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PRODUCT IDENTIFICATION IN THE ABSENCE OF TRADEMARKS

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Product Identification in the Absence of Trademarks*

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Abstract

This paper looks at whether it is possible for firms to differentiate their products from each other by choosing meaningless, worthless 'tags' in the absence of exogenous rules prohibiting imitation. It shows that even when there is general consensus regarding the ranking of firms' products (from worse to better sell), 'endogenous' differentiation might result. Moreover, it shows that for differentiation of this sort to obtain, the quality distribution must not be biased upwards.

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1 Introduction

In many situations, agents have a degree of control over their 'appearance', and can thus influence the ability of an observer to distinguish them from others, i.e., to 'identify' them. In particular, businesses choose the 'name' or 'appearance' of their products. In doing so, one of their main aims is to differentiate their products vis-á-vis those of their competitors. While businesses have a degree of control over the identity of their products, they do not have full control: The ability of an observer to differentiate between products will hinge crucially on the ability of other businesses to 'imitate' this agent's choices, making effective identification an eminently strategic undertaking.

A vast body of law aimed at preventing imitation and ensuring effective identification of products exists, and enormous efforts are made in order to implement it. While it is not surprising that product identification should be such a priority - key mechanisms on which markets rely to guarantee product quality, namely, reputation and competition, work only if products are well identified- this paper asks whether this might not be a case of 'too much of a good thing', i.e., whether in the absence of such explicit prohibitions, some degree of differentiation could nevertheless arise, so to say, endogenously, **even when market participants fully agree on the quality ranking of products** (e.g., Nike regarded by everyone as superior to Adidas, which in turn is generally regarded as superior to Reebok). The 'conventional wisdom' is that such endogenous differentiation cannot obtain (hence trademark laws). The analysis in this paper suggests that the logic underlying this conventional wisdom is not quite as compelling as might at first appear. This paper shows that even in a very elementary setup in which businesses only choose the 'appearance' of their products (no choice of product 'attributes'), and in which a product's appearance is simply a meaningless, costless 'tag value', there turns out to be scope for endogenous differentiation (though not for full differentiation). Besides showing that differentiation of this sort is possible, the paper makes a very preliminary attempt at characterizing the scope for such differentiation.

More formally, this paper presents a game in which businesses selling vertically differentiated products simultaneously send costless and intrinsically meaningless public signals to buyers who, on the basis of these signals alone, form beliefs regarding the quality of the product being offered by each sender. These beliefs, in turn, determine the price they are willing to pay for the product of the business they are (randomly) matched with. In effect, this is essentially a cheap-talk sender-receiver game á la Crawford and Sobel 1982 (CS), but one with multiple senders, unconventional (non-CS) preferences, an intrinsically meaningless language, and a quite different structure of information[1].

Two key requirements for differentiation emerge: One, that the market be minimally transparent, in the sense that buyers should be able to survey the 'tag values' adopted by all businesses in the market, i.e., that the messages should be public. Second, that the quality distribution in the economy not be biased upwards. The first allows imitating deviations to be spotted, so that, in principle, they could be deterred by specifying out-of-equilibrium

[1] For a more detailed discussion of how the present work relates to the existing cheap-talk literature, see next section .

beliefs appropriately. The second condition guarantees that it is possible to specify out-of-equilibrium beliefs in such a way as to effectively deter such deviations. This means that in this environment bad qualities are not so bad, in as far as they have the 'beneficial'[2] effect of facilitating identification.

The relevance of the present work for trademark regulation seems clear. Though the tentative and abstract analysis offered here hardly allows for any definite policy recommendations, it does suggest that there might be scope for a more 'laissez faire' attitude towards trademark regulation. Regulatory capacity being an extremely scarce resource, this is a line of enquiry that seems worthwhile pursuing.

Less evident is the connection to marketing. I think it could be reasonably argued that marketing is about how products are perceived, rather than what they are. And a key aspect of the way products are perceived is clearly whether they are identified or not. Perhaps it is appropriate then to see this work as an attempt to bring game theory (more precisely, the study of costless signalling) to bear on a marketing subject (product identification).

2 Relation to the Literature

The model in this paper differs in various substantial ways from CS's standard treatment: First and foremost, as mentioned already, there are multiple senders here[3]. There is a literature on cheap-talk with multiple senders, see,

[2] The issue of the welfare implications of endogenous differentiation is tricky in the present model. See discussion in Section 6.

[3] It might at first glance seem as if there are multiple receivers as well. In fact, there are, but the model is easily seen to be 'isomorphic' to one in which one buyer is randomly

for example, Krishna and Morgan 2000a,b, and references therein. Those references restrict attention to the case of 2 senders, while in the present work the number of senders is unrestricted (in fact, the 2 sender case does not admit informative pure strategy equilibria).

But the important difference between the present work and the CS-literature lies elsewhere, seems to me: It concerns the sorting role of preferences in CS. While this model also embodies conflicting interests between receivers and senders (the latter always preferring higher prices to lower ones), and diverging senders' preferences over receivers' actions (taking care to define a receiver's action as a mapping from tag values to prices), that preference configuration is **not** the key to separation here, unlike what happens in the CS-literature[4]. In the original CS-setup imitating deviations are not registered by receivers, which means that senders deviations cannot be punished. If they are to be deterred, it must then be that it does not pay ex-ante for sender-types to imitate each other (hence the need for some kind of sorting structure). In the present model, on the other hand, imitating deviations are always registered, and so, punishment for such deviations is possible. The problem is rather that, given the specific information structure in the model (more precisely, the fact that the distribution of qualities is assumed known), the consistency requirement of sequential equilibrium restricts out-of-equilibrium beliefs, and it is by no means guaranteed that admissible beliefs supporting appropriate deterring actions can be found (hence the need for the downward bias just mentioned).

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matched with many sellers.

[4]For a discussion of alternative preference configurations in a CS-framework, see Seidman 1990.

Now, with regard to the multiple senders' papers quoted above, one has to distinguish between the model in which messages are being sent simultaneously (Krishna and Morgan 2000b, Gilligan and Krehbiel 1989), and the one in which they are sent sequentially (Krishna and Morgan 2000a). In the simultaneous case, any individual deviation must be registered, making it invariably feasible to punish such behavior. In this case, consistency again restricts out-of-equilibrium beliefs[5], and conditions on preferences might have to be imposed in order to deliver separation. It seems, though, that the information structure of the CS-literature imposes less stringent conditions on out-of-equilibrium beliefs than the one used here[6]. The sequential case is less clear, but ultimately the logic at work appears to be basically the same as in the simultaneous case: An improvement in communication relative to the one sender case can only be achieved if the second senders' reaction 'denounces' the first sender's deviation in the eyes of the receiver, i.e., allows the receiver to register imitating deviations by the first sender[7]. Again, the key issue here

[5]Specifically, since simultaneous deviations are excluded by consistency, the support of the beliefs of a receiver who registers a deviation must only include states which could have led in equilibrium to the observed play (in case such states exist -else beliefs are unrestricted).

