Centro de Estudios Económicos
www.colmex.mx
El Colegio de México, A.C.

Serie documentos de trabajo

ON THE CORRECT USE OF OMNIBUS TESTS FOR NORMALITY

Carlos Manuel Urzúa Macias

DOCUMENTO DE TRABAJO

Núm. VIII - 1995

ON THE CORRECT USE OF OMNIBUS TESTS FOR NORMALITY

Carlos M. Urzúa

El Colegio de México*

This version: May, 1995

ABSTRACT

This paper warns about the incorrect use of the popular Jarque-Bera test for normality of residuals in the case of small and medium-size samples. It also provides a natural modification of the test that mitigates the problem.

Keywords: Normality test; Omnibus test

JEL classification: C10, C20

*Address correspondence to: Centro de Estudios Económicos
El Colegio de México
Camino al Ajusco No. 20
México, D.F. 01000
MEXICO
Fax (52-5) 645-0464
Tel (52-5) 645-5955 ext. 4171
e-mail curzua@colmex.mx

1. INTRODUCTION

The test for univariate normality of observations and residuals introduced by Jarque and Bera (1980, 1987) has gained great acceptance among economists. It is an omnibus test based on the standardized third and fourth moments:

$$LM = n[(\sqrt{b_1})^2/6 + (b_2-3)^2/24] \tag{1}$$

where n is the number of observations, $\sqrt{b_1} = m_3/m_2^{3/2}$, $b_2 = m_4/m_2^2$, and m_i is the i-th central moment of the observations [i.e., $m_i = \Sigma(x_j-\bar{x})^i/n$]. Asymptotically, the hypothesis of normality is rejected at some significance level if the value of LM exceeds the critical value of a chi-squared with two degrees of freedom. In the more usual case of a regression, (1) is calculated using the estimated residuals.

As shown by Jarque and Bera (1987), the test performs quite well compared to others available in the literature. This is not surprising since they proved that, if the alternatives to the normal distribution are in the Pearson family, LM is the corresponding Lagrange multiplier test for normality. Urzúa (1989) also showed the same when the alternatives are the maximum-entropy ("most likely") distributions with finite moments defined in Urzúa (1988).

However, the good performance of the test is highly dependent on the use, through a Montecarlo simulation, of empirical significance points (something, by the way, that is almost never done in studies where (1) is used). This is so because of the slow convergence in distribution to the chi-squared.

Interestingly enough, (1) has been known among statisticians since the work of Bowman and Shenton (1975). They derived it after noting that, under normality, the asymptotic means of $\sqrt{b_1}$ and b_2 are 0 and 3, the asymptotic variances are $6/n$ and $24/n$, and the asymptotic covariance is zero. Thus, LM is just the sum of squares of two asymptotically independent standardized normals.

Yet, there are few (if any) instances in the statistics literature where the Bowman-Shenton-Jarque-Bera test has been used. As one author flatly states in a comprehensive survey of tests for normality: "Due to the slow convergence of b_2 to normality this test is not useful." (D'Agostino, 1986, p. 391).

2. A NEW, BETTER-BEHAVED TEST STATISTIC

This section presents a new, better-behaved omnibus test for normality that is a natural extension of the Jarque-Bera test. The idea is straightforward: instead of the asymptotic means and variances of the standardized third and fourth moments, use their *exact* means and variances. Under normality, the latter can be easily computed using results already known to Fisher (1930).

Fisher's results were stated in terms of the so-called k-statistics, which can be expressed in terms of moments as (see Stuart and Ord, 1987, pp. 392, 422):

$$k_2 = nm_2/(n-1)\,,\ k_3 = n^2m_3/(n-1)(n-2)\,,\ k_4 = n^2[(n+1)m_4-3(n-1)m_2^2]/(n-1)(n-2)(n-3)$$

For our purposes, his relevant derivations are that, under normality, k_2 is independent of $k_p/k_2^{p/2}$ for p=3,4,..., and that

$$var(k_3/k_2^{3/2}) = 6n(n-1)/(n-2)(n+1)(n+3)\,,\ var(k_4/k_2^2) = 24n(n-1)^2/(n-3)(n-2)(n+3)(n+5)$$

But then we can use those results to easily show that, under normality, the exact mean and variance of the standardized third and fourth moments are

$$E(\sqrt{b_1}) = 0,\quad var(\sqrt{b_1}) = 6(n-2)/(n+1)(n+3) \tag{2}$$

$$E(b_2) = 3(n-1)/(n+1),\quad var(b_2) = 24n(n-2)(n-3)/(n+1)^2(n+3)(n+5) \tag{3}$$

