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A FUNCTION FOR THE LORENZ CURVE

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This paper presents a function for approximating the Lorenz curve. The function fits adequately the curve and has an interesting intuitive explanation. Only three parameters are used and they can be related to specific aspects of the income distribution.

JEL Classification number **D31** (Personal Income and Wealth Distribution)

A FUNCTION FOR THE LORENZ CURVE

I. Introduction

Approximating the Lorenz curve has been a major issue in empirical work on income distribution. Since Pareto [1897], a considerable amount of effort has been spent in finding a function that approximates this curve (for example, Aitchinson and Brown [1957], Champernowne [1953]). In particular Basmann, Hayes, Slottje and Johnson [1990] and Basmann, Hayes and Slottje [1991] present a function that approximates the Lorenz curve and can also be interpreted in terms of the *perceived* difficulty of moving up on the income distribution.

This note presents a different function that has also these properties and makes a somewhat better approximation to the curve. Besides, this function has an interesting intuitive explanation and may be easily incorporated into a dynamic growth model.

II. The Lorenz curve

We define the Lorenz curve function as

$$L(z) = z^{\alpha}\left(\frac{\gamma - z}{\gamma - 1}\right)^{\beta} \tag{1}$$

where $\alpha \geq 1$, $\beta \leq 0$ and $\gamma > 1$. The function maps the percentage of population z to the cumulative income perceived, $L(z)$. This function (which will be called the "gamma function" hereafter) has the following derivative with respect to z:

$$\frac{\partial L(z)}{\partial z} = L(z)\left[\frac{\alpha}{z} - \frac{\beta}{\gamma - z}\right] \tag{2}$$

The interesting theoretical properties come from the intuition behind the parameters. α is the major determinant of the shape of the lower portion of teh curve, when $z \to 0$, while β and γ are more influential in the upper part of the curve, that is, when $z \to 1$. The derivative of L(z) with respect to α is

$$\frac{\partial L(z)}{\partial \alpha} = L(z)\ln(z) \leq 0 \tag{3}$$

which means that an increase in α will reduce the income share received by all agents (except for $z=1$) since (5) is non-positive, but with a greater impact (in absolute terms) on the middle part of the distribution. Nevertheless, the relative effect of a change in α will be more important in the lower part of the curve, declining around $z>0.3$. β and γ are the major determinants of the upper portion of the distribution. The derivatives with respect to β and γ are

$$\frac{\partial L(z)}{\partial \beta} = L(z)\ln\left(\frac{\gamma - z}{\gamma - 1}\right) \geq 0 \tag{4}$$

$$\frac{\partial L(z)}{\partial \gamma} = L(z)\left(\frac{\beta}{\gamma - z}\right)\left(\frac{z - 1}{\gamma - 1}\right) \geq 0 \tag{5}$$

which are always positive and monotonically increasing. An increase in β ($\beta \to 0$, since $\beta \leq 0$) will increase the income share for all agents except for $z=1$; that is, income distribution becomes more egalitarian. An increase in γ produces the same result, a more egalitarian distribution of income.

III. Empirical Results

The function L(z) is non-linear. A very important challenge for empirical analysis of income distribution data is that we normally have only five, or at most ten data points to estimate the function. It is very difficult to have initialization values good enough to get meaningful non-linear regression results using only ten data points. However, an interesting empirical property of the function L(z) is that using $\gamma=1$ and then estimating a linearized version of L(z), gives us these initialization values. We can apply an interative method using linear regressions to obtain the non-linear parameters as follows:

1. Set $\gamma=1$.
2. Run linear regression to estimate

   $$\ln L(z) = \ln A + \alpha \ln z + \beta \ln(\gamma - z).$$

3. Set $\gamma_n = \exp\left(\frac{1}{\beta} \ln\left(\frac{1}{A} \right) \right) + 1$
4. If $|\gamma_n - \gamma| > \varepsilon$ then $\gamma = \gamma_n$, goto 2. else finish.

In fact, α and β are very stable parameters. Linear regressions for a sample of thirty countries resulted in alphas between 1.5 and 1.8, and betas in the interval -0.1 to -0.2. Using these initialization values, we obtain very good non-linear regression results.

Using this function to estimate income distribution data gives us results which are comparable to those obtained by Basmann *et al.* [1990, 1991]. Table I presents the results using the gamma function with the data provided by Basmann *et al.* along with their results and the actual values of the Lorenz curve using U.S. income distribution data for 1977. As is clearly seen, the estimated gamma function lies closer to the real values (the table only presents ten points). The first two columns contain both estimates, the middle columns present the absolute residuals, and the last two columns the relative residuals. Fig. 1 presents a graphical comparison among the absolute residuals obtained by Basmann *et al.* and by using the gamma function. Fig. 2 shows the relative residuals of the two procedures for estimating gamma (linear and non-linear), along with the estimates using the Basmann *et al.* model.

IV. Conclusions

A function was presented which fits very closely the Lorenz curve, while having some attractive properties. This function was compared with the one presented by Basmann et al (1990, 1991) with some advantages. An additional interesting point of the gamma function is that the parameters may be related to socio-economic factors such as work and education. Preliminary results show that α and β will increase (decrease) with more education and higher workers' share. Further research will proceed in this direction.