[6]If one ignores the restrictions consistency imposes on out-of-equilibrium beliefs (as Krishna and Morgan 2000b appear to do), it is not at all clear to me what role the opposing biases feature plays in the equilibrium under open rule (see Krishna and Morgan 2000b, p.11). It would seem possible to sustain full revelation even when biases coincide (simply by specifying sufficiently extreme beliefs in case of disagreement).

[7]This since the presence of a second sender does not directly affect the first sender's payoffs, except via the effect of the former's actions on receiver's beliefs. So, if there exists an equilibrium pattern of responses that induce a given level of communication in the setup with two senders, it should always be possible to specify an equilibrium pattern of

is identifying appropriate out-of-equilibrium beliefs; specifically, beliefs that generate a receiver's response that punishes the first sender's deviation while 'rewarding' the second sender's reaction. It is here where the opposing biases are needed. One can think of the receiver as playing off one sender against the other[8].

Another difference between this paper and the CS-literature is that the discussion is specialized to the case of an 'intrinsically meaningless' language. This implies that the refinements for cheap-talk games originating in the notion of 'neologism proofness' (Farrell 1993) do not apply.

Finally, it would seem that despite all these differences, the basic 'flavor' of the results in CS remains: The only informative equilibria require 'messages' to be imprecise (some degree of pooling). On the other hand, the downward bias condition parallels the opposing bias condition of Krishna and Morgan 2000a, in so far as both conditions represent restrictions on senders' preferences, and basically fulfill the same function: To make sure out-of-equilibrium beliefs exist that support receivers' responses that make imitating deviations unprofitable.

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responses in the one sender case which is just as informative.

[8]Note that in the model of this paper the receiver hardly has any margin to try and play off one sender against the other, as sellers make take-it-or-leave-it offers to buyers (receivers), and sequential rationality then pins down the latter responses. (A caveat: This so long as buyers play pure strategies.)

2.1 A Simple Trade Game

There are N buyers and an equal number of sellers. Each seller s_n, $n = 1, ..., N$, offers a distinct product valued by buyers as $v(s_n)$[9]. Each seller s_n will carry a 'tag-value', t_{s_n}[10], chosen from a set V, say, the positive integers. Buyers will not be able to tell sellers apart, except by observing the tag value carried by the seller. Tags will be assumed to be devoid of literal meaning and payoff irrelevant ('cheap-talk').

The game proceeds as follows: At the start, sellers simultaneously choose their tag-value. Then sellers and buyers are randomly matched 1:1. Buyers observe sellers' tag values, and then the seller proceeds to charge buyers the expected value of the product, and trade takes place at that price. The (expected) payoff to the seller is simply the price, while that to the buyer is the expected value of the product, given what he or she can observe, minus the price. By construction, this payoff will always be zero.

Note that the only way to identify a seller is via tags, as matching is random. Also noteworthy is the fact that sellers do not care about the identity of buyers directly (in contrast to what happens in so called 'marriage' models). Finally, note that the game is quasi-static in so far as the interesting stage is the simultaneous choice of tag values. Dynamics only come into play because sellers' choice of tag values affects buyers' beliefs, and, through this channel, the price sellers can extract.

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[9] Note that all the results in this paper would remain valid if instead of assuming total agreement in buyers' valuations, one would allow valuations to vary across buyers, i.e., write $v(b, s)$ instead of simply $v(s)$.

[10] Note that the single tag value case is equivalent to the case of many tags when there is full observability, i.e., every buyer can observe every tag.

2.1.1 Equilibrium and Some Basic Notation

The solution concept will be sequential equilibrium (Kreps and Wilson 1982). In particular, the consistency requirement on beliefs will be exploited heavily[11].

Only pure strategies will be considered here[12].

I now introduce some basic concepts and notation.

Definition 1 *Tags are private if buyers can only observe the tag-value of the seller they are matched with. Tags are public if buyers can simultaneously observe the tag-values of all sellers.*

For the private tags case, the beliefs of buyer i are represented by a mapping

$$\Psi_i : V \rightarrow \triangle\, S$$

with V representing the set of all tag-values, and $\triangle\, S$ representing lotteries over the set of sellers.

For the public tags case, the beliefs of buyer i are represented by a mapping going from a profile of tag-values, $\tau \in V^{\#S}$, and the tag-value of the seller buyer is matched with, $t \in T_{s(i)}$, to the set of lotteries on $S, \triangle\, S$, i.e.,

$$\Psi_i : V^{\#S} \times T_{s(i)} \rightarrow \triangle\, S$$

It will be assumed that beliefs are symmetric across buyers, in the sense that permuting buyers should not affect beliefs, that is, $\Psi_i(\tau, t) = \Psi_j(\tau, t)$.

[11] This requirement states that an equilibrium system of beliefs must correspond to the limit of the beliefs' sequence generated by a sequence of completely mixed strategies converging to equilibrium strategies.

[12] See the discussion below regarding this selection.

Along the equilibrium path, this is an implication of consistency. Off the equilibrium, this need not be.

A pure strategy of a seller s will be a tag-value t_s,and so, a pure strategy sequential equilibrium will be given by a profile of tag-values, τ , and a beliefs' mapping, Ψ,such that τ is sequentially rational given Ψ, while Ψ is consistent given τ[13].

Since attention will be restricted to pure strategies, it is convenient to define a 'pool' $P(t)$ as the set of all sellers choosing one and the same tag-value t . For on-the-equilibrium-path tag lists, consistency implies that

$$\Psi_i(\tau, t)(s) = \begin{cases} \frac{1}{\#P(t)} & for\ s \in P(t) \\ \\ 0 & otherwise \end{cases}$$

The expected value to a buyer who has been matched with a seller s_n, carrying tag-value t_{s_n} in τ, is then given by

$$\sum_{s \in S} \Psi(\tau, t_{s_n})(s)\, v(s)$$

Denoting by $pr(b)$ the probability that a seller gets matched to buyer b (which here is simply $\frac{1}{N}$), the 'identification payoff' of a seller s is given by

$$\sum_{b \in B} pr(b)\, v(s) = v(s)$$

This corresponds to the ex-ante (i.e., prior to matching) payoff that a seller can expect to obtain if all buyers can identify her. More generally, the payoff

[13] Buyers will be assumed to buy with probability 1. Since payoff to a buyer by definition zero, it is always a best response for him or her to buy with probability 1.

a seller s_n can expect to obtain when carrying tag-value t_{s_n} in τ, is given by

$$\sum_{s \in S} \Psi\left(\tau, t_{s_n}\right)(s)\, v(s)$$

3 Examples

This section presents examples illustrating the basic intuition behind endogenous differentiation.

