



CEEE

Centro de Estudios Económicos

www.colmex.mx

El Colegio de México, A.C.

Serie documentos de trabajo

**REAL INDETERMINACY AND THE TIMING
OF MONEY IN OPEN ECONOMIES**

Stephen McKnight
El Colegio de México

DOCUMENTO DE TRABAJO
Núm. I – 2011

Real Indeterminacy and the Timing of Money in Open Economies

Stephen McKnight*
El Colegio de México

March 2011[†]

Abstract

Should central banks target producer price inflation or consumer price inflation in the setting of monetary policy? Previous studies suggest that in order to avoid real indeterminacy and self-fulfilling fluctuations, the interest rate rule for open economies should react to producer price inflation. However, as this paper shows, the preference towards a particular inflation index crucially depends upon the timing assumption on money employed in the determinacy analysis. This timing assumption importantly determines the transactions-facilitating services of money. It is shown that the conclusions of the existing literature, that advocate targeting producer price inflation, is a by-product of adopting end-of-period timing, i.e. what matters for transactions purposes is the money one *leaves* the goods market with. However, we find that the conditions for equilibrium determinacy change significantly once cash-in-advance timing is adopted, i.e. what matters for current transactions is the money one *enters* the goods market with. Thus in stark contrast to previous studies, we show that under cash-in-advance timing, targeting consumer price inflation is preferable to targeting producer price inflation in preventing self-fulfilling expectations.

JEL Classification Number: E32; E43; E53; E58; F41

Keywords: Real Indeterminacy; Open Economy Monetary Models; Trade Openness; Interest Rate Rules.

*Correspondence address: Centro de Estudios Económicos, El Colegio de México, Camino al Ajusco 20, Col. Pedregal de Sta. Teresa, México D.F., C.P. 10740, México. E-mail: smckni@hotmail.com.

[†]I am grateful to Roy Bailey, Joao Miguel Ejarque, Timothy Fuerst, Fabio Ghironi, Aditya Goenka, Mark Guzman, Campbell Leith and two anonymous referees for helpful comments on an earlier version. However, the usual disclaimer applies.

1 Introduction

Over recent years the defining characteristic in the conduct of monetary policy has been the adoption of inflation-targeting policies by central banks that explicitly target consumer price inflation, while allowing the exchange rate to float freely (see e.g. De Fiore and Liu (2005)). However a number of recent studies have questioned this choice of the consumer price index, as the indicator of inflation targeted by central banks, for open economies. One branch of the theoretical literature suggests that the choice of the inflation index targeted has important consequences in terms of local equilibrium determinacy.¹ For example, Linnemann and Schabert (2006) and Llosa and Tuesta (2008) have advocated the targeting of producer price inflation, rather than consumer price inflation, in order to prevent monetary policy introducing real indeterminacy and sunspot fluctuations into the economy. A second related branch has attempted to characterize the optimal monetary policy for open economies. In an important contribution Clarida *et al.* (2002) find that for open economies the optimal monetary policy is to target producer price inflation. Using the criteria of equilibrium determinacy, the aim of this paper is to reinvestigate which inflation index should be targeted in policy rules for open economies. We will show that whether the interest rate rule should target producer price inflation, or consumer price inflation, crucially depends on how money is introduced into the analysis. In contrast to the existing literature, a key policy implication of this paper is that central banks may be justified in their adoption of inflation-targeting policies that focus on consumer price inflation.

A key issue in the design of monetary policy is that the interest rate rule adopted by a central bank should ensure a determinate equilibrium. That is, the policy rule should be designed to avoid generating real indeterminacy which can destabilize the economy through the emergence of sunspot equilibria and self-fulfilling fluctuations.² Such fluctuations are completely unrelated to economic fundamentals and can result in large reductions in the welfare of the economy. It has been well established in the closed economy literature that

¹This is in stark contrast to closed economy models. For example, Carlstrom *et al.* (2006), using a two-sector closed economy model, demonstrate that the price index targeted is irrelevant for (in)determinacy.

²Our focus is on real indeterminacy instead of price-level (or nominal) indeterminacy. By real indeterminacy we mean that there exists a continuum of equilibrium paths, starting from the same initial conditions, which converge to the steady state. Price-level indeterminacy on the other hand, is where for any equilibrium sequence there exists an infinite number of initial price levels consistent with a perfect-foresight equilibrium. Furthermore, our focus of attention rests solely with the consideration of local (real) determinacy as opposed to global determinacy. For further discussion of these issues see Woodford (2003).

under the Taylor Principle, i.e. a policy that adjusts the nominal interest rate by proportionally more than the increase in inflation, a central bank can easily prevent the emergence of indeterminacy and thus welfare-reducing self-fulfilling fluctuations, provided the central bank is not overly aggressive.³ Recently, a number of studies have investigated whether policies consistent with equilibrium determinacy in the closed economy are necessary and sufficient to preclude indeterminate equilibrium for open economies.⁴ One crucial factor upon which this depends is the inflation index targeted by central banks. Using a small open economy framework, Linnemann and Schabert (2006) and Llosa and Tuesta (2008) both find that the Taylor Principle guarantees equilibrium determinacy under plausible parameter constellations if the central bank reacts to future producer price inflation. This is in stark contrast to a policy rule that responds to future consumer price inflation, where the Taylor Principle may not be able to prevent indeterminacy, since the upper bound on the inflation response coefficient is more likely to bind with a sufficient degree of trade openness. Similarly, using a two-country framework, Batini *et al.* (2004) and Leith and Wren-Lewis (2009) also find that indeterminacy is exacerbated if the policy rule is based on consumer price inflation rather than producer price inflation.⁵

However a common characteristic of all these studies is that they either assume a cashless economy or employ a traditional money-in-the utility function framework (MIUF) in which end-of-period money balances enter the utility function in a separable way.^{6,7} But the ability of the Taylor Principle to ensure equilibrium determinacy in closed-economy models has been shown to crucially depend on the timing assumption on real money balances specified when using the popular money-in-the-utility-function (MIUF) approach. In an important contribution Carlstrom and Fuerst (2001) compare the determinacy implications under the traditional “cash-when-I’m-done” (CWID) timing convention, which assumes that end-of-period money balances enter the utility function, with “cash-in-advance” (CIA)

³See for example, Bernanke and Woodford (1997), Clarida *et al.* (2000) and Woodford (2003).

⁴For example, Zanna (2003), Batini *et al.* (2004), De Fiore and Liu (2005), Linnemann and Schabert (2006), Llosa and Tuesta (2008), Bullard and Schaling (2009) and Leith and Wren-Lewis (2009).

⁵Batini *et al.* (2004) consider the determinacy implications of inflation forecast rules that can be more than one-period into the future. Leith and Wren-Lewis (2009) consider the appropriateness of the Taylor Principle when consumers are assumed to be finite-lived.

⁶The assumption of a cashless economy is isomorphic to the traditional MIUF approach with end-of-period money balances, provided the utility function is separable between consumption and real money balances.

⁷A notable exception is De Fiore and Liu (2005) who employ a strict cash-in-advance constraint to introduce money into their small open economy model. However they only focus on the determinacy properties of policy rules that react to consumer price inflation.

timing, where the money held before engaging in goods market trading enters into the utility function. The essential difference between the CWID and CIA-timing assumptions is that in the latter what matters for current transactions is the money one *enters* the goods market with, whereas for the former what matters is the money one *leaves* the goods market with. A corollary of this is that under CIA-timing the nominal interest rate is scrolled forward one period in the intertemporal IS equation. Consequently with separable preferences between consumption and real money balances, Carlstrom and Fuerst (2001) find the following *timing equivalence* result: a current-looking (backward-looking) rule with CIA-timing has the same determinacy properties as a forward-looking (current-looking) rule with CWID-timing.

In this paper we utilize a two-country, sticky-price, MIUF model where monetary policy is characterized by an interest rate rule that can target either producer price inflation or consumer price inflation. In a two-country model the optimizing decisions of the *foreign* country can affect prices and allocations in the *home* country. This differs from the small open economy frameworks of Linnemann and Schabert (2006) and Llosa and Tuesta (2008), where the foreign sector is exogenously given. The conditions for equilibrium determinacy are analyzed for forward and current-looking versions of the interest rate rule for the two alternative timing assumptions on money. The main findings of the paper are as follows.

First, this paper shows that the timing equivalence result obtained from the closed economy literature holds for open economies only under a very restrictive preference specification. For the case when the elasticity of substitution between cross-country tradeable goods and the intertemporal substitution elasticity of consumption are equal, production spillover effects between the two countries are absent. Only in this special case, where the two economies are insular, does the timing equivalence result hold. However, under more general preference specifications, then the timing equivalence result breaks down in the presence of international spillover effects. The explanation behind this breakdown of the timing equivalence result for open economies arises from the fact that alternative assumptions on how money balances enter the utility function, have no impact on the uncovered interest parity condition. Thus scrolling forward the nominal interest rate can no longer equate the intertemporal IS equations for the two timing assumptions because of the presence of the exchange rate.

Second, with the breakdown of the timing equivalence result, this paper shows that different timing assumptions on money that have no consequences for equilibrium determinacy in a closed economy, can have potentially non-trivial implications for indeterminacy in open economies. For policy rules that target producer price inflation, we find that the regions of indeterminacy crucially depends on the sign of international spillover effects in production. In the presence of negative international spillover effects then indeterminacy is greater under a forward-looking rule with CWID-timing than under a current-looking rule with CIA-timing. However for positive international spillover effects, then indeterminacy under a current-looking rule with CIA-timing is greater than a forward-looking rule with CWID-timing. These differences arise because in the open economy different timing assumptions on money have important consequences for the aggregate supply equation, which governs the dynamics of producer price inflation.

Third, this paper shows that the timing assumption employed has important implications for policymakers concerning which inflation index the policy rule should target. Under CWID-timing, this paper shows that targeting producer price inflation is always preferable to targeting consumer price inflation regardless of the sign of international spillover effects. However, under CIA-timing, we show that targeting consumer price inflation is generally preferable to targeting producer price inflation in minimizing equilibrium indeterminacy. While there is little practical difference between policy rules that target producer price inflation or consumer price inflation in the presence of negative international spillover effects, we find it is particularly important for policymakers to target consumer price inflation under CIA-timing, in the presence of positive international spillover effects, in order to minimize self-fulfilling fluctuations.

Our results contribute to the recent literature that considers the consequences for equilibrium determinacy of designing interest rate rules for countries open to international trade. In relation to the key policy question of which index of inflation central banks should target we show that the policy conclusion of the existing literature, advocating producer price inflation over consumer price inflation, is a by-product of imposing the traditional CWID-timing assumption. Indeed, if one accepts Carlstrom and Fuerst (2001) argument that the most appropriate way to model money is to employ CIA-timing, then our results suggest that the existing inflation-targeting policies of central banks that explicitly target consumer

price inflation, is appropriate to avoid self-fulfilling expectations.⁸

In addition this paper can be viewed as a determinacy based complement to the current debate on optimal policy for open economies. A number of studies have argued that the optimal monetary policy for open economies is to target producer price inflation (e.g. Clarida *et al.* (2002) and Gali and Monacelli (2005)). This stems from the fact that optimality requires both open and closed economies to mimic the flexible price equilibrium. However this result has been recently challenged. For example, Benigno and Benigno (2003) show that for the flexible price allocation to be optimal for open economies, this requires a very restrictive preference specification in terms of the elasticity of substitution between cross-country tradeable goods and the intertemporal substitution elasticity of consumption. Furthermore, as shown by Benigno and Benigno (2006), the optimal cooperative outcome can be achieved if each central bank targets consumer price inflation. This paper also challenges the appropriateness of targeting producer price inflation but on the grounds of equilibrium (in)determinacy.

The remainder of the paper is organized as follows. Section 2 develops the two-country model. Section 3 shows the breakdown of the timing equivalence result for open economies and outlines the implications for equilibrium determinacy under policy rules that target producer price inflation and consumer price inflation. Finally, Section 4 briefly concludes.

