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**THE BORDER GAME: FENCES AND “HAPPY MEALS”**

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# The Border Game: Fences and “Happy Meals”

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## Abstract

This paper presents a strategic analysis of the U.S.-Mexico border. Specifically, it looks at the welfare implications and the effects on border fatalities of a) reducing the probability of fatalities conditional on crossing through the desert, and b) erecting border barriers. It is shown that reducing the risk of death (conditional on crossing through the desert) will often -but not always- lead to a fall in overall fatalities. It is also shown that there is, in general, no definite relation between changes in the probability of fatalities and either the welfare of immigrants or overall welfare. Barriers, on the other hand, can never lead to Pareto superior outcomes, though they might lead to lower fatalities.

**Keywords:** Border, Games, Patrolling

**JEL Codes:** F22; R23

“Twelve illegal immigrants who crossed the Mexican border perished as they tried to traverse barren Arizona desert in 115-degree heat and reach a highway, the Border Patrol said yesterday.”

NYT, May 24, 2001

“Repartirá el gobierno 200 mil kits de sobrevivencia entre personas que crucen a EU sin documentos.-Se busca evitar que mueran en el viaje, dice Juan Hernandez a un diario de San Francisco.-El kit o *cajita feliz*, como en tono de broma se bautizó al programa, incluirá medicina antidiarréica, vendas, aspirinas, acetaminophen (Tylenol), antídotos contra la picadura de alacranes, sustancias para prevenir deshidratación, agua, sal, carne seca, atún y granola. Además, a las mujeres indocumentadas que busquen llegar a Estados Unidos se les proporcionará un paquete de píldoras anticonceptivas y a los hombres 25 condones.”

La Jornada, May 18, 2001

## 1 Introduction

The above quotes illustrate the dramatic stakes facing illegal immigrants attempting to cross the U.S.-Mexico border, as well as the inadequacy (cynicism?) of the authorities' responses. Now, contrary to what the previous quotes seem to suggest, this helplessness is by no means restricted to the Mexican side. American policy is certainly erratic on a large scale, as is often pointed out : It spends enormous efforts in trying to prevent people from crossing the border, but then does nothing (or very little) to prosecute those employing them (despite the fact that prevailing law - IRCA 1986 - does contemplate such sanctions)<sup>1</sup>. It has been claimed that the INS de facto policy since 1997 has been one of “once you are in, you are in” (Orrenius 2001)<sup>2</sup>. Less noticed is the fact that American policy is just as erratic in pursuing the narrower goal of blocking the border: The Border Patrol focused for a long time on apprehensions, and then, for no obvious reasons,

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<sup>1</sup>Moreover, the focus on the border is in itself quite puzzling, given that a great proportion of the illegal aliens (40-50% by some estimates, see Andreas 2000, p.100) enter the country legally. The INS does not make a major effort to go after the overstayers.

<sup>2</sup>Another author (Andreas 2000) goes even further and claims that the whole U.S. border enforcement policy is basically a public relations exercise geared towards creating an impression of control, rather than the actually preventing illegal entry.

switched to a more deterrence-oriented, localized, barrier-based approach<sup>3</sup>. As this approach was pursued with ever increasing conviction, immigration flows shifted to more inaccessible areas, creating a public opinion backlash over increased border casualties. This is what prompted the Mexican government to propose the “happy meals” program (which, by the way, was aborted before it got started -probably as a result of American pressure). In the face of the enormous cost that physically blocking the whole border would entail, I conjecture this will eventually lead back to more emphasis on patrolling. Indeed, already a so-called NIMBY (“not-in-my-backyard”) effect has been noted: “As crackdowns in one area pushed migrants to neighboring areas, officials and residents in those areas predictably lobbied for more border patrols and resources” (Andreas 2000, p.94). Moreover, the crackdown has led to the development of very sophisticated border-crossing technologies (e.g., tunnels -it is estimated that there are between 50 and 60 tunnels in operation in the border city of El Paso alone, see Andreas p.97; and “coyote”-organizations -whose fees fell for a long time before they started to rise a few years ago; see Orrenius 2001, and Andreas 2000, p.97).

Whatever the particulars, the overall picture is that of a highly strategic interaction between a number of players (the U.S. federal government, the border states, the INS and the Border Patrol, the immigrant themselves, the Mexican government, etc.), whose objectives hardly ever fully coincide. It is rather hard to predict the result of specific measures in such environment -certainly without having recourse to some sort of formal analysis. I think that, at least in part, the lack of such analysis explains the zigzagging border policies.

This paper presents a first attempt at analyzing the situation at the border using game-theoretic tools. To do this, it uses a simple game, a sort of extended inspection-game (see, for example, Fudenberg and Tirole 1992, p.17-18). The game is played between a Border Patrol who chooses the level of patrolling, and a representative immigrant who chooses where to cross from amongst two basic types of locations, desert and non-desert. The difference between these is simply that, when crossing through the desert, the immigrant risks death. While the game evidently does not capture the full complexity of the border interaction, I hope this type of game will prove useful in thinking about various policy questions.

