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POTENTIAL GAINS FROM INTEGRATION OF INCOMPLETE MARKETS

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Potential Gains from Integration of Incomplete Markets

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Abstract

In a economy without a complete set of security markets, we analyse the possibility of obtaining potential Pareto gains when a supply-side policy reform is introduced. As a particular case, we consider the formation of a Customs Union in which a group of countries frees trade among them setting a common external tariff. In either case we construct a mechanism for distributing the gains from productive efficiency. This mechanism has the government fix consumer prices, after tax returns to assets and capital gains whereas producer prices and asset prices are kept free to clear markets. With incomplete markets public policy will affect the distribution of risks among the population so changing risk taking behavior. General conclusions about its effectiveness, in economies with a stock market, require conditions to be specified that imply the unanimity of national consumers in the evaluation of the reform together with the compensatory policies used. In these kind of economies potential gains are possible. Our results require however strong assumptions to make governments able to compensate the losers from the reform. It will be very difficult to achieve Pareto gains.

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1 Introduction

There is no doubt that allowing for the incompleteness of asset markets is one step toward the real world. Bounded rationality of agents impedes, for instance, forecasting all the possible states of the world. Incentive constraints create missing capital markets which cause the failure of many other asset markets. When output is not observable or is costly to monitor, the stock market does not work well. Therefore, the appropriate setting is incomplete markets theory rather than the Arrow-Debreu framework of complete state contingent commodity markets where a consumer makes simultaneously, at one moment in time, all consumption decisions for each state and moment in time.

Once one realizes that there are missing markets, the interesting properties that can be found in the Arrow-Debreu framework are rather hard to ensure. Hart (1975) and Bhattacharya (1987) provide examples where no equilibrium exists in economies with incomplete markets satisfying the other sufficient conditions for existence in the Arrow-Debreu world. They also have examples of Pareto ranked equilibria.

There have been two technical problems in showing existence of rational expectations equilibrium in economies with incomplete markets. The first comes from the fact that when asset markets are incomplete, consumers are consuming in a restricted set so that demands may not be proper (when the price of a commodity goes to zero, its demand may not go to infinity). Also, when the asset payoff matrix depends on prices (that is when real assets are considered), this dependence makes it change rank which leads to discontinuities in demand functions. These problems are sorted out in Cass (1984) with the "Cass trick" (the idea of the normalized no arbitrage equilibrium) which has been used widely after that, and by replacing contingent commodity markets by markets for assets promising delivery in fiat money (that is, contingent claims of units of account independent of spot prices). Geanakoplos and Polemarchakis (1986) show existence with *numéraire* securities. Probably the most important contribution to this literature on existence proofs is Duffie and Shafer's (1985, 1986) introduction of the Grassmanian manifold. They prove generic existence of equilibrium with incomplete real asset markets.

Concerning efficiency, Diamond (1980), Newbery and Stiglitz (1982), Stiglitz (1982), Greenwald and Stiglitz (1986), Geanakoplos and Polemarchakis (1986) present results to the effect that equilibrium is generically not even constrained efficient (that is, efficient relative to the existent market structure) when the market structure is incomplete. Welfare can be improved with the intervention of a policy-maker.

Grossman (1977) and Grossman and Hart (1979) have a different analysis based on the concept of Social Nash Optimality. The idea is that the policy-maker tries to achieve a Pareto Optimum, but is subject to the same coordination constraints as the rest of the agents in the economy in allocating resources across time and states of nature. It is as if there exists one policy-maker in each date and state of nature acting without coordination with other policy-makers in other dates and states. Each designs its own policy taking actions of the others as given. A Social Nash Optimum occurs when each policy-maker chooses on optimal policy given the actions of all the other policy-makers. They show that a stock market equilibrium without completeness is a Social Nash Optimum. This concept is extended in Repullo (1988) to cover all possible coordination schemes.

Geanakoplos and Polemarchakis (1990) have a different focus. In order to improve the market allocation the policy-maker needs to know the preferences of individuals. Recovering people's preferences in economies with incomplete markets requires knowledge of demands for assets and commodities, contemporaneous first derivatives of these functions, first derivatives of the demand for assets with respect to asset prices and revenue while commodity prices are fixed, first derivatives of spot commodity demand with respect to spot commodity prices, and first derivatives of asset demands with respect to future spot commodity prices. If the policy-maker does not have all this information, the market allocation is constrained Pareto efficient because there is no sure way to generate a Pareto improvement.

In economies with incomplete markets, through extensive use of lump-sum taxation a well informed government could allocate resources perfectly between different states of nature, thus substituting for asset markets and therefore producing a trivial efficiency gain in models where only open-loop strategies are used.[1] Therefore we will keep generalized state contingent

[1] but this would create other sorts of problems in models where feedback strategies are allowed, as shown in Hammond (1990).

lump-sum compensation out of the government's set of feasible instruments, restricting the use of this sort of taxation to the spot economy or using taxation so that the span of markets is not changed.