2 The Model

Consider a global economy that consists of two-countries denoted *home* and *foreign*, where an asterisk denotes *foreign* variables. Within each country there exists a representative infinitely-lived agent, a representative final good producer, a continuum of intermediate good producing firms, and a monetary authority. The representative agent owns all domestic intermediate good producing firms and supplies labor to the production process. Intermediate firms operate under monopolistic competition and use domestic labor as inputs to produce tradeable goods which are sold to the *home* and *foreign* final good producers. The labor market is assumed to be competitive. Each representative final good producer is a

⁸As discussed by Carlstrom and Fuerst (2001), it is very difficult to justify CWID-timing on theoretical grounds since what aids in current transactions is the money one *leaves* the goods market with and not the money one *entered* the market with.

competitive firm that bundles domestic and imported intermediate goods into non-tradeable final goods which are consumed by the domestic agent. Preferences and technologies are symmetric across the two countries. The following presents the features of the model for the *home* country on the understanding that the *foreign* case can be analogously derived. Finally since we are concerned with issues of local determinacy the following discussion is limited to a deterministic framework.

2.1 Final Good Producers

The *home* final good (Z) is produced by a competitive firm that uses a composite of *home* (Z_H) and *foreign* (Z_F) intermediate goods as inputs according to the following CES aggregation technology index:

$$Z_t = \left[a^{\frac{1}{\theta}} Z_{H,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} Z_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (1)$$

where the constant elasticity of substitution between aggregate *home* and *foreign* intermediate goods is $\theta > 0$ and the relative share of domestic and imported intermediate inputs used in the production process is $0.5 < a < 1$ and the parameter $(1-a)$ is a measure of the degree of trade openness.^{9,10} The inputs Z_H and Z_F are defined as the quantity indices of domestic and imported intermediate goods respectively:

$$Z_{H,t} = \left[\int_0^1 z_{H,t}(i)^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}}, \quad Z_{F,t} = \left[\int_0^1 z_{F,t}(j)^{\frac{\lambda-1}{\lambda}} dj \right]^{\frac{\lambda}{\lambda-1}},$$

where the elasticity of substitution across domestic (*foreign*) intermediate goods is $\lambda > 1$, and $z_H(i)$ and $z_f(j)$ are the respective quantities of the domestic and imported type i and j intermediate goods. Let $p_H(i)$ and $p_F(j)$ represent the respective prices of these goods in *home* currency. Cost minimization in final good production yields the aggregate demand

⁹The analysis only considers this empirically relevant home bias case and ignores the case when $0 < a \leq 0.5$.

¹⁰Symmetrically,

$$Z_t^* = \left[a^{\frac{1}{\theta}} Z_{F,t}^{*\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} Z_{H,t}^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

is the production technology of the *foreign* final good (Z^*).

conditions for *home* and *foreign* goods:

$$Z_{H,t} = a \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} Z_t, \quad Z_{F,t} = (1-a) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} Z_t, \quad (2)$$

where the demand for individual goods is given by

$$z_{H,t}(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\lambda} Z_{H,t}, \quad z_{F,t}(j) = \left(\frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\lambda} Z_{F,t}. \quad (3)$$

Furthermore, since the final good producer is competitive it sets its price equal to marginal cost

$$P_t = \left[a P_{H,t}^{1-\theta} + (1-a) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (4)$$

where P is the consumer price index (CPI) and P_H and P_F are the respective price indices of *home* and *foreign* intermediate goods, all denominated in the home currency:

$$P_{H,t} = \left[\int_0^1 p_{H,t}(i)^{1-\lambda} di \right]^{\frac{1}{1-\lambda}}, \quad P_{F,t} = \left[\int_0^1 p_{F,t}(j)^{1-\lambda} dj \right]^{\frac{1}{1-\lambda}}.$$

We assume that there are no costs to trade between the two countries and the law of one price holds, which implies that

$$P_{H,t} = e_t P_{H,t}^*, \quad P_{F,t}^* = \frac{P_{F,t}}{e_t}, \quad (5)$$

where e denotes the nominal exchange rate. Letting $Q = \frac{e P^*}{P}$ denote the real exchange rate, under the law of one price, the CPI index (4) and its *foreign* equivalent imply:

$$\left(\frac{1}{Q_t} \right)^{1-\theta} = \left(\frac{P_t}{e_t P_t^*} \right)^{1-\theta} = \frac{a P_{H,t}^{1-\theta} + (1-a) (e_t P_{F,t}^*)^{1-\theta}}{a (e_t P_{F,t}^*)^{1-\theta} + (1-a) P_{H,t}^{1-\theta}} \quad (6)$$

and hence the purchasing power parity condition is satisfied only in the absence of any bias between *home* and *foreign* intermediate goods (i.e. $a = 0.5$). The relative price T , the terms of trade, is defined as $T \equiv \frac{e P_F^*}{P_H}$.

2.2 Intermediate Goods Producers

Intermediate firms hire labor to produce output given a (real) wage rate w_t . A firm of type i has a linear production technology

$$y_t(i) = L_t(i). \quad (7)$$

Given competitive prices of labor, cost minimization yields

$$mc_t = w_t \frac{P_t}{P_{H,t}} \quad (8)$$

where $mc_t \equiv \frac{MC_t}{P_{H,t}}$ is real marginal cost.

Firms set prices according to Calvo (1983), where in each period there is a constant probability $1 - \psi$ that a firm will be randomly selected to adjust its price, which is drawn independently of past history. A domestic firm i , faced with changing its price at time t , has to choose $p_{H,t}(i)$ to maximize its discounted value of profits, taking as given the indexes P , P_H , P_F , Z and Z^* .¹¹

$$\max_{p_{H,t}(i)} \sum_{s=0}^{\infty} (\beta\psi)^s X_{t,t+s} \{ [p_{H,t}(i) - MC_{t+s}(i)] [z_{H,t+s}(i) + z_{H,t+s}^*(i)] \}, \quad (9)$$

where

$$z_{H,t+s}(i) + z_{H,t+s}^*(i) \equiv \left(\frac{p_{H,t}(i)}{P_{H,t+s}} \right)^{-\lambda} [Z_{H,t+s} + Z_{H,t+s}^*]$$

and the firm's discount factor is $\beta^s X_{t,t+s} = \beta^s [U_c(C_{t+s})/U_c(C_t)][P_t/P_{t+s}]$.¹² Firms that are given the opportunity to change their price, at a particular time, all behave in an identical manner. The first-order condition to the firm's maximization problem yields

$$\tilde{P}_{H,t} = \frac{\lambda}{\lambda - 1} \sum_{s=0}^{\infty} q_{t,t+s} MC_{t+s}. \quad (10)$$

The optimal price set by a domestic *home* firm $\tilde{P}_{H,t}$ is a mark-up $\frac{\lambda}{\lambda-1}$ over a weighted

¹¹While the demand for a firm's good is affected by its pricing decision $p_{H,t}(i)$, each producer is small with respect to the overall market.

¹²Under the assumption that all firms are owned by the representative agent, this implies that the firm's discount factor is equivalent to the individual's intertemporal marginal rate of substitution.

average of future nominal marginal costs, where the weight $q_{t,t+s}$ is given by

$$q_{t,t+s} = \frac{(\beta\psi)^s X_{t,t+s} P_{H,t+s}^\lambda (Z_{H,t+s} + Z_{H,t+s}^*)}{\sum_{s=0}^{\infty} (\beta\psi)^s X_{t,t+s} P_{H,t+s}^\lambda (Z_{H,t+s} + Z_{H,t+s}^*)}.$$

Since all prices have the same probability of being changed, with a large number of firms, the evolution of the price sub-indexes is given by

$$P_{H,t}^{1-\lambda} = \psi P_{H,t-1}^{1-\lambda} + (1-\psi) \tilde{P}_{H,t}^{1-\lambda} \quad (11)$$

since the law of large numbers implies that $1-\psi$ is also the proportion of firms that adjust their price each period.

2.3 Representative Agent

The representative agent chooses consumption C_t , domestic real money balances A_t/P_t , and labor L_t , to maximize utility:

$$\max \sum_{t=0}^{\infty} \beta^t [U(C_t) + V(A_t/P_t) - H(L_t)] \quad (12)$$

where the discount factor is $0 < \beta < 1$, subject to the period budget constraint

$$\Gamma_{t,t+1} B_{t+1} + M_t + P_t C_t \leq B_t + M_{t-1} + P_t w_t L_t + \int_0^1 \Pi_t d(h) - \Upsilon_t. \quad (13)$$

The agent carries M_{t-1} units of money, and B_t nominal bonds into period t . Before proceeding to the goods market, the agent visits the financial market where a state contingent nominal bond B_{t+1} can be purchased that pays one unit of domestic currency in period $t+1$ when a specific state is realized at a period t price $\Gamma_{t,t+1}$. Letting R_t denote the gross nominal yield on a one-period discount bond, then in the absence of uncertainty, $R_t^{-1} \equiv \Gamma_{t,t+1}$. During period t the agent supplies labor to the intermediate good producing firms, receiving real income from wages w_t , nominal profits from the ownership of domestic intermediate firms Π_t and a lump-sum nominal transfer Υ_t from the monetary authority. The agent then uses these resources to purchase the final good.

Following Carlstrom and Fuerst (2001), we will consider two alternative measures of

money which may appear in the utility function: the traditional *cash-when-i'm-done* (CWID)-timing and the alternative *cash-in-advance* (CIA)-timing. Under CWID-timing, end of period money balances enter into the utility function:

$$\text{CWID: } A_t = M_t. \quad (14)$$

Here the stock of money that yields utility to the representative agent is the amount of money he leaves the goods market with. However, under CIA-timing, the stock of money that yields utility is the value of money holdings after bonds have been purchased in the financial markets, but before income has been received or final goods have been purchased:¹³

$$\text{CIA: } A_t = M_{t-1} - \Upsilon_t + B_t - \frac{B_{t+1}}{R_t}. \quad (15)$$

The first-order conditions from the *home* agent's maximization problem yield:

$$\beta R_{t+i} \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{P_t}{P_{t+1}} = 1 \quad (16)$$

$$\frac{H_L(L_t)}{U_c(C_t)} = w_t \quad (17)$$

$$\text{CWID: } \frac{V_m(m_t)}{U_c(C_t)} = \frac{R_t - 1}{R_t} \quad (18)$$

$$\text{CIA: } \frac{V_m(m_t)}{U_c(C_t)} = R_t - 1. \quad (19)$$

Equation (16) is the consumption Euler equation for the holdings of domestic bonds, which must hold for each possible state, where $i = 0$ represents CWID-timing and $i = 1$ corresponds to CIA-timing, with the respective money demand equation given by equations (18) and (19). Thus, the first key difference between the two timing assumptions is that under CIA-timing the nominal interest rate is scrolled forward one period in (16). Changes in real holdings of money directly influence the real interest rate under CIA-timing, whereas they only have an indirect effect on the real interest rate under CWID-timing.¹⁴ The labor

¹³Here, as in Carlstrom and Fuerst (2001), the agent engages in asset market trading in advance of consumption trading under CIA-timing. Hence the money balances that enter into the utility function include the net gains from asset trading. As discussed by Kurozumi (2006), an alternative CIA-timing approach is to assume that asset market trading follows consumption trading.

¹⁴Under CWID-timing, the representative agent's maximization problem yields the familiar bond-pricing

supply decision is determined by equation (17). Optimizing behavior implies that the budget constraint (13) holds with equality in each period and the appropriate transversality condition is satisfied. Analogous conditions apply to the *foreign* agent.