Two specific issues are considered. For starters, whether measures such

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<sup>3</sup>See, Andreas 2000, p.92.

as the “happy meals” program could really lead to fewer deaths? This is by no means obvious: Even granting that such a program might be effective in directly reducing the probability of dying in the desert conditional on not being apprehended, it will likely also induce more people to attempt desert crossings. Besides, it will affect patrolling patterns. The issue is whether patrolling responses will be such so as to fully neutralize any additional incentives to attempt crossing through the desert<sup>4</sup>.

The other issue studied is border barriers. Again, the main focus is their effect on the probability of death. In addition, we ask whether these policy interventions enhance efficiency (i.e., increase the expected utilities of all parties).

The results are mixed: Reducing the (conditional) probability of dying will often result in fewer deaths overall, but not always. Similarly, erecting barriers at non-desert locations might or might not reduce fatalities.

It is shown that, in general, there is no systematic connection between efficiency and changes in the unconditional probability of dying when crossing the border. Only if one ignores costs of patrolling, does a connection of sorts emerge: Any welfare improving fall in the conditional probability of dying must lead to a reduction in the probability of fatalities conditional on an unsuccessful crossing attempt.

Finally, it is shown that erecting border barriers can never lead to efficiency gains, as the immigrants will always be harmed.

The paper is organized as follows: After a brief discussion of the literature, the model is presented. Then its basic equilibrium features are characterized (Section 3). Section 4 focuses on comparative statics. The first part of section 4 considers a change in the conditional probability of dying. The second looks at the effects of erecting border barriers. A subsection at the end of section 4 briefly discusses how this model could be modified to study other policy issues. Section 5 concludes.

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<sup>4</sup>It will be shown that desert patrolling will surely increase. The only issue is then whether it will increase by enough to reduce desert crossings. Note that, under the assumption that risking death is worthwhile (an assumption that will be maintained here), increases in desert patrolling, even if more patrolling in the desert should actually reduce the probability of immigrants dying (an assumption that will also be maintained in what follows), will invariably reduce the attractiveness of crossing through the desert.

## 2 The Basic Model

There will be two players, the Border Patrol,  $P$ , and the illegal immigrant,  $I$ .

The Border Patrol chooses the probability of patrolling each one of  $n(> 3)$  crossing points, i.e., chooses  $\{p_k\}_{k=1,\dots,n}$ , with  $p_k \in [0, 1]$ . The illegal immigrant chooses with which probability to cross at each point<sup>5</sup>, i.e., chooses  $\{\sigma_l\}_{l=1,\dots,n+1}$ , with  $\sigma_l \in [0, 1]$ , and,  $\sum_{l=1}^{n+1} \sigma_l = 1$  (note that a restriction analogous to the latter does not apply to the  $p_k$ 's -  $P$  can choose to patrol all crossing points simultaneously). These decisions are taken simultaneously. Crossing points fall into two categories: Desert, and Non-Desert. I will differentiate desert from non-desert crossing points by a  $d$ , resp., a  $nd$  superscript (e.g.,  $t_i^d$ , resp.,  $t_i^{nd}$ ). There will be  $m(< n)$  desert crossing points, with  $m > 1$  and  $m - n > 1$  (in words, there are at least two desert points and two non-desert ones). The operational difference between desert and non-desert crossing points is that, when crossing through the former, the immigrant risks dying with probability  $p_x$ , **conditionally on not getting caught**. In other words, if the desert crossing point  $t_{k'}^d$  is being patrolled with probability  $p_{k'}$ , then the probability of dying when using this crossing is given by  $(1 - p_{k'}) p_x$ . The players are risk-neutral, and their payoff are as follows: An illegal immigrant who stays in Mexico derives a lifetime utility of  $u_I$ , while life in the U.S. generates a lifetime utility of  $u_I^*$ . If the immigrant should die trying to cross the border, then his utility is  $u_I^-$ . It is convenient to set this equal to 0. Then,  $u_I^* > u_I > 0$ . Note that the utility of dying is bounded below. This must be, as otherwise no one will ever risk death. There will also be a cost for the immigrant of attempting to cross the border,  $c$ . The border patrol will get a utility of  $u_P^*$  if no crossing takes place, and a utility of  $u_P^-$  if it does. Obviously,  $u_P^* > u_P^-$ . Again it will be convenient to set  $u_P^- = 0$ . At each crossing point  $t_i$ , it will incur a cost of patrolling,  $c_P$ , which will depend on the level of patrolling,  $p_k$ ,  $c_P(p_k)$ , with  $c_P(0) = 0$ ,  $c_P'(\cdot) \geq 0$ ,  $c_P''(\cdot) > 0$ .

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<sup>5</sup>Note that it is being implicitly assumed that  $I$  crosses with probability one. Allowing  $I$  not to cross would be more natural, but it would change the analysis only in the case where immigrants are exactly indifferent between crossing and not. The equilibrium described in the text will remain one in this extended model so long as the immigrant's utility from crossing exceeds  $u_I$ .

### 3 A Border Equilibrium

It is natural to assume in an exercise of this nature that crossing the border is worthwhile for the potential immigrant, just as it is natural to assume that it is worthwhile deterring such crossings. The following pair of assumptions captures these features.