3.1 Two Sellers

Private Tag Under differentiation, the payoff to seller s_1 would be the identification payoff of that seller, $v(s_1)$. Similarly for the other seller. So, if

$$v(s_1) \neq v(s_2)$$

then either one or the other seller would have an incentive to deviate, and adopt their rival's tag-value. Such a deviation will not be registered by buyers, as tags are assumed private. The only way there can be differentiation is then if both identification-payoffs coincide. If these magnitudes do not coincide, then the only pure strategy equilibrium entails pooling (non-imitating deviations, i.e., deviations that do not aim to imitate an opponent, can always be made unprofitable by specifying appropriate out-of-equilibrium beliefs, these being unrestricted here -say, buyers believe that the deviator is the seller with the lowest quality).

This establishes the following proposition,

Proposition 2 *With a single private tag and two sellers there cannot be strict differentiation.*

Public Tag Now imitating deviations will be registered. So, whether such deviations are profitable or not will depend on whether one can find consistent out-of-equilibrium beliefs that result in prices which deter such deviations (non-imitating deviations can invariably be made unprofitable by specifying, for example, that buyers assume that the deviator is the seller with the lowest valuation in his or her pool[14]). It can be shown that consistency pins down out-of-equilibrium beliefs after an imitating deviation at $\frac{1}{2}$ (see Appendix). But then, in order to deter imitating deviations in either direction, it must be that both the following inequalities hold,

$$\left(\frac{1}{2}v(s_1)+\frac{1}{2}v(s_2)\right)\leq v(s_1)$$

This is the condition guaranteeing that s_1 has no incentives to imitate s_2.

$$\left(\frac{1}{2}v(s_1)+\frac{1}{2}v(s_2)\right)\leq v(s_2)$$

This guarantees that s_2 has no incentives imitating s_1.

But this implies that

$$v(s_1)=v(s_2)$$

The conclusion is the same as with private tags,

Proposition 3 *With a single public tag and two sellers, there cannot be strict differentiation.*

As before in the case of a single private tag, the only equilibria with

$$v(s_1)\neq v(s_2)$$

will be pooling.

[14]Note that buyers will always be in a position to identify the equilibrium pool of the deviator, as tags are assumed public.

3.1.1 Mixed Strategies and Differentiation

The focus on pure strategy equilibria excludes **perfectly** informative equilibria of the following sort: Denote the probability of a purchase conditional on the seller carrying tag-value t by $p_b(t)$. Let (t_1, t_2) be a candidate tag-profile in a fully informative equilibrium. It is straightforward to check that such a profile forms an equilibrium iff

$$p_b(t_1) = \frac{v(s_2)}{v(s_1)} p_b(t_2)$$

Note that there is a continuum of such equilibria, the best one having $p_b(t_2) = 1$. In this setup though, all such equilibria are Pareto Dominated by equilibria in which buyers buy with certainty (regardless of how informative those are)[15]. It can be shown that there are no Pareto Optimal mixed-strategy equilibria in this two-seller case[16].

In a sense then, the restriction to pure strategies is equivalent to restricting attention to Pareto Optimal equilibria, a 'conventional' selection criterium in the cheap-talk literature. Note that more informative equilibria are not 'better' than less informative ones. On this issue, see the discussion in Section 6.

[15] This construction generalizes easily to cases with more than two sellers, with the probability of purchase decreasing strictly with identification payoffs.

[16] Whether this is also the case for more than two sellers is something I have not shown, but which I conjecture to be true.

3.2 Nested Partial Pooling with 3 Sellers and a Single Public Tag

In the previous elementary cases, one recovers the 'intuition' that with ranked sellers there cannot be strict differentiation. That intuition survives the introduction of more sellers in the case of private tags (by exactly the same reasoning as before). As the following example shows, with public tags a degree of strict differentiation can be obtained even with only 3 sellers.

Definition 4 *'Nested partial pooling' refers to an equilibrium with at least 2 pools in which there is no pool such that all the identification-payoffs of its members are above all the identification-payoffs of the members of another pool.*

Proposition 5 *With a single public tag and 3 sellers, each with a distinct identification-payoff, the only possible form of differentiation is via 'nested partial pooling'.*

Proof. It can be easily seen that full differentiation (each seller on his or her own) is not possible, as the lowest-ranked seller will always have an incentive to imitate a higher ranked one (since out-of-equilibrium beliefs will be restricted to assign probability $\frac{1}{2}$ to the deviator in the newly formed pool -see Appendix).

Now, assume that in equilibrium two differentiated pools exist, and denote them by P and P' (of course, not both singletons). Now, if a member of, say, P, adopts the tag-value corresponding to the other pool, assume that buyers (who will notice the deviation as tags are public) believe that the deviator is the seller in P with the lowest identification-payoff. Note that this is the beliefs' specification most conducive to differentiation (i.e., it induces the highest levels of deterrence).

Denote by $v(P)$ the average identification-payoff in pool P (which I shall refer to as the 'value of the pool'); by $\underline{s}(P)$ the seller with the lowest identification-payoff in pool P; and by $\#P$ the cardinality of pool P. Under consistency and the beliefs' restriction just described, the following inequalities must be satisfied,

$$\frac{\#P'}{\#P'+1}v(P') + \frac{1}{\#P'+1}v(\underline{s}(P)) \leq v(P)$$

$$\frac{\#P}{\#P+1}v(P) + \frac{1}{\#P+1}v(\underline{s}(P')) \leq v(P')$$

If all identification payoffs of sellers in a pool are above all those in the other, then one of the two preceding inequalities must be violated: Without loss of generality, let $v(P) > v(P')$. Now, either $\#P = 1$ and $\#P' = 2$,or $\#P = 2$ and $\#P' = 1$. In the first case, the second inequality is violated as

$$\frac{1}{2}v(P) + \frac{1}{2}v(\underline{s}(P')) > \frac{1}{2}v(s(P')) + \frac{1}{2}v(\underline{s}(P'))$$

In the second case, it is also violated as

$$\frac{2}{3}v(P) + \frac{1}{3}v(\underline{s}(P')) > v(\underline{s}(P'))$$

■

Here is an example of a game with a nested-partial-pooling equilibrium:

$$v(3) > v(2) > v(1)$$

$$v(2) > \frac{1}{3}v(3) + \frac{1}{3}v(1) + \frac{1}{3}v(2)$$

A 'nested partial pooling' equilibrium has the sellers with the highest and lowest identification-payoffs pooled, while the middle seller stands alone. To see that this is an equilibrium, start by noting that the expected payoff to a seller in a pool is a convex combination of the identification-payoffs of the members of the pool, with the weights given by buyers' beliefs. If the middle seller imitates the pool of top and bottom sellers, she will obtain the payoff given by the RHS of the second inequality above (consistency pins down beliefs here completely, as the middle seller stands alone). Whether imitating the middle seller is profitable for sellers in the pool will depend on the exact specification of out-of-equilibrium beliefs. Those beliefs, though restricted by consistency to assign a weight of $\frac{1}{2}$ to the middle seller (see Appendix), are nevertheless not completely pinned down. This because buyers cannot tell which member of the pool is the deviator. If buyers assume that the deviator is for sure the lowest-ranked seller, then clearly the deviation will not be profitable for either member of the pool.