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Table I. Estimation Results and Comparison with Basmann et al.

	Estimation			*Abs. Residuals*		*Rel. Residuals*	
z	*L(z)*	*Basmann*	*Gamma*	*Basmann*	*Gamma*	*Basmann*	*Gamma*
0.01	0.0004	0.0004	0.0004	0.0000	0.0000	0.00%	-8.04%
0.10	0.0180	0.0180	0.0162	0.0000	-0.0018	0.00%	-9.89%
0.20	0.0528	0.0537	0.0513	0.0009	-0.0015	1.70%	-2.91%
0.30	0.1015	0.1021	0.1013	0.0006	-0.0002	0.59%	-0.21%
0.40	0.1644	0.1631	0.1653	-0.0013	0.0009	-0.79%	0.57%
0.50	0.2424	0.2385	0.2435	-0.0039	0.0011	-1.61%	0.46%
0.60	0.3364	0.3317	0.3368	-0.0047	0.0004	-1.40%	0.12%
0.70	0.4481	0.4471	0.4475	-0.0010	-0.0006	-0.22%	-0.14%
0.80	0.5814	0.5910	0.5802	0.0096	-0.0012	1.65%	-0.21%
0.90	0.7459	0.7717	0.7464	0.0258	0.0005	3.46%	0.06%
0.99	0.9596	0.9746	0.9556	0.0150	-0.0040	1.56%	-0.42%

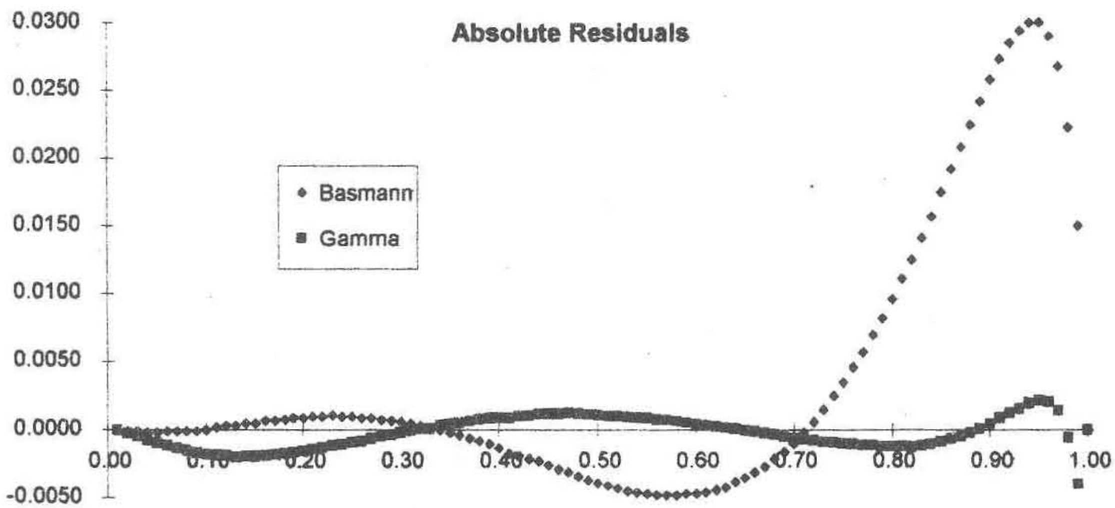

Fig. 1 Absolute Residuals

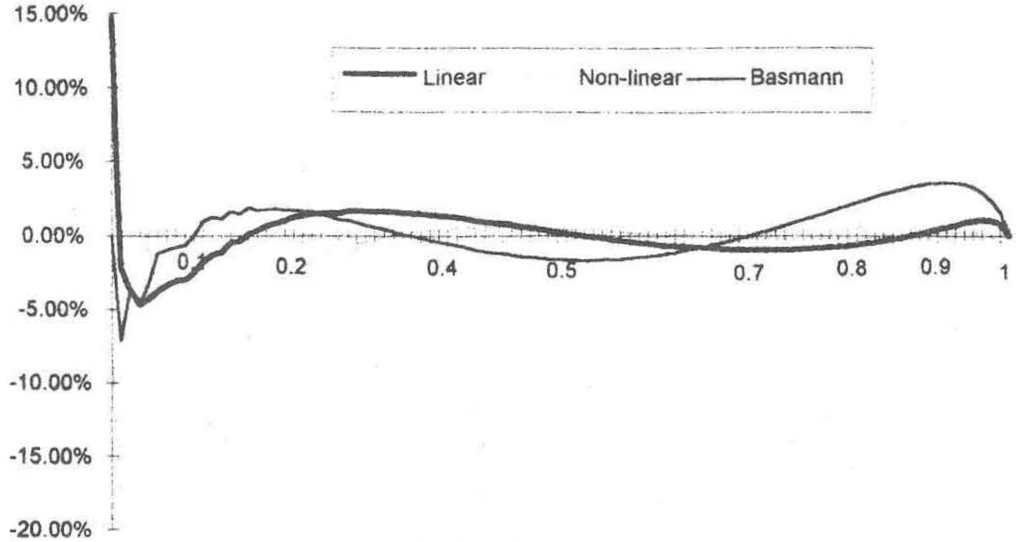

Fig. 2 Relative Residuals

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