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THE LAW OF SUPPLY AND DEMAND IN THE PROOF OF EXISTENCE OF GENERAL COMPETITIVE EQUILIBRIUM

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INTRODUCTION

During the 1950's results of the proof of existence of a general equilibrium for a walrasian economy wer published by several authors (McKenzie 1954, Arrow and Debreu 1954, Gale 1955, Nikaido 1956 Debreu 1959). The proof of existence of a general competitive equilibrium (GCE) soon became to b known as a solid bedrock of economic theory, and a result capable of providing a foundation for libera economic policies worldwide. Since then, the proof of existence of equilibrium, in strong contrast witl price formation theory, has traditionally been considered a flawless and reliable result. This pape identifies and examines a problem, hitherto ignored, in the standard set-theoretical proof of existence o equilibrium. We call into question the predominant view that the mappings used in the proofs of existence of a GCE represent what Arrow and Debreu (1954:274-5) called "the classical law of supply and demand".

A state of equilibrium for an economy is characterized by the fact that there are no endogenous forces to modify it. In this situation all markets clear simultaneously and agents have no incentive to change their choices. In the context of general equilibrium theory, this is associated with a price vector such that excess supply and demand at those prices generate the same price vector. Mathematically, this can be represented as a fixed point of a suitable mapping.

If an equilibrium is a fixed point, it does not follow that every fixed point is an economic equilibrium.

This depends on the nature of the intervening variables and the definition of the mapping used in the proof of existence of equilibrium. This is why the usual presentations of the proof of existence of a general competitive equilibrium highlight the economic meaning of the mappings being utilized. This is done by stating that the mappings f which transform price vectors $\mathbf{p} \in S_n$ into price vectors $f(\mathbf{p}) \in S_n$ (where S_n is the unit simplex) and which are used in the proof of existence in Arrow-Debreu models satisfy the law of supply and demand. These assertions are quite natural. It would be rather surprising to use a mapping which did not represent the law of supply and demand to demonstrate, by means of its fixed point, the existence of an equilibrium between supply and demand.

The relevance of this point justifies a more careful study, in spite of the apparent agreement regarding the proofs of existence of GCE. The analysis centers on the economic interpretation of these mappings which describe a sequence analogous to that of the law of supply and demand: at prices $\mathbf{p}$ agents calculate their supplies and demands which, in turn, determine a vector of aggregate excess demands $\mathbf{Z}(\mathbf{p})$ from which a new price vector $f(\mathbf{p})$ is obtained. But this is where the analogy ends. As we shall see below, in mapping f, prices are related with aggregate excess demand in a way which *contradicts*, in general, the law of supply and demand. Thus, the fixed point in this mapping lacks the economic sense unanimously attributed to it.

Our argument is that, contrary to what the authors of these proofs maintain, the mappings used are not the expression of an economically meaningful price adjustment rule. This is an important point: for the authors who worked out the proof of existence of equilibrium using a fixed point theorem, the mappings must possess an economic sense for the proof to be meaningful. Given the nature of the task at hand, the rest point determined by the fixed point theorem must be an *economic* rest point representing a state of the economy in which economic forces intervening in price formation cease to operate and all markets clear simultaneously at the equilibrium prices. It is thus required that the *same* economic forces that are at rest in the fixed point, are represented by the mapping in all other points.[1] The search for a mapping with an economic meaning is thus a legitimate concern. It follows that if the mapping lacks a meaningful economic interpretation, this entails the lack of validity of the proof of existence from the economic viewpoint, whatever the mathematical properties of the intervening sets and mappings.

We reject the idea that only the mathematical properties of the proof should be taken into account (see for example, Hildenbrand and Kirman, 1988:106). It is not possible to obtain the proof of existence of an economic equilibrium from the existence of the fixed point of a mapping having the desired mathematical properties and expressing the non-negative solution of an abstract and unspecified system of equations. Inevitably, the proof must rely on a system of equations in which prices play a role as the key allocative mechanism. This is why the system of equations must be clearly specified, and the mapping must relate to them in a manner consistent with their nature.[2] Thus, the authors of the proof of existence of GCE correctly insist on the economic interpretation of the mappings.

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[1] This problem is analogous to Hahn's appraisal of Archibald and Lipsey's model of stationary monetary equilibrium. Because these authors recognize that "out of equilibrium [the model] does not make economic sense", Hahn (1960:42) asks "how can one set up a system of equations which only makes sense in equilibrium in order to solve for equilibrium? It is like solving for the equilibrium price for apples by the use of a demand and supply function for apples which only holds when supply is equal to demand".

[2] The work of Nash (1950) provided a breakthrough, but was still insufficient for the proof of existence of GCE. In his analysis, the correspondence of each n-tuple of strategies, with its set of countering n-tuples, gives a one-to-many mapping of the product space into itself. The product space and the mapping having the required mathematical properties, the mapping has a fixed point. The fixed point is associated to a self-countering n-tuple, i.e., one in which the strategies yield the highest "obtainable expectation" for all the players involved. But the highest obtainable expectation may be related to many kinds of gratification, depending on the nature of the payoff functions. This is why the equilibrium point associated with the fixed point of this extremely general mapping can be associated with *any* game, whether it involves economic magnitudes or not. This explains why there are no references to an economic equilibrium in Nash's paper.