Recognizing that stock markets are incomplete, we analyse the welfare effects of the introduction of supply-side policies that increase aggregate real income. As a particular case, we study the formation of a customs union with commodity and asset market integration under uncertainty. With incomplete security markets, public policies also change the distribution of risks among the society, so changing aggregate risk-taking behavior. For the same reason that competitive equilibrium is not constrained optimal, changes in relative prices will change relative prices of risk affecting the final allocation. As shown in Newbery and Stiglitz (1984a), the latter could bring about some surprising effects and it is necessary to take this into account at the time of designing policies. At this stage some extra assumptions to make the evaluation of the reform unambiguous are needed. First there is a problem for defining what is meant by improvements in productive efficiency. Then the criterion we will use is that some consumers agree that present value of aggregate profits is greater than before. Second there is a problem evaluating the compensatory policies used. To make this unambiguous the compensatory policies should not create more insurance opportunities than those existing before the reform. Compensatory policies should not change the span of markets.

We construct a mechanism which makes possible a potential Pareto improvement. This is possible even without lump-sum transfers. In this sense, we follow Diamond and Mirrlees' (1971) idea. In order to convert an increase in production efficiency into a Pareto improvement, their idea involves the losers being compensated by movements in commodity and profit taxes. They assume unlimited commodity taxation (in particular, their argument requires the government to have the power to freeze consumer prices and dividends while producer prices are left free to vary so that supply and demand are balanced in each market), a positive direction of commodity tax reform, 100 per cent taxation of profits (or of incremental profits, at least), and a sort of free disposal in the public sector to balance the system completely. This kind of reasoning was generalized in Hammond and Sempere (1992), where discrete Pareto improvements were obtained by assuming only the first of their four important assumptions.

The paper is organized as follows: Section 2 presents the model with the general assumptions. Section 3 establishes our main results concerning general supply-side policies. Section 4 analyzes the consequences of forming a customs union and Section 5 concludes with some final comments.

2 The Model: General Assumptions

The general result in models with incomplete security markets is that producers maximize discounted profit but not at the right discount factors. Present values are not well defined unless there is a correct discount rate. With indeterminacy in discount coefficients caused by incompleteness of markets, actually there are not correct prices to determine production plans in different states. This implies that marginal rates of transformation between production in different states will differ among the set of firms. Discounted prices provide incorrect signals and their distribution in the population of firms is sensitive to changes in the range of the dividend matrix.

2.1 Commodities, Consumers and Producers

We consider a subset of the world formed by K countries indexed by k. Each country is composed of I_k consumers indexed by i ($I = \cup_{k \in K} I_k$), J_k firms indexed by j ($J = \cup_{k \in K} I_k$). Let L be the set of goods. All the agents have common information according to an event tree Ξ with a finite number of nodes $S+1$. Node 0 will denote the unique node with no predecessor or root node. For any other node s, s_+ will denote the set of successor nodes of s. The number of successor nodes of $s \in \Xi$ will be denoted by $\#s$. A terminal node will be s such that $\#s = 0$.

Each consumer i is characterized by: (i) a consumption set $X^i \subset \Re^{L(S+1)}$ of possible consumptions through the event tree which is assumed to be convex, closed, bounded below and to satisfy $0 \in X^i$; (ii) a process of initial endowment of goods w^i which represents spot endowments $w^i_s \in \Re^L$ at each node $s \in \Xi$; and (iii) a preference order $\succsim_i$ on X^i which is assumed to be convex, continuous and strictly monotonic. These conditions ensure the existence of non-empty, upper hemicontinuous individual demand functions when markets are complete and the individual has income enough so that there are cheaper points in the consumption set. He chooses a consumption vector process $x^i \in X^i$ representing spot consumption x^i_s at each generic node $s \in \Xi$.

Each firm is characterized by a production set $Y^j \subset \Re^{L(S+1)}$ representing feasible productions through the event tree. Each Y^j is assumed to be closed, convex, and $0 \in Y^j$. This ensures the existence of non-empty upper hemi-continuous supply correspondences when profits are bounded. Each firm chooses a production process y^j, which represents spot production y^j_s at each node $s \in \Xi$. Inputs appear with a negative sign and outputs with a positive sign. We also assume that the set

$$Y^K(\underline{y}) = \{y^K \in \prod_{k \in K} Y^k \mid \sum_{k \in K} y_k \geq \underline{y}\}$$

of restricted international productions for any aggregate lower bound on net outputs is bounded.

2.2 Market Structure

At each node $s \in \Xi$, there is a spot market for each of the L goods. Let p be a spot price process, where p_s is the associated vector of spot prices at a generic node $s \in \Xi$.

We suppose that trading with securities only happens at the root node so financial markets open just once. There are M financial assets issued by the agents. The first J securities are the shares of the firms. The other securities are unrestricted, except that the value of their payoff at any node must be a linear function of spot prices.

There are security markets for the assets. Each asset m is a claim to a dividend process denoted by A^m, where $A^m_s \in \Re^L$ is the payoff of asset m at the generic node $s \in \Xi$. What consumer i receives as a result of purchasing θ^{im} units of asset m at the root node is the income process $\theta^{im}(p \Box A^m) = \theta^{im}(p_s A^m_s)^S_{s=0}$. Thus, if the vector of asset prices is v, he spends $\theta^i v$ for the purchase of new assets at the root node. To save notation we assume that securities do not pay dividends at the root node.