It is easy to see that there is a full pooling equilibrium outcome as well. This and the partial pooling situation just described are the only two possible (pure strategy) equilibrium outcomes.

More generally, assuming that buyers invariably believe the deviator to be the seller with the lowest identification-payoff, the following proposition characterizes the mapping from valuations to (pure strategy) equilibria in this 3 sellers case:

Proposition 6 *With a single public tag and three sellers with distinct identification payoffs, the game will have a nested-partial-pooling equilibrium in pure strategies iff*

$$\frac{1}{2}v(3) + \frac{1}{2}v(1) \leq v(2)$$

Else, it will have only a pooling equilibrium.

Proof. If the inequality is violated, it will be profitable in a separating equilibrium for the middle seller to imitate the two others, as the payoff to a member of the resulting pool will be given by the LHS of the following expression,

$$\frac{2}{3}\left(\frac{1}{2}v(1) + \frac{1}{2}v(3)\right) + \frac{1}{3}v(2) > v(2)$$

If the inequality is satisfied, then the middle seller will have no incentive to imitate the pooled sellers; while if, under the specified out-of-equilibrium beliefs, a member of the pool imitates the middle seller, he or she will obtain a payoff

$$\frac{1}{2}v(2) + \frac{1}{2}v(1)$$

which is smaller than the payoff of the pool. ■

3.3 Stratified Differentiation with More Than 3 Sellers

Here is an example of a game with more than 3 sellers in which it is possible to have pools made up only of sellers whose identification-payoffs are all strictly above or below the identification-payoffs of sellers in any other pool. I will refer to such differentiation as 'strict stratification'.

There are N sellers. The N sellers are divided into three classes: A top seller with identification-payoff $v(1)$; a large intermediate group of $N-2$ sellers, each with identification-payoff $v(2)$; and a bottom seller with identification-payoff $v(3)$. Assume that

$$v(3) > v(2) > v(1)$$

$$\frac{1}{2}v(1) + \frac{1}{2}v(3) < v(2)$$

Then the top seller on her own, and all the others in a pool, represents an equilibrium outcome if buyers believe that a deviator from the pool must be the bottom type. This follows since the second inequality above implies that, for sufficiently large N,

$$\frac{1}{2}v(1) + \frac{1}{2}v(3) < \frac{N-2}{N-1}v(2) + \frac{1}{N-1}v(1)$$

4 Conditions for Differentiation in Vertical Economies

This section presents conditions for endogenous differentiation in 'vertical economies', i.e., games in which no two sellers have the same identification-payoffs. The example with strictly stratified differentiation suggests that the specialization to 'vertical' economies has substantial analytical implications[17]. Its 'real life' relevance, though, turns on whether 'product clusters' can reasonably be modelled as consisting of almost perfect substitutes, rather than perfect substitutes. Hardly a clear cut issue[18].

The basic aim here is to identify conditions sufficient and/or necessary for this type of games to display 'minimal', resp. a 'maximal' degree of differentiation. 'Minimal' differentiation obtains when there are at least two pools in equilibrium. The notion of 'maximal' differentiation is problematic. It is not obvious in the present context which differentiation pattern should be considered more 'informative'. The 'natural' approach would be to simply consider the number of pools in an equilibrium. But then again, a strictly stratified equilibrium might be reasonably regarded as more 'informative' than a more scrambled one with a larger number of pools, in as far as the former allows buyers to associate prices with well defined quality intervals, while the latter does not.

The 'obvious' criterium to decide this question, namely, welfare, being of no help here (all equilibria considered being Pareto Optimal)[19], I will present results along both lines.

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[17]Another example pointing in this direction: If one adds an extra lowest seller to an economy with three sellers that admits differentiation, the resulting game will not have an equilibrium in which the original population of sellers is pooled exactly as in the original game (as the additional lowest seller cannot stand on his or her own in any equilibrium).

[18]Moreover, it seems that certain forms of replication might leave some of the equilibrium predictions from the vertical scenario unchanged. For example, adding replicas of existing *pools* will always allow one to construct an equilibrium in which the original population of sellers is pooled exactly as before (with the newcomers grouped in the replicated pools).

[19]On the issue of why more differentiated equilibria should be considered 'better' than less differentiated ones, see the remarks in Section 6.

4.1 A Necessary and Sufficient Condition for Minimal Differentiation

The question here is which (vertical) economies admit equilibria with more than one pool? The following proposition provides a concise and relatively simple answer:

Proposition 7 *A vertical economy admits no differentiation iff*

$$v\left(P_{\underline{N}}\right) > v\left(S\right)$$

with $v\left(S\right)$ denoting the average identification-payoff in the economy, while $v\left(P_{\underline{N}}\right)$ stands for the value of the lowest value pool amongst all the pools containing the seller with the highest valuation (but S).

Proof. Order sellers so that

$$v\left(s_N\right) > ... > v\left(s_1\right)$$

I start by showing that the condition implies that there cannot be differentiation. Let $v\left(P\right)$ stand for the average valuation of pool $P \subseteq S$. It is immediate that

$$v\left(P\right) > v\left(S\right) \Rightarrow v\left(S/P\right) < v\left(S\right)$$

If P_N denotes a pool containing the seller with the highest valuation, then it can be shown that, if the condition in the proposition is satisfied, $\{P_N, S/P_N\}$ cannot form part of an equilibrium[20]. To see this, note that by

[20]It is meant that there is no (pure strategy) equilibrium in which all members of P_N choose one tag, while all members of S/P_N choose another.

the condition in the proposition, it must be that

$$v\left(P_N \cup v\left(s\right)\right) > v\left(S\right) \quad \forall\, s \in S/P_N$$

So, we have

$$v\left(P_N \cup v\left(s\right)\right) > v\left(S\right) > v\left(S/P_N\right) \quad \forall\, s\; \in S/P_N$$

In other words, it pays for a member of S/P_N to imitate P_N.

Now, given any partition $\mathcal{P}$ of $P \subseteq S$, if there a $p \in \mathcal{P}$ such that

$$v\left(p\right) > v\left(P\right)$$

then there must exist another element of the partition $p' \in \mathcal{P}$ such that

$$v\left(p'\right) < v\left(P\right)$$

This implies that no matter how S/P_N is partitioned, there will always be a pool whose members will have an incentive to imitate P_N (i.e., a pool with value less than $v\left(P_N\right)$).