Following standard usage, the law of supply and demand can be expressed in two equivalent forms. Let $\Delta p_i = p'_i - p_i$, and $p_i = P_i/\Sigma P_i$, $p'_i = P'_i/\Sigma P'_i$, where P_i and P'_i are prices for a unit of commodity i, in an abstract unit of account and all price vectors are strictly positive; $z_i(\mathbf{p})$ denotes the excess demand function for commodity i. In its first form, the law of supply and demand can be stated as follows:

i) for all $z_i(\mathbf{p}) \neq 0$, $\Delta p_i = F_i[z_i(\mathbf{p})]$ such that $z_i(\mathbf{p}) \cdot \Delta p_i > 0$;

ii) for all $z_i(\mathbf{p}) = 0$, $\Delta p_i = F_i[z_i(\mathbf{p})]$ with $\Delta p_i = 0$.

Thus, F_i is a sign-preserving function of $z_i(\mathbf{p})$, with $F_i(0) = 0$.

In its second form, the law of supply and demand is written as follows:

i) for all $z_i(\mathbf{p}) > 0$, $\Delta p_i = F_i[z_i(\mathbf{p})]$ such that

$z_i(\mathbf{p}) \cdot \Delta p_i > 0$;

ii) in all other cases, $\Delta p_i = F_i[z_i(\mathbf{p})]$ where $\Delta p_i = 0$.

In both formulations, relative prices vary according to the law of supply and demand. Consequently, this is the economic rule that must be respected by the mappings used in the proof of existence of equilibrium.

In the first section we summarize the economic interpretation of the mappings as described by the most important authors and we present our central critique. In the second section we demonstrate that the three main mappings in the literature are inconsistent with the law of supply and demand. We do not examine the proofs of existence which rely on the results of welfare theory (Arrow and Hahn, 1971), nor do we consider the existence results which rely on assumptions of differentiability of individual supply and demand functions.[3] Thus, our paper is concerned with proofs of existence of general equilibrium in the

[3] It is true that, in the context of general equilibrium theory, global analysis represents an approach which is closer to the older traditions (Smale, 1987). Nonetheless, the crucial point for our

more general setting. The third section analyzes the performance of the mappings for special two commodity economies. The fourth and fifth sections contain respectively a synthesis of the results and a conclusion. We assume the reader is familiar with the techniques used in the proof of existence of general competitive equilibrium.

I. The Economic Interpretation of the Mappings and its Critique

1. Basic References

The 1956 papers by Nikaido and Debreu stressed the idea that the mappings used in the proof of existence of equilibrium were the mathematical expression of the law of supply and demand. This view was already present in Arrow and Debreu (1954) and Gale (1955). The title and contents of Gale's paper suggested a close relation between the proof of existence and the law of supply and demand, defined as the mechanism by which "prices eventually regulate themselves to values at which supply and demand exactly balance, these being the prices at economic equilibrium". In Gale's terms, the proof of existence itself was a "rigorous investigation (...) as to the conditions under which such a balance is possible". Actually, Gale did not explicitly claim that the mapping involved in his proof of existence was the expression of the supply and demand mechanism.[4]

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purposes is that work along these lines (Smale, 1981; Mas-Collel, 1985) imposes assumptions which are more restrictive than those required by Arrow-Debreu models.

[4] The main result in Gale's paper was obtained independently by Debreu and Nikaido, who did assert that the mapping represented the law of supply and demand, as explained above. As to Debreu's approach, Hildenbrand (1983:20) describes it as follows: "Debreu used another method of proof in his further work on competitive equilibrium analysis (...), i.e. the 'excess demand approach' because he thought that this method of proving existence is more in line of traditional economic thinking". Debreu (1956) himself concludes that "the purpose of this note was to give a general market equilibrium theorem with a simple and economically meaningful proof".

Nikaido and Debreu proclaimed that the mapping used in the proof of existence of GCE represents the law of supply and demand in its broad sense, including not only the rule of price variation as a function of the sign of excess demand, but also the adjustment mechanism, i.e., encompassing stability. A few quotations of relevant passages suffice to make this point.

The economic interpretation of mapping Θ outside of the fixed point is advanced by Nikaido (1968:268 and 1970:321-2) in the following terms:

> "The mapping Θ..... which appears in the proof of Theorem 16.6 may be interpreted as representing the behavior of the auctioneer who proposes a modification of prices responding to a nonequilibrium market situation."

> "Goods are exchanged in the market according to their prices (...). If their demand and supply are not equal, current prices are induced to change under the influence of the "Invisible Hand". If new prices do not equate demand and supply, another round of price changes follows. Successive changes in prices which alterations in demand and supply continue until demand and supply are equated for all goods. In place of the Invisible hand, we may suppose a fictitious auctioneer who declares prices p in the market. Participants in the market then cry out quantities they buy and sell. If their demand and supply do not match, the auctioneer declares a new set of prices p. Θ defined above may be interpreted as an adjustment mechanism of demand and supply that associates new prices with current prices and excess [demand]".

This interpretation first appeared in Nikaido (1956). Consider a non negative price vector.