A1 : Worth Dying in the Pursuit of Happiness

$$(1 - p_x) u_I^* - c > u_I$$

A2 : Happiness is Worth Patrolling but Not Too Much

$$c'_P(p_k) = 0 \Leftrightarrow p_k = 0$$

$$c'_P(p_k) \rightarrow \infty \text{ as } p_k \rightarrow 1$$

The following result is intuitive,

**Proposition 1** *There is no Nash equilibrium in which the immigrant crosses at a point with probability one.*

**Proof.** Take a desert point,  $t_l^d$ . If  $\sigma_l = 1$ , then in any best response,  $P$  will choose  $p_k = 0$ ,  $k \neq l$ . This follows since the unique best response of  $P$  to  $\sigma_k = 0$  is not to patrol that point, as  $c_P(p_k) = 0$  iff  $p_k = 0$ . But then it is better for the immigrant to deviate and cross at a non-desert point.

If  $I$  crosses through a non-desert point,  $t_i^{nd}$ , then again the best response at any other non-desert point is not to patrol. But then it is best for the immigrant to cross there. ■

Next, we show that  $I$  enters with the same probability at all desert, resp., non-desert points.

**Proposition 2** *At all desert points,  $I$  enters with the same non-negative probability. At all non-desert points,  $I$  crosses with the same positive probability.*

**Proof.** That  $I$  crosses at non-desert points with positive probability is a corollary of the previous result. Take any two non-desert points,  $t_l^{nd}, t_{l'}^{nd}$ . Assume that  $\sigma_l > \sigma_{l'} (> 0)$ . At non-desert points,  $P$  will solve

$$\max_{p_l \in [0,1]} \sigma_l u_P^* p_l - c_P(p_l)$$

This is a concave problem, hence F.O.C. characterize the optimum. Moreover, given the assumptions on  $c_P(\cdot)$ , the solution will always be interior. Hence, at an optimum, it satisfies

$$\sigma_l u_P^* = c_P'(p_l)$$

If  $\sigma_l > \sigma_{l'}$ , then, from the strict convexity of  $c_P$ , it must be that  $p_l > p_{l'}$ . But if  $p_l > p_{l'}$ , then the payoff from crossing at  $t_l^{nd}$  must be lower than that from crossing at  $t_{l'}^{nd}$ , which contradicts the fact that both points belong to the support.

Take any two desert points,  $t_l^d, t_{l'}^d$ . At desert points,  $P$  will solve

$$\max_{p_l \in [0,1]} \sigma_l u_P^* (1 - p_x) p_l - c_P(p_l)$$

Again, the solution to this concave program will be interior, and so, it satisfies

$$\sigma_l u_P^* (1 - p_x) = c_P'(p_l)$$

If  $\sigma_l > \sigma_{l'}$ , it must be that  $p_l > p_{l'}$ , which implies that the payoff from crossing at  $t_l^d$  is lower than that from crossing at  $t_{l'}^d$ . This contradicts the fact that both points belong to the support of  $\sigma$ . ■

**Corollary 3** *All desert points are patrolled with the same probability. All non-desert points are patrolled with the same probability.*

Finally, it can be shown that the probability of patrolling is lower in the desert. This is intuitive as well: The probability is lower since the risk of death helps deter crossings here, and patrolling tends to reduce the fatality rate.

**Proposition 4**  $p^d < p^{nd}$ .

**Proof.** If crossing takes place with positive probability both at deserts and non-desert points, then it must be that

$$\begin{aligned} \{p^d u_I + (1 - p^d) (1 - p_x) u_I^*\} = \\ \{p^{nd} u_I + (1 - p^{nd}) u_I^*\} \end{aligned}$$



If one sets  $p^d = p^{nd}$  in the above expression, the LHS is smaller than the RHS. Hence, since the LHS is the convex combination of  $(1 - p_x) u_I^*$  and  $u_I$ , and, since by A1,

$$(1 - p_x) u_I^* > u_I$$

the only way to equate the LHS to the RHS is by lowering  $p^d$ , thus giving the higher term in the LHS (i.e.,  $(1 - p_x) u_I^*$ ) more relevance.

If  $\sigma^d = 0$ , then it must be that  $p^d = 0$ , while  $p^{nd}$  is invariably positive. ■

The relation between  $\sigma^d$  and  $\sigma^{nd}$ , on the other hand, is not straightforward, and will depend on the properties of  $c_P$ .

For the conditions for existence and uniqueness, see Appendix.

### 3.1 Efficiency

It is easy to see that the equilibrium will always be inefficient, in the sense that it will be possible to make one party strictly better off (i.e., increase her expected utility) without making the other worse off. The easiest way to see this is to construct an outcome which always dominates any given equilibrium. Given an equilibrium  $(\sigma^d, \sigma^{nd}, p^d, p^{nd})$ , let  $P_c$  be the probability of a successful crossing, i.e.,

$$\sigma^d m (1 - p_x) (1 - p^d) m + (1 - \sigma^d m) (1 - p^{nd}) \quad (**)$$

Then the following outcome is efficient and dominates the expected outcome of this equilibrium:

$${}^e \sigma^{nd} = P_c^e; \quad \sigma^{nd} = 0; \quad {}^e p^d = 0; \quad {}^e p^{nd} = 0$$

Note that one could lower  $P_c$  considerably and still obtain an outcome dominating the equilibrium.