From the first-order conditions for the *home* and *foreign* agent, the following risk-sharing conditions can be derived:

$$R_t = R_t^* \left[\frac{e_{t+1}}{e_t} \right] \quad (20)$$

$$\text{CWID:} \quad Q_t = q_0 \frac{U_c(C_t^*)}{U_c(C_t)} \quad (21)$$

$$\text{CIA:} \quad Q_t = q_0^* \frac{U_c(C_t^*) + V_m(m_t^*)}{U_c(C_t) + V_m(m_t)} = q_0^* \frac{U_c(C_t^*) R_t^*}{U_c(C_t) R_t} \quad (22)$$

where the constants $q_0 = Q_0 \left[\frac{U_c(C_0)}{U_c(C_0^*)} \right]$ and $q_0^* = Q_0 \left[\frac{U_c(C_0) + V_m(m_0)}{U_c(C_0^*) + V_m(m_0^*)} \right]$. Equation (20) is the standard uncovered interest parity condition, whereas equations (21) and (22) follow from the assumption of complete asset markets, under CWID and CIA-timing respectively.¹⁵ Hence, the second key difference between the timing assumptions relates to the risk sharing condition which equates the real exchange rate Q with the marginal utilities of consumption. Under CIA-timing, the marginal utilities of money are also included in (22), reflecting the fact that under CIA-timing a bond sale for consumption purposes, increases the utility from current consumption and current liquidity.

2.4 Monetary Authority

The monetary authority can adjust the nominal interest rate in response to changes in producer price inflation (PPP) π_{t+v}^h or to changes in consumer price inflation (CPI) π_{t+v} , according to the rules:

$$\text{PPI:} \quad R_t = \mu (\pi_{t+v}^h) = \bar{R} \left(\frac{\pi_{t+v}^h}{\bar{\pi}^h} \right)^\mu, \quad (23)$$

equation: $\beta R_t \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{P_t}{P_{t+1}} = 1$. However, under CIA-timing the bond-pricing equation is given by:

$$\frac{V_m(m_t) + U_c(C_t)}{P_t} = \frac{\beta R_t [V_m(m_{t+1}) + U_c(C_{t+1})]}{P_{t+1}}.$$

Hence under CIA-timing the real interest rate is influenced by the marginal utilities of consumption *and* real money balances. Using the money demand equation (19) to eliminate $V_m(m_t)$ and $V_m(m_{t+1})$ from the above yields $\beta R_{t+1} \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{P_t}{P_{t+1}} = 1$.

¹⁵Under CIA-timing the money demand equation (19) and its *foreign* equivalent can be used to eliminate $V_m(m_t)$ and $V_m(m_t^*)$ from equation (22).

$$\text{CPI: } R_t = \mu(\pi_{t+v}) = \bar{R} \left(\frac{\pi_{t+v}}{\bar{\pi}} \right)^\mu, \quad (24)$$

where $\bar{R} > 1$ and $\mu \geq 0$. The timing-index v represents the inflation-targeting behavior of the monetary authority. If $v = 0$, the monetary authority targets current inflation, whereas $v = 1$ corresponds to forward-looking inflation targeting.

2.5 Market Clearing and Equilibrium

Market clearing for the *home* goods market requires

$$Z_{H,t} + Z_{H,t}^* = Y_t. \quad (25)$$

Total *home* demand must equal the supply of the final good,

$$Z_t = C_t, \quad (26)$$

and the labor, money and bond markets all clear:

$$\Upsilon_t = M_t - M_{t-1} \quad B_t + B_t^* = 0. \quad (27)$$

Definition 1 (Perfect Foresight Equilibrium): *Given an initial allocation of $B_{t_0}, B_{t_0}^*$, and $M_{t_0-1}, M_{t_0-1}^*$, a perfect foresight equilibrium is a set of sequences $\{C_t, C_t^*, M_t, M_t^*, L_t, L_t^*, B_t, B_t^*, R_t, R_t^*, MC_t, MC_t^*, w_t, w_t^*, Y_t, Y_t^*, e_t, Q_t, P_t, P_t^*, P_{H,t}, \widetilde{P}_{H,t}, \widetilde{P}_{F,t}^*, P_{H,t}^*, P_{F,t}, P_{F,t}^*, Z_t, Z_t^*, Z_{H,t}, Z_{F,t}, Z_{H,t}^*, Z_{F,t}^*\}$ for all $t \geq t_0$ characterized by: (i) the optimality conditions of the representative agent, (16) to (17) and the appropriate money demand equation (18) or (19); (ii) the intermediate firms' first-order condition (8), price-setting rules, (10) and (11), and the aggregate version of the production function (7); (iii) the final good producer's optimality conditions, (2) and (4); (iv) all markets clear, (25) to (27); (v) the representative agent's budget constraint (13) is satisfied and the transversality conditions hold; (vi) the monetary policy rule is satisfied, (23) or (24); along with the *foreign* counterparts for (i)-(vi) and conditions (5), (6), (20) and either (21) if CWID-timing is adopted or (22) if CIA-timing is adopted.*

2.6 Local Equilibrium Dynamics

In order to analyze the equilibrium dynamics of the model, a first-order Taylor approximation is taken around the steady state.¹⁶ In what follows, a variable \widehat{X}_t denotes the percentage deviation of X_t with respect to its steady state value \bar{X} (i.e. $\widehat{X}_t = \frac{X_t - \bar{X}}{\bar{X}}$). Given the (gross) producer price inflation (π_t^h) and consumer price inflation (π_t) rates for the *home* country are defined respectively as $\pi_t^h \equiv \frac{\pi_t^h}{\pi_{t-1}^h}$ and $\pi_t \equiv \frac{\pi_t}{\pi_{t-1}}$, then linearizing the consumption Euler equation (16) and noting from (26) that $\widehat{Z}_t = \widehat{C}_t$ yields the linearized IS equation for the *home* country:

$$\widehat{Z}_{t+1} = \widehat{Z}_t + \sigma \widehat{R}_{t+1} - \sigma \pi_t \quad (28)$$

where $\sigma > 0$ measures the intertemporal substitution elasticity of consumption. Linearizing the price-setting equations (10) and (11) results in the linearized aggregate supply condition

$$\widehat{\pi}_t^h = \kappa \widehat{m}c_t + \beta \widehat{\pi}_{t+1}^h \quad (29)$$

where $\kappa \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi} > 0$ is the real marginal cost elasticity of inflation. Combining the linearized versions of (4), (7), (8) and (17) yields the following expression for real marginal cost:

$$\widehat{m}c_t = (1-a)\widehat{T}_t + \frac{1}{\sigma}\widehat{Z}_t + \phi\widehat{Y}_t \quad (30)$$

where $\phi > 0$ is the inverse of the elasticity of labor supply. Combining the linearized versions of (2), (4) and their *foreign* equivalents with (25) gives domestic tradeable output

$$\widehat{Y}_t = 2a\theta(1-a)\widehat{T}_t + a\widehat{Z}_t + (1-a)\widehat{Z}_t^*. \quad (31)$$

From the definitions of the terms of trade and the real exchange rate and using equations (5) and (6) yields the following linearized equations for the CPI inflation differential and

¹⁶To be precise the model is linearized around a symmetric steady state in which inflation is zero ($\bar{\pi} = \bar{\pi}^* = 1$) and prices in the two countries are equal ($\bar{P}_H = \bar{P}_F = \bar{P} = \bar{P}^* = \bar{P}_H^* = \bar{P}_F^*$). Then by definition the steady state terms of trade and nominal and real exchange rates are $\bar{T} = \bar{e} = \bar{Q} = 1$.

the real exchange rate:

$$\widehat{\pi}_t - \widehat{\pi}_t^* = (2a - 1) \left[\widehat{\pi}_t^h - \widehat{\pi}_t^{*f} \right] + 2(1 - a) \Delta \widehat{e}_t \quad (32)$$

$$\widehat{Q}_t = (2a - 1) \widehat{T}_t \quad (33)$$

where $\Delta \widehat{e}_t \equiv \widehat{e}_t - \widehat{e}_{t-1}$. Finally, linearizing the remaining equations (20)-(24) yields:

$$\widehat{R}_t - \widehat{R}_t^* = \Delta \widehat{e}_{t+1} \quad (34)$$

$$\text{CWID: } \widehat{Q}_t = \frac{1}{\sigma} \left[\widehat{Z}_t - \widehat{Z}_t^* \right] \quad (35)$$

$$\text{CIA: } \widehat{Q}_t = \frac{1}{\sigma} \left[\widehat{Z}_t - \widehat{Z}_t^* \right] + \widehat{R}_t^* - \widehat{R}_t \quad (36)$$

$$\text{PPI: } \widehat{R}_t = \mu \widehat{\pi}_{t+v}^h \quad (37)$$

$$\text{CPI: } \widehat{R}_t = \mu \widehat{\pi}_{t+v}. \quad (38)$$

The *foreign* country equivalents to (28)-(31) complete the linearized system.¹⁷

Before proceeding it will be helpful in what follows to consider the international spillover effects of the model. These effects are intuitively best illustrated by considering a version of the model where $a = 0.5$ (i.e. no home bias and perfect trade integration). Then using equation (31) to eliminate \widehat{T}_t from (30) and noting that $\widehat{Z}_t + \widehat{Z}_t^* = \widehat{Y}_t + \widehat{Y}_t^*$ gives:

$$\widehat{mc}_t = \frac{\widehat{Y}_t - \widehat{Y}_t^*}{2\theta} + \frac{\widehat{Y}_t + \widehat{Y}_t^*}{2\sigma} + \phi \widehat{Y}_t, \quad (39)$$

and thus

$$\frac{\partial \widehat{mc}_t}{\partial \widehat{Y}_t^*} = \frac{1}{2} \left[\frac{1}{\sigma} - \frac{1}{\theta} \right]. \quad (40)$$

Inspection of (40) suggests that the sign of the international spillovers crucially depends on the relative size of the intertemporal substitution elasticity of consumption (σ) and elasticity of substitution between *home* and *foreign* goods (θ). If $\sigma < \theta$ then *home* and *foreign* goods are substitutes in the utility function and there is a negative spillover effect. Thus terms of

¹⁷The money demand equations are omitted from the linearized system since the remaining conditions determine local equilibrium determinacy in the absence of real balance effects.

Table 1: Linearized system of equations

| <i>Difference System</i> | |
|--|------------------|
| $\widehat{Z}_{t+1}^R = \widehat{Z}_t^R + \sigma \widehat{R}_{t+i}^R - \sigma \widehat{\pi}_{t+1}^R$ | IS ^R |
| $\widehat{R}_t^R = \Delta \widehat{e}_{t+1}$ | UIP |
| $\widehat{\pi}_t^{R(h-f^*)} = \kappa 2(1-a)[1 + \phi 2\theta a] \widehat{T}_t + \kappa [\phi(2a-1) + \frac{1}{\sigma}] \widehat{Z}_t^R + \beta \widehat{\pi}_{t+1}^{R(h-f^*)}$ | AS ^R |
| $\widehat{R}_t^R = \mu \widehat{\pi}_{t+v}^{R(h-f^*)}$ | Taylor rule: PPI |
| $\widehat{R}_t^R = \mu \widehat{\pi}_{t+v}^R$ | Taylor rule: CPI |
| $\widehat{\pi}_t^R = (2a-1) \widehat{\pi}_t^{R(h-f^*)} + 2(1-a) \Delta \widehat{e}_t$ | Inflation |
| $\widehat{Q}_t = \frac{1}{\sigma} \widehat{Z}_t^R = (2a-1) \widehat{T}_t$ | RER(CWID-timing) |
| $\widehat{Q}_t = \frac{1}{\sigma} \widehat{Z}_t^R - \widehat{R}_t^R = (2a-1) \widehat{T}_t$ | RER(CIA-timing) |
| <i>Aggregate System</i> | |
| $\widehat{Z}_{t+1}^W = \widehat{Z}_t^W + \sigma \widehat{R}_{t+i}^W - \sigma \widehat{\pi}_{t+1}^W$ | IS ^W |
| $\beta \widehat{\pi}_{t+1}^W = \widehat{\pi}_t^W - \kappa [\phi + \frac{1}{\sigma}] \widehat{Z}_t^W$ | AS ^W |
| $\widehat{R}_t^W = \mu \widehat{\pi}_{t+v}^W$ | Taylor rule |

