) + \frac{\kappa\alpha}{\Lambda_1} > 0, \tag{28}$$

where $\Lambda_1 = 1 - \beta(1-\delta)$.

Proof. See Appendix A.1. □

The determinacy conditions in Proposition 1 are isomorphic to the conditions obtained by Carlstrom and Fuerst (2005) for the closed economy.[19] For the baseline parameter values, (27) is violated only if $\psi \geq 0.75$. Thus if prices are sufficiently sticky, indeterminacy can arise for some values of $\mu > 1$ provided condition (28) is violated. However the region of indeterminacy is small. For example if $\psi = 0.8$ then indeterminacy arises provided $1.1 \leq \mu < 1.71$, whereas if $\psi = 0.75$ condition (28) is satisfied $\forall \mu > 1$ and thus indeterminacy is not possible.

3.2 Difference System

3.2.1 Domestic Price Inflation

If domestic price inflation is the policy indicator, then the set of linearized conditions for cross-country differences yields a system of the form:

$$\mathbf{b}_{t+1}^R = \mathbf{A}_{PPI}^R \mathbf{b}_t^R, \quad \mathbf{b}_t^R = \left[\widehat{mc}_t^R \ \widehat{x}_t^R \ \widehat{\pi}_t^{R(h-f^*)} \ \widehat{K}_t^R\right]',$$

$$\mathbf{A}_{PPI}^R \equiv \begin{bmatrix} 1-\alpha(2a-1)+\frac{\kappa}{\beta}J_1 & \alpha^2(2a-1)-\alpha & \left(\mu-\frac{1}{\beta}\right)J_1 & 0 \\ -(2a-1)+\frac{\kappa}{\beta}J_2 & \alpha(2a-1) & \left(\mu-\frac{1}{\beta}\right)J_2 & 0 \\ -\frac{\kappa}{\beta} & 0 & \frac{1}{\beta} & 0 \\ -\left[\sigma(2a-1)\frac{\overline{C}}{\overline{K}}+\frac{4\theta a(1-a)}{(2a-1)}\frac{\overline{Z}}{\overline{K}}\right] & J_3 & 0 & 1+\frac{\frac{\overline{C}}{\overline{K}}+\delta 2(1-a)}{(2a-1)} \end{bmatrix},$$

[19] However, Carlstrom and Fuesrt (2005) do not analytically derive condition (28).

where $J_1 = \left[1 + \frac{\alpha(1-\Lambda_1)(2a-1)}{\Lambda_1}\right]$ $J_2 = \frac{(1-\Lambda_1)(2a-1)}{\Lambda_1}$ and $J_3 = \frac{(1-\alpha)}{(2a-1)}\frac{\overline{Z}}{\overline{K}} + \frac{\alpha 4\theta a(1-a)}{2a-1}\frac{\overline{Z}}{\overline{K}} + \sigma\alpha(2a-1)\frac{\overline{C}}{\overline{K}}$. By inspection , one of the eigenvalues is given by $1+\frac{1}{(2a-1)}\left[\frac{\overline{C}}{\overline{K}} + \delta 2(1-a)\right] > 1$. For determinacy, two of the remaining eigenvalues of $\mathbf{A}_{PPI}^{R}$ are required to lie outside the unit circle and one eigenvalue is inside the unit circle.

Proposition 2 *If the policy rule reacts to contemporaneous domestic price inflation, then the necessary and sufficient conditions for local equilibrium determinacy of the difference system is $\mu > 1$ and either:*

$$(2\beta - 1)\Lambda_1 < \kappa\left[1 - \beta(1-\delta)(1-(2a-1)\alpha)\right] \quad (29)$$

or

$$\frac{\mu\Lambda_3}{\Lambda_1\beta}\left[\frac{\Lambda_3(\mu-1)}{\Lambda_1} + \Lambda_3 - (1+\kappa+\Lambda_1\beta)\right] + (1-\beta) + \kappa\mu + \frac{\Lambda_3}{\Lambda_1} > 0; \quad (30)$$

where $\Lambda_1 = 1 - \beta(1-\delta)$ and $\Lambda_3 = \kappa\alpha(2a-1)$.

Proof. See Appendix A.2. □

Proposition 2 suggests that the problem of indeterminacy can be more severe in the open economy under a domestic price inflation rule than compared to a closed economy. For example, if conditions (28) and (30) are not satisfied then from direct comparison of condition (29) of Proposition 2 with condition (27) of Proposition 1, determinacy in the

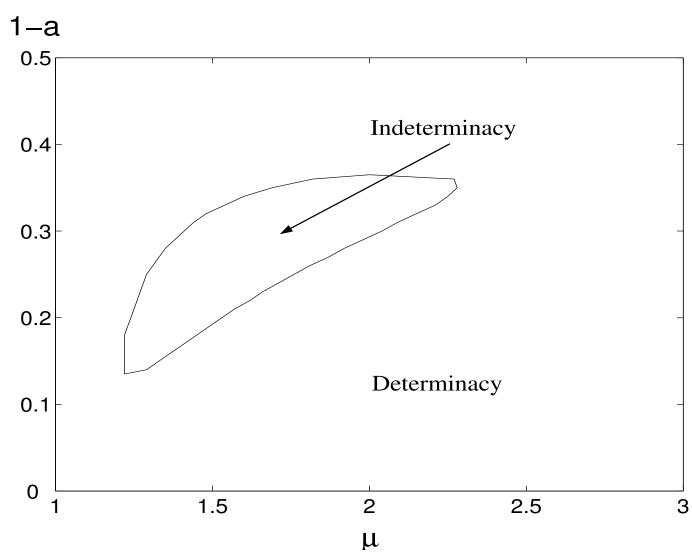

Fig. 1: Region of indeterminacy under a domestic price inflation rule ($\psi = 0.75$)

closed economy does not preclude determinacy in the open economy since:

$$(2\beta - 1)\Lambda_1 < \kappa \left[1 - \beta(1-\delta)(1-(2a-1)\alpha)\right] < \kappa \left[1 - \beta(1-\delta)(1-\alpha)\right].$$