Let v be the vector of security prices and define W, the $(S+1) \times M$ matrix of security returns processes, as

$$W = \begin{pmatrix} -v^1 & \dots & -v^M \\ p_{0+}\Box A^1 & \dots & p_{0+}\Box A^M \end{pmatrix}$$

Now we have enough information to define the consumer budget correspondences. For suppose that each agent correctly foresees at the root node spot prices in each $s \in 0+$ and the corresponding returns of the securities. Then the budget set of agent i is

$$B^i(p, v, W, w^i) = \{x^i \in X^i,\ \ \theta^i \mid p\Box(x^i - w^i) \leq W\theta^i\}$$

Faced with this budget set, consumer i's demand correspondence is

$$\xi^i(p, v, W, w^i) = \{x^i \in B^i(p, v, W, w^i) \mid \hat{x}^i \succ_i x^i \Longrightarrow \forall \hat{\theta}^i \ \ p\hat{x}^i > W\hat{\theta}^i\}$$

In this section, and only for the sake of intuition in the interpretation of the parameters, we assume that this problem has a solution. Assume also, for the same reasons as before, that the budget set always contains some point x^i in the relative interior of X^i for which $p\Box(x^i - w^i) < W\theta^i$ for some θ^i. Then we can find the appropriate Lagrange multipliers. Thus the $S+1$ constraints implied by B^i give rise to a vector

$$\lambda^i = (\lambda^i_0 \dots \lambda^i_{S+1})$$

of marginal utilities of income for each i. Thus λ^i induces a vector of marginal rates of substitution between states of nature. From this, the vector of present value coefficients of consumer i is

$$\pi^i = (\pi^i_s)^S_{s=0} = (\lambda^i_s/\lambda^i_0)^S_{s=0}$$

From the first order conditions for an optimal portfolio we get

$$\pi^i W(v, p) = 0$$

or, in another formulation,

$$\pi^i_1 \, V(p) = v$$

where V is obtained from W by substracting the first row. This is a no free lunch condition (which implies that no gain is obtained without a positive investment) which is not only necessary for the existence of continuous demand but also sufficient because it implies the compactness of consumers' budget sets.

From this expression, we can discuss the problem which arises when the market structure is incomplete.

When $M \geq S$ and Rank $V = S$ then the columns of the asset returns matrix span the whole consumption set. We say the security markets are generically complete.[2] In this case, there is generically a unique solution to the non-arbitrage equation. The present value coefficients coincide for all the consumers and this leads to a well defined present value function for each firm's profit process $p\Box y^j$. Thus all firms are maximizing present value at the same discounted prices and production efficiency is ensured generically. And there is generically also full Pareto efficiency. These present value coeficients can also be used for the unambiguous evaluation of any supply-side change.

If $M < S$ then the security markets are incomplete. In fact, Rank $V \leq M < S$ and the degree of indeterminacy of the non-arbitrage system is at least $(S - M)$. The spanned set will be a proper subset of the consumption set. The equilibrium, if it exists, will be a restricted equilibrium in any case. Assuming a smooth economy, two different consumers will generically have different present value coefficients (Geanakoplos, Magill, Quinzii and Drèze (1990)) and so the appropriate objective function of the firm is not so clear. The problem of setting firm's objectives has been studied as a collective decision problem (see, for instance Milne (1981)). Most of the literature focusses on unanimity of shareholders when firms are using a particular decision rule (no veto). Then spanning conditions (see Ekern and Wilson (1974), Leland (1974), Radner (1974)) or competitive conditions in the stock market (Hart (1979), Makowski (1983)) are needed. When different decision rules are used, different conclusions are obtained. Nonetheless, it is clear that the firm should consider as one of its arguments shareholders' preferences. The criterion proposed by Drèze (1974) is to use as a discount factor the weighted average of the present value coefficients of the new or "ex post" shareholders, where the weights are the shares of each shareholder. Grossman and Hart (1979) propose to use the weighted average of the present value coefficients of the initial shareholders. In any case, different firms will generally face different discounted prices and this will lead to production inefficiency. An additional problem is the absence of unambiguous prices to evaluate supply-side changes, unless we make some additional strong assumptions.

[2]The rank of V depends on spot prices. There could be situations in which Rank $V \leq S$ but this will not be generic.

3 Potential Pareto Gains

As is well known, even in economies with complete markets, supply side policies which improve aggregate production efficiency do not ensure by themselves a Pareto gain. Such policies will benefit some people but they will generally also cause some workers to lose their jobs, entrepeneurs to lose their firms, etc. To find a Pareto gain requires the design of a redistribution mechanism so that the losers can be compensated.

It is also clear that a very important point in our analysis will be the concept of relevant producer prices. As remarked in the last section, there are a lot of different criteria firms can follow to determine production plans (and so a lot of different relevant producer prices). So there is ambiguity about what is meant by improvements in productive efficiency. We define an improvement in aggregate productive efficiency as a situation in which the firms make more aggregate discounted profits following some consumer's criterion. If we want this feature to happen for all the consumers each time it happens for one of them —that is, if we want to ensure unanimity of consumers in evaluating reforms affecting the production sector —then we can either assume that firms and national production sectors are negligible in the market (Hart (1979)) or assume there is spanning for firms and also for national production sectors (Ekern and Wilson (1974), Leland (1974), Radner (1974)). The latter being because we want to evaluate policies which will affect the whole production sector. Or, as we will see later, in our case we can rather assume conditions ensuring that people agree on the evaluation of the compensatory policies.

Assuming that the spanning condition is satisfied for the aggregate production sector means that

$$\Delta y^k \in < y >$$

where $< y >$ is the space generated by existing world production. This implies that the national production sector cannot influence the span of markets. Each possible change in national production can be obtained as a linear combination of existing world production. It has as immediate consequence that

$$\pi^i \sum_{j \in J_k} p \Box \Delta y^j$$

can be expressed as a linear combination of existing firms' market values and so is independent of i.