In an ex-post sense, the equilibrium is obviously inefficient (since whenever  $I$  is caught, it would have been better for him not to attempt crossing in the first place).

## 4 Policy Issues

Before looking at some comparative statics, a caveat seems in order: As will become apparent, these are not particularly intuitive. There is a reason for

this: The equilibrium entails mixed strategies. As is well known in game theory, mixed play is not intuitive: The players mixing (here, the immigrants) are indifferent between choosing any action in the support of their strategy. The question is then what determines the probabilities with which they choose their actions (here, cross through desert, cross through non-desert)? The answer is the need to make the other players' action optimal. In a sense, the actions of the mixing player are determined by the interests of the opponent, rather than his or her own. Note though, that in this sort of model (a kind of inspection game), mixed strategies result rather naturally, unlike what happens in other games.

## 4.1 Happy Meals

### 4.1.1 What Do “Happy Meals” Do?

The apparent objective of the “cajita feliz” (“happy meal”) was to reduce the probability of deaths at the border. Let us grant that the measure does in fact reduce  $p_x$  (the risk of death conditional on crossing through the desert and on not being apprehended). This does not mean that the program reduces the unconditional probability of border deaths, though. This probability, denoted by  $P_x$ , is given by

$$m\sigma^d(1 - p^d)p_x$$

So, even if  $p_x$  falls, one has to evaluate the effect of this fall on  $\sigma^d$  and  $p^d$ .

**Proposition 5** *A fall in  $p_x$  will lead to a rise in  $p^d$ , the probability of patrolling the desert.*

**Proof.** See Appendix.

Here is a verbal argument: If  $p^d$  would remain unchanged, c.p.,  $I$  would strictly prefer to cross through the desert (since, to start with,  $I$  was indifferent between crossing through desert and crossing through non-desert). In order to restore the indifference between crossing through both types of terrain (a requirement for equilibrium),  $p^{nd}$  must fall or  $p^d$  must increase. Since the marginal cost of patrolling ( $c'(p^{nd})$ ) must equal the marginal reward ( $\sigma^{nd}u^*$ ), a fall in  $p^{nd}$  implies a fall in  $\sigma^{nd}$ . This fall, in turn, implies an increase in  $\sigma^d$ . This last increase reinforces the effect of the original fall in  $p_x$ , further

increasing the marginal reward from patrolling the desert ( $\sigma^d u_P^* (1 - p_x)$ ), thus leading  $P$  to invest additional resources in patrolling the desert.

It is not possible to sign in general the effect of a change of  $p_x$  on  $\sigma^d$ . As the condition below makes clear, this effect can go either way, depending on the specific parametrization chosen.

**Proposition 6**  $p^{nd}$  will rise (and, hence,  $\sigma^{nd}$  will rise, while  $\sigma^d$  falls) iff

$$\frac{c'_P(p^d)}{c''_P(p^d)} < \frac{(1 - p^d)(1 - p_x)u_I^*}{(1 - p_x)u_I^* - u_I}$$

**Proof.** An equilibrium in which  $\sigma^d > 0$  solves the following system of equations,

$$i) (n - m)\sigma^{nd} + m\sigma^d = 1$$

$$ii) \sigma^{nd}u_P^* = c'_P(p^{nd})$$

$$iii) \sigma^d u_P^* (1 - p_x) = c'_P(p^d)$$

$$iv) p^{nd} = \frac{[u_I - (1 - p_x)u_I^*]}{(u_I - u_I^*)}p^d - \frac{p_x u_I^*}{(u_I - u_I^*)}$$

From equation *ii*), it follows that if  $p^{nd}$  rises so will  $\sigma^{nd}$ . It then follows (from equation *i*)) that  $\sigma^d$  must fall. Differentiating totally the previous system of equations (see Appendix), it can be shown that  $p^{nd}$  will rise only if the above condition is satisfied. ■

The condition shows that when the curvature of the cost schedule is sufficiently high, the adjustment must take place in part through an increase in  $p^{nd}$ . Otherwise, a fall in  $p_x$  will not necessarily decrease  $\sigma^d$ , and so, will not necessarily lead to a fall in  $P_x$ .

The following diagram illustrates these comparative statics (for the case of one desert point, and one non-desert point): In each quadrant, one of the equations in the system of the preceding proof is presented (*i*) is in the northwest quadrant; *ii*) is in the northeast quadrant; *iii*) is in the southwest quadrant; *iv*) in the southeast quadrant). Each box corresponds to a solution to the system. The diagram shows two equilibria. The centered one ( $p^{nd} =$

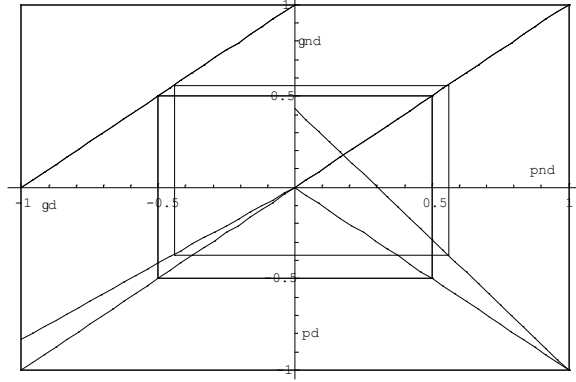


Figure 1:

$p^d = \sigma^d = \sigma^{nd} = .5$ ) corresponds to the following parametrization

$$u_P^* = 1; p_x = 0; u_I^* = 3; u_I = 0.7; c = 0$$

$$c'(x) = x$$

The parametrization for the second equilibrium ( $p^{nd} = \sigma^{nd} = .56; \sigma^d = .44; p^d = .37$ ) is

$$u_P^* = 1; p_x = .2; u_I^* = 3; u_I = 0.7; c = 0$$

$$c'(x) = x$$

Both these satisfy the restriction that  $(1 - p_x)u_I^* - u_I > c$ , as well as the condition for existence. It is easy to calculate that this condition is satisfied at all  $p_x$  in  $[0, \bar{p}_x]$ , with  $\bar{p}_x > 0.2$ , so that  $p^{nd}$  will rise as  $p_x$  increases in this range.

#### 4.1.2 Welfare Analysis: Who is Happy with a “Happy Meal”?

**Proposition 7** *If  $p^{nd}$  rises as a consequence of a fall in  $p_x$ , both, the expected utility of  $I$ ,  $Eu^I$ , and the unconditional probability of death,  $P_x$ , fall.*

**Proof.**

The previous proposition showed that if  $p^{nd}$  rises,  $\sigma^{nd}$  rises while  $\sigma^d$  falls. Hence,  $P_x = m\sigma^d(1 - p^d)p_x$  must fall, as  $p^d$  always rises.

In any equilibrium of the sort being considered here (i.e., such that  $\sigma^d > 0$ ), it must be that

$$p^d u_I + (1 - p^d)(1 - p_x) u_I^* = p^{nd} u_I + (1 - p^{nd}) u_I^* \quad (*)$$

It follows that  $Eu^I$  does not depend on  $\sigma$ . Hence, if  $p^{nd}$  rises,  $Eu^I$  must fall.

■

In other words, reducing  $p_x$ , even when this succeeds in lowering  $P_x$ , will not necessarily be efficiency-enhancing. In fact, in this case, even though the policy succeeds in lowering  $P_x$ , it actually harms the immigrant.

An interesting corollary of the previous result is that the effect of lowering  $p_x$  on the immigrant's well-being can be evaluated (ex post) by looking at  $p^{nd}$ : From (\*), it is immediate that only if patrolling in non-desert areas falls, will the immigrant have benefited.

Can a reduction in  $p_x$  decrease  $P_x$  and enhance efficiency?

**Proposition 8** *If  $p^{nd}$  stays constant when  $p_x$  falls, then  $P_x$  must fall, while efficiency might be enhanced. If  $p^{nd}$  decreases when  $p_x$  falls, then efficiency might be enhanced but effectiveness is not guaranteed.*

**Proof.** If  $p^{nd}$  stays constant then  $\sigma^d$  must stay constant as well. But then  $P_x = m\sigma^d(1 - p^d)p_x$  must fall (as  $p^d$  increases when  $p_x$  falls). On the other hand,  $(1 - p^d)(1 - p_x)$  must fall (from (\*)), hence, so must  $P_c$ . The cost effects on  $Eu^P$  are negative ( $p^d$  rises), but the effect of the fall in  $P_c$  is positive. Depending on the relative magnitude of these effects,  $Eu^P$  might rise or not ( $Eu^I$  must stay constant since  $p^{nd}$  does).

If  $p^{nd}$  decreases then  $\sigma^d$  must increase, so that now  $P_x$  might increase. The cost effects on  $Eu^P$  are now ambiguous ( $p^d$  increases, but  $p^{nd}$  decreases). The effect on  $P_c$  is also ambiguous ■

Even this cursory analysis suffices to make an important point: There is no systematic relationship between reducing  $P_x$  and immigrants well-being, much less between this magnitude and efficiency.

By the way, the argument in the proof of the previous proposition also makes it plain that one cannot rely on the probability of successful crossing to evaluate the effect of a fall in  $p_x$  on the well-being of the immigrant (even though this magnitude can be shown to decrease when  $p^{nd}$  rises - see Appendix).

A more definite relationship between  $P_x$  and welfare emerges if in evaluating welfare, one considers only the benefits from deterring immigration (i.e., one disregards the costs of patrolling). This might be justified on the grounds that, while costs of patrolling play an important role in determining the Border Patrol's behavior (and its welfare), the overriding national objective is deterring illegal immigration. Define such an alternative welfare criterium as  $(1 - P_c) u_P^*$ , and denote it,  $Eu^{US}$ .