The determinacy conditions in Proposition 2 are illustrated using the baseline parameter values summarized in Table 2. Figure 1 depicts the regions in the parameter space (a, μ) that are associated with determinacy and indeterminacy given $\psi = 0.75$. Recalling that with these parameter constellations indeterminacy is not possible in the closed economy (aggregate system) $\forall \mu > 1$, it is apparent from Fig. 1 that the range of indeterminacy is greater in the open economy relative to the closed economy. Figure 2 illustrates the regions of indeterminacy for combinations of the inflation response coefficient (μ) and the degree of price rigidity (ψ), for three alternative values of $a = 0.7, 0.8, 0.9$. The area of indeterminacy for the difference system is greater relative to the aggregate system and this area increases in magnitude as the degree of trade openness increases (i.e. as the value of a deceases). Consequently, we can conclude that under domestic price inflation the range of indeterminacy is greater in the open economy relative to the closed economy.

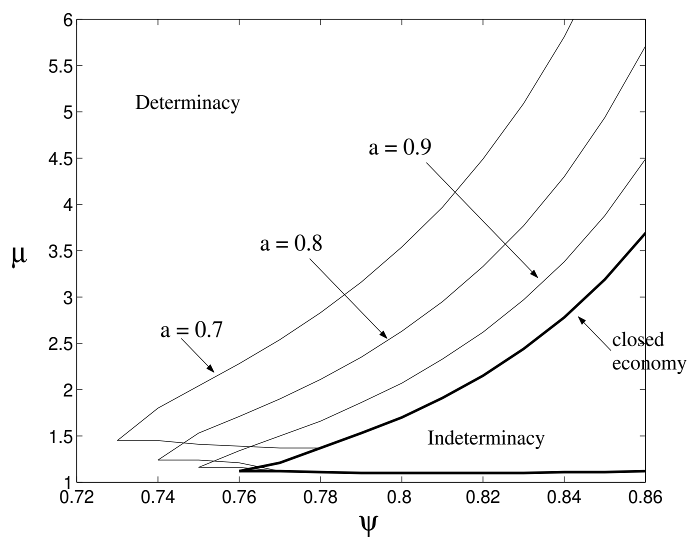

Fig. 2: Regions of indeterminacy under a domestic price inflation rule: variations in ψ

3.2.2 Consumer Price Inflation

If consumer price inflation is the policy indicator, then the set of linearized conditions for cross-country differences yields the five-dimensional system of the form:

$$\mathbf{b}_{t+1}^R = \mathbf{A}_{CPI}^R \mathbf{b}_t^R, \quad \mathbf{b}_t^R = \left[\widehat{mc}_t^R \; \widehat{x}_t^R \; \widehat{\pi}_{t-1}^R \; \widehat{\pi}_t^R \; \widehat{K}_t^R\right]', \text{ and } \mathbf{A}_{CPI}^R \equiv$$

$$\begin{bmatrix} 1-\alpha(2a-1)+\frac{\kappa(1+\alpha J_1)}{\beta} & \alpha^2(2a-1)-\alpha & J_2(1+\alpha J_1) & J_3(1+\alpha J_1) & 0 \\ -(2a-1)+\frac{\kappa}{\beta}J_1 & \alpha(2a-1) & J_2 J_1 & J_3 J_1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{\kappa(2a-1)}{\beta} & 0 & -\frac{2(1-a)\mu}{\beta} & \frac{1+2(1-a)\beta\mu}{\beta} & 0 \\ -\left[\frac{\sigma(2a-1)\overline{C}}{\overline{K}}+\frac{4\theta a(1-a)}{(2a-1)}\frac{\overline{Z}}{\overline{K}}\right] & J_4 & 0 & 0 & 1+\frac{\frac{\overline{C}}{\overline{K}}+\delta 2(1-a)}{(2a-1)} \end{bmatrix},$$

where $J_1 = \frac{(1-\Lambda_1)(2a-1)}{\Lambda_1}$, $J_2 = \frac{2(1-a)\mu}{(2a-1)\beta}$, $J_3 = \mu - \frac{1}{\beta(2a-1)}$ and $J_4 = \frac{(1-\alpha)}{(2a-1)}\frac{\overline{Z}}{\overline{K}} + \frac{\alpha 4\theta a(1-a)}{2a-1}\frac{\overline{Z}}{\overline{K}} + \sigma\alpha(2a-1)\frac{\overline{C}}{\overline{K}}$. As before, one eigenvalue is given by $1 + \frac{1}{(2a-1)}\left[\frac{\overline{C}}{\overline{K}} + \delta 2(1-a)\right] > 1$. Since there are now two predetermined variables $\widehat{K}_t^R$ and $\widehat{\pi}_{t-1}^R$, equilibrium determinacy requires that two of the remaining eigenvalues of $\mathbf{A}_{CPI}^R$ lie outside the unit circle and two eigenvalues are inside the unit circle.