If we want to achieve a Pareto gain, we will also need unanimous evaluations of the changes due to the compensatory policies. A way to ensure this unanimity is to assume that the compensatory policies do not change the insurance opportunities available before the reform. That is, they should not change the span of markets. For the case of the compensatory policies that we will use, we have unanimity in the evaluation of the production sector reform if and only if we also have unanimity in the evaluation of the compensatory policies. So we will only assume conditions for the unambiguous evaluation of the compensatory policies without imposing any spanning condition on production sectors directly.

A possible additional justification of the requirement of conditions for unambiguous evaluation of the reform is based on credibility of the policy. If there is ambiguity about the ex ante evaluation of the reform, and if enough people are dissatisfied, there could exist a coalition of consumers who would vote to overthrow the government and return to the status quo. In this case the reform would lack credibility. If the government is concerned with people's preferences about the reform, as it could be because it is concerned about possible reelection, there could be clear incentives for not acting if conditions for unanimity do not hold.

Denote by q the vector of consumer prices, its dependence with respect to producer prices being given by commodity taxation. Then, define an ex ante Pareto gain as a situation in which there exists a vector of government transfers $\phi_k = \phi_k(p, v)$ and a consumer price process $q_k = q_k(p, v)$ which imply the existence of a competitive equilibrium $\{\hat{x}, \hat{y}, p, v\}$ such that:

(i) $\pi^i p \Box \hat{x}^i > \pi^i p \Box \bar{x}^i \ \forall i \in I$

(ii) $\hat{x}^i \in \xi^i(q, v, w^i, V, \phi_k) \ \forall i \in I \ \forall k \in K$

(iii) $\hat{y}^j \in \arg\max_{y^j \in Y^j} \pi^j(p \Box y^j) \ \forall j \in J$

(iv) $\sum_{k \in K} \sum_{i \in I_k} (\hat{x}^i - w^i) = \sum_{j \in J} \hat{y}^j$

(v) $p \Box \sum_{i \in I_k} (\hat{x}^i - w^i) = p \Box \sum_{j \in J_k} \hat{y}^j + (p - w) \Box \bar{z}_k + \bar{b}^k \ \forall k \in K$

(vi) $\sum_{i \in I} \hat{\theta}^{im} = 1$ for every $m = 1, ..., J$, and $\sum_{i \in I} \hat{\theta}^{im} = 0$ for every $m = J, ..., M$.

The first condition is the requirement that each consumer has more discounted income, using as discount factors the consumer's own present value coefficients. If all of them get the same income transfer, we will need obviously conditions for unanimity in evaluating the direction of the change. The fifth condition implies that each government only distributes income coming from national firms, tariff revenue and external borrowing. The sixth represents the condition of equilibrium in asset markets. It has this particular form because shares of the firms are in unit net supply whereas the rest of the assets are assumed to be in zero net supply.

Apart from the characteristics mentioned above, we will also impose a *market structure compatibility* condition. The mechanism has to respect the structure of asset markets in the sense that the government cannot create a trivial efficiency gain by simply introducing new assets or by substituting state contingent lump-sum compensation for missing asset markets. The consequence is that the redistribution mechanism will be restricted to use the existing asset structure so it does not change the span of markets.

4 Potential Gains from Supply-Side Policies

This section derives sufficient conditions for potential Pareto gains when a reform increasing productive efficiency is introduced. Instances of this are policies improving competition or the introduction of a public sector project with net positive value at relevant producer prices. We extend to our financial economy with incomplete markets the mechanism to distribute aggregate gains devised in Hammond and Sempere (1992) for an Arrow-Debreu economy. We abstract from the international sector by focussing on a closed economy so that we will not need to use subindices to refer to different countries.

Suppose that in the starting position q is the consumer price vector process and $\bar{v}$ are the asset prices. Let $\bar{y} = (\bar{y}^1, ..., \bar{y}^J)$ be the vector of nation wide production processes, V the matrix of asset payoffs in period 1, and

$\bar{x} = (\bar{x}^1, ..., \bar{x}^I)$ the vector of nation wide demand processes. The latter satisfies

$$\bar{x}^i \in \arg\max \succsim_i \ s.t. \ x^i \in B^i(q, \bar{v}, w^i, V) \ \forall i \in I$$

It is also assumed that the pre-reform allocation is balanced. That is, it satisfies $\sum_{i \in I} \bar{x}^i = \sum_{j \in J} \bar{y}^j$.

Now assume that a reform which is intended to make firms produce more efficiently is introduced. This policy reform could also have a risk distribution effect that translates into a change in the weights with which each consumer discounts the risk. Our assumption of improved efficiency in production will imply that in the aggregate, when their outputs are valued with the present value coefficient of some of the consumers, firms make more profit by adjusting production plans than by producing the pre-reform plans. That is

$$\pi^i \sum_{j \in J} p \Box \hat{y}^j > \pi^i \sum_{j \in J} p \Box \bar{y}^j$$

for some $i \in I$, where $\hat{y}^j \in \arg\max_{y^j \in Y^j} \pi^j (p \Box y^j)$. If there were only one representative consumer in our economy, equilibrium of plans would exist generically. Assuming that the reform is such that at any vector of positive prices, the aggregate production sector reacts to the policy change by increasing present value, according to the representative consumer's criterion, above what it would have been if production would have remained unchanged, then a Pareto gain would be ensured.