The implication in the opposite direction: It will be shown that

$$v\left(P_{\underline{N}}\right) \leq v\left(S\right) \Rightarrow \textit{ Differentiation}$$

(rather than the equivalent statement, *No Differentiation* $\Rightarrow v\left(S\right) < v\left(P_{\underline{N}}\right)$). Define P_N^k as the pool consisting of the seller with the highest valuation plus sellers $s_1, ..., s_k$. Then it can be shown that $v\left(P_{\underline{N}}\right) \leq v\left(S\right)$ implies that either there exists a $k \in \{0, ..., N\}$ (with $P_N^0 = \{s_N\}$), such that

$$v\left(P_N^k\right) > v\left(S\right) > v\left(S/P_N^k\right) \textit{ and } v\left(P_N^{k+1}\right) < v\left(S\right) < v\left(S/P_N^{k+1}\right)$$

or there is a $k'' \in \{1, ..., N\}$ such that

$$v\left(P_N^{k''}\right) = v(S) = v\left(S/P_N^{k''}\right)$$

To see this: Since $v(P_N^0) > v(S)$, it must be that $v(S/P_N^0) < v(S)$. As $P_{\underline{N}} = P_N^{k'}$, some $k' \in \{1, ..., N\}$, by the condition we have $v\left(P_N^{k'}\right) < v(S) < v\left(S/P_N^{k'}\right)$. Clearly then, either one of the situations above must result.

Say there exists a $k'' \in \{1, ..., N\}$ such that

$$v\left(P_N^{k''}\right) = v(S) = v\left(S/P_N^{k''}\right)$$

then $\left\{P_N^{k''}, S/P_N^{k''}\right\}$ is an equilibrium outcome under the beliefs that a deviator is the lowest valuation seller in the pool.

In the other situation, two cases must be distinguished:

Case 1: $v\left(S/P_N^k\right) > v\left(P_N^{k+1}\right)$. It is claimed that $\left\{P_N^k, S/P_N^k\right\}$ is an equilibrium outcome under the beliefs that a deviator is the lowest seller. The incentives to imitate go only from the lower value pool to the higher, so, here, from S/P_N^k to P_N^k. Now, the value of the imitated pool equals $v\left(P_N^{k+1}\right)$, which by hypothesis is lower than $v\left(S/P_N^k\right)$. In other words, imitation does not pay.

Case 2: $v\left(S/P_N^k\right) \leq v\left(P_N^{k+1}\right)$. It is claimed that $\left\{P_N^{k+1}, S/P_N^{k+1}\right\}$ is an equilibrium outcome under the beliefs that a deviator from the lower value pool is that pool's second-highest valuation seller (in case of deviations from the higher value pool, beliefs are that the deviator is the lowest valuation seller). Again, incentives to imitate go from the lower value pool to the higher, so, here, from P_N^{k+1} to S/P_N^{k+1}. The value of the imitated pool is then $v\left(S/P_N^k\right)$, and again it does not pay to imitate. ■

While the condition in the previous proposition might not seem intuitive at first, upon some reflection it might be read as excluding 'right-skewness' in the quality map of the economy: Take a vertical economy in which the condition is satisfied. Increase the top identification-payoff, keeping all others constant. This will raise the value of any given pool containing the top seller as well as the mean valuation. But the mean valuation will increase at a constant rate which is smaller or equal to the rate at which the value of any pool containing the top seller increases. Hence, eventually the condition must be violated[21]. Note that reducing the bottom valuation will relax the relevant inequality, by exactly an analogous logic.

Putting it differently, the condition tells us that in order for differentiation to obtain, the quality map might be 'skewed' to the left but not to the right. The origin of this asymmetry is not difficult to make out: A 'left-outlier' is not so problematic as a 'right-outlier', since the former allows for a worsening of out-of-equilibrium beliefs, unlike the latter. In other words, while the temptation to imitate grows with the distance from the mean in either direction, greater distance to the left enhances punishment possibilities, while distance to the right does not.

Stratified Pooling If one restricts attention to specific patterns of differentiation, then the condition above can take a substantially simpler form.

[21] Note that $v\left(P_{\underline{N}}\right)$ cannot asymptote to the mean: If the rate at which $v\left(P_{\underline{N}}\right)$ increases coincides with that at which $v\left(S\right)$ increases, then their levels must coincide as well. Else, $v\left(P_{\underline{N}}\right)$ must increase at a strictly faster pace than $v\left(S\right)$. Since the rate at which $v\left(P_{\underline{N}}\right)$ increases can only take a finite number of values, it must be that eventually $v\left(P_{\underline{N}}\right)$catches up with $v\left(S\right)$.

An interesting example concerns strict stratification, a form of differentiation which, as already pointed out, might be of particular interest, since it allows consumers to associate quality intervals with tag values.

The question that will be addressed is, given a vertical economy, when does there exist an equilibrium with strictly stratified pooling?

Proposition 8 *A strictly stratified equilibrium exists in a vertical economy with* $N \geq 2$ *iff*

$$\frac{1}{2}v\left(s_N\right)+\frac{1}{2}v\left(s_1\right)\leq v\left(S/s_N\right)$$

Proof. Necessity: Represent the pool made up of all sellers with identification-payoffs between $v\left(s_k\right)$ and $v\left(s_{k+n}\right)$, inclusive, as $\left[s_k,s_{k+n}\right]$. It is claimed that there cannot be a strictly stratified equilibrium if the following system of inequalities holds,

$$\frac{\#\left[s_k,s_N\right]}{\#\left[s_k,s_N\right]+1}v\left(\left[s_k,s_N\right]\right)+\frac{1}{\#\left[s_k,s_N\right]+1}v\left(s_1\right)>v\left(\left[s_1,s_{k-1}\right]\right)$$

$$\textit{with}\ \ k\leq N\ \textit{and}\ \begin{cases}\geq \min\left(\frac{N}{2}+3,N\right) & \textit{if N is even}\\ \\ \geq \min\left(\frac{N+1}{2}+3,N\right) & \textit{if N is odd}\end{cases}$$

To see this: In any strictly stratified equilibrium, if $v\left(P\right)>v\left(P'\right)$, it must be that $\#P+1<\#P'$. This follows from the fact any such a pair of pools must satisfy

$$\frac{\#P}{\#P+1}v\left(P\right)+\frac{1}{\#P+1}v\left(\underline{s}\left(P'\right)\right)\leq v\left(P'\right)$$
$$=\frac{\#P'-1}{\#P'}v\left(P'/\underline{s}\left(P'\right)\right)+\frac{1}{\#P'}v\left(\underline{s}\left(P'\right)\right)$$

As the equilibrium is taken to be strictly stratified, $v\left(P\right)>v\left(P'/\underline{s}\left(P'\right)\right)$. It follows that the only way the inequality can hold is if $\#P+1<\#P'$. Hence the size of the pool containing the top seller in any stratified equilibrium can never exceed the bounds displayed above (as $\#[s_k, s_N] = N - k + 1$).