> "If the corresponding total demand $X = \Sigma X_i$ does not match with the total available bundle A, the referee must try to set up a new price constellation which will be effective enough to let the individuals adjust their demands in such a way that the deviation of the total demand from A may be reduced. This scheme of the referee will be most effectively achieved by making the excess of the total monetary value PX to be paid by the individuals for X over their total available income PA as large as possible, i.e., by setting up a price constellation belonging to $\Theta(X) = \{P \mid P(X\text{-}A) = \max Q(X\text{-}A) \text{ over all } Q \in S^k\}$. This function is multivalued and will be called the price manipulating function" (Nikaido, 1956:139).

At the time, Debreu (1956) was stating the same thing, mainly that his mapping had "a simple economic interpretation: in order to reduce the excess demand, the weight of the price system is brought to bear on those commodities for which the excess demand is the greatest". He would later (Debreu 1959:83 and 1974:219) restate this as follows:

"[A]n increase in the price of a commodity increases, or leaves unchanged, the total supply of tha commodity. This hints at a tendency for an increase in the price of a commodity to decrease the corresponding excess demand. It prompts one, when trying to reduce positive excess demand, to pu the weight of the price system on those commodities for which the excess demand is the greatest"

"According to a commonly held view of the role of prices, a natural reaction of a price-setting agenc to this disequilibrium situation [i.e. a price vector with non-zero excess demands] would be to selec a new price vector so as to make the excess demand F(p) as expensive as possible".

The explicit reference to stability only disappears in Debreu's entry to the *New Palgrave* (1989) when he states that the mapping "carries to one extreme the idea that the price-setter should choose high price for commodities that are in excess demand and low prices for the commodities that are in excess supply"

We will show that this is an erroneous interpretation of the mapping. First, it expresses a radica confusion between existence and stability of equilibrium, a confusion which is inexplicable in the writing of these authors. Second, once this confusion is dispelled so that we only keep the price variation rul as a function of the sign of excess demand, we demonstrate that it is not true that the mappings are consistent with the law of supply and demand in the narrow sense.

2. Stability and the Law of Supply and Demand

To avoid possible confusions, it is necessary to insist on the differences between the law of supply and demand and the general question of stability. The problem examined in this paper is not whether an equilibrium point is stable or not, but rather if the equilibrium represented by a fixed point of the mappings used in the proofs of existence can be effectively interpreted as an economic equilibrium, independently of the stability properties it may possess. In plain language, for the mapping to have an economic meaning it is <u>sufficient</u> for it to express an economically meaningful relation between excess demand and prices (the law of supply and demand), leaving aside the effects of variations in prices over excess demands (the problem of stability).

The question of stability is radically different. It involves a dynamic process, i.e., one that includes a time dimension. The fundamental relation is $z_{i,t+1}(p) = G_i(\Delta p_{i,t})$ and for equilibrium to be stable it is required that $\Delta z_{i,t+1}(\mathbf{p}) \cdot \Delta p_{i,t} < 0$. The question raised by stability analysis is the following: given the law of supply and demand, under which conditions can we obtain a relation of this kind guaranteeing stability? In the terms of Negishi (1962:637) "The stability problem is concerned with the question of what happens to the time paths of economic variables, such as prices and outputs, which are generated from certain dynamic adjustement processes". If they converge towards a position of equilibrium, these dynamic processes are defined as stable. For this reason, stability analysis depends on the "form" of the supply and demand functions. The purpose of the standard hypotheses of gross substitutability, dominant diagonal or the weak axiom of revealed preferences is to obtain an adequate form that guarantees the stability of the dynamic process.

In contrast, the mappings used in the proof of existence of equilibrium do not constitute a price adjustment process as defined in the context of the analysis of equilibrium stability. The mappings generate price vectors as a function of the matrix of excess demands but they do not allow to run the entire cycle and determine the evolution of excess demands resulting from these price variations. Therefore, they do not describe a dynamic path for prices and excess demands which could possibly be studied from the point of view of its convergence towards an equilibrium point.

3. Existence and the Law of Supply and Demand

The nature of the problem occupying our attention is clearly revealed if we follow the different stages of the construction of the mappings as exemplified in Arrow and Hahn (1971:25-27) procedure. The starting point is a two-commodity economy for which four price-variation rules, valid also in the general case of a n-commodity economy, are adopted:

"(1) Raise the price of the good in positive excess demand.

(2) Lower or at least do not raise the price of the good in excess supply, but never lower the price below zero.

(3) Do not change the price of a good in zero excess demand.

(4) Multiply the resulting price vector by a scalar, leaving relative prices unchanged, so that the new price vector you obtain is in S_n".

In the construction of the correspondence

"[W]e first seek for a continuous function $M_i(\mathbf{p})$ with the following three properties:

P1 $M_i(\mathbf{p}) > 0$ if and only if $z_i(\mathbf{p}) > 0$

P2 $M_i(\mathbf{p}) = 0$ if $z_i(\mathbf{p}) = 0$

P3 $p_i + M_i(\mathbf{p}) \geq 0$

It is intended that $M_i(\mathbf{p})$ represent an adjustment to an existing price so that a price vector $\mathbf{p}$ is transformed into a new price vector with components $p_i + M_i(\mathbf{p})$."