And hence, using (2) and (3), we can finally define the new test, to be called the *adjusted Lagrange multiplier* test for normality, as:

$$ALM = n[(\sqrt{b_1})^2/var(\sqrt{b_1}) + (b_2-E(b_2))^2/var(b_2)] \tag{4}$$

This new test statistic converges to the chi-squared with two degrees of freedom faster than the Jarque-Bera statistic, as can be glimpsed from the Montecarlo simulations reported in Table 1 (a more complete table is available upon request). Incidentally, the estimated significance points in that table can be used to test for normality of observations, but they *cannot* be used in the case of regression residuals, since, for each particular regression, the significance points depend on the design (regressor) matrix and the distribution of the residuals (see, e.g., Weisberg, 1980).

3. ESTIMATED POWER OF THE TESTS

This section compares the power of *ALM* and *LM* when used as tests for normality of regression residuals. The Montecarlo simulation procedures used by us were, on purpose, identical to the ones employed by White and MacDonald (1980) in their much quoted paper on the subject. As in there, the five alternatives to the normal distribution of the residuals were: Student's t with five degrees of freedom; heteroscedastic normal; chi-squared with two degrees of freedom; Laplace (double exponential); and lognormal (all of them standardized to have mean zero and variance 25). Furthermore, for the generation of pseudo-random numbers we followed in each case the same computational procedure as in their paper.

Also following White and MacDonald (1980), the design matrices for the regressions were constructed adding to a column of ones three columns of uniform random numbers with mean zero and variance 25. The number of rows in each design matrix (i.e., the sample size) was given by n = 20,35,50,100.

As a first exercise, we estimated the power of both tests when, as is incorrectly done in almost all empirical studies, the significance point is taken to be $\chi^2_{2,0.10} = 4.61$, even though the sample sizes are not large. The number of replications in each Montecarlo simulation was 10000 (instead of 200 in White and MacDonald, 1980), and the results are presented in Table 2.

As can be appreciated there, the results are overwhelmingly in favor of the new *ALM* test. It comes first in all the distributions considered and all the sample sizes. Furthermore, in the case of the smaller samples the power of *ALM* is significantly larger than the power of *LM*.

Naturally, the next question to ask is whether the same relative performance is obtained when, prior to applying the tests, "correct" significance points are found for each test using Montecarlo simulations (this is actually the procedure that is explicitly suggested in Jarque and Bera, 1987). The results obtained that way are presented in Table 3. Once again, *ALM* outperforms *LM*. Interestingly enough, the only five cases (out of 20) where the *LM* test comes first correspond to the distributions that are farther apart from the normal.

4. CONCLUDING REMARKS

This paper has presented a new omnibus test for normality of residuals and observations: the adjusted Lagrange multiplier test *ALM*. As shown here, the *ALM* test outperforms in terms of power the traditional Jarque-Bera *LM* test, both, when significance points are directly taken from a chi-squared, or when the "correct" significance points are obtained through simulations. Thus, the use of *ALM* over *LM* seems warranted in both circumstances.

As a final point, a similar adjustment to the one suggested here can be extended to the multivariate tests for normality that are also based on third and fourth standardized moments, such as the one proposed in Urzúa (1989).

REFERENCES

Bowman, K. O. and L. R. Shenton, 1975, Omnibus contours for departures from normality based on $\sqrt{b_1}$ and b_2, Biometrika 62, 243-250.

D'Agostino, R. B., 1986, Tests for the normal distribution, in: R. B. D'Agostino and M. A. Stephens, eds., Goodness of fit techniques (Marcel Dekker, New York), 367-419.

Fisher, R. A., 1930, The moments of the distribution for normal samples of measures of departure from normality, Proceedings of the Royal Statistical Society A 130, 16-28.

Jarque, C. M. and A. K. Bera, 1980, Efficient tests for normality, homoscedasticity and serial independence of regression residuals, Economics Letters 6, 255-259.

Jarque, C. M. and A. K. Bera, 1987, A test for normality of observations and regression residuals, International Statistical Review 55, 163-172.

Stuart, A. and J. K. Ord, 1987, Kendall's advanced theory of statistics, vol. 1 (New York: Oxford University Press).

Urzúa, C. M., 1988, A class of maximum-entropy multivariate distributions, Communications in Statistics, Theory and Methods 17, 4039-4057.