The fact that it is not possible to obtain an equilibrium configuration from a non-equilibrium partition of sellers, $\{P, \overline{P}\}$, by partitioning the imitating pool (i.e., the pool with the lower value), then implies that if all these inequalities hold, there cannot be a strictly stratified equilibrium (in a strictly stratified equilibrium the top seller must belong to $\#[s_{k'}, s_N]$, some k').

It is now claimed that if the last inequality holds ($k = N$, which corresponds to the negation of the condition in the statement of the proposition), all others must as well. This is done by induction: If the last condition holds, then the second-to-last holds as well. To see this, just rewrite the LHS (left-hand-side) of the second-to-last condition to read

$$\frac{2}{3}\left(\frac{1}{2}v(s_N) + \frac{1}{2}v(s_1)\right) + \frac{1}{3}v(s_{N-1}) \tag{1}$$

As $\left(\frac{1}{2}v(s_N) + \frac{1}{2}v(s_1)\right)$ is just the LHS of the last inequality, it exceeds $v(S/s_N)$. This latter magnitude is, in turn, larger than $v(S/s_{N,}s_{N-1})$. So, the convex combination (1) must be larger than $v(S/s_{N,}s_{N-1})$. Now, assume the $k = m$ inequality holds, then it is immediate that the $k = m - 1$ one also does, as one can rewrite the LHS of the latter inequality as

$$\frac{\#[s_{m-1}, s_N]}{\#[s_{m-1}, s_N] + 1} v([s_m, s_N] \cup s_1) + \frac{1}{\#[s_{m-1}, s_N] + 1} v(s_{m-1})$$

which, as a convex combination of two magnitudes larger than $v([s_1, s_{m-1}])$ ($v([s_m, s_N] \cup s_1)$ corresponding to the LHS of the $k = m$ inequality), must be larger than $v([s_1, s_{m-1}])$. But $v([s_1, s_{m-1}]) > v([s_1, s_{m-2}])$, and so the induction goes through.

Sufficiency is obvious: If the condition holds, then there is at least one stratified equilibrium with the top seller on his or her own, and all the remaining sellers grouped in a lower pool. ■

A useful reference in trying to interpret the conditions identified above is the case of a perfectly symmetric vertical economy, i.e., a vertical economy centered around its mean,

$$v(s_k) - v(S) = d > 0 \quad \Leftrightarrow \quad v(S) - v(s_{k'}) = d$$

It is easy to see that such an economy always admits differentiation, yet it does not admit strict stratification as

$$v(S/s_N) = \frac{1}{2}v(s_{N-1}) + \frac{1}{2}v(s_1) < \frac{1}{2}v(s_N) + \frac{1}{2}v(s_1) = v(S)$$

This suggests that in order to have a strictly stratified equilibrium, the distribution of identification payoffs must be 'skewed' to the left. Note that while for differentiation it is enough that the quality map not be skewed to the right , for strict stratification a 'low quality' bias is needed (the reason why a symmetric vertical economy admits differentiation but no stratification).

It might be of some interest to say something about the maximal number of strata that an economy that admits strict stratification might support. While it seems hard to say anything general and precise in this respect, it is easy to provide a bound for this number as a function of the number of sellers in the economy.

Proposition 9 *In a vertical economy with N sellers, the number of strata in a strictly stratified equilibrium is bounded above by $\sqrt{N}$.*

Proof. As argued in the proof of the previous proposition, for two pools, P, P', such that $v(P) > v(P')$, it must be that $P' > P + 1$. So, if a vertical economy with N sellers admits a strictly stratified equilibrium with K strata, K must satisfy

$$\sum_{k=1}^{K} [1 + (k-1)\,2] \leq N$$

(Since each strata must exceed the preceding one by at least 2). This expression can be rewritten as

$$\sum_{k=1}^{K} (2k-1) = 2\,(K+1)\,\frac{K}{2} - K = K^2$$

■

This bound does not say much for economies with large numbers of sellers, but in small economies it can prove useful; for example, it implies that an economy with less than 9 sellers can sustain at most two strata; and one with less than 4 sellers cannot sustain stratification at all.

5 Maximal Number of Pools

>From the fact that in any equilibrium there can be at most one singleton pool[22], it follows immediately that no equilibrium in an economy with N sellers can have more than $N/2$ pools (if N is even) or $(N+1)/2$ (if N is odd). This section presents a necessary and sufficient condition for the existence of equilibria with that maximum possible number of pools ('maximal' equilibria).

[22] Else, the low singleton could imitate the other; and it would be profitable to do so, since consistency pins down beliefs for the pool that forms at exactly $\frac{1}{2}$

Definition 10 *Ordering sellers by their identification-payoffs from lowest (1) to highest (N), a strictly nested maximal pooling of a vertical economy, $\mathcal{P}_{s.n.}$, is a partition of sellers of the form*

$$\left\{\left(s_{1,}s_{N}\right),\left(s_{2},s_{N-1}\right),...,\left(s_{\overline{m}+1},s_{\underline{m}-1}\right),\left(s_{\underline{m}},s_{\overline{m}}\right)\right\}$$

where $s_{\underline{m}}$ ($s_{\overline{m}}$) stands for the lower (upper) median seller (when the number of sellers is odd, $s_{\overline{m}}=s_{\underline{m}}=s_{m}$).

Proposition 11 *A maximal equilibrium exists in a vertical economy iff*

$$\max_{P\in\mathcal{P}_{s.n.}} v\left(P\right)\leq\min_{P\in\mathcal{P}_{s.n.}}\left[\frac{3}{4}v\left(\overline{s}\left(P\right)\right)+\frac{1}{4}v\left(\underline{s}\left(P\right)\right)\right]$$

Proof. Given any maximal partition $\mathcal{P}$ (i.e.e, a partition of sellers into pairs -and one singleton if the number of sellers is odd), the condition for no-imitation between pairs can be rewritten

$$v\left(P\right)\leq\frac{3}{4}v\left(\overline{s}\left(P'\right)\right)+\frac{1}{4}v\left(\underline{s}\left(P'\right)\right)$$

So, if satisfied for $\arg\max_{P\in\mathcal{P}}v\left(P\right)$ and $\arg\min_{P\in\mathcal{P}}\left[\frac{3}{4}v\left(\overline{s}\left(P\right)\right)+\frac{1}{4}v\left(\underline{s}\left(P\right)\right)\right]$, it will be satisfied for all other pairs. If violated, then one of the pairs involved will have an incentive to imitate the other.