Proposition 3 *If the policy rule reacts to contemporaneous consumer price inflation, then the necessary and sufficient conditions for local equilibrium determinacy of the difference system is $\mu > 1$ and either:*

$$\mu > \frac{(2\beta-1)\Lambda_1 - \kappa\left[\Lambda_1 + \alpha(1-\Lambda_1)(2a-1)\right]}{\Lambda_1 \beta 2(1-a)}, \tag{31}$$

or

$$\frac{\mu}{\beta}\left[2(1-a)+\frac{\Lambda_3}{\Lambda_1}\right]\left[2(1-a)(1-\beta)\mu + \frac{\Lambda_3[\mu-(1-\Lambda_1)]}{\Lambda_1} - (1+\kappa)\right] + (1-\beta) + \mu\left[\kappa - \Lambda_3\right] + 2(1-a)\mu\beta + \frac{\Lambda_3}{\Lambda_1} > 0; \tag{32}$$

where $\Lambda_1 = 1-\beta(1-\delta)$ and $\Lambda_3 = \kappa\alpha(2a-1)$.

Proof. See Appendix A.3. □

Proposition 3 suggests that the indeterminacy problem is less severe in the open economy if the policy rule targets consumer price inflation rather than domestic price inflation. For example, if conditions (30) and (32) are not satisfied then this conclusion follows from direct comparison of condition (29) of Proposition 2 with condition (31) of Proposition 3:

$$(2\beta - 1)\Lambda_1 < \Gamma_1 < 2\beta\Lambda_1(1-a)\mu + \Gamma_1,$$

where $\Gamma_1 \equiv \kappa\left[1 - \beta(1-\delta)(1-(2a-1)\alpha)\right]$. Indeed, given the baseline values of α, β and δ, the numerical analysis suggests that for any combination of the parameter triplets (a, ψ, μ) the open economy under consumer price inflation targeting does not introduce any additional areas of indeterminacy. To date, we have yet to find a combination of parameter values that generate determinacy in the aggregate system without also generating determinacy in the difference system. Therefore, the numerical analysis suggests that the determinacy conditions for closed and open economies are approximately analogous under a consumer price inflation rule.

3.3 Discussion

The previous subsections found that for parameter constellations consistent with equilibrium determinacy for a closed economy, preclude indeterminacy in open economies if the policy rule reacts to consumer price inflation. However, under a domestic price inflation rule the range of indeterminacy is greater in open economies relative to a closed economy. These findings are in stark contrast to recent studies that have considered the impact of trade openness for equilibrium determinacy where production is assumed to be linear in labour (i.e. $\alpha = 0$). De Fiore and Liu (2005), Linnemann and Schabert (2006) and Llosa and Tuesta (2008), using a small open economy framework, and McKnight (2007a) using a two country setup, all find that the Taylor Principle is validated under contemporaneous policy rules regardless of the magnitude of the degree of trade openness. Why does the criteria for equilibrium determinacy differ for alternative measures of inflation with the inclusion of capital? First consider a labour-only economy. In a closed economy an active monetary policy ($\mu > 1$) increases the real interest rate. From the aggregate demand channel of monetary policy this reduces (real) marginal cost such that current inflation rises by less than expected inflation via the Phillips curve. In an open economy the consumer price inflation rate depends on both the domestic price inflation rate and the terms of trade:

$$\widehat{\pi}_t = \widehat{\pi}_t^h + (1-a)\left(\widehat{T}_t - \widehat{T}_{t-1}\right), \tag{33}$$

where $\widehat{T}_{t-1}$ is predetermined. An increase in the real interest rate not only reduces domestic price inflation via a fall in marginal cost but in addition results in an improvement in the terms of trade ($\widehat{T}_t \downarrow$). From (33) this trade channel of monetary policy generates additional downward pressure on consumer price inflation. Consequently for both closed and open economies and for each inflation target, self-fulfilling inflation expectations cannot be supported.

Now consider an economy with capital accumulation. Here the initial increase in inflationary expectations can be self-fulfilling if there is a further rise in expected inflation. This can occur since an increase in the real interest rate puts upward pressure on the expected future rental price of capital (from the investment condition (18)). This leads to an increase in expected future marginal cost and thus an additional rise in expected future inflation, which via the Phillips curve, generates higher inflation today. Therefore, indeterminacy is generated if the effect of this cost channel of monetary policy is sufficiently strong to counteract the downward pressure on inflation arising from the aggregate demand channel. In the open economy an increase in expected future inflation also results in a current deterioration in the terms of trade ($\widehat{T}_t \uparrow$). This in turn puts upward pressure on current real marginal cost, which is exacerbated as the degree of trade openness increases and hence from the Phillips curve, upward pressure on domestic price inflation. Consequently, this cost channel can dominate the aggregate demand channel for a broader range of parameter constellations in the open economy than the closed economy, thus making a rise in domestic price inflation self-fulfilling. However from (33) there are two opposing effects on consumer price inflation. While the cost channel generates upward pressure in domestic price inflation and the terms of trade ($\widehat{T}_t \uparrow$), the trade channel of monetary policy puts downward pressure on the terms of trade ($\widehat{T}_t \downarrow$). Thus, since any increase in domestic price inflation can be offset by the trade channel of monetary policy, this reduces the likelihood of indeterminacy if the policy rule reacts to consumer price inflation.