Urzúa, C. M., 1989, Tests for multivariate normality of observations and residuals, Documento de trabajo No. III-89, Centro de Estudios Económicos, El Colegio de México, Mexico City. Presented at the IX Latin American Meeting of the Econometric Society held in Chile, 1989.

Weisberg, S., 1980, Comment to some large-sample tests for nonnormality in the linear regression model, Journal of the American Statistical Association 75, 28-31.

White, H. and G. M. MacDonald, 1980, Some large-sample tests for nonnormality in the linear regression model, Journal of the American Statistical Association 75, 16-28.

Table 1

Significance points for two tests for normality of observations

n:	20	50	100	200	400	800	∞
ALM							
α=.10	3.95	4.00	4.12	4.30	4.39	4.47	4.61
α=.05	7.01	6.60	6.29	6.17	6.04	5.97	5.99
LM							
α=.10	2.13	2.90	3.14	3.48	3.76	4.32	4.61
α=.05	3.26	4.26	4.29	4.43	4.74	5.46	5.99

Sources: For *LM* Jarque and Bera (1987, table 2), and for *ALM* own simulations using 10000 replications.

Table 2

Tests for normality of residuals; estimated power with 10000 replications, using as significance point $\chi^2_{2,0.10}$

n		t_5	Heteroskedastic Normal	χ^2_2	Laplace	Lognormal
20	*ALM*	0.231	0.091	0.493	0.290	0.808
	LM	0.140	0.039	0.380	0.181	0.727
35	*ALM*	0.362	0.116	0.829	0.464	0.985
	LM	0.293	0.077	0.782	0.374	0.978
50	*ALM*	0.467	0.128	0.963	0.595	1.000
	LM	0.406	0.093	0.950	0.513	0.999
100	*ALM*	0.694	0.161	1.000	0.835	1.000
	LM	0.658	0.135	1.000	0.797	1.000

Table 3

Tests for normality of residuals; estimated power with 10000 replications, using estimated significance points (α = .10)

n		t_5	Heteroskedastic Normal	χ^2_2	Laplace	Lognormal
20	*ALM*	0.254	0.477	0.533	0.317	0.831
	LM	0.247	0.456	0.586	0.306	0.856
35	*ALM*	0.391	0.724	0.864	0.494	0.989
	LM	0.376	0.697	0.896	0.470	0.992
50	*ALM*	0.493	0.852	0.973	0.624	1.000
	LM	0.474	0.831	0.982	0.595	1.000
100	*ALM*	0.712	0.984	1.000	0.849	1.000
	LM	0.698	0.981	1.000	0.836	1.000

SERIE DOCUMENTOS DE TRABAJO
Centro de Estudios Económicos
EL Colegio de México

Los siguientes documentos de trabajo de publicación reciente pueden solicitarse a:

The following working papers from recent year are still available upon request from:

Rocío Contreras,
Centro de Documentación, Centro de Estudios Económicos, El Colegio de México A.C.,
Camino al Ajusco # 20 C.P. 01000 México, D.F.

90/I Ize, Alain. *Trade liberalization, stabilization, and growth: some notes on the mexican experience.*

90/II Sandoval, Alfredo. *Construction of new monetary aggregates: the case of Mexico.*

90/III Fernández, Oscar. *Algunas notas sobre los modelos de Kalecki del ciclo económico.*

90/IV Sobarzo, Horacio. *A consolidated social accounting matrix for input-output analysis.*

90/V Urzúa, Carlos. *El déficit del sector público y la política fiscal en México, 1980 - 1989.*

90/VI Romero, José. *Desarrollos recientes en la teoría económica de la unión aduanera.*

90/VII García Rocha, Adalberto. *Note on mexican economic development and income distribution.*

90/VIII García Rocha, Adalberto. *Distributive effects of financial policies in Mexico.*

90/IX Mercado, Alfonso and Taeko Taniura. *The mexican automotive export growth: favorable factors, obstacles and policy requirements.*

91/I Urzúa, Carlos. *Resuelve: a Gauss program to solve applied equilibrium and disequilibrium models.*

91/II Sobarzo, Horacio. *A general equilibrium analysis of the gains from trade for the mexican economy of a North American free trade agreement.*

91/III Young, Leslie and José Romero. *A dynamic dual model of the North American free trade agreement.*

91/IV Yúnez, Antonio. *Hacia un tratado de libre comercio norteamericano; efectos en los sectores agropecuarios y alimenticios de México.*

91/V Esquivel, Gerardo. *Comercio intraindustrial México - Estados Unidos.*

91/VI Márquez, Graciela. *Concentración y estrategias de crecimiento industrial.*