**Proposition 9** *A reduction in  $p_x$  that does not reduce either  $Eu^I$  or  $Eu^{US}$ , necessarily reduces  $P_{x|nc}$ , i.e., the probability of death conditional on an unsuccessful crossing.*

**Proof.** Let  $P_{nc}$  be the probability of an unsuccessful crossing. We have

$$Eu^I = P_c u_I^* + (1 - P_c) (1 - P_{x|nc}) u_I - c$$

and

$$Eu^{US} = (1 - P_c) u_P^*$$

Clearly,  $Eu^I$  can only increase if  $P_c$  falls. But then  $Eu^I$  can only increase or stay constant if  $P_{x|nc}$  falls. ■

This last result is intuitive: In the absence of a positive probability of death, the only way to increase the expected utility of one party would be to reduce that of the other. The probability of death introduces a wedge between the expected utilities, so that it becomes possible to reduce  $P_c$  without necessarily reducing  $Eu^I$ , namely, through a reduction in  $P_{x|nc}$ . By the way, this also helps in clarifying the role of costs of patrolling in the preceding analysis: They provide yet another channel through which gains can be achieved by one party without at the same time reducing the welfare of the remaining party.

## 4.2 Fences

The effects of border barriers is an issue that can be studied relying substantially on the previous analysis.

We model barriers as a cost differential between the area where the barrier is located and the remaining areas. In this section we consider how such a cost-gap modifies the equilibrium. Assume that the 'barrier' is installed in non-desert areas, thus raising the cost of crossing through non-desert points,

$c^{nd}$ , vis á vis the cost of crossing through desert points,  $c^d$  (for necessary and sufficient conditions for existence in this case, see Appendix).

The system of equations characterizing the comparative statics -with respect to parameters other than cost- remain the same, as all what changes is that a term  $\frac{c^d - c^{nd}}{u_I - u_I^*}$  is added to equation *iv*) in proposition 6. Since this term is constant, it disappears when differentiating.

That additional term results because the cost of crossing can no longer be cancelled out in the expression equating the payoffs from crossing at desert and non-desert points,

$$\begin{aligned} \left\{ p^d u_I + (1 - p^d) (1 - p_x) u_I^* \right\} - c^d = \\ \left\{ p^{nd} u_I + (1 - p^{nd}) u_I^* \right\} - c^{nd} \end{aligned} \quad (+)$$

Since  $c^d > c^{nd}$ , it is no longer necessarily the case that  $p^d < p^{nd}$ . This, of course, might change the direction of some of the comparative statics.

In what follows, I concentrate on analyzing the effects of raising the cost of crossing through non-desert, starting out from a situation in which the cost of crossing was the same for desert and non-desert areas.

**Proposition 10** *An increase in the cost of crossing through non-desert areas, starting from a situation in which costs of crossing are equal, leads to an increase in  $p^d$ , a increase in  $\sigma^d$  and a fall in  $p^{nd}$ .*

**Proof.** See Appendix.

It is now easy to show that erecting barriers cannot be efficiency enhancing.

**Proposition 11**  *$Eu^I$  will fall.*

**Proof.** Since (+) must hold,  $Eu^I$  does not depend on  $\sigma$ . Totally differentiating the RHS of (+),

$$-1 + \frac{dp^{nd}}{dc^{nd}} (u_I - u_I^*)$$

Substituting for  $\frac{dp^{nd}}{dc^{nd}}$ , this expression reduces to

$$-1 + \left\{ \frac{-\frac{c_P''(p^{nd})}{u_P^*} \left[ \frac{U_I - (1-p_x)u_I^*}{u_I - u_I^*} \right]}{\frac{c_P''(p^{nd})}{u_P^*} \left[ \frac{U_I - (1-p_x)u_I^*}{u_I - u_I^*} \right] + \frac{m}{n-m} \frac{c_P''(p^d)}{u_P^*(1-p_x)}} + 1 \right\}$$

Which is negative. ■

Can such a measure reduce  $P_x$ ?

**Proposition 12** *A rise in  $c^{nd}$  will reduce  $P_x$  iff*

$$\frac{c'_P(p^d)}{c''_P(p^d)} > (1 - p^d)$$

**Proof.** Differentiating  $P_x$  with respect to  $c^{nd}$ , one obtains

$$\left[ (1 - p^d) c''_P(p^d) - c'_P(p^d) \right] \frac{dp^{nd}}{dc^{nd}} \frac{p_x m}{u_P^* (1 - p_x)}$$

Since  $\frac{dp^{nd}}{dc^{nd}} > 0$ , the condition follows. ■

Note that if one assumes, for example, that  $c''' > 0$ , then, for any  $p^d \leq \frac{1}{2}$ , the condition will be violated (since for any convex function such that  $f(0) = 0$ , one has that  $f'(x)/x \leq f''(x)$ ). Moreover, if  $c''(1) < \infty$ , then there exists a critical  $p^{d*}$  such that the policy is effective for  $p^d$  in  $[p^{d*}, 1]$ , ineffective otherwise. In words, for this parametrization, the intensity of desert patrols can be taken as a good indicator of whether erecting barriers will reduce deaths.

**Possible Extensions: Coyotes, Jail Penalties, Nimby Effects** Other issues can also be analyzed using this sort of framework, though some might require a substantial reworking of the analysis.

One that could be analyzed without much further work, is the issue of jail penalties for apprehended immigrants. This could be captured by letting  $I$ 's payoff be  $u_I - j$  (instead of just  $u_I$ ), whenever an he or she gets caught ( $j$  standing for the penalty).