4 Ameliorating the Indeterminacy Problem

In this section we consider three potential policy rules, advocated by Taylor (1993, 2001), that can help mitigate the problem of indeterminacy under domestic price inflation targeting. First, we consider policy rules that respond to either the real or nominal exchange rate, and then we consider a policy rule that responds to current output.

4.1 Policy Response to the Real Exchange Rate

Suppose the monetary authority reacts to both current domestic price inflation and the real exchange rate. In this case the linearized policy rule for the home country is given by:

$$\widehat{R}_t = \mu_\pi \widehat{\pi}_t^h + \mu_Q \widehat{Q}_t \tag{34}$$

where $\mu_Q > 0$.[20] Consequently, the set of linearized conditions for cross-country differences yields a system of the form:[21]

$$\mathbf{b}_{t+1}^R = \mathbf{A}_{PPI}^R \mathbf{b}_t^R, \quad \mathbf{b}_t^R = \left[\widehat{mc}_t^R \ \widehat{x}_t^R \ \widehat{\pi}_t^{R(h-f^*)} \ \widehat{K}_t^R\right]', \ \mathbf{A}_{PPI}^R \equiv$$

$$\begin{bmatrix} 1-\alpha(2a-1)+\left[\frac{\kappa}{\beta}+J_3\right]J_1 & \alpha^2(2a-1)-J_1J_3 & \left(\mu_\pi-\frac{1}{\beta}\right)J_1 & 0 \\ -(2a-1)+\left[\frac{\kappa}{\beta}+J_3\right]J_2 & \alpha(2a-1)[1-2\mu_Q J_2] & \left(\mu_\pi-\frac{1}{\beta}\right)J_2 & 0 \\ -\frac{\kappa}{\beta} & 0 & \frac{1}{\beta} & 0 \\ -\left[\sigma(2a-1)\frac{\overline{C}}{\overline{K}}+\frac{4\theta a(1-a)}{(2a-1)}\frac{\overline{Z}}{\overline{K}}\right] & J_4 & 0 & 1+\frac{\frac{\overline{C}}{\overline{K}}+\delta 2(1-a)}{(2a-1)} \end{bmatrix},$$

where $J_1 = \left[1+\frac{\alpha(1-\Lambda_1)(2a-1)}{\Lambda_1}\right]$, $J_2 = \frac{(1-\Lambda_1)(2a-1)}{\Lambda_1}$, $J_3 = 2(2a-1)\mu_Q$ and $J_4 = \frac{(1-\alpha)}{(2a-1)}\frac{\overline{Z}}{\overline{K}} + \frac{\alpha 4\theta a(1-a)}{2a-1}\frac{\overline{Z}}{\overline{K}} + \sigma\alpha(2a-1)\frac{\overline{C}}{\overline{K}}$. By inspection, one eigenvalue is given by $1+\frac{1}{(2a-1)}\left[\frac{\overline{C}}{\overline{K}}+\delta 2(1-a)\right] > 1$. For determinacy, two of the remaining eigenvalues of $\mathbf{A}_{PPI}^R$ are required to lie outside the unit circle and one eigenvalue must lie inside the unit circle.

Proposition 4 *If the policy rule reacts to contemporaneous domestic price inflation and the real exchange, then for an active monetary stance ($\mu > 1$) the necessary and sufficient*

[20] For the foreign country's rule, the response coefficient for the real exchange rate is the negative of that for the home country.

[21] Note that the aggregate system presented in Section 3.1. is unchanged under (34). Hence, the determinacy conditions of the aggregate system summarized in Proposition 1 are still appropriate.

condition for local equilibrium determinacy of the difference system requires either:

$$(2\beta-1)\Lambda_1 < \kappa\left[1-\beta(1-\delta)(1-(2a-1)\alpha)\right] + 2\beta(2a-1)\Lambda_1\mu_Q \tag{35}$$

or

$$\frac{\mu_\pi\Lambda_3}{\Lambda_1\beta}\left[\frac{\Lambda_3(\mu_\pi-1)}{\Lambda_1} + \Lambda_3 - (1+\kappa+\Lambda_1\beta)\right] + (1-\beta) + \kappa\mu_\pi + \frac{\Lambda_3}{\Lambda_1} + 2(2a-1)\mu_Q\left[1-\frac{\Lambda_3\mu_\pi}{\Lambda_1}\right] > 0; \tag{36}$$

where $\Lambda_1 = 1-\beta(1-\delta)$ *and* $\Lambda_3 = \kappa\alpha(2a-1)$.