There are correspondences with properties P1-P3, for example:

$M_i(\mathbf{p}) = \max(-p_i, k_i \cdot z_i(\mathbf{p}))$, where $k_i > 0$.

"[I]f we interpret $(p_i + M_i(\mathbf{p}))$ as the *i*th component of the new price vector that the mapping produces, given $\mathbf{p}$, the procedure for finding these new prices satisfies the rules discussed earlier. However, while all $(p_i + M_i(\mathbf{p}))$ are certainly non-negative, there is nothing to ensure that they will add up to one. In other words, (...) there is no reason to suppose that $(\mathbf{p} + \mathbf{M}(\mathbf{p}))$ is in S_n when $\mathbf{p}$ is in S_n. Since we seek a mapping of S_n into itself, we must modify the mapping".

This is where the price normalisation implied by rule (4) intervenes and the result is correspondence

$$\mathbf{T}(\mathbf{p}) = \frac{\mathbf{p} + \mathbf{M}(\mathbf{p})}{[\mathbf{p} + \mathbf{M}(\mathbf{p})]\mathbf{e}}$$

According to Arrow and Hahn this is an "obvious way" of solving the difficulty they identified (see also (Arrow, 1968:117). But this assertion is incorrect because rule (4) modifies the initial mapping so as to make it <u>non-compliant with the first three rules</u>.

Our analysis of the most important mappings used in the proof of existence of GCE (following section) reveals that, under these conditions, the adjustment of price p_i does not depend so much on the sign of $z_i(\mathbf{p})$ as on the relation between $z_i(\mathbf{p})$ and the other $z_j(\mathbf{p})$ for $j \neq i$. It is the relative weight of $z_i(\mathbf{p})$ within the set of excess demands that has an influence on the direction of the change in p_i. This is the source of the strange price adjustment mechanism established by these correspondences: in a market with positive excess demand the price can increase or decrease depending on the relative importance of the excess demands on the markets $j \neq i$.

It is important to note that this "structural" rule which brings to bear the relative weight of excess demands in other markets on the direction of price variations in one market has nothing to do with the type of interdependencies commonly considered in general equilibrium theory, such as substitution and income effects. The latter concern the effects of the changes in prices on the excess demands and not the effects of changes in excess demands on prices. None of these interdependencies can explain why the price of one commodity decreases (increases) when its excess demand is positive (negative). We shall see that the interdependencies acting on the direction of the price variation in the mappings is a direct consequence of the normalisation of the price system.

The predicament can be stated as follows. In order to avoid falling outside of the price simplex, one leaves the law of supply and demand: we either have a fixed point and the mapping is devoid of economic sense; or we use a correspondence with an economic meaning, but loose the fixed point.[5]

[5] Would it be possible to avoid this predicament? This would imply seeking for a fixed point in a correspondence consistent with the law of supply and demand, for example $p_i + M_i(\mathbf{p}) \geq 0$. To our knowledge this has not been attempted. The reason for this probably lies in the additional restrictions that would have to be imposed on the supply and demand correspondences. As is well known from the work of Sonnenschein, Mantel and Debreu, there is no economic justification for such restrictions. Moreover, such additional constraints on these correspondences would limit the generality which is commonly attributed to the proof of existence in Arrow-Debreu models.

II. MAPPINGS AND THE LAW OF SUPPLY AND DEMAND

This section examines the three most important mappings used in the proof of existence of a general competitive equilibrium. The mappings examined here are from Nikaido (1968, 1970 and 1989), Arrow and Hahn (1971) and, finally, Arrow and Debreu (1954) and Debreu (1956, 1959).

II.1 Nikaido's Mapping

Nikaido (1968, 1970, 1989) proves the existence of a general equilibrium by using the following mapping:

$$\Theta_i(p) = \frac{p_i + \max(z_i, 0)}{1 + \Sigma \max(z_j, 0)} \qquad (i = 1,\ldots,n) \qquad (1)$$

where p_i and z_i are the price and the excess demand of commodity i respectively. The mapping transforms points in the unit simplex P_n into price vectors **p** contained in the unit simplex. Each element of the unit simplex P_n is a normalized vector of prices such that $\Sigma_i p_i = 1$. Homogeneity of degree 0 of the excess demand and supply functions in all prices allows to limit the search of equilibrium price vectors to the unit simplex of $\mathbb{R}_n$.

Once we have eliminated the confusion between existence and stability, the main point is that in the context of a competitive economy, the law of supply and demand states that when a commodity has a positive (resp. negative) excess demand it is necessary to increase (resp. reduce) its price. To determine if mapping (1) satisfies the law of supply and demand, we will examine successively the following three cases: $z_i > 0$, $z_i < 0$ y $z_i = 0$.

a) **Positive Excess Demand**

In the case of $z_i > 0$, the fictitious auctioneer must increase

the price of commodity i. This implies $\Theta_i(\mathbf{p}) > p_i$ and, in turn, according with mapping (1) this means that we must have

$$z_i > p_i \cdot z_i + p_i \cdot \Sigma_j \max (z_j, 0). \qquad (2)$$

In this case, because $p_i < 1$, then $z_i \cdot p_i < z_i$. Condition (2) is verified if for all other commodities $j \neq i$ excess demands are negative or null. If one commodity $j \neq i$ has a positive excess demand, condition (2) may not be satisfied and $\Theta_i(\mathbf{p})$ may not be consistent with the law of supply and demand.