An issue that would seem harder to analyze without substantially modifying the model is that of 'coyotes' or 'polleros'. Incorporating this is not difficult. One could, for example, introduce a function  $\lambda$  mapping 'evasion effort'  $e$  into the probability of being arrested ( $\lambda(e)$  would multiply the probabilities of patrolling,  $p^d$  and  $p^{nd}$ ), as well as a cost to evasion effort,  $c(e)$ . It would then seem natural to let  $I$  choose  $e$ , in addition to choosing points of crossing. However, this additional choice variable would make a complete reworking of the analysis necessary.



Yet other issues might require a rather different model. For example, studying the “nimby”-effects mentioned in the introduction. Here one would, at the very least, have to introduce sequential moves, as with simultaneous moves no equilibrium with barriers would appear possible (it is always convenient for the agents erecting the barriers to refrain from doing so, if moves are simultaneous).

## 5 Conclusions

The results are decidedly mixed. While reducing  $p_x$  will often lead to a fall in  $P_x$ , this will not always be the case. Moreover, there is no definite connection between a fall in  $P_x$  and immigrants’ well-being, much less between this magnitude and efficiency. A connection of sorts emerges only when one ignores costs of patrolling, in which case a welfare improvement as a result of a fall in  $p_x$  necessarily reduces  $P_{x|nc}$ . Less surprising is that barriers can never lead to Pareto superior outcomes, though they might lead to lower fatalities. Also, the analysis gives some hints as to how the policy maker might judge (ex post) the effects of these measures (look at  $p^{nd}$  for ‘happy meals’ interventions; look at the intensity of desert patrols in the case of fences).

But, in my opinion, what this model offers, mainly, is a cautionary tale: The absence of strong, unambiguous, qualitative predictions suggests that policymakers should tread carefully when implementing measures such as the “happy meals” program. I conjecture that given the nature of the equilibrium (the mixed strategy feature, in particular), most interventions will tend to have rather unpredictable effects.

# A Appendix

## A.1 Existence and Uniqueness

**Proposition 13** *A necessary and sufficient condition for existence is*

$$c_P'^{-1} \left( \frac{u_P^*}{n-m} \right) > -\frac{p_x u_I^*}{(u_I - u_I^*)}$$

*The equilibrium, if it exists, is unique.*

**Proof.** The following system of equations characterizes an equilibrium with  $\sigma^d > 0$ :

$$i) (n-m)\sigma^{nd} + m\sigma^d = 1$$

$$ii) \sigma^{nd} u_P^* = c_P'(p^{nd})$$

$$iii) \sigma^d u_P^* (1-p_x) = c_P'(p^d)$$

$$iv) p^{nd} = \frac{[u_I - (1-p_x)u_I^*]}{(u_I - u_I^*)} p^d - \frac{p_x u_I^*}{(u_I - u_I^*)}$$

Substituting *i*), *iii*) and *iv*) into *ii*), one obtains

$$\left\{ c_P' \left[ \frac{u_P^*}{n-m} - \frac{m}{n-m} \frac{c_P'(p^d)}{(1-p_x)} \right] + \frac{p_x u_I^*}{(u_I - u_I^*)} \right\} \frac{(u_I - u_I^*)}{(u_I - (1-p_x)u_I^*)} = p^d$$

Evaluating the LHS at  $p^d = 0$ ,

$$\left\{ c_P' \left[ \frac{u_P^*}{n-m} \right] + \frac{p_x u_I^*}{(u_I - u_I^*)} \right\} \frac{(u_I - u_I^*)}{(u_I - (1-p_x)u_I^*)}$$

This is positive if

$$c_P' \left[ \frac{u_P^*}{n-m} \right] > -\frac{p_x u_I^*}{(u_I - u_I^*)} \quad (\#)$$

Now, the LHS is monotone decreasing in  $p^d$ , as  $c'' > 0$ . Hence, if (#) is satisfied, it must intersect the 45° degree line (i.e., have a fix point). On the other hand, if (#) is not satisfied, then, since the schedule is downward sloping, the schedule cannot intersect the 45° degree line.

## A.2 Comparative Statics of a Fall in $p_x$ : Propositions 5 and 6

Totally differentiating the system in the previous section,

$$\begin{aligned}
 i') \quad \frac{d\sigma^{nd}}{dp_x} &= -\frac{m}{(n-m)} \frac{d\sigma^d}{dp_x} \\
 ii') \quad \frac{d\sigma^{nd}}{dp_x} &= \frac{c_P''(p^{nd})}{u_P^*} \frac{dp^{nd}}{dp_x} \\
 iii') \quad \frac{d\sigma^d}{dp_x} &= \frac{\sigma^d}{(1-p_x)} + \frac{c_P''(p^d)}{u_P^*(1-p_x)} \frac{dp^d}{dp_x} \\
 iv') \quad \frac{dp^{nd}}{dp_x} &= \left[ 1 + \frac{p_x u_I^*}{(u_I - u_I^*)} \right] \frac{dp^d}{dp_x} + \frac{u_I^*}{(u_I - u_I^*)} (p^d - 1)
 \end{aligned}$$

Solving for  $\frac{dp^d}{dp_x}$ ,

$$\frac{dp^d}{dp_x} = \frac{-\frac{n-m}{m} \frac{c_P''(p^{nd})}{u_P^*} \frac{u_I^*}{(u_I - u_I^*)} (1 - p^d) - \frac{\sigma^d}{(1-p_x)}}{\frac{c_P''(p^d)}{u_P^*(1-p_x)} + \frac{n-m}{m} \frac{c_P''(p^{nd})}{u_P^*} \left[ 1 - \frac{p_x u_I^*}{(u_I^* - u_I)} \right]}$$

Since the denominator is positive, and both terms in the numerator are negative, this expression is negative.