In our economy there is more than one consumer. The reform can benefit some of them but it can also harm some others. To show the existence of Pareto gains we have to find a redistribution mechanism so that the losers are compensated by the winners from the reform. This redistribution mechanism has to be compatible with the information the policy-maker has available. With more than one consumer, in an economy with incomplete markets we also need conditions to make the evaluation of both the reform and the compensatory policies unambiguous.

In order to avoid anyone being harmed by a change in consumer prices or in assets payoffs, assume that the government can freeze consumer prices by movements in commodity taxation and that it also can do the same with assets payoffs by taxing the returns to securities. To ensure that nobody is harmed by losses of capital, assume that all gains from capital are taxed away and that they are given back to people in proportion to their pre-reform demand for assets. That is, each one is taxed $-v\theta^i$ and given back $v\bar{\theta}^i$. This does not affect the government's budget if asset markets clear because $\sum_{i\in I} v\theta^i = \sum_{i\in I} v\bar{\theta}^i$. If there is a surplus in the government's budget, this is given to people in the form of a poll subsidy ϕ_s at each of the nodes. If the total present value of the ϕ_s's is positive for every consumer, all the consumers are better off. Their pre-reform allocation is still feasible and they have more discounted income. In cases where there is not more income in each of the states, unanimity in the evaluation of the change in income is needed. A *spanning* condition imposed on the poll subsidy, meaning that the poll subsidy does not change the insurance possibilities available before the reform, is sufficient for that purpose. Then the poll subsidy would be unanimously valued as a linear combination of marketed asset prices.

In order to prove the possibility of Pareto gains, we have to show existence of an equilibrium of plans when consumer prices and asset returns are frozen at their pre-reform levels, when gains from capital are taxed away and redistributed so that each can afford his pre-reform portfolio, and when asset and production prices are left free to clear markets.

An *equilibrium of plans* for this economy is $\{\hat{x}, \hat{y}, \phi, p, v\}$ such that:

(i) $\hat{x}^i \in \xi^i(q, v, w^i, \phi, V)\ \ \forall i \in I$

(ii) $\hat{y}^j \in \arg\max\ \pi^j(p\Box y^j)\ \ \forall j \in J$

(iii) $\sum_{i\in I}(\hat{x}^i - w^i) = \sum_{j\in J}\hat{y}^j$

(iv) $\sum_{i\in I}\hat{\theta}^{im} = 1$ for every $m = 1, ..., J$, and $\sum_{i\in I}\hat{\theta}^{im} = 0$ for every $m = J, ..., M$.

The first condition means that all consumers maximize utility over the corresponding budget sets. The second implies that each firm is maximizing the present value of its stream of profits following a criterion accepted by all its shareholders. The third is a commodities market clearing condition. The fourth is an aggregate budget equation which, if satisfied and by Walras' law, will imply public sector budget balance in each of the states. The last is a market clearing condition of the securities market.

We need an additional assumption to be able to prove existence of equilibrium. What is needed is the translation to our framework of Duffie's (1987) *wealth accessibility*'s condition. This could be formulated as,

WEALTH ACCESSIBILITY: *For each consumer, there exists θ^i such that $\theta^i W + w^i + \phi > qx^i$ for at least one $x^i \in X^i$ for all p.*

A sufficient condition for this version of wealth accessibility is that $\phi_s > 0$ for each $s \in \Xi$ and every producer price vector p. What we need is the existence of a portfolio transferring enough income to each node so that at each node the consumer can avoid the possibility of being stuck at cheaper points of the consumption set.

We prove existence of such an equilibrium with the following proposition.

Proposition 1 *Maintain the assumptions of Section 2 and assume also that asset returns do not allow arbitrage possibilities but do satisfy Wealth Accesibility. Then a pair (p, v) exists such that markets clear and every consumer is better off than before the reform.*

PROOF: We first eliminate asset prices from our system by requiring that they are related to spot prices in a way that excludes any arbitrage opportunities. That is, if v is an acceptable vector of asset prices, there must not be a portfolio θ with positive return ($W\theta > 0$). This implies that there exists $\beta \in \Re^{S+1}_{++}$ such that $v = \beta V$ (where we have normalized the β's so that $\beta_0 = 1$).

The budget set of consumer i is

$$B^i(q, v, w^i, V, \phi) = \left\{ \begin{array}{c} x^i \in X^i,\ \theta^i \mid \\ q_0(x^i_0 - w^i_0) \leq -v\theta^i + (v\theta^i - v\bar{\theta}^i) + \phi_0 \\ q_{0+}\Box(x^i_{0+} - w^i_{0+}) \leq \theta^i V + \phi_{0+} \end{array} \right\}$$

We can insert the non arbitrage condition into the constraint for the root node. Multiplying the second set of budget constraints by β and adding to the first, we have

$$q'(x^i - w^i) \leq \beta V(\theta^i - \bar{\theta}^i) + \phi'$$

where $q' = \beta q$ and $\phi' = \beta\phi$. So we could normalize the budget constraint by β and it would be equivalent. That means that the budget constraint is independent of asset prices, or that the asset market clearing condition is redundant once the non-arbitrage condition is imposed.