Proof. See Appendix A.4. □

Proposition 4 suggests that the indeterminacy problem associated with domestic inflation targeting can be mitigated if the policy rule also responds to the real exchange rate. For example, provided conditions (30) and (36) are not satisfied, then this directly follows from comparing condition (35) of Proposition 4 with condition (29) of Proposition 2:

$$(2\beta-1)\Lambda_1 < \kappa\left[1-\beta(1-\delta)(1-(2a-1)\alpha)\right] < \kappa\left[1-\beta(1-\delta)(1-(2a-1)\alpha)\right] + 2\beta(2a-1)\Lambda_1\mu_Q.$$

The determinacy conditions summarized in Proposition 4 are illustrated using the baseline parameter values for a degree of trade openness $a = 0.7$. Figure 3 illustrates the regions of indeterminacy for combinations of the inflation response coefficient (μ_π) and the degree

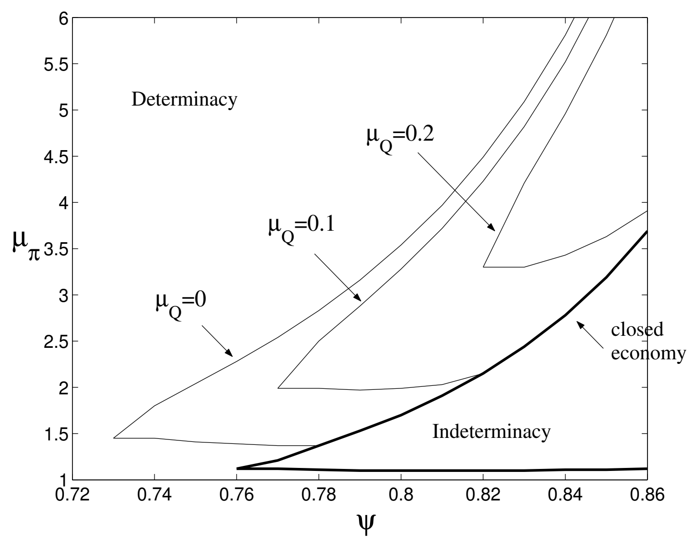

Fig. 3: Regions of indeterminacy when reacting to the real exchange rate ($a = 0.7$)

of price rigidity (ψ) for three alternative values of the real exchange rate coefficient $\mu_Q = 0, 0.1, 0.2$. If the monetary authority does not respond to the real exchange rate ($\mu_Q = 0$) then the area of indeterminacy for the difference system is substantially greater relative to the aggregate system. However this area of indeterminacy significantly decreases in magnitude as the value of μ_Q increases. Indeed for our baseline calibration, there are no additional regions of indeterminacy in the open-economy for any $\psi \leq 0.86$ if $\mu_Q = 0.3$.

Why is the indeterminacy problem under domestic price inflation targeting mitigated if policy also reacts to the real exchange rate? Recall that indeterminacy is generated in the presence of investment activity provided that the cost channel of monetary policy, which puts upward pressure on inflation, outweighs the downward pressure on inflation arising from the aggregate demand channel. From the linearized version of the risk sharing condition (21), the real exchange rate moves in tandem with relative consumption: $\widehat{Q}_t = \frac{1}{\sigma}\left(\widehat{C}_t - \widehat{C}_t^*\right)$. Hence by also reacting to the real exchange rate, an increase in the real interest rate yields additional downward pressure on real marginal cost thereby strengthening the aggregate demand channel and making self-fulfilling inflation expectations less likely.

4.2 Policy Response to the Nominal Exchange Rate

Now suppose the monetary authority reacts to both current domestic price inflation and to changes in the nominal exchange rate. In this case the linearized policy rule for the home country is given by:

$$\widehat{R}_t = \mu_\pi \widehat{\pi}_t^h + \mu_e \Delta \widehat{e}_t \tag{37}$$

where $\mu_e > 0$.[22] Under this policy specification the set of linearized conditions for cross-country differences yields a five-dimensional system: $\mathbf{b}_{t+1}^R = \mathbf{A}_{PPI}^R \mathbf{b}_t^R$, where $\mathbf{b}_t^R = \left[\widehat{mc}_t^R \ \widehat{x}_t^R \ \widehat{\pi}_t^{R(h-f^*)} \ \widehat{K}_t^R \ \widehat{R}_{t-1}^R\right]'$. One eigenvalue is outside the unit circle and another is zero. Since there are two predetermined variables, $\widehat{K}_t^R$ and $\widehat{R}_{t-1}^R$, equilibrium determinacy requires that two of the remaining eigenvalues of $\mathbf{A}_{PPI}^R$ lie outside the unit circle and one eigenvalue must lie inside the unit circle. The determinacy analysis is carried out using a numerical investigation for the baseline parameter values. Setting the degree

[22]For the foreign country's rule, the response coefficient for the nominal exchange rate is the negative of that for the home country.

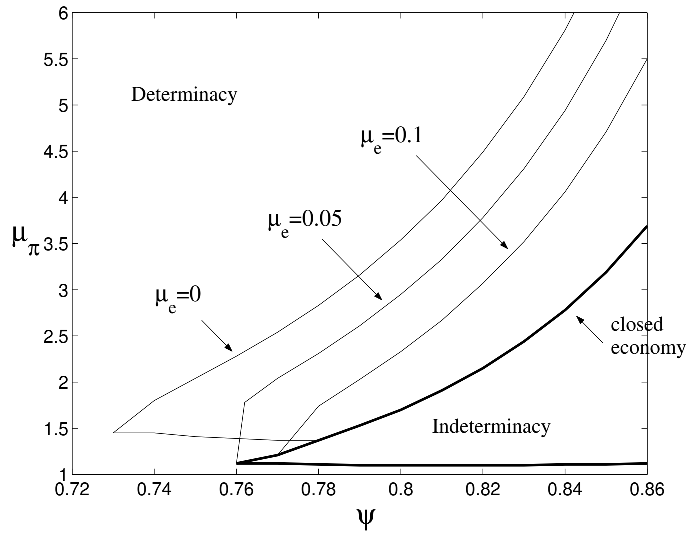

Fig. 4: Regions of indeterminacy when reacting to the nominal exchange rate ($a = 0.7$)