Notes: The index R refers to the difference between *home* and *foreign* variables e.g. $\widehat{C}_t^R \equiv (\widehat{C}_t - \widehat{C}_t^*)$, $\widehat{\pi}_t^{R(h-f^*)} \equiv (\widehat{\pi}_t^h - \widehat{\pi}_t^{*f})$. The index W refers to world aggregates where $\pi^W = \frac{\pi^h + \pi^*}{2} = \frac{\pi^h + \pi^{*f}}{2}$ and $\Delta \widehat{e}_t \equiv \widehat{e}_t - \widehat{e}_{t-1}$.

trade changes will lead to different production responses in the two countries. For example, a deterioration in the *foreign* terms of trade ($\widehat{T}_t \uparrow$) increases real marginal cost in the *foreign* country ($\widehat{mc}_t^* \uparrow$) and from the *foreign* equivalents to (29) and (31), *foreign* producer price inflation ($\widehat{\pi}_t^{*f} \uparrow$) and output ($\widehat{Y}_t^* \uparrow$) both rise. From (40) a rise in *foreign* output, implies a decrease in the real marginal cost of *home* producers which from (29) and (31) results in a decline in *home* producer price inflation ($\widehat{\pi}_t \downarrow$) and output ($\widehat{Y}_t \downarrow$). However if $\sigma > \theta$ then *home* and *foreign goods* are complements and there is a positive spillover effect. Thus in response to changes in the terms of trade, *home* (\widehat{Y}_t) and *foreign* (\widehat{Y}_t^*) output will expand or contract together. Only in the special case of $\sigma = \theta$ are production spillover effects absent.¹⁸

Since the two countries are symmetric, we employ the Aoki (1981) decomposition approach in order to analyze the determinacy properties of the model. The Aoki decomposition decomposes the model into two decoupled dynamic systems: the aggregate system that cap-

¹⁸As discussed by Benigno and Benigno (2003) when $\theta = \sigma$ no spillover effects on production exist as the two economies are insular.

tures the properties of the closed world economy and the difference system that portrays the open economy dimension. Thus, we solve both for cross-country differences $X^R \equiv \widehat{X} - \widehat{X}^*$ and worldwide aggregates¹⁹ $X^W \equiv \frac{\widehat{X}}{2} + \frac{\widehat{X}^*}{2}$. Determinacy of the aggregate and difference systems implies determinacy at the individual country level since $\widehat{X} = X^W + \frac{X^R}{2}$ and $\widehat{X}^* = X^W - \frac{X^R}{2}$. The complete linearized system of equations is summarized in Table 1.

2.7 Parameterization

In order to illustrate the conditions for determinacy, the ensuing analysis uses the following baseline parameter values summarized in Table 2. Parameter β is standard in the literature, ϕ is taken from Woodford (2003) and ψ from Taylor (1999). Setting $\psi = 0.75$ constitutes an average price duration of one year and this implies that the real marginal cost elasticity of inflation $\kappa = 0.08$. Rotemberg and Woodford (1997) estimate $\sigma = 6.37$ for the US economy. We follow Chari *et al.* (2002) and Llosa and Tuesta (2008) and initially set a slightly lower value of $\sigma = 5$. Setting $\sigma = 5$ implies a value of the risk aversion coefficient of $1/\sigma = 0.2$. This value is lower than the range of estimates obtained from micro-level studies (e.g. Vissing-Jorgensen (2002)) that typically suggest a risk aversion coefficient $1/\sigma \geq 1$. Thus an alternative choice of $\sigma = 1$ is also examined.²⁰ Empirical studies offer no clear conclusion on the magnitude of θ . Micro-level studies (e.g. Harrigan (1993)) suggest a value of around 5 whereas macro-level studies (e.g. Bergin (2006)) suggest a much lower value of around 1. Thus we compute the numerical eigenvalues of the model for alternative values of $\theta \in [1, 5]$. Finally, three alternative values for the degree of trade openness ($1 - a$) are also chosen, which are roughly consistent with the ratio of imports to GDP of the USA ($a = 0.85$), UK ($a = 0.7$) and Canada ($a = 0.6$).

¹⁹The determinacy conditions for the aggregate system are identical to comparable closed-economy New Keynesian models (e.g. Carlstrom and Fuerst (2001)). Note the measure of inflation targeted is irrelevant in the aggregate system since producer and consumer price inflation are the same concept. i.e. $\pi^W = \frac{\pi + \pi^*}{2} = \frac{\pi^h + \pi^{*f}}{2}$.

²⁰Woodford (2003) argues that a low risk aversion coefficient is justified on the grounds that the intertemporal substitution elasticity of consumption is significantly higher once investment in capital and consumer durables are considered.

Table 2: Baseline parameter values

| | | |
|----------|---|---------------------|
| β | Discount factor | 0.99 |
| ϕ | Inverse of the elasticity of labor supply | 0.47 |
| ψ | Degree of price rigidity | 0.75 |
| κ | Real marginal cost elasticity of inflation | 0.08 |
| σ | Intertemporal substitution elasticity of consumption | 1 or 5 |
| θ | Elasticity of substitution between home and foreign goods | $\theta \in [1, 5]$ |
| $1 - a$ | Degree of Trade Openness | 0.15, 0.3 or 0.4 |

3 Equilibrium Determinacy

This section considers the issue of local equilibrium determinacy. A key conclusion to arise from the analysis is that the timing equivalence result does not generally hold for open economies. As a consequence, whether monetary policy should react to producer price inflation or consumer price inflation, in order to minimize policy-induced real indeterminacy, crucially depends on the measure of money that enters into the utility function. The analysis proceeds as follows. First, the breakdown of the timing equivalence result for open economies is established by considering interest rate rules that react only to producer price inflation. Here the conditions for equilibrium determinacy are examined when monetary policy is characterized by a forward-looking interest rate rule under CWID-timing or a current-looking rule under CIA-timing. After examining the indeterminacy implications of targeting producer price inflation, the analysis then considers how the determinacy conditions differ when monetary policy reacts to consumer price inflation under both timing assumptions.

3.1 Breakdown of the Timing Equivalence Result

Carlstrom and Fuerst (2001) show that for a standard New Keynesian closed-economy model, the determinacy conditions for a forward-looking rule with CWID-timing is analogous to the determinacy conditions for a current-looking rule with CIA-timing: i.e.

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)}. \quad (41)$$

This subsection shows the breakdown of this timing equivalence result for the open economy under producer price inflation targeting.

Proposition 1 *Suppose that monetary policy reacts to forward-looking (current-looking) producer price inflation under CWID (CIA) timing. Then the necessary and sufficient conditions for equilibrium determinacy are:*

(a) Forward-looking rule (CWID-timing)

- *Aggregate System / Closed Economy*

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)}$$

- *Difference System*

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa[1 + \phi\sigma + 4\phi a(1 - a)(\theta - \sigma)]}$$

- *Open Economy*

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)} \quad \text{if } \theta \leq \sigma, \quad \text{or} \quad (42a)$$

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa[1 + \sigma\phi + 4\phi a(1 - a)(\theta - \sigma)]} \quad \text{if } \theta > \sigma. \quad (42b)$$

(b) Current-looking rule (CIA-timing)

- *Aggregate System / Closed Economy*

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)}$$

- *Difference System*

i. $2a\theta \geq \sigma(2a - 1)$ and

$$\mu > 1 \quad \text{if} \quad 4a(1 - a)\phi\theta \geq \phi\sigma(2a - 1)(3 - 2a) + 1, \quad \text{or}$$

$$1 < \mu < \frac{2(1 + \beta) + \kappa\Lambda_1}{\kappa[1 + \sigma\phi(2a - 1)(3 - 2a) - 4a(1 - a)\phi\theta]} \quad \text{if} \quad 4a(1 - a)\phi\theta < \phi\sigma(2a - 1)(3 - 2a) + 1.$$

ii. $2a\theta < \sigma(2a - 1)$ and

$$1 < \mu < \frac{1 - \beta}{2\kappa\phi(1 - a)[\sigma(2a - 1) - 2a\theta]} \quad \text{if } 4a(1 - a)\phi\theta \geq \phi\sigma(2a - 1)(3 - 2a) + 1, \quad \text{or}$$

$$1 < \mu < \min \left\{ \frac{2(1 + \beta) + \kappa\Lambda_1}{\kappa[1 + \sigma\phi(2a - 1)(3 - 2a) - 4a(1 - a)\phi\theta]}, \frac{1 - \beta}{2\kappa\phi(1 - a)[\sigma(2a - 1) - 2a\theta]} \right\}$$

if $4a(1 - a)\phi\theta < \phi\sigma(2a - 1)(3 - 2a) + 1,$

where $\Lambda_1 \equiv 1 + \sigma\phi + 4a\phi(1 - a)(\theta - \sigma)$.

- *Open Economy*

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)} \quad \text{if } 2a\theta \geq \sigma(2a - 1), \quad \text{or} \quad (43a)$$

$$1 < \mu < \min \left\{ \frac{(1 - \beta)}{2\kappa\phi(1 - a)[2a(\sigma - \theta) - \sigma]}, 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)} \right\} \quad \text{if } 2a\theta < \sigma(2a - 1). \quad (43b)$$

Proof. See Appendix 5.1. \square

The following remark directly follows from Proposition 1.

Remark 1 (*Timing Equivalence Result for Open Economies*) *The determinacy conditions for a forward-looking rule under CWID-timing are analogous to a current-looking rule under CIA-timing for the open economy, if and only if, $\theta = \sigma$, i.e.*

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)}.$$

As summarized by Remark 1, the timing equivalence result holds for the open economy, if and only if, the intertemporal substitution elasticity of consumption is equal to the elasticity of substitution between *home* and *foreign* goods ($\sigma = \theta$). In this case the determinacy conditions for CWID and CIA-timing are analogous i.e. (42a) = (43a). As discussed in Section 2.6, with $\sigma = \theta$ there are no international spillover effects in production as the two countries are insular. Hence, this also explains why with $\sigma = \theta$ the determinacy conditions for the open and closed economy are the same i.e. (41) = (42a) = (43a). However if $\theta \neq \sigma$ then the timing equivalence result breaks down and the timing assumption on money balances adopted has important qualitative implications for the potential range of

indeterminacy.

First consider the case when the goods produced in the two countries are substitutes ($\sigma < \theta$) and thus there are negative spillover effects between the two countries. Inspection of condition (42b) highlights that under CWID-timing the upper bound on the inflation coefficient is reduced relative to (41) and this upper bound gets progressively smaller the greater the difference between $\theta - \sigma > 0$ and the higher the degree of trade openness:

$$\frac{\partial(42b)}{\partial a} = \frac{8(1 + \beta)\kappa\phi(\theta - \sigma)(2a - 1)}{\kappa^2 [1 + \sigma\phi + 4\phi a(1 - a)(\theta - \sigma)]^2} > 0 \quad \text{for any } \theta > \sigma. \quad (44)$$

This is in stark contrast to CIA-timing where from (43a), if the goods are substitutes the same upper bound on the inflation coefficient exists for both the open and closed-economy i.e. (41) = (43a).

Now consider the case when the goods produced in the two countries are complements $\sigma > \theta$, thereby implying positive spillover effects between the two countries. Under CWID-timing then from (42a) the determinacy conditions for the open and closed-economy correspond exactly. However under CIA-timing, if $\theta < \frac{\sigma(2a-1)}{2a}$, inspection of condition (43b) highlights that the potential range of indeterminacy is greater in the open economy provided that $\frac{(1-\beta)}{2\kappa\phi(1-a)[2a(\sigma-\theta)-\sigma]} < 1 + \frac{2(1+\beta)}{\kappa(1+\sigma\phi)}$. If this is satisfied then:

$$\frac{\partial(43b)}{\partial a} = \frac{2(1 - \beta)\kappa\phi [4a(\sigma - \theta) - \sigma]}{[2\kappa\phi(1 - a) [2a(\sigma - \theta) - \sigma]]^2} \geq 0$$

which implies that as the degree of trade openness increases, the potential range of indeterminacy increases if $4a(\sigma - \theta) - \sigma > 0$ and decreases if $4a(\sigma - \theta) - \sigma < 0$.