The second step is to replace the possibly unbounded production and consumption sets by suitable bounded ones. By convexity of production sets, of preferences and of consumption sets, and given also that V is fixed and so the set of feasible trades is a linear subspace, following an argument of Debreu (1959) (see also Hammond and Sempere (1992)), an equilibrium corresponding to these artificial sets is an equilibrium corresponding to the original ones. Call these artificial sets $\hat{X}^i$ $\forall i$, and $\hat{Y}^j$ $\forall j$.

Now we prove the continuity properties of demand with the following lemmas.

LEMMA 1: *The budget constraints are bounded, closed, convex, and continuous in ϕ.*

PROOF: The boundedness follows from the non-arbitrage condition given that $\beta >> 0$ and $q >> 0$, V is fixed because dividends are frozen, and from boundedness below of X^i. Convexity is obvious. In order to prove continuity, we first prove lower hemicontinuity, then applying a standard argument due to Hildenbrand (1974, lemma 1, p. 33) the other properties of budget correspondences will imply continuity.

To show lower hemi-continuity, we follow an argument due to Hildenbrand (1974, p. 99). Consider first the correspondence defined by

$$\hat{B}^i(q, v, w^i, V, \phi) = \left\{ \begin{array}{c} x^i \in \hat{X}^i,\ \theta^i \mid \\ \beta q(x^i - w^i) < \beta V(\theta^i - \bar{\theta}^i) + \beta\phi \\ q_{0+}\Box(x^i_{0+} - w^i_{0+}) < \theta^i V + \phi_{0+} \end{array} \right\}$$

If $\hat{B}^i(\phi)$ is non-empty, there is a vector $x^i \in \hat{B}^i(\phi)$. Let $\{(x^i_n, \phi_n)\}$ be a sequence converging to (x^i, ϕ). Obviously, $x^i \in \hat{B}^i(\phi)$ implies $x^i_n \in \hat{B}^i(\phi_n)$ for n large enough. The latter implies that $\hat{B}^i(\phi)$ is lower hemicontinuous. The closure of a lower hemicontinuous correspondence in a compact set is lower hemicontinuous. Thus $B^i(\phi)$ is lower hemicontinuous. □

LEMMA 2: *The demand correspondences have closed graphs and convex values.*

PROOF: Convexity is obvious, given that both budget correspondences and preferences are convex sets. To show the closedness of their graphs, it is enough to apply the maximum lemma, given that budget correspondences and preferences are continuous. □

With the regularity conditions of demand shown in Lemmas 1 and 2, we can now show existence of equilibria by a straightforward extension of the existence proof of Hammond and Sempere (1992), so we will omit the rest of the argument.

Now, it remains to be shown that each consumer is better off at the equilibrium whose existence we just proved. This will require our assumption of increased efficiency in production. We assumed that at any positive price vector, the pre-reform production was not value maximizing. Then the production sector reacts to the reform by increasing present value, following some consumer's criterion, above what it would have been if production would have remained unchanged. From the economy's total budget constraint that is obtained once the non arbitrage condition has been used to eliminate asset prices from our system, we have

$$\begin{aligned}\pi^h p\Box \sum_{i\in I}[x^i(q,w^i,V,\phi)-w^i] &= \pi^h p\Box \sum_{j\in J}\hat{y}^j > \pi^h p\Box \sum_{j\in J}\bar{y}^j \\ &= \pi^h p\Box \sum_{i\in I}[x^i(q,w^i,V,0)-w^i]\end{aligned}$$

for some $h \in I$. This implies that $\pi^h \phi > 0$. The spanning condition on the poll subsidy would imply that the latter is satisfied for every i. □

It is important to notice that this proof has used the fact that, because of the frozen asset returns, the matrix V does not change rank with changes in producer prices. This fact allows for a proof technique involving a standard fixed point argument without requiring smoothness assumptions. That means that there is an equilibrium of plans for all w. If we allow V to change rank, only a generic existence result would be obtained.

Concerning our mechanism of compensation, it is worth realizing that the personalized taxation of capital gains requires knowing the purchases of assets that each consumer would have made in absence of the reform. It could be argued that, in most countries transactions in asset markets are registered, personalized and taxed, so it is fairly easy for the tax administration to get all the relevant information about trade in this markets for each consumer.

It is relevant to be aware of the fact that securities' demands are made by consumers only for the purpose of ensuring for themselves a certain process of real consumption. So they are a function of what the possible process of real demands would have been in absence of reform. This implies that the government has to know preferences and endowments of individuals. In case people were asked about their demand for assets they would declare a number infinitely large so budget constraints would not be bounded. In this sense, once incentive constraints are introduced this mechanism would disappear from the government feasible set. Nevertheless, it is also important to notice that this incentive incompatible compensation is only used during the first period. Thus it does not substitute for missing asset markets, so our compensatory mechanism is market structure compatible.

5 Integrating Markets

5.1 Background

In this section we show conditions sufficient to get Pareto improvements when a set of countries free trade among themselves while setting a external common tariff.

The classical argument to show the superiority of free trade regimes against autarky was based on the reasoning that market integration produces an improvement in terms of productive efficiency. The aggregate production possibility set becomes enlarged. Then more aggregate comsumption is available. If adequate redistribution is possible, all consumers in the economy could be made better off. The Kaldor-Hicks compensation test would be passed.