92/I Twomey, Michael. *Macroeconomic effects of trade liberalization in Canada and Mexico.*

92/II Twomey, J. Michael. *Multinational corporations in North America: Free trade intersections.*

92/III Izaguirre Navarro, Felipe A. *Un estudio empírico sobre solvencia del sector público: El caso de México.*

92/IV Gollás, Manuel y Oscar Fernández. *El subempleo sectorial en México.*

92/V Calderón, Angel. *The dynamics of real exchange rate and financial assets of privately financed current account deficits.*

92/VI Esquivel, Gerardo. *Política comercial bajo competencia imperfecta: Ejercicio de simulación para la industria cervecera mexicana.*

93/I Fernández, Jorge. *Debt and incentives in a dynamic context.*

93/II Fernández, Jorge. *Voluntary debt reduction under asymmetric information.*

93/III Castañeda, Alejandro. *Capital accumulation games.*

93/IV Castañeda, Alejandro. *Market structure and innovation a survey of patent races.*

93/V Sempere, Jaime. *Limits to the third theorem of welfare economics.*

93/VI Sempere, Jaime. *Potential gains from market integration with individual non-convexities.*

93/VII Castañeda, Alejandro. *Dynamic price competition in inflationary environments with fixed costs of adjustment.*

93/VIII Sempere, Jaime. *On the limits to income redistribution with poll subsidies and commodity taxation.*

93/IX Sempere, Jaime. *Potential gains from integration of incomplete markets.*

93/X Urzúa, Carlos. *Tax reform and macroeconomic policy in Mexico.*

93/XI Calderón, Angel. *A stock-flow dynamic analysis of the response of current account deficits and GDP to fiscal shocks.*

93/XII Calderón, Angel. *Ahorro privado y riqueza financiera neta de los particulares y de las empresas en México.*

93/XIII Calderón, Angel. *Política fiscal en México.*

93/XIV Calderón, Angel. *Long-run effects of fiscal policy on the real levels of exchange rate and GDP.*

93/XV Castañeda, Alejandro. *On the invariance of market innovation to the number of firms. The role of the timing of innovation.*

93/XVI Romero, José y Antonio Yúnez. *Cambios en la política de subsidios: sus efectos sobre el sector agropecuario.*

94/I Székely, Miguel. *Cambios en la pobreza y la desigualdad en México durante el proceso de ajuste y estabilización.*

94/II Calderón, Angel. *Fiscal policy, private savings and current account deficits in Mexico.*

94/III Sobarzo, Horacio. *Interactions between trade and tax reform in Mexico: Some general equilibrium results.*

94/IV Urzúa, Carlos. *An appraisal of recent tax reforms in Mexico.* (Corrected and enlarged version of DT. núm. X-1993)

94/V Sáez, Raúl and Carlos Urzúa. *Privatization and fiscal reform in Eastern Europe: Some lessons from Latin America.*

94/VI Feliz, Raúl. *Terms of trade and labour supply: A revision of the Laursen-Metzler effect.*

94/VII Feliz, Raúl and John Welch. *Cointegration and tests of a classical model of inflation in Argentina, Bolivia, Brazil, Mexico, and Peru.*

94/VIII Sempere, Jaime. *Condiciones para obtener ganancias potenciales de liberalización de comercio.*

94/IX Sempere, Jaime y Horacio Sobarzo. *La descentralización fiscal en México: Algunas propuestas.*

94/X Sempere, Jaime. *Are potential gains from economic integration possible with migration?.*

94/XI Gollás, Manuel. *México 1994. Una economía sin inflación, sin igualdad y sin crecimiento.*

95/I Schettino, Macario. *Crecimiento económico y distribución del ingreso.*

95/II Schettino, Macario. *A function for the Lorenz curve.*

95/III Székely, Miguel. *Economic Liberalization, Poverty and Income Distribution in Mexico.*

95/IV Taylor, Edward y Antonio Yúnez. *Impactos de las reformas económicas en el agro mexicano: Un enfoque de equilibrio general aplicado a una población campesina.*

95/V Schettino, Macario. *Intuition and Institutions: The Bounded Society.*

95/VI Bladt, Mogens. *Applied Time Series Analysis.*

95/VII Yúnez Naude, Antonio y Fernando Barceinas. *Modernización y el mantenimiento de la biodiversidad genética en el cultivo del maíz en México.*

95/VIII Urzúa, Carlos M. *On the Correct Use of Omnibus Tests for Normality.*