of trade openness $a = 0.7$, Fig. 4 illustrates the regions of indeterminacy for three alternative values of the nominal exchange rate coefficient $\mu_e = 0, 0.05, 0.1$. By inspection of Fig. 4, the regions of indeterminacy decrease significantly as the value of μ_e increases. Indeed, for our baseline calibration there are no additional regions of indeterminacy in the open-economy for any $\psi \leq 0.86$ if $\mu_e = 0.18$.

Why does reacting to the nominal exchange rate help induce equilibrium determinacy? The inclusion of the nominal exchange rate in the policy rule in effect means a policy response to the lagged interest rate and consequently currently-looking policy now also responds to past domestic price inflation. By using the linearized version of interest parity condition (20), the policy rule (37) and its foreign equivalent, the Taylor rule for the difference system can be expressed as: $\widehat{R}_t^R = \mu_\pi^R \widehat{\pi}_t^{(h-f^*)} + 2\mu_e \widehat{R}_{t-1}^R$. Hence by reacting to the nominal exchange rate this introduces policy inertia or interest-rate smoothing into the dynamic system, which it is now well established from the closed-economy literature that policy inertia helps in preventing equilibrium indeterminacy (e.g. Woodford, 2003; Kurozumi and Zandweghe, 2008).

4.3 Policy Response to Current Output

Now suppose that the monetary authority reacts to both current domestic price inflation and current output. In this case the linearized policy rule for the home country is given by:

$$\widehat{R}_t = \mu_\pi \widehat{\pi}_t^h + \mu_y \widehat{Y}_t \tag{38}$$

where $\mu_\pi, \mu_y > 0$. Under this policy specification the characteristic equations for both the aggregate and difference systems are fourth-order polynomials. Given the analytical complexity in obtaining the necessary and sufficient conditions for this case we alternatively carry out a numerical investigation. We use the baseline parameter values given in Table 2 and set the degree of trade openness $a = 0.7$. With (38) the capital stock dynamics can no longer be decoupled from the rest of the system. Hence we need to set values for the intertemporal substitution elasticity of consumption, σ, and the elasticity of substitution between aggregate home and foreign goods, θ. We set $\sigma = 2$ and $\theta = 1$ which are consistent with Gali *et al.* (2007) and Trefler and Lai (1999) respectively.[23]

Figure 5 illustrates the regions of indeterminacy for combinations of the inflation response coefficient (μ_π) and the degree of price rigidity (ψ) for two alternative values of the output coefficient $\mu_y = 0, 0.02$. The top half of Fig. 5 illustrates the indeterminacy regions for the closed economy and the bottom half illustrates the open economy case when $a = 0.7$. By inspection of Fig. 5, the area of indeterminacy is relatively greater in the open economy than the closed economy under (38). Furthermore, by responding slightly to output when $\mu_\pi > 1$, substantially reduces the indeterminacy problem in both economies. Indeed, with a very small response to output, the numerical analysis suggests that indeterminacy can be eliminated $\forall \mu_\pi > 1$ in both the closed and open economy. For example, if $\psi = 0.8$ then indeterminacy is eliminated if $\mu_y \geq 0.013$ for the closed economy. For the open economy indeterminacy is eliminated if $\mu_y \geq 0.034$ when $a = 0.7$; $\mu_y \geq 0.027$ when $a = 0.8$ and $\mu_y \geq 0.019$ when $a = 0.9$. Thus indeterminacy is easily preventable! All that is required is that the monetary authority targets output slightly more aggressively than what would be needed for a closed economy.

[23] Note that the sensitivity analysis suggests that while changes in σ and θ generate different quantitative results, the qualitative conclusions are the same.