To show necessity (i.e., that maximal differentiation implies the condition), start by noting that any maximal equilibrium partition must be 'centered'. In the even case, this means that the upper-median seller and all sellers above must be paired with sellers strictly below the upper-median one. In the odd case, this means that all sellers above the median seller must be matched with sellers below that seller, with that seller herself standing on her own. This follows simply from noting that, otherwise, there must exist two pairs, P, P', such $v(\overline{s}(P')) < v(\underline{s}(P))$, which would lead the lower pool to imitate the higher one.

Order the pools by their span, i.e., $v(\overline{s}(P)) - v(\underline{s}(P))$, from the smallest to the largest, P_1 to P_K (with K denoting the number of pools). Now, if the condition is violated, it must be that there exist two pools P_k, P_j, such that

$$v(P_k) > \frac{3}{4} v(\overline{s}(P_j)) + \frac{1}{4} v(\underline{s}(P_j))$$

It will be presently shown that it is not possible to reverse this inequality by 'un-nesting' pools. Say $1 < k < K$. If $j > k$, and one attempts to do this by rematching $\overline{s}(P_k)$ with a seller with a value below $v(\underline{s}(P_k))$, the latter must belong to a pool $P_{k+l},\ l = 1, ..., K-k$ (evidently, if $k = K$, this cannot be done). Denote this seller by $\underline{s}(P_{k+l})$. A 'top' seller with a value above $v(\overline{s}(P_k))$ must be 'released'. Denote this seller by $\overline{s}(P_{k+l})$. If this seller is rematched with $\underline{s}(P_k)$, then one obtains two pools

$$\{\overline{s}(P_k), \underline{s}(P_{k+l})\}, \{\underline{s}(P_k), \overline{s}(P_{k+l})\}$$

which, again, violate the condition, as

$$v(\{\overline{s}(P_k), \underline{s}(P_{k+l})\}) < v(P_j) < \frac{3}{4} v(\overline{s}(P_j)) + \frac{1}{4} v(\underline{s}(P_j))$$

and

$$v(\{\underline{s}(P_k), \overline{s}(P_{k+l})\}) > \frac{3}{4} v(\overline{s}(P_j)) + \frac{1}{4} v(\underline{s}(P_j))$$

If, instead, one tries to avoid this last match by rematching $\underline{s}(P_k)$, so to say, 'backwards', i.e., with a 'top' seller $\overline{s}(P_{k-l})$, $l = 1, ..., k-1$, eventually one would have to rematch the top seller released before with a 'bottom' seller, $\underline{s}(P_{k-l})$, such that $v(\underline{s}(P_{k-l})) > v(\underline{s}(P_k))$.

If $j < k$, and one attempts to solve the problem by rematching $\underline{s}(P_j)$ with a seller with a value above $\overline{s}(P_j)$, i.e., necessarily a seller $\overline{s}(P_{j+l})$, $l = 1, ..., K-j$, again, one has to 'release' a seller with a value above $v(\underline{s}(P_j))$, which, if rematched to $\overline{s}(P_j)$, will form two pools violating the condition.

As in the previous case, trying to avoid violating the condition by matching the initially released seller 'backwards', will nevertheless result in two pools that violate the condition.

The arguments for the remaining cases are analogous.

Sufficiency is obvious: If the condition is satisfied then a strictly nested maximal equilibrium exists. ■

An immediate corollary of the previous result is that if a maximal equilibrium exists, then there must exist a strictly nested maximal equilibrium. This helps interpret the proposition: While this condition, just as the condition for differentiation in general, precludes a rightward bias in the quality distribution (it can be shown that $v_{\overline{m}} \geq v(S)$ is necessary for maximal differentiation[23]), it also limits an eventual leftward bias, unlike the condition for differentiation; in other words, some degree of 'symmetry' around the median (value/range) is needed. This can be seen from the link to strict nestedness. It is easy to see from a four sellers example that, for given 'spans', the inside pool cannot veer too far up or down within the outside pool (more precisely,

[23] If, on the contrary, $v_{\overline{m}} < v(S)$, then there must exist a P in any maximal partition such that $v(P) > v_{\overline{m}}$. This cannot be, as by 'centering', $s_{\overline{m}}$ must be paired with a seller whose value is below hers, so that

$$v(P) > \frac{3}{4}v_{\overline{m}} + \frac{1}{4}v(\underline{s}(P_{v_{\overline{m}}}))$$

the value of the inside pool cannot exceed $\frac{3}{4}v(\overline{s}(P_2)) + \frac{1}{4}v(\underline{s}(P_2))$, nor go below $v(P_2) - \frac{1}{4}(v(\overline{s}(P_1)) - v(\underline{s}(P_1))))$[24].

6 Why is Differentiation Desirable?

As pointed out in the introduction, this work selects the Pareto Optimal equilibria in assuming that buyers always buy with probability one. In so far as all equilibria under consideration are Pareto Optimal (buyers are indifferent between buying and not, while the sum of sellers surplus[25] is constant and equal 0 across equilibria), regardless of how differentiated they might be, this model cannot 'internally' provide an answer to the question being asked, and, must, in so far, be regarded as incomplete. And while a complete model in the previous sense would obviously be better, my guess is that such a model would prove substantially harder to analyze. It would then be

[24] More indirectly, one can point to the fact that a perfectly symmetric odd vertical economy admits only a strictly nested maximal equilibrium: From 'centering' it follows that the median seller must stand alone. Then it is easy to see that if there is a pair whose value exceeds v_m, the median seller will have an incentive to imitate it. In other words, $\max v(P) = v_m$. In a strictly symmetric vertical economy, i.e., one where

$$v_{m+l} - v_m = v_m - v_{m-l}, \ \forall\, l = 0, ..., \frac{N-1}{2}$$

it must then be that if a seller s_{m+l} is matched with a seller s_{m-j}, with $j \neq l$, either $v(\{s_{m-j}, s_{m+l}\}) > v_m$, or there exists another pair such that its value is strictly above the median value. In an even economy, though, symmetry is not enough to obtain a unique maximal equilibrium which is strictly nested.

[25] Where by 'surplus' I mean the extra revenues that sellers earn or lose relative to a situation where they can be perfectly identified.

justified to study this very basic setup in a first approach to the problem.

The model being incomplete, all what one can do is to invoke outside considerations in order to argue in favor of specific equilibrium constellations. In this spirit, in the introduction to Section 4 a (vague) notion of 'informativeness' was brought into play. A way of making that notion precise is to argue that individuals (or regulators?) care about the probability that paying a higher price will yield higher quality (ex post)[26]. In a strictly stratified equilibrium that probability is 1; in a weakly stratified maximal equilibrium[27] of an even economy with $N = 4$, it is $\frac{3}{4}$; while in a strictly nested maximal equilibrium of an economy with $N = 5$, it will be only $\frac{1}{8}$ -for a buyer matched with the median seller (and will differ according to the pool one is matched with, so that one would have to somehow weight each individual evaluation, and then 'sum' in order to obtain a workable criterium). According to this criterium then, a strictly stratified equilibrium would be best.