When markets are incomplete, the same argument generally will not work. Newbery and Stiglitz (1984a) have an example in which free trade could be Pareto inferior to autarky in one economy without the possibility of risk sharing. Their example is based on individual producers, who take production decisions based on a utility function and on the autarkic demand having unit elasticity. When autarky is replaced by free trade, expected aggregate profits are the same as before, but incorporate a greater variance. The risk that originally was fully borne by consumers, is now fully passed on to producers. For some values of producers' risk aversion, everyone feels worse off in the new regime. Newbery and Stiglitz (1984b) show, using the same model, that the optimal distribution of risk is obtained as one intermediate between autarky and free trade.

It has been for some time a known weakness (Dixit (1987), (1989a), (1989b), (1991)) of their model that it does not allow for any possibility of risk sharing (which is, certainly extreme). It does not explain either the cause of security market failure nor take into account the possibility of using any kind of taxation. In cases where there is a stock market, there would be linear risk sharing. In fact, in an economy with only one good, and with bond and equity markets, even though markets may be incomplete, there are no individual risks beyond those implied by the ownership of equity. This implies that if either investors have von Neumann-Morgenstern additively separable utility functions or producers face multiplicative uncertainty, the market allocation is constrained Pareto efficient. In this case, free trade would still be Pareto superior to autarky. Grinols (1987) considers the gains from trade theorem within a framework of incomplete markets. He shows the superiority of the free trade regime in a economy with one consumer-shareholder when the balance of capital is positive. The idea in all these cases is that the present value coeficients are collinear so that all agents' preferences for risk point in the same direction. Then the evaluation of the direction of change in aggregate production would be unambiguous. In our case, the same result is obtained by assuming the *spanning* condition on the compensatory policies.

5.2 Gains from Forming Customs Unions with Market Structure Compatible Compensation

We consider a set of countries K which forms a customs union. We will extend the mechanism devised in section 3 to cover this case.

Suppose that at the starting position q_k is the consumer prices process in country k (which differ among the countries forming the union because each one is using commodity taxes and tariffs in a different way), w is the world prices process, $\bar{v}_k$ are asset prices in country k, V is the matrix of asset payoffs of the group of countries forming the union, $\bar{y}_k = (\bar{y}^j)_{j \in J_k}$ is the vector of production processes in country k, $\bar{x}_k = (\bar{x}^i)_{i \in I_k}$ is the vector of demand processes in country k, and $\bar{z}_k = \sum_{i \in I_k} \bar{x}^i - \sum_{j \in J_k} \bar{y}^j$. Then

$$\bar{x}^i \in \arg\max U^i \;\; \text{s.t.} \;\; x^i \in B^i(q_k, \bar{v}_k, w^i, V) \; \forall i \in I_k$$

Assume also that each country has balanced trade, so that

$$w \square \bar{z}_k = \bar{b}^k$$

where $\bar{b}^k$ is a vector whose components could be interpreted as the maximum allowable deficit process. Thus $\bar{b}^k_s$ could be interpreted as the maximum allowable trade deficit at the generic node $s \in \Xi$.[3]

Notice that V is the same for every country even before forming the Union but asset prices are different. The idea is that before the union is formed a consumer belonging to any country could trade assets issued by firms of any other country, but only in the consumer's own national asset market. Arbitrage is not possible because of capital controls. We assume implicitly that, if there is taxation on asset payoffs, this is based on the source principle (so the payoff net of taxes is the same everywhere), and international agreements impede double taxation of these incomes. Then an additional source of gains could be the possibility of trading financial assets with people having different risk aversion.

The argument to show that nobody in the world is worse off after the creation of the union rests on the ability to set an external tariff so that both world prices and the amount of trade of the union with the rest of the world are frozen. In our model, the union external tariff will be endogenous and will depend on producer prices inside the union so that this objective is fulfilled.

The tariff reform that occurs in each country which joins the union could have been negative (in terms of revenue) for some countries and positive for others. This could lead some countries to become worse off after joining the union. In order to avoid this effect, we follow the literature (see, for instance, Grinols (1981)) and postulate intergovernamental transfers which compensate for the losses of tariff revenue. It is assumed that all tariff revenue forms a fund of the union. This gets divided between the members following the pre-reform patern of trade of each country. Each country member k gets a

[3] Notice that it could be different for different nodes of the event tree.

transfer process of $(p-w)\Box\bar{z}_k$. Here $(p_s-w_s)\bar{z}_{ks}$ is the transfer corresponding to node $s \in \Xi$.

Suppose a reform creating a customs union is implemented. Assuming the aggregate spanning condition is satisfied, we now assume that the reform improves aggregate production efficiency in the sense that, for any vector of prices, and according to the criterion of some consumer i the sum of present discounted aggregate profits which the production sector would make by adjusting production plans is greater than the present value that the aggregate of firms would make by remaining at the pre-reform plan. We assume that this happens for each country belonging to the union. That is, for all $k \in K$,

$$\pi^i \sum_{j\in J_k} p\Box\hat{y}^j + > \pi^i \sum_{j\in Jk} p\Box\bar{y}^j$$

for some $i \in I$ where $\hat{y}^j \in \arg\max_{y^j\in Y^j} \pi^j p\Box y^j$ This assumption implies that the reform would make possible more aggregate discounted consumption in each country. This is related to the assumption, used by Grinols (1987), that the change in the balance of capital had to be positive. What would be required is that the change in the discounted balance of payments has to be positive if we keep national demands fixed at original levels. That the sum of national aggregate capital gains will be zero is something that will be obtained from our compensatory mechanism. If there was only one consumer in each country, generic existence of an equilibrium of plans would be ensured by our assumptions of section 2. With our assumption of improved production efficiency, this consumer would obviously be better off. This would imply the existence of an ex-ante Pareto improvement. When there is more than one consumer in each national economy, it is necesary to find a redistribution mechanism so that those who benefit from the reform could compensate the losers. This redistribution mechanism has to be compatible with the policy-maker's information set. It is also necessary to make assumptions to be able to make both the reform and the compensatory policies unambiguous.