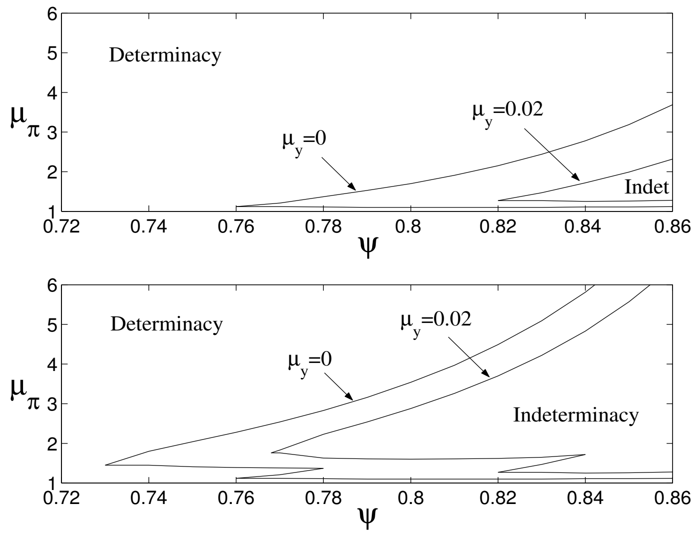

Fig. 5: Regions of indeterminacy when reacting to output: closed vs open economy

5 Conclusion

Using a two country, sticky-price model, this paper has examined the role of trade openness in generating indeterminate equilibria when monetary policy is characterized by an interest rate rule that can respond to either domestic or consumer price inflation. Recent studies have found that the determinacy conditions for closed and open economies are analogous if the policy rule is contemporaneous. Only in the case of forecast-based policy rules does the index of inflation targeted appear to matter for equilibrium determinacy, in which case domestic price inflation is deemed to be preferable to consumer price inflation. However, the existing literature has ignored the role of capital and investment spending. This paper has demonstrated that a policy rule that responds to domestic price inflation increases the potential range of indeterminacy in comparison to consumer price inflation. However, we have highlighted three prescriptions for the indeterminacy problem under domestic price inflation. By either responding to the real or nominal exchange rate, or to current output, can help to prevent self-fulfilling inflation expectations and hence indeterminacy.

One empirically unappealing feature of our analysis is the absence of adjustment costs to capital and investment. However, Carlstrom and Fuerst (2005) find for a closed economy that adding capital adjustment costs quantitatively affects the range of indeterminacy only if these adjustments costs are unrealistically high. One potential area of future research is to examine the impact of capital adjustment costs for equilibrium determinacy in open economies to check the validity of the results presented here. In addition, this paper has assumed throughout that both fundamental preferences and policy are identical in both countries. However, in a model without investment, Bullard and Schaling (2009) consider the implications for local determinacy if the degree of trade openness is asymmetric between the two countries. This study suggests that one topic of future research is to examine the determinacy implications of policy rules when the steady state is asymmetric between the two countries and/or employing a multi-country analysis.

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A Appendix

A.1 Proof of Proposition 1

For the coefficient matrix $\mathbf{A}^W$ one eigenvalue is given by $1+\frac{\overline{C}}{K}>1$. The remaining three eigenvalues are solutions to the cubic equation $r^3+a_2r^2+a_1r+a_0=0$, where

$$a_2 = -1-\frac{1}{\beta}-\frac{\kappa}{\beta}\left[1+\frac{\alpha(1-\Lambda_1)}{\Lambda_1}\right]$$
$$a_1 = \frac{1}{\beta}+\frac{\kappa\mu(1-\alpha)}{\beta}+\frac{\kappa\alpha(1+\mu)}{\Lambda_1\beta}$$
$$a_0 = -\frac{\mu\alpha\kappa}{\Lambda_1\beta}.$$

For determinacy two of these three eigenvalues must be outside the unit circle and one eigenvalue must lie inside the unit circle. By Proposition C.2 of Woodford (2003) this is the case if and only if either of the following two cases are satisfied:

(Case 1): $1+a_2+a_1+a_0<0$, $-1+a_2-a_1+a_0>0$;

(Case 2): $1+a_2+a_1+a_0>0$, $-1+a_2-a_1+a_0<0$, & $|a_2|>3$ or $a_0^2-a_0a_2+a_1-1>0$;

where $1+a_2+a_1+a_0=\frac{\kappa(\mu-1)(1-\alpha)}{\beta}$ and $-1+a_2-a_1+a_0=-\frac{2(1+\beta)}{\beta}-\frac{\kappa(1+\mu)(1-\alpha)}{\beta}-\frac{\kappa\alpha(1+2\mu)}{\beta\Lambda_1}$.

By inspection, Case (1) is not obtainable since the second inequality is never satisfied. The first inequality of Case (2) requires $\mu>1$, the second inequality is always satisfied and the final two inequalities yield (27) and (28) respectively.

A.2 Proof of Proposition 2

For the coefficient matrix $\mathbf{A}^R_{PPI}$ one eigenvalue is given by $1+\frac{1}{(2a-1)}\left[\frac{\overline{C}}{K}+2\delta(1-a)\right]>1$. The remaining three eigenvalues are solutions to the cubic equation $r^3+a_2r^2+a_1r+a_0=0$, where

$$a_2 = -1-\frac{1}{\beta}-\frac{\kappa}{\beta}\left[1+\frac{\alpha(1-\Lambda_1)(2a-1)}{\Lambda_1}\right]$$
$$a_1 = \frac{1}{\beta}+\frac{\kappa\mu}{\beta}\left[1+\frac{\alpha(1-\Lambda_1)(2a-1)}{\Lambda_1}\right]+\frac{\alpha\kappa(2a-1)}{\Lambda_1\beta}$$
$$a_0 = -\frac{\kappa\mu\alpha(2a-1)}{\Lambda_1\beta}.$$

For determinacy two of these three eigenvalues must be outside the unit circle and one eigenvalue must lie inside the unit circle. By Proposition C.2 of Woodford (2003) this is the case if and only if either of the following two cases are satisfied:

(Case 1): $1 + a_2 + a_1 + a_0 < 0$, $-1 + a_2 - a_1 + a_0 > 0$;

(Case 2): $1 + a_2 + a_1 + a_0 > 0$, $-1 + a_2 - a_1 + a_0 < 0$, & $|a_2| > 3$ or $a_0^2 - a_0 a_2 + a_1 - 1 > 0$; where $1+a_2+a_1+a_0 = \frac{(\mu-1)\kappa[1-(2a-1)\alpha]}{\beta}$ and $-1+a_2-a_1+a_0 = -\frac{2(1+\beta)}{\beta} - \frac{(\mu+1)\kappa}{\beta}\left[1 + \frac{(2a-1)\alpha(2-\Lambda_1)}{\Lambda_1}\right]$. By inspection, Case (1) is not obtainable since the second inequality is never satisfied. The first inequality of Case (2) requires $\mu > 1$, the second inequality is always satisfied and the final two inequalities yield (29) and (30) respectively.