Here it is interesting to ask what kind of complete model might deliver such ordering? Perhaps a version of the previous model with risk-averse buyers. Or perhaps one would have to postulate regretful buyers (whose utility from consuming the good is discounted if -ex post- it should turn out that they paid a price exceeding their valuation). In any case, the analysis of both such models promises to be considerably harder than that of the simple

[26]More precisely: The probability that no buyer paying a lower price will get ex post higher quality.

[27]That is, a maximal partition such that

$$\overline{s}(P) > \overline{s}(P') \Rightarrow \underline{s}(P) > \underline{s}(P')$$

setup I deal with.

7 Conclusions

The existence conditions presented have in common that they all exclude a rightward bias in the quality distribution. One can take this to mean that an economy with an upward quality bias need not be 'better' than one with a downwards bias, at least as long as differentiation is regarded as desirable. In other words, bad qualities play a 'good' role here: Supporting pessimistic beliefs, and, through this channel, helping to enforce differentiation.

On the other hand, the condition for stratification actually requires a leftward bias in the distribution, while that for maximal differentiation not only excludes a rightward bias, but actually limits the degree to which the distribution might be slanted to the left. This suggests that there might be a trade-off between maximal differentiation and stratification. Of course, this presumes that the more 'centered' a quality distribution, the greater the maximum degree of equilibrium differentiation. In the absence of results characterizing the most differentiated equilibrium of a given vertical economy generally, this must remain conjecture.

In any case, these results, however sketchy, do make the point that endogenous differentiation is possible, at least in sufficiently transparent markets, and that, moreover, the quality constellation in the market will determine the degree to which such differentiation is feasible. And some of the specific predictions might even strike some as plausible descriptions of what happens in the real world: For example, the market leader (Nike) being victim of imitation by low quality producers, with ‘middle brow’ brands being left unimitated, does not seem at odds with the broad picture prevailing in some developing countries in which trademark laws are not effectively enforced.

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A Appendix

For the case of two sellers, and pure strategies: Let the candidate equilibrium be given by

$$\sigma_1(t_1) = 1$$

$$\sigma_2(t_2) = 1$$

The posterior probability of being matched with seller s_1 given an observation of (t_2, t_2), and given that this buyer's seller bears a tag value t_2, is given by

$$p(b(s_1) \mid (t_2, t_2), \text{“}t_2\text{”}) = \frac{p(\text{“}t_2\text{”} = s_1 \mid (t_2, t_2))}{\sum p(\text{“}t_2\text{”} = s_j \mid (t_2, t_2))} = \frac{1}{2}$$

This follows since $p(\text{“}t_2\text{”} = s_j \mid (t_2, t_2))$ (the probability that the bearer of a tag value t_2 who is currently matched with this buyer is actually seller s_j, given that there are two such tag values), is the same for both sellers, and equals $\frac{1}{2}$ (from random matching). Note that

$$p(\text{“}t_2\text{”} = s_j \mid (t_2, t_2))$$

$$= p(b(s_1))\, p(\text{“}t_2\text{”} = s_1 \mid b(s_1), (t_2, t_2))$$

i.e., the product of the probability of this buyer being matched with seller $s_1, p(b(s_1))$ $(= \frac{1}{2})$, and the probability that s_1 bears a tag value t_2, given that seller s_1 is matched with this buyer, and given (t_2, t_2),

$$p(\text{“}t_2\text{”} = s_1 \mid b(s_1), (t_2, t_2)) = 1.$$

Note further that since there is only one possible mapping of realized tag-values to sellers here, in this two sellers' case the mixing probabilities do not enter the picture[28].

The argument for assigning a weight of $\frac{1}{2}$ to the middle seller in any specification of out-of-equilibrium beliefs after an imitating deviation from the pool $\{s_1, s_3\}$ is similar:

For the purposes of this argument, it suffices to specify that any fully mixed profile converging to equilibrium strategies is of the form,

$$\widetilde{\sigma}_1(t_{1,3}) = 1 - \varepsilon$$

$$\widetilde{\sigma}_3(t_{1,3}) = 1 - \alpha$$

$$\widetilde{\sigma}_2(t_2) = 1 - \gamma$$

The probability that a buyer matched with a seller bearing a tag value t_2 is actually matched with seller s_2, given observed tag profile $(t_{1,3}, t_2, t_2)$, is given by

$$\frac{p(\text{“}t_2\text{”} = s_2 | s_2 = t_2, s_1 = t_2, s_3 = t_{1,3}) + p(\text{“}t_2\text{”} = s_2 | s_2 = t_2, s_1 = t_{1,3}, s_3 = t_2)}{\sum p(\text{“}t_2\text{”} = s_j | s_j = t_2, s_k = t_2, s_l = t_{1,3})}$$

where the sum in the denominator is over all assignments of sellers to observed tags. Substituting the mixing probabilities above (with, for example, ε standing for any distribution of residual probability weight over non-equilibrium

[28]Strictly speaking, a state of the world is given by both, an assignment of observed tags to sellers, and an assignment of sellers to buyers.

imitating tag values),

$$\frac{(1-\gamma)\,\varepsilon\,(1-\alpha)+(1-\gamma)\,\alpha\,(1-\varepsilon)}{2\,(1-\gamma)\,\varepsilon\,(1-\alpha)+2\,(1-\gamma)\,\alpha\,(1-\varepsilon)+2\varepsilon\gamma\alpha}$$

Divide through by the expression in the numerator,

$$\frac{1}{2+\frac{2\gamma\varepsilon\alpha}{(1-\gamma)\varepsilon(1-\alpha)+(1-\gamma)\alpha(1-\varepsilon)}}$$

The term $2\gamma\alpha\varepsilon$ is second order of magnitude, and, hence, does not affect the limit of this expression as residual probabilities converge to zero[29].

[29] The easiest, though perhaps not the most transparent way, to arrive at the desired conclusion, is just to note that

$$p\,(s_1=t_2=s_3, s_2=t_{1,3}|\,(t_2,t_{2,}t_{1,3}))=\alpha\gamma\varepsilon$$

which obviously converges to 0 as $\alpha,\gamma,\varepsilon\to 0$. Hence, the complementary event

$$(s_2=t_2, s_1=t_2, s_3=t_{1,3}) \text{ or } (s_2=t_2, s_1=t_2, s_3=t_{1,3})$$

must have probability one in the limit. By random matching, it must then be that a buyer paired with a t_2-seller assigns exactly probability $\frac{1}{2}$ to this seller being s_2.