b) **Negative Excess Demand**

If $z_i < 0$ the fictitious auctioneer must reduce the price of commodity i: $\Theta_i(p) < p_i$. Because $\max (z_i, 0) = 0$, this inequality implies

$$p_i < p_i + p_i \cdot \Sigma_j \max (z_j, 0). \qquad (3)$$

This condition is verified if there is at least one commodity $j \neq i$ with a positive excess demand, which is guaranteed by Walras' Law. In this case, the price adjustment rule expressed by the mapping $\Theta_i(\mathbf{p})$ is the law of supply and demand.

c) **Zero Excess Demand**

When $z_i = 0$ the auctioneer must not modify price p_i, thus $\Theta_i(\mathbf{p}) = p_i$. But once again, we have problems to interpret mapping Θ as consistent with the law of supply and demand. What are the conditions under which this equality is verified? Because $\max (z_i, 0) = 0$, we have

$$p_i = p_i + p_i \cdot \Sigma_j \max (z_j, 0). \qquad (4)$$

This condition is verified if the second term in the right hand side is zero, and this is the case when for all $j \neq i$, $z_j \leq 0$. Because of Walras' Law, this is not possible except in general equilibrium. Outside of equilibrium points, there exists at least one commodity $j \neq i$ with positive excess demand. The price adjustment rule established in mapping Θ makes the auctioneer reduce price p_i. This is in contradiction with the law of supply and demand.

II.2 The Arrow-Hahn Mapping

As we have already seen, Arrow and Hahn (1971) use the following mapping:

$$\mathbf{T(p)} = \frac{\mathbf{p} + \mathbf{M(p)}}{[\mathbf{p} + \mathbf{M(p)}]\mathbf{e}} \tag{5}$$

where **e** is the n-dimensional unit vector. For the i-th component the mapping is

$$T_i(\mathbf{p}) = \frac{p_i + \max(-p_i, z_i(\mathbf{p})}{1 + \Sigma_j \max(-p_j, z_j(\mathbf{p}))} .$$

Although it may be a bit monotonous, an analysis similar to the previous one is required.

a) **Positive Excess Demand**

The price p_i must rise, that is $T_i(\mathbf{p}) > p_i$. This can be expressed as follows:

$$p_i + z_i(p) > p_i\,[1 + \Sigma_j \max(-p_j, z_j(p))]$$

$$z_i(p) > p_i\, \Sigma_j \max(-p_j, z_j(p))$$

$$z_i(p) > p_i \cdot z_i(p) + p_i \cdot \Sigma_{j \neq i} \max(-p_j, z_j(p))] \tag{6}$$

If there exists a commodity $j \neq i$ with a positive excess demand, condition (6) is verified only if the value of $z_i(\mathbf{p})$ is sufficiently large to prevail over the positive value of $z_j(\mathbf{p})$. The price variation rule imposed by mapping $T(\mathbf{p})$ does not respect the law of supply and demand.

b) **Negative Excess Demand**

Price p_i must decrease, that is $T_i(\mathbf{p}) < p_i$. Hence,

$$p_i + \max(-p_i, z_i(p)) < p_i\,[1 + \Sigma_j \max(-p_j, z_j(p))]$$

$$\max(-p_i, z_i(p)) < p_i \cdot \Sigma_j \max(-p_j, z_j(p)). \tag{7}$$

Obviously, the possibility of reducing the price of commodity i depends on the absolute values of p_i, $z_i(\mathbf{p})$, p_j and $z_j(\mathbf{p})$. Thus, condition (7) may not be verified. According to the values of these variables, we can obtain $T_i(\mathbf{p}) > p_i$; this means that, in spite of the excess supply for commodity i, the price imposed by $T_i(\mathbf{p})$ may increase.

c) **Zero Excess Demand**

When $z_i(\mathbf{p}) = 0$, we should have $T_i(\mathbf{p}) = p_i$. Thus,

$$p_i + \max(-p_i, z_i(p)) = p_i\,[1 + \Sigma_j \max(-p_j, z_j(p))]$$

$$p_i = p_i + p_i \cdot \Sigma_j \max(-p_j, z_j(p))]. \tag{8}$$

Equality (8) is verified only if $z_i(\mathbf{p}) = 0$ for all commodities $j \neq i$, i.e. when we are in general equilibrium. Outside of this point mapping $T_i(\mathbf{p})$ does not obey the law of supply and demand.