A.3 Proof of Proposition 3

For the coefficient matrix $\mathbf{A}_{CPI}^{R}$ one eigenvalue is given by $1 + \frac{1}{(2a-1)}\left[\frac{\overline{C}}{\overline{K}} + 2\delta(1-a)\right] > 1$ and another eigenvalue is zero. The remaining three eigenvalues are solutions to the cubic equation $r^3 + a_3 r^2 + a_2 r + a_1 = 0$, where

$$a_3 = -1 - \frac{1}{\beta} - \frac{\kappa}{\beta}\left[1 + \frac{\alpha(1-\Lambda_1)(2a-1)}{\Lambda_1}\right] - 2(1-a)\mu$$

$$a_2 = \frac{1}{\beta} + \frac{\kappa\mu}{\beta}\left[1 + \frac{\alpha(1-\Lambda_1)(2a-1)}{\Lambda_1}\right] + \frac{\alpha\kappa(2a-1)}{\Lambda_1\beta} + \frac{2(1-a)\mu(1+\beta)}{\beta}$$

$$a_1 = -\frac{\mu}{\beta}\left[2(1-a) + \frac{\kappa\alpha(2a-1)}{\Lambda_1}\right].$$

For determinacy two of these three eigenvalues must be outside the unit circle and one eigenvalue must lie inside the unit circle. By Proposition C.2 of Woodford (2003) this is the case if and only if either of the following two cases are satisfied:

(Case 1): $1 + a_3 + a_2 + a_1 < 0$, $-1 + a_3 - a_2 + a_1 > 0$;

(Case 2): $1 + a_3 + a_2 + a_1 > 0$, $-1 + a_3 - a_2 + a_1 < 0$, & $|a_3| > 3$ or $a_1^2 - a_1 a_3 + a_2 - 1 > 0$; where $1+a_3+a_2+a_1 = \frac{(\mu-1)\kappa[1-(2a-1)\alpha]}{\beta}$ and $-1+a_3-a_2+a_1 = -\frac{2(1+\beta)}{\beta} - \frac{4\mu(1-a)(1+\beta)}{\beta} - \frac{(\mu+1)\kappa}{\beta}\left[1 + \frac{(2a-1)\alpha(2-\Lambda_1)}{\Lambda_1}\right]$. By inspection, Case (1) is not obtainable since the second inequality is never satisfied. The first inequality of Case (2) requires $\mu > 1$, the second inequality is always satisfied and the final two inequalities yield (31) and (32) respectively.

A.4 Proof of Proposition 4

For the coefficient matrix $\mathbf{A}_{PPI}^{R}$ one eigenvalue is given by $1+\frac{1}{(2a-1)}\left[\frac{\overline{C}}{\overline{K}}+2\delta(1-a)\right]>1$. The remaining three eigenvalues are solutions to the cubic equation $r^3+a_2r^2+a_1r+a_0=0$, where

$$a_2 = -1-\frac{1}{\beta}-\frac{\kappa}{\beta}\left[1+\frac{\alpha(1-\Lambda_1)(2a-1)}{\Lambda_1}\right]-2(2a-1)\mu_Q$$

$$a_1 = \frac{1}{\beta}+\frac{\kappa\mu_\pi}{\beta}\left[1+\frac{\alpha(1-\Lambda_1)(2a-1)}{\Lambda_1}\right]+\frac{\alpha\kappa(2a-1)}{\Lambda_1\beta}+\frac{2(2a-1)\mu_Q}{\beta}$$

$$a_0 = -\frac{\kappa\mu_\pi\alpha(2a-1)}{\Lambda_1\beta}.$$

For determinacy two of these three eigenvalues must be outside the unit circle and one eigenvalue must lie inside the unit circle. By Proposition C.2 of Woodford (2003) this is the case if and only if either of the following two cases are satisfied:

(Case 1): $1+a_2+a_1+a_0<0$, $-1+a_2-a_1+a_0>0$;

(Case 2): $1+a_2+a_1+a_0>0$, $-1+a_2-a_1+a_0<0$, & $|a_2|>3$ or $a_0^2-a_0a_2+a_1-1>0$;

where $1+a_2+a_1+a_0=\frac{(\mu_\pi-1)\kappa[1-(2a-1)\alpha]}{\beta}+\frac{2(2a-1)(1-\beta)\mu_Q}{\beta}$ and $-1+a_2-a_1+a_0=-\frac{2(1+\beta)}{\beta}-\frac{(\mu_\pi+1)\kappa}{\beta}\left[1+\frac{(2a-1)\alpha(2-\Lambda_1)}{\Lambda_1}\right]-\frac{2(2a-1)(1+\beta)\mu_Q}{\beta}$.

By inspection, Case (1) is not obtainable since the second inequality is never satisfied. Assume $\mu>1$, since otherwise the aggregate system would be indeterminate. Then the first two inequalities of Case (2) are always satisfied and the final two inequalities yield (35) and (36) respectively.