Solving now for  $\frac{dp^{nd}}{dp_x}$ ,

$$\frac{dp^{nd}}{dp_x} = \left\{ \frac{\frac{c_P''(p^d)}{u_P^*(1-p_x)} \frac{u_I^*}{(u_I - u_I^*)} (1 - p^d) - \left[ 1 - \frac{p_x u_I^*}{(u_I^* - u_I)} \right] \frac{\sigma^d}{(1-p_x)}}{\frac{c_P''(p^d)}{u_P^*(1-p_x)} + \frac{n-m}{m} \frac{c_P''(p^{nd})}{u_P^*} \left[ 1 - \frac{p_x u_I^*}{(u_I^* - u_I)} \right]} \right\}$$

Since the denominator is positive, this expression is positive iff

$$\frac{c_P''(p^d)}{u_P^*(1-p_x)} \frac{u_I^*}{(u_I - u_I^*)} (1 - p^d) > \left[ 1 - \frac{p_x u_I^*}{(u_I^* - u_I)} \right] \frac{\sigma^d}{(1-p_x)}$$

Substituting for  $\sigma^d$  from *iii)*, one obtains the expression in the proposition.

### A.3 Effect of a Fall in $p_x$ on $P_c$

**Proposition 14** *If  $p^{nd}$  rises as a consequence of a fall in  $p_x$ , the probability of a successful crossing,  $P_c$  falls.*

**Proof.** The probability of a successful crossing,  $P_c$ , can be written

$$\sigma^d m (1 - p_x) (1 - p^d) + (1 - \sigma^d m) (1 - p^{nd}) \quad (**)$$

A necessary condition for  $P_c$  to fall is

$$(1 - p^{nd}) < (1 - p_x) (1 - p^d)$$

To see why this condition is necessary, note that, from the argument establishing that  $E u^I$  falls, it follows that  $(1 - p_x) (1 - p^d)$  falls as  $p_x$  falls. The expression (\*\*) is a convex combination of the RHS and LHS of the preceding inequality. Now, we know that  $\sigma^d$  falls as  $p^{nd}$  increases. Should the preceding inequality hold, weight is shifted from the larger expression to the lower one. Concurrent with this shift, both expressions are falling, so, the overall expression must fall as well.

To establish the inequality, rewrite expression (\*) as

$$(p^d - p^{nd}) \frac{u_I}{u_I^*} + (1 - p^d) (1 - p_x) = (1 - p^{nd})$$

Since  $p^d < p^{nd}$ , the inequality follows. ■

### A.4 Existence and Uniqueness with Fences

By an argument analogous to the one presented in A.1, it can be shown that the necessary and sufficient condition for existence has to be strengthened to

$$c_P'^{-1} \left( \frac{u_P^*}{n - m} \right) > - \frac{p_x u_I^*}{(u_I - u_I^*)} + \frac{c^d - c^{nd}}{u_I - u_I^*}$$

### A.5 Comparative Statics of An Increase in $c^{nd}$ : Proposition 12

Equation *iv*) is modified to

$$i v'' ) p^{nd} = \frac{[u_I - (1 - p_x) u_I^*]}{(u_I - u_I^*)} p^d - \frac{p_x u_I^*}{(u_I - u_I^*)} - \frac{c^d - c^{nd}}{(u_I - u_I^*)}$$

Totally differentiating with respect to  $c^{nd}$ ,

$$i'') \frac{dp^{nd}}{dc^{nd}} = \left[ 1 + \frac{p_x u_I^*}{(u_I - u_I^*)} \right] \frac{dp^d}{dc^{nd}} + \frac{1}{u_I - u_I^*}$$

$$ii'') \frac{d\sigma^{nd}}{dc^{nd}} = -\frac{m}{(n-m)} \frac{d\sigma^d}{dc^{nd}}$$

$$iii'') \frac{d\sigma^{nd}}{dc^{nd}} = \frac{c_P''(p^{nd})}{u_P^*} \frac{dp^{nd}}{dc^{nd}}$$

$$iv'') \frac{d\sigma^d}{dp_x} = \frac{c_P''(p^d)}{u_P^*(1-p_x)} \frac{dp^d}{dc^{nd}}$$

Solving,

$$\frac{dp^d}{dc^{nd}} = \frac{-\frac{c_P''(p^{nd})}{u_P^*} \frac{1}{u_I - u_I^*}}{\frac{m}{n-m} \frac{c_P''(p^d)}{u_P^*(1-p_x)} + \frac{c_P''(p^{nd})}{u_P^*} \left[ 1 - \frac{p_x u_I^*}{(u_I^* - u_I)} \right]} > 0$$

Hence,

$$\frac{d\sigma^d}{dp_x} = \frac{c_P''(p^d)}{u_P^*(1-p_x)} \frac{dp^d}{dc^{nd}} > 0$$

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