The results presented above suggest that Carlstrom and Fuerst's (2001) observation that a forward-looking rule with CWID-timing is equivalent to a current-looking rule with CIA-timing does not typically hold in an open economy setting. The explanation for why this timing equivalence breaks down in the open economy follows because the timing convention adopted has no effect on the uncovered interest parity (UIP) condition (20). Intuitively this can be most evidently seen by inspecting the linearized IS condition for the difference system under each timing convention. The linearized IS equation for the *home* country (28) and its *foreign* equivalent imply $\widehat{Z}_{t+1}^R = \widehat{Z}_t^R + \sigma\widehat{R}_{t+i}^R - \sigma\widehat{\pi}_{t+1}^R$. Using the linearized UIP (34)

and CPI inflation differential (32) equations, the linearized IS conditions for the difference system can be expressed as:

$$\text{CWID: } \widehat{Z}_{t+1}^R = \widehat{Z}_t^R + \sigma(2a - 1) \left(\widehat{R}_t^R - \widehat{\pi}_{t+1}^{R(h-f^*)} \right) \quad (45)$$

$$\text{CIA: } \widehat{Z}_{t+1}^R = \widehat{Z}_t^R + \sigma \widehat{R}_{t+1}^R - \sigma(2a - 1) \widehat{\pi}_{t+1}^{R(h-f^*)} - 2\sigma(1 - a) \widehat{R}_t^R. \quad (46)$$

Note that under CIA-timing, the last term of (46) enters as a direct result of the UIP condition. In a closed economy this last term disappears (i.e. $a = 1$) and since $\sigma(2a - 1)$ becomes σ , the only difference in a closed economy between (45) and (46) is that the nominal interest rate in the latter is scrolled forward one period. Thus the timing equivalence result for a closed economy directly follows. However in the presence of international spillover effects ($\sigma \neq \theta$), there can be no timing equivalence in the open economy without the interest rate in the UIP term also being scrolled forward one period. Hence with the regular UIP condition the timing equivalence of (45) and (46) breaks down since scrolling the interest rate in the former no longer replicates the IS condition under CIA-timing.

3.2 CWID vs. CIA-timing: Indeterminacy Implications of Targeting Producer Price Inflation

Lets now illustrate the regions of indeterminacy using the baseline parameter values summarized in Table 2 of Section 2.7 for policy rules that react to producer price inflation. As discussed in the previous subsection, if there are negative (positive) international spillover effects then there is more likely to be indeterminacy in an open economy compared to a closed economy under CWID (CIA) timing. First, suppose that the policy rule reacts to forward-looking producer price inflation under CWID-timing.²¹ Table 3 summarizes the relevant upper bounds in the inflation response coefficient (μ) when $\sigma = 1$.²² In accordance with (44), the upper bounds computed for the open-economy decrease the higher is $\theta - \sigma > 0$ and the greater the degree of trade openness (lower is a). However, while these upper bounds are considerably lower in the open economy relative to the closed economy for all cases of $\sigma < \theta$, they are still of a sizable magnitude to be deemed very unlikely to bind. Hence, for

²¹The determinacy conditions of this policy rule are given by (42a) and (42b) of Proposition 1.

²²We do not report the case when $\sigma = 5$ since that would require values of θ much greater than the empirical estimates discussed in Section 2.7.

Table 3: Upper bound computations on the inflation response coefficient (μ) for determinacy under CWID-timing ($\sigma = 1$)

| | $\theta = 1$ | $\theta = 2$ | $\theta = 3$ | $\theta = 4$ | $\theta = 5$ |
|-----------------|---------------|---------------|---------------|---------------|---------------|
| Closed economy: | $\mu < 32.54$ | $\mu < 32.54$ | $\mu < 32.54$ | $\mu < 32.54$ | $\mu < 32.54$ |
| Open economy: | | | | | |
| $a = 0.85$ | $\mu < 32.54$ | $\mu < 28.12$ | $\mu < 24.79$ | $\mu < 22.18$ | $\mu < 20.09$ |
| $a = 0.70$ | $\mu < 32.54$ | $\mu < 25.87$ | $\mu < 21.52$ | $\mu < 18.47$ | $\mu < 16.21$ |
| $a = 0.60$ | $\mu < 32.54$ | $\mu < 25.14$ | $\mu < 20.55$ | $\mu < 17.42$ | $\mu < 15.16$ |

both open and closed economies, a policy rule that targets future producer price inflation under CWID-timing does not seem to matter for equilibrium determinacy at a practical level.

Now suppose that the policy rule targets contemporaneous producer price inflation under CIA-timing.²³ In the presence of significant positive spillover effects between the two countries, such that $2a\theta < \sigma(2a - 1)$, then from (43b) of Proposition 1 the inflation coefficient is constrained by two upper bounds: $1 < \mu < \min \{\Gamma^1, \Gamma^2\}$ where $\Gamma^1 \equiv \frac{(1-\beta)}{2\kappa\phi(1-a)[2a(\sigma-\theta)-\sigma]}$ and $\Gamma^2 \equiv 1 + \frac{2(1+\beta)}{\kappa(1+\sigma\phi)}$. The second upper bound Γ^2 is identical to the determinacy requirements for a closed economy and this bound is unlikely to bind for reasonable parameter values. For example, using the baseline parameter values outlined in Table 2 of Section 2.7, if $\sigma = 5$ then $\Gamma^2 \approx 14.84$. However, under the baseline parameterization the first bound Γ^1 is much more likely to bind. Figure 1 depicts the regions in the parameter space $(1 - a, \mu)$ that are associated with determinacy (D) and indeterminacy (I) when $\sigma = 5$ for three alternative values of $\theta = 1, 1.5, 2$. The dashed-lines in Figure 1 illustrate the value of a required for $2a\theta = \sigma(2a - 1)$ and thus this upper bound Γ^1 ceases to apply.²⁴ Figure 1 suggests that the upper bound on the inflation coefficient μ given by Γ^1 can be small. For example, if $\theta = 1$ then only for a very low degree of trade openness or a sufficiently high degree of trade openness is Γ^1 unlikely to bind. However as θ increases the range of determinacy widens significantly. Thus for values of θ consistent with Bergin (2006) equilibrium indeterminacy is a potential problem when the policy rule targets contemporaneous producer price inflation under CIA-timing.²⁵

²³The determinacy conditions of this policy rule are given by (43a) and (43b) of Proposition 1.

²⁴As shown by (43a) of proposition 1, determinacy in this case requires that $1 < \mu < \Gamma^2$.

²⁵For higher values of θ consistent with Harrigan (1993) then the indeterminacy problem is alleviated.

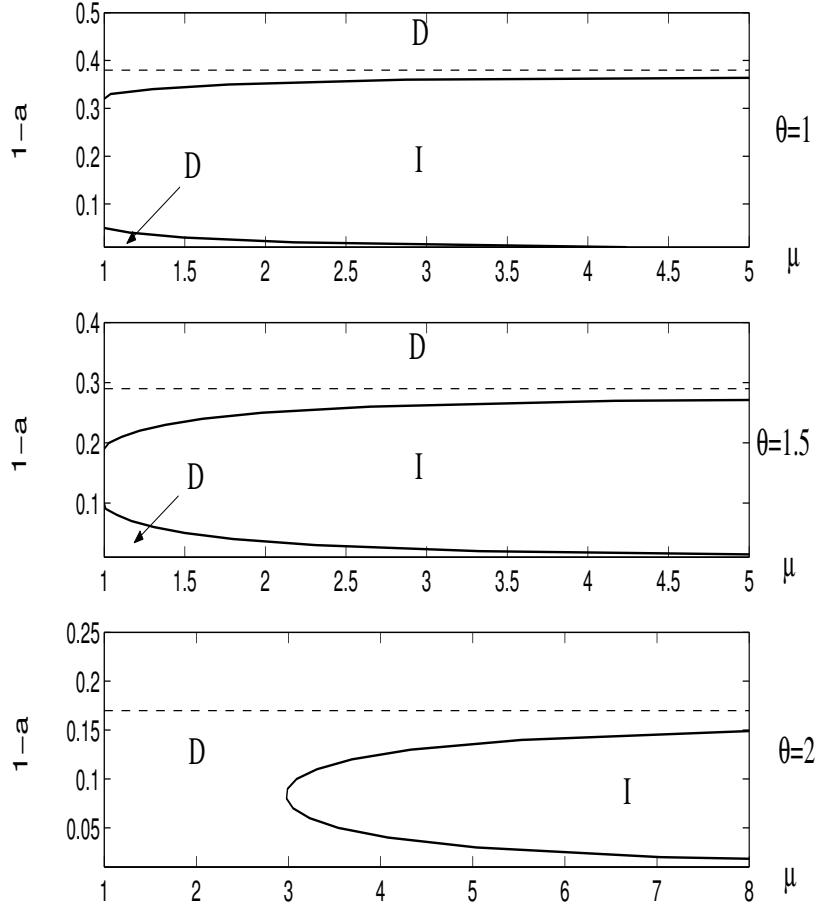


Figure 1: Regions of indeterminacy under a current-looking producer price inflation rule with CIA-timing ($\sigma = 5$)

The preceding analysis suggests that in the absence of a timing equivalence result for open economies, the problem of indeterminacy increases under producer price inflation targeting as we replace CWID-timing with CIA-timing. The explanation behind this finding can be seen by comparing the Aggregate Supply (AS) condition implied by each timing convention. Using equations (29), (30) and (31) and their *foreign* equivalents, implies the following linearized Aggregate Supply (AS) condition for the difference system: $\widehat{\pi}_t^{R(h-f^*)} = \kappa 2(1-a)[1 + \phi 2\theta a]\widehat{T}_t + \kappa [\phi(2a-1) + \frac{1}{\sigma}]\widehat{Z}_t^R$. Using (33) and the respective linearized risk sharing conditions (35) and (36) to eliminate \widehat{T}_t , the linearized AS equation for the difference system can be expressed as:

$$\text{CWID: } \widehat{\pi}_t^{R(h-f^*)} = (\kappa_T \zeta + \kappa_Z)\widehat{Z}_t^R + \beta \widehat{\pi}_{t+1}^{R(h-f^*)} \quad (47)$$

$$\text{CIA: } \widehat{\pi}_t^{R(h-f^*)} = (\kappa_T \zeta + \kappa_Z) \widehat{Z}_t^R + \beta \widehat{\pi}_{t+1}^{R(h-f^*)} - \sigma \kappa_T \zeta \widehat{R}_t^R \quad (48)$$

where $\kappa_T \equiv 2\kappa(1-a)[1+2a\phi\theta] > 0$, $\kappa_Z \equiv \kappa[\phi(2a-1)+1/\sigma] > 0$ and $\zeta \equiv [\sigma(2a-1)]^{-1} > 0$. First note that in a closed economy the AS equations are the same under the two timing conventions i.e. if $a = 1$ then κ_T is zero and (47) and (48) collapse to $\widehat{\pi}_t^{R(h-f^*)} = \kappa_Z \widehat{Z}_t^R + \beta \widehat{\pi}_{t+1}^{R(h-f^*)}$. However, for open economies, inspection of the above equations suggest that the dynamics of producer price inflation crucially depends on the terms of trade, which in turn depends on how money is introduced into the model. Under CWID-timing the dynamics of the terms of trade are embed into the dynamics of the output gap since they are proportional to one another from the RER condition: $\frac{1}{\sigma} \widehat{Z}_t^R = (2a-1) \widehat{T}_t$. In contrast under CIA-timing the interest rate drives a wedge between the terms of trade and the output gap since $\frac{1}{\sigma} \widehat{Z}_t^R - \widehat{R}_t^R = (2a-1) \widehat{T}_t$. Thus under CIA-timing the nominal interest rate also enters into the AS equation for open economies as a negative cost shock. Consequently there are now two channels where the terms of trade affect producer price inflation and these channels can yield opposite effects on the local dynamics of the economy. Given the policy rule $\widehat{R}_t^R = \mu \widehat{\pi}_t^{R(h-f^*)}$ then under CIA-timing, (48) can be alternatively expressed as

$$\text{CIA: } \widehat{\pi}_t^{R(h-f^*)} = \left(\frac{\kappa_T \zeta + \kappa_Z}{1 + \mu \sigma \kappa_T \zeta} \right) \widehat{Z}_t^R + \left(\frac{\beta}{1 + \mu \sigma \kappa_T \zeta} \right) \widehat{\pi}_{t+1}^{R(h-f^*)}. \quad (49)$$

By comparing the coefficients for \widehat{Z}_t^R and $\widehat{\pi}_{t+1}^{R(h-f^*)}$ given in the AS equations (47) and (49), a by-product of CIA-timing is that current domestic inflation ($\widehat{\pi}_t^{R(h-f^*)}$) is less responsive to changes in domestic demand and future domestic inflation.