II.3 Debreu's Approach

Arrow-Debreu (1954) and more clearly Debreu (1959) consider a price vector $\mathbf{p}$ in the unit simplex $P = \{p \in \mathbb{R}_n^+ \mid p \geq 0, \Sigma_i p_i = 1\}$, and the set of possible excess demands Z. Then $\zeta(p) = \xi(p) - \eta(p) - \{\omega\}$ (where $\xi(p)$ is the aggregate demand correspondence, $\eta(p)$ the correspondence of aggregate supply and $\{\omega\}$ the vector of initial endowments of the economy) associates to each price vector $\mathbf{p} \in P$ a vector $\mathbf{z} \in Z$. A new correspondence $\mu(z)$ then associates to z a vector of prices within P such that $\mathbf{p}\cdot\mathbf{z}$ is maximized:

$$\mu(z) = \{p \in P \mid \mathbf{p}\cdot\mathbf{z} = \mathrm{Max}\ P\cdot\mathbf{z}\}.$$

Debreu then defines a new correspondence ψ of set $P \times Z$ on itself $\psi(\mathbf{p},\mathbf{z}) = \mu(\mathbf{z}) \times \zeta(\mathbf{p})$. This mapping $\psi(z,p)$ implies that to each vector $\mathbf{z}$ a price vector $\mathbf{p}$ is associated in order to maximize $\mathbf{p}\cdot\mathbf{z}$. This is what

Debreu (1959: 83) calls "the central idea in the proof" which is then described in the following terms: "Let H be the set of commodities for which the component of **z** is the greatest. Maximizing $\mathbf{p}\cdot\mathbf{z}$ on P amounts to taking $\mathbf{p} \geq 0$ such that $p_h = 0$ if $h \notin H$, and $\Sigma_{h\in H}p_h = 1$".

The price adjustment rule is the following: the commodity with the highest excess demand in vector **z** is chosen, say k such that $z_k \geq z_i$, $\forall z_i \in Z$, $i \neq h$. The new price vector resulting from correspondence $\mu(\mathbf{p})$ has all of its components $p_{i\neq k} = 0$ and component $p_k = 1$ (because no linear combination of the price vector and the excess demand vector results in a higher value than $p_k \cdot z_k$). That is to say, outside of the fixed point, the prices of commodities with positive excess demands (at positive prices) inferior to the largest excess demand are reduced to zero. Their prices are brought to zero for the simple reason that their excess demand is not superior to the other excess demands.

What is the justification of this strange price adjustment rule according to which the price-setting agency chooses the new price vector **p** in P so as to make "the excess demand as expensive as possible"? (Debreu, 1983:219) According to Debreu (1982:708) the economic interpretation of this mapping is quite clear, which may explain his allegiance to this mapping over the years: "the maximization with respect to **p** of this [excess demand] function agrees with a commonly held view of the way in which prices perform their market-equilibrating role by making commodities with positive excess demand more expensive and commodities with negative excess demand less expensive, thereby increasing the value of excess demand".

Two different points are mixed up here. The first is related to the law of supply and demand, that is, the price variation rule as a function of excess demand. The second one invokes the effects which common opinion ascribes to this variation, mainly the "market equilibrating role" of prices. This last assertion concerns the stability of equilibrium and, as such, has nothing to do with the existence question. We shall examine the two assertions separately.

1. Let **p** be the price vector, **z** the vector of excess demands calculated at these prices and **p'** the new price vector resulting from the law of supply and demand. Necessarily we have $\mathbf{p'}\cdot\mathbf{z} > \mathbf{p}\cdot\mathbf{z}$: the consequence of this law is that, outside the fixed point, the aggregate value of excess demand must increase. But the economic meaning of this result stems from the same reason advanced by Debreu: the increase (resp. decrease) of the prices of commodities with positive (resp. negative) excess demand. Thus, contrary to Debreu's assertion, the value of $\mathbf{p}\cdot\mathbf{z}$ cannot be a maximum without contradicting the law of supply and demand. This is self evident: to reach this maximum, the prices of commodities with excess demands which are, both positive and inferior to the largest, must be reduced to zero; in the case several commodities have the same largest excess demand, all of their prices, except one, can be reduced to zero, reserving $p = 1$ for the exception.[6] There is here a brazen contradiction with the law of supply and demand.[7]

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[6] "[T]otal prices must add up to one, but this total is to be distributed only over those commodities with maximum excess demand" (Arrow, 1972:219). (Our emphasis). The mapping used in Arrow and Debreu (1954) and Debreu (1959) finds its origins in the hypotheses of the Maximum Theorem. According to Takayama (1988:254), although Debreu used the maximum theorem in his Theory of Value (1959) in order to establish the upper semicontinuity of the demand and supply functions, no explicit mention of the literature on the theorem (in particular, the seminal work of C. Berge) was made by him. Debreu (1982) does make an explicit reference to Berge's maximum theorem. This theorem can be used to prove the upper semicontinuity of multivalued correspondences (Klein, 1973) and it is thus employed to establish this property for the supply and demand correspondences. Although the correspondence max $\mathbf{p}\cdot\mathbf{x}$ does exhibits this property, the difficulty is that in order to ensure the property of upper semicontinuity, the proof relies on a correspondence lacking a reasonable economic meaning. The predicament here is that the property of upper semicontinuity is guaranteed at the cost of rendering the correspondence incompatible with the law of supply and demand.

[7] In Arrow and Debreu (1954:275) a "market participant" with a price-setting role is introduced. This agent, rebaptized by Debreu (1982:134) as the "fictitious price-setting agent" and endowed with a "utility function" which "is specified to be $\mathbf{p}\cdot\mathbf{z}$", chooses a price vector **p** in P for a given **z** and "receives $\mathbf{p}\cdot\mathbf{z}$". As we have seen, this new price vector **p** maximizes $\mathbf{p}\cdot\mathbf{z}$, which implies, outside the fixed point,

These considerations should help explain Arrow's (1972:219) reservations ("this rule is somewhat artificial") and later, Debreu's (1987:134) previously quoted assertion: "Maximizing the function $p \rightarrow p.z$ over P carries to one extreme the idea that the price-setter should choose high prices for the commodities that are in excess demand, and low prices for the commodities that are in excess supply". But these calls for caution are hopeless: the mapping which maximizes $\mathbf{p} \cdot \mathbf{z}$ is <u>totally</u> artificial, and it does not carry to one extreme the law of supply and demand, but utterly <u>contradicts</u> it.[8]

2. The impossibility of interpreting the mapping in a manner consistent with the law of supply and demand may also help to explain the futile attempt to rely on common sense prejudice ("the commonly held view") in relation to the stability of competitive equilibrium. As we observed before this attempt was introduced with the first appearance of the mapping in Nikaido (1956) and later, the literature systematically followed the explanation advanced by this author.

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that all prices are zero except the price of the commodity with the largest excess demand. Arrow and Debreu (1954:274-5) continue: "Suppose the market participant does not maximize instantaneously but, taking other participants' choices as given, adjusts his choice of prices so as to increase his pay-off. For given **z**, **pz** is a linear function of **p**; it can be increased by increasing p_h for those commodities for which $z_h > 0$, decreasing $z_h < 0$ (provided p_h is not already zero). <u>But this is precisely the classical 'law of supply and demand'</u>, and so the motivation of the market participant corresponds to one of the elements of the competitive equilibrium" (our emphasis). This behavior, which is totally artificial, reinforces our conclusion. Instead of abruptly contradicting the law of supply and demand, the contradiction is obtained gradually. In this case, the law holds as long as the market participant does not maximize his utility function, and ceases to hold when this agent at last behaves according to the rationality which is assigned to him.

[8] Nikaido (1968:267) also presents this type of correspondence as an alternative way to ascertain the existence of a competitive equilibrium. Correspondence η yields equilibrium solutions for the excess-supply correspondence χ as fixed points of mapping

$$(\mathbf{u},\mathbf{p}) \longrightarrow \chi(\mathbf{p}) \times \eta(\mathbf{u}): \Gamma \times P_n \longrightarrow 2^{\Gamma \times P_n}$$

where **u** represents the vector of excess supplies, and

$$\eta(\mathbf{u}) = \{r \mid \text{minimizes } <\mathbf{u},\mathbf{q}> \text{ for all } \mathbf{q} \in P_n\}.$$

Our remarks on the Arrow-Debreu mapping apply *mutatis mutandis* to this approach to the proof of existence of GCE.

We want to stress once more that it is not possible to rest on arguments relative to stability in order to find an economic justification to the mapping used in the proof of existence of equilibrium. This line of reasoning is even more astonishing if we consider that these claims are present in the later works of Debreu (see for example, Debreu 1982). If in the 1950's it was still conceivable to expect satisfactory results in stability theory, these expectations were shattered in the 1970's due to the work of Sonnenschein (1972), Mantel (1974) and Debreu himself (1974) who contributed in a resolute manner to demonstrate that the "commonly held view" on the "market equilibrating role" of prices in the Arrow-Debreu model is utterly unjustified. In other terms, it is upon the *weakest* result of general equilibrium theory, that Debreu aspires to justify the economic interpretation of the proof of existence of equilibrium, considered to be one of the *strongest* results of the last forty years.[9]

III. THE SPECIAL CASE OF A TWO-COMMODITY ECONOMY

In the special case of a two-commodity economy the correspondences that we have examined are consistent with the law of supply and demand. Their analysis reveals the limits of the proof of existence as it exposes the restrictive conditions within which the fixed point can be rightfully interpreted as an economic equilibrium.

Consider a two-commodity economy with p_1, p_2 and z_1, z_2, the prices and excess demands of commodities 1 and 2 respectively, and suppose all customary conditions for the existence of equilibrium are verified. By virtue of the Law of Walras, $\mathbf{p} \cdot \mathbf{z} = \mathbf{0}$, and thus $z_1 \cdot z_2 < 0$.

[9] As we explain in Section I, consistency with the law of supply and demand is required *both* in the proof of existence *and* stability of equilibrium. Arrow and Hahn (1971:304) state this unambiguously when they underline that the price adjustment rule adopted in stability analysis using Newton's method "is not a process that mimicks the invisible hand". The reason for this is that "the price of a good may be raised even though it is in excess supply", and this means that the rule of price adjustment contradicts the law of supply and demand.

1. Consider Nikaido's correspondence:

$$\Theta_i(p) = \frac{p_i + \max(z_i, 0)}{1 + \Sigma_j \max(z_j, 0)}$$

with $z_1 > 0$. Because $p_1 z_2 < 0$ we have $z_1 > p_1 z_1 + p_1 z_2$. If $z_1 < 0$, we have $p_1 z_2 > 0$. These inequalities are necessarily verified: the price of commodity 1 increases in the first case and decreases in the second in accordance with the law of supply and demand.