3.3 Producer Price Inflation Targeting vs. Consumer Price Inflation Targeting

A key question for monetary policy setting in open economies is whether producer price inflation might be a better target than consumer price inflation. As this subsection shows, in terms of equilibrium determinacy, whether the policy rule should react to producer or consumer price inflation crucially depends on the timing convention on money assumed.

3.3.1 Forward-Looking Rules Under CWID-timing

The criteria for determinacy when the monetary authority reacts to future consumer price inflation is summarized in Proposition 2.

Proposition 2 *Suppose that monetary policy reacts to forward-looking consumer price inflation under CWID timing. Then the necessary and sufficient conditions for equilibrium determinacy are:*

- *Aggregate System / Closed Economy*

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)} \quad (50)$$

- *Difference System*

$$1 < \mu < \min \left\{ \frac{1}{2(1 - a)}, \frac{2(1 + \beta) + \kappa\Lambda_1}{\kappa\Lambda_1 + 4(1 + \beta)(1 - a)} \right\}$$

- *Open Economy*

$$1 < \mu < \frac{2(1 + \beta) + \kappa\Lambda_1}{\kappa\Lambda_1 + 4(1 + \beta)(1 - a)} \quad (51)$$

where $\Lambda_1 \equiv 1 + \sigma\phi + 4\phi a(1 - a)(\theta - \sigma)$.

Proof. See Appendix 5.2. \square

Proposition 2 shows that the indeterminacy problem is more severe in open economies with CWID-timing under a forward-looking consumer price inflation rule. This follows from comparing the upper bound on the inflation response coefficient (μ) of condition (51) with (50).²⁶ The impact that the degree of trade openness has on the upper bound in condition (51) is given by:

$$\frac{\partial(51)}{\partial a} = \frac{4(1 + \beta) [\kappa [1 + 2\phi\theta - \phi\sigma - 4\phi(\theta - \sigma)a(1 - a)] + 2(1 + \beta)]}{[\kappa\Lambda_1 + 4(1 + \beta)(1 - a)]^2} > 0 \quad (52)$$

and thus the greater the degree of trade openness, the higher the range of indeterminacy.

It is also important to note that the relative size of σ and θ have little significance for

²⁶Is straightforward to verify that $\frac{2(1+\beta)+\kappa\Lambda_1}{\kappa\Lambda_1+4(1+\beta)(1-a)} < 1 + \frac{2(1+\beta)}{\kappa(1+\sigma\phi)}$.

Table 4: Upper bound computations on the inflation response coefficient (μ) for determinacy under a forward-looking CPI rule with CWID-timing

| | | $\theta = 1$ | $\theta = 2$ | $\theta = 3$ | $\theta = 4$ | $\theta = 5$ |
|-----------------|------------|---------------|---------------|---------------|---------------|---------------|
| Closed economy: | | | | | | |
| $\sigma = 1$ | | $\mu < 32.54$ | $\mu < 32.54$ | $\mu < 32.54$ | $\mu < 32.54$ | $\mu < 32.54$ |
| $\sigma = 5$ | | $\mu < 14.84$ | $\mu < 14.84$ | $\mu < 14.84$ | $\mu < 14.84$ | $\mu < 14.84$ |
| Open economy: | | | | | | |
| $\sigma = 1$ | $a = 0.85$ | $\mu < 3.11$ | $\mu < 3.08$ | $\mu < 3.05$ | $\mu < 3.02$ | $\mu < 2.99$ |
| | $a = 0.70$ | $\mu < 1.63$ | $\mu < 1.62$ | $\mu < 1.62$ | $\mu < 1.61$ | $\mu < 1.60$ |
| | $a = 0.60$ | $\mu < 1.24$ | $\mu < 1.24$ | $\mu < 1.23$ | $\mu < 1.23$ | $\mu < 1.23$ |
| $\sigma = 5$ | $a = 0.85$ | $\mu < 2.99$ | $\mu < 2.96$ | $\mu < 2.93$ | $\mu < 2.91$ | $\mu < 2.88$ |
| | $a = 0.70$ | $\mu < 1.63$ | $\mu < 1.62$ | $\mu < 1.61$ | $\mu < 1.60$ | $\mu < 1.59$ |
| | $a = 0.60$ | $\mu < 1.24$ | $\mu < 1.24$ | $\mu < 1.23$ | $\mu < 1.23$ | $\mu < 1.23$ |

determinacy when the policy rule targets consumer price inflation, a stark contrast to when the policy rule targets producer price inflation. For example, in the case when production spillover effects are absent between the two countries ($\sigma = \theta$), the upper bound given in (51) collapses to

$$1 < \mu < \frac{2(1 + \beta) + \kappa [1 + \sigma\phi]}{\kappa [1 + \sigma\phi] + 4(1 + \beta)(1 - a)}.$$

Comparing this upper bound with (50) it is straightforward to see that determinacy is still relatively lower in the open economy because of the presence of the degree of trade openness.

We illustrate the regions of indeterminacy using the baseline parameter values summarized in Table 2 of Section 2.7. Table 4 summarizes the relevant upper bounds in the inflation response coefficient (μ) for values of $\sigma = 1$ and $\sigma = 5$. In accordance with (52), the upper bounds computed for the open-economy decrease the greater the degree of trade openness (lower is a). For all combinations of θ and σ and for all values of a , the upper bounds are not only considerably lower in the open economy relative to the closed economy, but they are of an empirically relevant magnitude to suggest that equilibrium indeterminacy could be a serious problem.

Comparing the determinacy condition (51) of Proposition 2 with conditions (42a) and (42b) of Proposition 1, one clear conclusion to emerge under CWID-timing is that in terms of equilibrium determinacy targeting producer price inflation is preferable to consumer price

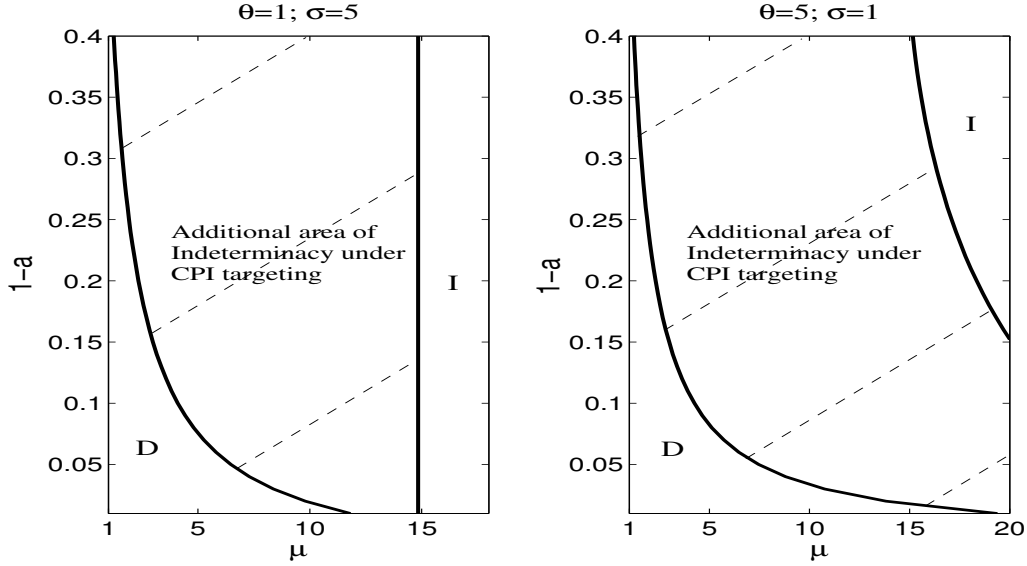


Figure 2: Regions of indeterminacy under forward-looking rules with CWID-timing

inflation.²⁷ This conclusion can be easily illustrated for the baseline parameterization. For these two alternative measures of inflation, Figure 2 depicts the regions in the parameter space (a, μ) that are associated with determinacy (D) and indeterminacy (I) for two alternative combinations of (σ, θ) . By inspection, reacting to consumer price inflation substantially increases the range of indeterminacy. This finding coincides with the conclusion of the existing literature (e.g. Linnemann and Schabert (2006) and Llosa and Tuesta (2008)) and is the basis for advocating that monetary policy should target producer price inflation, rather than consumer price inflation, in order to minimize policy-induced indeterminacy for open economies.

3.3.2 Current-Looking Rules Under CIA-timing

The criteria for determinacy when the monetary authority reacts to contemporaneous consumer price inflation under CIA-timing is summarized in Proposition 3.

Proposition 3 *Suppose that monetary policy reacts to current-looking consumer price inflation under CIA timing. Then the necessary and sufficient conditions for equilibrium*

²⁷By comparing the upper bounds on the inflation coefficient it is straightforward to show that this upper bound is relatively lower under consumer price inflation targeting: i.e. $\frac{2(1+\beta)+\kappa\Lambda_1}{\kappa\Lambda_1+4(1+\beta)(1-a)} < 1 + \frac{2(1+\beta)}{\kappa(1+\sigma\phi)}$ and $\frac{2(1+\beta)+\kappa\Lambda_1}{\kappa\Lambda_1+4(1+\beta)(1-a)} < 1 + \frac{2(1+\beta)}{\kappa[1+\sigma\phi+4\phi a(1-a)(\theta-\sigma)]}$.

determinacy are:

- *Aggregate System / Closed Economy*

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)} \quad (53)$$

- *Difference System*

$$(a) \ 4(1 - a)(1 + \beta) \geq \kappa [\phi\sigma + 4a - 3 - 4\phi a(1 - a)(\theta + \sigma)]:$$

$\mu > 1$ and either

$$(i) \ |a_2| > 3 \quad \text{or} \quad (ii) \ a_0^2 - a_0a_2 + a_1 > 1$$

$$(b) \ 4(1 - a)(1 + \beta) < \kappa [\phi\sigma + 4a - 3 - 4\phi a(1 - a)(\theta + \sigma)]:$$

$$1 < \mu < \frac{2(1 + \beta) + \kappa\Lambda_1}{\kappa [\phi\sigma + 4a - 3 - 4\phi a(1 - a)(\theta + \sigma)] - 4(1 - a)(1 + \beta)}$$

and either

$$(i) \ |a_2| > 3 \quad \text{or} \quad (ii) \ a_0^2 - a_0a_2 + a_1 > 1$$

where $\Lambda_1 \equiv 1 + \sigma\phi + 4a\phi(1 - a)(\theta - \sigma)$.

- *Open Economy*

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)} \quad (54a)$$

and either

$$(i) \ |a_2| > 3 \quad \text{or} \quad (ii) \ a_0^2 - a_0a_2 + a_1 > 1 \quad (54b)$$

where $a_j, j = 0, 1, 2$ are given in Appendix 5.3.