2. We arrive at the same conclusion considering the correspondence of Arrow-Hahn:

$$T_i(p) = \frac{p_i + \max(-p_i, z_i(p))}{1 + \Sigma_j \max(-p_j, z_j(p))}$$

Suppose $z_1 > 0$. According to the law of supply and demand p_1 must rise. Because $p_1 < 1$, we have $(1 - p_1)z_1 > 0$. Since $z_2 < 0$, $p_1[(\max(-p_2, z_2)] < 0$, thus $(1 - p_1)z_1 > p_1[(\max(-p_2, z_2)]$. The conditions for increasing p_1 are satisfied.

Consider now $z_1 < 0$. Then $(1 - p_1)z_1 < 0$, $z_2 > 0$ and $\max(-p_2, z_2) = z_2$. Thus, $p_1(\max(-p_2, z_2)) > 0$ and $(1 - p_1)z_1 < p_1(\max(-p_2, z_2)$. The conditions for the reduction of p_1 are verified.

3. Finally, the price adjustment rule imposed by the Arrow-Debreu mapping which maximizes the value of $\mathbf{p} \cdot \mathbf{z}$ yields the following result. If $z_1 > 0$, we have $z_2 < 0$ and p_1 is increased until it is equalled to 1. If $z_1 < 0$, p_1 is reduced until it becomes 0. Thus, it is only in the case of a two-commodity economy that it is possible to admit Debreu's assertion that this correspondence "carries to one extreme" the price variation rule dictated by the law of supply and demand.

IV. SYNTHESIS OF RESULTS

Following Arrow and Debreu (1954: 271) the "classical law of supply and demand" can be stated expressed in terms of necessary and sufficiency conditions:

i) when $z_i > 0$ (resp. $z_i < 0$, $z_i = 0$), p_i increases (resp. decreases, remains unchanged): necessary condition;

ii) when p_i increases (resp. decreases, remains unchanged) we have $z_i > 0$ (resp. $z_i < 0$, $z_i = 0$): sufficiency condition.

Our analysis shows that the correspondences used in the proof of existence are not consistent with the law of supply and demand. A synthesis of the results follows.

1. $z_i > 0$

   a) $z_i > 0 \Rightarrow p_i$ increases

   b) p_i increases $\Rightarrow z_i > 0$

For correspondences $\Theta_i(p)$ and $T_i(p)$ statement a) is false and b) is true. Therefore, $z_i > 0$ is the necessary condition, but not sufficient, for the increment in p_i.

2. $z_i < 0$

   a) $z_i < 0 \Rightarrow p_i$ decreases

   b) p_i decreases $\Rightarrow z_i < 0$

For correspondence Θ_i statement a) is true, but statement b) is false. Thus, $z_i<0$ is the sufficient condition, but not the necessary condition for the reduction of p_i.

For correspondence $T_i(\mathbf{p})$ both statements are false: $z_i < 0$ is neither the sufficient, nor the necessary condition for the reduction of p_i.

3. $z_i = 0$

   a) $z_i = 0 \Rightarrow p_i = \Theta_i(p)$

b) $p_i = \Theta_i(p) \Rightarrow z_i = 0$

For correspondence $\Theta_i(p)$, a) is false, but b) is true. Thus, we have that $z_i = 0$ is a sufficient, but not a necessary condition for $p_i = 0$.

For correspondence $T_i(\mathbf{p})$, a) and b) are both false. Thus, $z_i = 0$ is neither the necessary, nor the sufficient condition for $T_i(\mathbf{p}) = p_i$.[10]

V. CONCLUSION

If we had a theory which explained, through a dynamic price adjustment process consistent with the law of supply and demand, how an equilibrium is attained which coincides with the fixed point, then we would be able to interpret the fixed point as an economic equilibrium. The only candidate to perform this task is the theory of the Walrasian *tâtonnement*. However, this theory embodies a dynamic process which, while being consonant with the law of supply and demand, is nonetheless generally <u>unstable</u>.

Thus, the state of orthodox value theory is very unsatisfactory. It has not been able to furnish an economically coherent explanation of both, the formation and the existence of equilibrium prices. More accurately, <u>in both cases</u> the results are obtained under conditions which are devoid of economic meaning. In the first case, these conditions are related to the well-known hypotheses which guarantee the stability

[10] If we consider relative prices of the form p_i/p_j, then
a) $z_i > 0$ and $z_j < 0 \Rightarrow p_i/p_j$ increases;
b) p_i/p_j increases $\Rightarrow z_i > 0$ and $z_j < 0$
whichever correspondence is considered, $\theta(p)$ or $T(p)$, a) is true and b) is false. Thus, $z_i > 0$ and $z_j < 0$ is the sufficient condition, but not the necessary condition for the increase of p_i/pj. The same conclusion applies in the opposite case ($z_i < 0$, $z_j > 0$). Evidently, the comparison of "relative prices" does not furnish indications about the state of supplies and demands which, through these correspondences, have generated the price variation. The only thing it reveals is that if, for example, $\theta_i(p)/\theta_j(p) > p_i/p_j$ then $z_i > z_j$. But these excess demands can be both positive, negative or of positive sign.

of the *tâtonnement*. In the second instance, in flagrant contradiction with what is commonly affirmed, this stems from the need to use a correspondence which is incompatible with the law of supply and demand in order to prove the existence of the equilibrium between supply and demand. There is no disagreement regarding the first case. We hope to contribute to clarify the second case.

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