Proof. See Appendix 5.3. \square

If the policy rule reacts to contemporaneous consumer price inflation, then Proposition 3 shows that under CIA-timing the upper bound on the inflation coefficient is the same for both closed and open economies i.e. (54a) = (53). Thus provided at least one of the conditions given in (54b) is satisfied then the determinacy requirements for the open

economy mirror the closed economy. For the baseline parameter values summarized in Table 2 in Section 2.7 the numerical analysis suggests that condition (ii) of (54b) is always satisfied for any $\mu > 1$. Noting that this determinacy condition can be expressed as:

$$\begin{aligned} & \frac{2(1-a)}{\beta} \mu [2(1-a)\mu(1-\beta) + \kappa\Lambda_1(\mu-1) - \kappa\mu 2(1-a)[1+2a\theta\phi] - 1] \\ & + (1-\beta) + 2\beta(1-a)\mu + \kappa\mu 2(1-a)[1+2a\theta\phi] > 0, \end{aligned}$$

then this becomes transparent by considering the case where $\beta \rightarrow 1$ since this condition collapses to:

$$\mu + 2a\phi\theta + (2a-1)\phi\sigma(\mu-1) > 0.$$

Therefore the determinacy properties of the closed and open economy are approximately the same under CIA-timing if policy reacts to contemporaneous consumer price inflation. However this is in stark contrast to producer price inflation targeting, where from the analysis presented in Section 3.2, equilibrium indeterminacy is a potentially more serious problem. Thus we can conclude that under CIA-timing, consumer price inflation is preferable to producer price inflation in order to minimize policy-induced indeterminacy for open economies.

3.3.3 Discussion

To summarize, the previous subsections showed that the preference towards a particular inflation index, suggested by the criteria for equilibrium determinacy, crucially depends upon the timing assumption on money employed. Under CWID-timing, it was shown that producer price inflation is preferable in preventing equilibrium indeterminacy, whereas under CIA-timing, targeting consumer price inflation is preferable. To decipher this result intuitively, the key is to understand why the problem of indeterminacy is mitigated as we replace CWID-timing with CIA-timing under consumer price inflation targeting.

Using the linearized equation for the CPI inflation differential (32) and the UIP condition (34), the interest rate rule under CPI targeting for the difference system can be expressed as:

$$\widehat{R}_t^R = \mu(2a-1)\widehat{\pi}_{t+v}^{R(h-f^*)} + 2\mu(1-a)\widehat{R}_{t+v-1}^R. \quad (55)$$

If the interest-rate rule is forward-looking ($v = 1$) then the reaction to future inflation may be negative for high μ and low a . Hence monetary policy activeness against consumer price inflation expectations and trade openness may provoke indeterminacy. However if the interest rate rule is current-looking ($v = 0$) from (55) this generates policy inertia which increases the likelihood of determinacy. This policy inertia appears as a result of the UIP condition and is not present in the closed economy (i.e. $a = 1$).

The intuition for why under CIA-timing, contemporaneous consumer price inflation rules exhibit policy inertia in the open economy, rests with how changes in the terms of trade affect the dynamics of the CPI inflation rate. In an open economy the *home* CPI inflation rate depends on both the rate of producer price inflation and the terms of trade:

$$\widehat{\pi}_{t+v} = \widehat{\pi}_{t+v}^h + (1 - a) \left(\widehat{T}_{t+v} - \widehat{T}_{t+v-1} \right) \quad (56)$$

where $v = 1$ under future inflation targeting and $v = 0$ under contemporaneous inflation targeting. Under CWID-timing, the policy rule reacts to forward-looking consumer price inflation ($v = 1$). Given that an increase in the real interest rate of the *home* country results in a current improvement in the terms of trade ($\widehat{T}_t \downarrow$), then in response to a non-fundamental shock, inflationary expectations are self-fulfilling and indeterminacy is generated provided $\widehat{\pi}_{t+1} \uparrow$. Whereas, indeterminacy depends on the sign of international spillover effects (i.e. the relative size of σ and θ) under producer price inflation targeting, as shown in (56) for consumer price inflation targeting, indeterminacy depends on the relative weight of changes in producer price inflation and adjustments in the terms of trade. For example, suppose that an increase in the real interest rate, lowers real marginal cost putting downward pressure on the producer price inflation rate $\widehat{\pi}_{t+1}^h \downarrow$ and from (56) downward pressure on the CPI inflation rate. With $v = 1$ the improvement in the terms of trade ($\widehat{T}_t \downarrow$) associated with an increase the real interest rate, from (56) generates upward pressure on the CPI inflation rate. Since the degree of trade openness determines the influence of the terms of trade on the CPI inflation rate, if this effect is strong enough, the CPI inflation rate can actually rise despite producer price inflation falling, thus validating the initial inflationary belief.

Under CIA-timing the policy rule reacts to contemporaneous-looking consumer price inflation ($v = 0$) and thus from (56) \widehat{T}_{t-1} is predetermined. In this case an improvement in

the terms of trade ($\widehat{T}_t \downarrow$) exerts downward pressure on CPI inflation. For example, suppose that an increase in the real interest rate results in putting upward pressure on the producer price inflation (which as discussed in Sections 3.1 and 3.2. requires positive international spillover effects). With $v = 0$ the improvement in the terms of trade generates upward pressure on the CPI inflation rate making indeterminacy less likely if the consumer price inflation is targeted. In other words, monetary policy, through targeting contemporaneous consumer price inflation, can help to prevent self-fulfilling inflation expectations by offsetting the negative cost shock to producer price inflation introduced through CIA-timing.

4 Conclusion

In the design of monetary policy it is imperative that a proposed policy rule does not introduce real indeterminacy and thus self-fulfilling fluctuations into the economy. For open economies, a key policy question relates to which index of inflation central banks should target in the policy rule. Recent research has advocated the targeting of producer price inflation, since the targeting of consumer price inflation may lead to welfare-reducing sunspot fluctuations. The contribution of this paper was to demonstrate that such policy recommendations are highly sensitive to the timing of money employed in the determinacy analysis.

This paper has shown that the timing equivalence result for a closed economy does not generally apply for open economies due to the presence, in the latter, of international spillover effects in production. A corollary of this is that different timing assumptions on money, that have no consequences for equilibrium determinacy in a closed economy, can have potentially non-trivial implications for indeterminacy in open economies. Using the criteria for equilibrium determinacy, we have shown that the preferred index of inflation in the policy rule is producer price inflation under CWID-timing, and consumer price inflation under CIA-timing.

Consequently, in contrast with the existing literature, the results presented in this paper suggest that on the grounds of equilibrium determinacy, central banks may be justified in their adoption of inflation-targeting policies that focus on consumer price inflation.

References

- [1] Aoki, M. 1981. *Dynamic Analysis of Open Economies*. New York: Academic Press.
- [2] Batini, N., P. Levine and J. Pearlman, 2004. Indeterminacy with Inflation-Forecast-Based Rules in a Two-Bloc Model. International Finance Discussion Paper No. 797, Board of Governors of the Federal Reserve System.
- [3] Benigno, G. and P. Benigno, 2003. Price Stability in Open Economies. *Review of Economic Studies* 70, pp743-764.
- [4] Benigno, G. and P. Benigno, 2006. Designing Targeting Rules for International Monetary Policy Cooperation. *Journal of Monetary Economics* 53, pp473-506.
- [5] Bergin, P., 2006. How Well Can the New Open Economy Macroeconomics Explain the Exchange Rate and Current Account. *Journal of International Money and Finance* 25, pp675-701.
- [6] Bernanke, B. and M. Woodford, 1997. Inflation Forecast and Monetary Policy, Part 2. *Journal of Money, Credit and Banking* 29, pp653-684.
- [7] Bullard, J. and E. Schaling, 2009. Monetary Policy, Determinacy and Learnability in a Two-Block World Economy. *Journal of Money, Credit and Banking* 41, pp1585-1612.
- [8] Calvo, G., 1983. Staggered Prices in a Utility Maximising Framework. *Journal of Monetary Economics* 12, pp983-998.
- [9] Carlstrom, C.T. and T.S. Fuerst, 2001. Timing and Real Indeterminacy in Monetary Models. *Journal of Monetary Economics* 47, pp285-298.
- [10] Carlstrom, C.T., T.S. Fuerst, and F. Ghironi, 2006. Does it Matter (for Equilibrium Determinacy) What Price Index the Central Bank Targets? *Journal of Economic Theory* 128, pp214-231.
- [11] Chari, V., P. Kehoe, and E. McGrattan, 2002. Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates? *Review of Economic Studies* 69, pp533-563.

- [12] Clarida, R., J. Gali, and M. Gertler, 2000. Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics* 115, pp147-180.
- [13] Clarida, R., J. Gali, and M. Gertler, 2002. A Simple Framework for International Monetary Policy Analysis. *Journal of Monetary Economics* 49, pp877-904.
- [14] De Fiore, F. and Z. Liu, 2005. Does Trade Openness Matter for Aggregate Instability? *Journal of Economic Dynamics and Control* 29, pp1165-1192.
- [15] Gali, J. and T. Monacelli, 2005. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economics Studies* 72, 707-734.
- [16] Harrigan, J., 1993. OECD Imports and Trade Barriers in 1983. *Journal of International Economics* 35, pp91-111.
- [17] Kurozumi, T., 2006. Determinacy and Expectational Stability of Equilibrium in a Monetary Sticky-Price Model with Taylor Rules. *Journal of Monetary Economics* 53, pp827-846.
- [18] Leith, C. and S. Wren-Lewis, 2009. Taylor Rules in the Open Economy. *European Economic Review* 53, pp971-995.
- [19] Linnemann, L. and L. Schabert, 2006. Monetary Policy and the Taylor Principle in Open Economies. *International Finance* 9, pp343-367.
- [20] Llosa, G. and V. Tuesta, 2008. Determinacy and Learnability of Monetary Policy Rules in Small Open Economies. *Journal of Money, Credit and Banking* 40, pp1033-1063.
- [21] Rotemberg, J. and M. Woodford, 1997. An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy. *NBER Macroeconomics Annual* 12, pp297-346.
- [22] Taylor, J.B., 1999. Staggered Price and Wage Setting in Macroeconomics, in: Taylor, J.B. and M. Woodford, eds; *Handbook of Macroeconomics*, Vol.1B, pp1009-1050, Amsterdam: Elsevier North-Holland.
- [23] Vissing-Jorgensen, A., 2002. Limited-Asset Market Participation and the Elasticity of Intertemporal Substitution. *Journal of Political Economy* 110, pp825-853.

- [24] Woodford, M., 2003. *Interest Rates and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.
- [25] Zanna, L., 2003. Interest Rate Rules and Multiple Equilibria in the Small Open Economy. mimeo, University of Pennsylvania.

5 Appendix

The linearized system of equations summarized in Table 1 consists of both the aggregate and difference systems. Equilibrium determinacy requires that there is a unique solution for both the aggregate system and the difference system, as only in this case is determinacy achieved at the individual country level. Since the aggregate system is analogous to Carlstrom and Fuerst (2001) closed-economy model it straightforward to show that under a forward-looking (current-looking) interest-rate rule with CWID (CIA) timing, a necessary and sufficient condition for determinacy of the aggregate system (or closed-economy) is:

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa(1 + \sigma\phi)}. \quad (\text{A1})$$

The difference system can be reduced to:

$$\begin{aligned} \text{IS: } \widehat{Z}_{t+1}^R &= \widehat{Z}_t^R - \sigma(2a - 1)\widehat{\pi}_{t+1}^{R(h-f^*)} + \varsigma_0\widehat{R}_{t+1}^R + \varsigma_1\widehat{R}_t^R \\ \text{AS: } \widehat{\pi}_t^{R(h-f^*)} &= (\kappa_T\zeta + \kappa_Z)\widehat{Z}_t^R + \beta\widehat{\pi}_{t+1}^{R(h-f^*)} + \varsigma_2\widehat{R}_t^R \\ \text{TR: } \widehat{R}_t^R &= \varrho_0\widehat{\pi}_{t+v}^{R(h-f^*)} + \varrho_1\widehat{R}_{t+v-1}^R \end{aligned} \quad (\text{A2})$$

where $\kappa_T \equiv 2\kappa(1-a)[1+2a\phi\theta] > 0$, $\kappa_Z \equiv \kappa[\phi(2a-1)+1/\sigma] > 0$ and $\zeta \equiv [\sigma(2a-1)]^{-1} > 0$. Under CWID-timing $\varsigma_0 = 0$, $\varsigma_1 = \sigma(2a-1)$ and $\varsigma_2 = 0$. Under CIA-timing $\varsigma_0 = \sigma$, $\varsigma_1 = -2\sigma(1-a)$ and $\varsigma_2 = -\kappa_T\zeta\sigma$. For producer price inflation targeting $\varrho_0 = \mu$ and $\varrho_1 = 0$. For consumer price inflation targeting $\varrho_0 = \frac{\mu(2a-1)}{1-2\mu(1-a)}$ and $\varrho_1 = 0$ when policy is forward-looking ($v = 1$), whereas $\varrho_0 = \mu(2a-1)$ and $\varrho_1 = 2\mu(1-a)$ when policy is contemporaneous ($v = 0$).

5.1 Proof of Proposition 1

If monetary policy targets forward-looking producer price inflation under CWID-timing then (A2) can be reduced to a two-dimensional system $\mathbf{x}_{t+1}^R = \mathbf{A}\mathbf{x}_t^R$ where \mathbf{x}^R is the column vector of non-predetermined endogenous variables $[\widehat{Z}^R, \widehat{\pi}^{R(h-f^*)}]$, and \mathbf{A} is the coefficient matrix:

$$\mathbf{A} \equiv \begin{bmatrix} 1 - \frac{(\mu-1)}{\beta} [\kappa_T + \sigma(2a-1)\kappa_Z] & \frac{\sigma(2a-1)(\mu-1)}{\beta} \\ -\frac{1}{\beta} [\kappa_T\zeta + \kappa_Z] & \frac{1}{\beta} \end{bmatrix}.$$

Equilibrium determinacy requires that both eigenvalues of \mathbf{A} are outside the unit circle. From Woodford (2003) this is the case if and only if: (i) $\det \mathbf{A} > 1$, (ii) $1 + \det \mathbf{A} - \text{tr} \mathbf{A} > 0$ and (iii) $1 + \det \mathbf{A} + \text{tr} \mathbf{A} > 0$; where $\det \mathbf{A} = \frac{1}{\beta}$ and $\text{trace} \mathbf{A} = 1 - \frac{(\mu-1)}{\beta} [\kappa_T + \sigma(2a-1)\kappa_Z] + \frac{1}{\beta}$. Condition (i) is always satisfied while condition (ii) is satisfied provided $\mu > 1$. Condition (iii) then implies that

$$1 < \mu < 1 + \frac{2(1 + \beta)}{\kappa[1 + \phi\sigma + 4\phi a(1-a)(\theta - \sigma)]}. \quad (\text{A3})$$

By comparison of the upper bounds on μ given by (A1) and (A3), it is straightforward to verify that $1 + \frac{2(1+\beta)}{\kappa(1+\sigma\phi)} \leq 1 + \frac{2(1+\beta)}{\kappa[1+\phi\sigma+4\phi a(1-a)(\theta-\sigma)]}$ if $\theta \leq \sigma$ and $1 + \frac{2(1+\beta)}{\kappa(1+\sigma\phi)} > 1 + \frac{2(1+\beta)}{\kappa[1+\phi\sigma+4\phi a(1-a)(\theta-\sigma)]}$ if $\theta > \sigma$. Hence (42a) and (42b) are the necessary and sufficient conditions for local determinacy of the aggregate and difference systems.

Now suppose that monetary policy targets current-looking producer price inflation under CIA-timing. In this case the coefficient matrix \mathbf{A} is given by:

$$\mathbf{A} \equiv \begin{bmatrix} 1 - \left(\frac{\mu-(2a-1)}{\beta}\right) [\sigma\kappa_T\zeta + \sigma\kappa_Z] & \sigma \left(\frac{\mu-(2a-1)}{\beta}\right) [1 + \mu\sigma\kappa_T\zeta] - 2\sigma\mu(1-a) \\ -\frac{1}{\beta} [\kappa_T\zeta + \kappa_Z] & \frac{1+\mu\sigma\kappa_T\zeta}{\beta} \end{bmatrix}.$$

As before equilibrium determinacy requires: (i) $\det \mathbf{A} > 1$, (ii) $1 + \det \mathbf{A} - \text{tr} \mathbf{A} > 0$ and (iii) $1 + \det \mathbf{A} + \text{tr} \mathbf{A} > 0$; where $\det \mathbf{A} = \frac{1}{\beta} + \frac{\mu\sigma}{\beta} [(2a-1)\kappa_T\zeta - 2(1-a)\kappa_Z]$ and $\text{tr} \mathbf{A} = 1 + \frac{1}{\beta} + \frac{(2a-1)\sigma\kappa_T\zeta}{\beta} - \left(\frac{\mu-(2a-1)}{\beta}\right) \sigma\kappa_Z$. Condition (ii) is satisfied provided $\mu > 1$. Condition (iii) can be expressed as

$$2(1+\beta) + \frac{\kappa\Lambda_1}{\beta} + \frac{\mu\kappa}{\beta} [4a(1-a)\phi\theta - \phi\sigma(2a-1)(3-2a) - 1] > 0 \quad (\text{A4})$$

where $\Lambda_1 \equiv 1 + \sigma\phi + 4a\phi(1-a)(\theta-\sigma) > 0$. (A4) is automatically satisfied if $4a(1-a)\phi\theta \geq \phi\sigma(2a-1)(3-2a) + 1$. Otherwise (A4) imposes the following upper bound on μ :

$$\mu < \frac{2(1+\beta) + \kappa\Lambda_1}{\kappa [1 + \sigma\phi(2a-1)(3-2a) - 4a(1-a)\phi\theta]}. \quad (\text{A5})$$

Comparing the upper bounds on μ given by (A5) and (A1) it is straightforward to verify that $\frac{2(1+\beta) + \kappa\Lambda_1}{\kappa[1 + \sigma\phi(2a-1)(3-2a) - 4a(1-a)\phi\theta]} > 1 + \frac{2(1+\beta)}{\kappa(1+\sigma\phi)}$ and thus (A5) is redundant here. Condition (i) is automatically satisfied if $2a\theta \geq \sigma(2a-1)$. Otherwise

$$1 < \mu < \frac{1-\beta}{2\kappa\phi(1-a)[\sigma(2a-1) - 2a\theta]}. \quad (\text{A6})$$

From comparison of the upper bounds given by (A6) and (A1) it follows that $\frac{1-\beta}{2\kappa\phi(1-a)[\sigma(2a-1) - 2a\theta]} \geq 1 + \frac{2(1+\beta)}{\kappa(1+\sigma\phi)}$ and hence (43a) and (43b) are the necessary and sufficient conditions for local determinacy of the aggregate and difference systems.

5.2 Proof of Proposition 2

If monetary policy targets forward-looking consumer price inflation under CWID-timing then (A2) can be reduced to a two-dimensional system $\mathbf{x}_{t+1}^R = \mathbf{A}\mathbf{x}_t^R$ where \mathbf{x}^R is the column vector of non-predetermined endogenous variables $[\widehat{Z}^R, \widehat{\pi}^{R(h-f^*)}]$, and \mathbf{A} is the coefficient matrix:

$$\mathbf{A} \equiv \begin{bmatrix} 1 - \frac{(\mu-1)}{\beta[1-2(1-a)\mu]} [\kappa_T + \sigma(2a-1)\kappa_Z] & \frac{\sigma(2a-1)(\mu-1)}{\beta[1-2(1-a)\mu]} \\ -\frac{1}{\beta} [\kappa_T\zeta + \kappa_Z] & \frac{1}{\beta} \end{bmatrix}.$$

Equilibrium determinacy requires that both eigenvalues of \mathbf{A} are outside the unit circle. From Woodford (2003) this is the case if and only if: (i) $\det \mathbf{A} > 1$, (ii) $1 + \det \mathbf{A} -$

$\text{tr}\mathbf{A} > 0$ and (iii) $1 + \det \mathbf{A} + \text{tr}\mathbf{A} > 0$; where $\det \mathbf{A} = \frac{1}{\beta}$ and $\text{trace}\mathbf{A} = 1 + \frac{1}{\beta} - \frac{(\mu-1)}{\beta[1-2(1-a)\mu]} [\kappa_T + \sigma(2a-1)\kappa_Z]$. Condition (i) is always satisfied while condition (ii) is satisfied for $\mu > 1$ provided

$$1 < \mu < \frac{1}{2(1-a)}. \quad (\text{B1})$$

Condition (iii) then implies that

$$1 < \mu < \frac{2(1+\beta) + \kappa\Lambda_1}{4(1+\beta)(1-a) + \kappa\Lambda_1}, \quad (\text{B2})$$

where $\Lambda_1 \equiv 1 + \sigma\phi + 4\phi a(1-a)(\theta - \sigma) > 0$. It is straightforward to show that the upper bound on μ given by (B2) is a stronger restriction than the upper bounds on μ given by either (B1) or (A1). Hence (51) is the necessary and sufficient condition for local determinacy of the aggregate and difference systems.

5.3 Proof of Proposition 3

If monetary policy targets current-looking consumer price inflation under CIA-timing then (A2) can be reduced to a three-dimensional system $\mathbf{x}_{t+1}^R = \mathbf{A}\mathbf{x}_t^R$ where \mathbf{x}^R is the column vector of endogenous variables $[\widehat{Z}_t^R, \widehat{\pi}_t^R, \widehat{\pi}_{t-1}^R]$. The three eigenvalues of \mathbf{A} are solutions to the cubic equation $r^3 + a_2r^2 + a_1r + a_0 = 0$, where

$$\begin{aligned} a_2 &= -1 - \frac{1}{\beta} - 2(1-a)\mu - \frac{\mu(2a-1)\sigma\kappa_T}{\beta} + \frac{(\mu-1)[\kappa_T\zeta + \kappa_Z]}{\beta} \\ a_1 &= \frac{1}{\beta} + \frac{2(1-a)\mu}{\beta} + 2(1-a)\mu + \frac{\mu(2a-1)\sigma\kappa_T}{\beta} \\ a_0 &= -\frac{2\mu(1-a)}{\beta}. \end{aligned}$$

With one predetermined variable, determinacy requires that two eigenvalues are outside the unit circle and one eigenvalue is inside the unit circle. By Proposition C.2 of Woodford (2003) this is the case if and only if either of the following two cases are satisfied:

(Case 1): $1 + a_2 + a_1 + a_0 < 0$, $-1 + a_2 - a_1 + a_0 > 0$;

(Case 2): $1 + a_2 + a_1 + a_0 > 0$, $-1 + a_2 - a_1 + a_0 < 0$, & $|a_2| > 3$ or $a_0^2 - a_0a_2 + a_1 - 1 > 0$;

Case (1) is not obtainable since the first inequality can never be satisfied for $\mu > 1$. The first inequality of Case (2) requires $\mu > 1$, and the second inequality is automatically satisfied if $4(1-a)(1+\beta) \geq \kappa[\phi\sigma + 4a - 3 - 4\phi a(1-a)(\theta + \sigma)]$ or otherwise the second inequality imposes the following upper bound on μ :

$$\mu < \frac{2(1+\beta) + \kappa\Lambda_1}{\kappa[\phi\sigma + 4a - 3 - 4\phi a(1-a)(\theta + \sigma)] - 4(1-a)(1+\beta)}. \quad (\text{C1})$$

By comparison of the upper bounds on μ given by (A1) and (C1), it is straightforward to show that $\frac{2(1+\beta) + \kappa\Lambda_1}{\kappa[\phi\sigma + 4a - 3 - 4\phi a(1-a)(\theta + \sigma)] - 4(1-a)(1+\beta)} > 1 + \frac{2(1+\beta)}{\kappa(1+\sigma\phi)}$ and thus (C1) is redundant here. The final two inequalities yield (54b). This completes the proof. \square