In order to avoid anyone being harmed by a change in consumer prices or asset returns, assume that each government can freeze consumer prices by changes in national commodity taxation and that it also can do the same with asset returns by taxing the returns to the securities issued in its particular country. To ensure that nobody is harmed by capital losses, assume that all gains from capital are taxed away, and all losses are compensated. The net proceeds are used to form a union fund that is given back to people in proportion to their pre-reform demands for assets. That is each one is taxed $-v\theta^i$ and given $v\bar{\theta}^i$. This is clearly feasible for the union as a whole if the union asset markets clear. This requires international lump-sum transfers.[4] If there is a surplus in government's budget, this is given to people in the form of a poll subsidy ϕ_{sk} in each one of the nodes of the event tree. If the present value of ϕ_{sk} is positive for every consumer and k, all consumers in the union are better off ex ante. Their pre-reform allocation is still feasible and they have more discounted income. To get an unambiguous evaluation of the reform we assume, as before, that the poll subsidy does not change the insurance opportunities which were available in the financial market before the reform.

In order to prove the possibility of Pareto gains, we have to show existence of an equilibrium of plans when consumer prices and asset returns are frozen at their pre-reform levels, when gains from capital are taxed away and redistributed so that each one can afford his pre-reform portfolio, and when asset and production prices are left free to clear markets.

An *equilibrium of plans* for this economy is a combination $(\hat{x}, \hat{y}, \phi, p, v)$ such that:

(i) $\hat{x}^i \in \xi^i(q_k, v, w^i, V, \phi_k)\ \ \forall i \in I\ \ \forall k \in K$

(ii) $\hat{y}^j \in \arg\max\ \pi^j(p\square y^j)\ \ \forall j \in J$

(iii) $\sum_{k\in K}\sum_{i\in I_k}(\hat{x}^i - w^i) = \sum_{j\in J}\hat{y}^j$

(iv) $p\square\sum_{i\in I_k}(\hat{x}^i - w^i) = p\square\sum_{j\in J_k}\hat{y}^j + \bar{z}_k\square(p-w) + \bar{b}^k + v\Delta\theta_k\ \ \forall k \in K$

(v) $\sum_{i\in I}\hat{\theta}^{im} = 1$ for every $m = 1, ..., J$, and $\sum_{i\in I}\hat{\theta}^{im} = 0$ for every $m = J, ..., M$.

[4]In this respect, Grinols (1987) did not need international transfers because he directly assumes that the change in the balance of capital is positive. This implies that the improvement in productive efficiency is enough to compensate any negative national aggregate capital gain for any country.

Now the fourth condition has as additional terms the international transfers corresponding to tariff revenue, and the net external borrowing. It implies that national governments can only redistribute income coming from profits of national firms, external borrowing and international transfers originated in tariff revenue and in capital gains.

We prove existence of equilibrium with the following proposition.

Proposition 2 *Maintain the assumptions of Section 2 and assume also that asset prices do not allow for arbitrage opportunities and that the Wealth Accessibility condition is satisfied. Then a pair (p, v) exists such that markets clear in the union, every consumer belonging to any member of the customs union is better off than in the unreformed economy, and the rest of the world remains as it was before the reform.*

PROOF: The proof is a slightly modified version of the proof of Proposition 1. We only point out the differences without developing the whole argument.

In this case the budget constraint of consumer i living in country k is

$$B^i(q_k, v, w^i, V, \phi_k) = \left\{ \begin{array}{c} x^i \in X^i,\ \theta^i \mid \\ q_{k0}(x_0^i - w_0^i) = -v\theta^i + (v\theta^i - v\bar{\theta}^i) + \phi_{k0} \\ q_{k0+}\Box(x_{0+}^i - w_{0+}^i) = \theta^i V + \phi_{k0+} \end{array} \right\}$$

Now consumer prices could be different in different countries because of different commodity taxation. The amount of poll subsidy could also be different because each country is paying out as much subsidy as it can afford.

To show the necessary regularity conditions of demand is a straightforward extension of Lemmas 1 and 2 of section 4. The rest of the existence proof follows the same argument as in Proposition 1. The rest of the world is unaffected by the formation of the union because border prices and the amount of trade with the whole union are frozen beacuse of compensating movements in the external common tariff. It remains to show that every consumer in the union could be made better off. Notice that, from (ii), with our assumption that the aggregate production sector of each country reacts to the reform in such a way that the present value of profits, according to the criterion of some consumer living in that country, lies above what the sum would have been if production plans would have remained unchanged. That is

$$\begin{aligned}\pi^h p\Box \sum_{i\in I_k}[x^i(q_k,w^i,V,\phi_k)-w^i] &= \pi^h p\Box \sum_{j\in J_k}[\hat{y}^j+\bar{z}_k\Box p] > \\ > \pi^h p\Box \sum_{j\in J_k}[\bar{y}^j+\bar{z}_k\Box p] &= \pi^h p\Box \sum_{i\in I_k}[x^i(q_k,w^i,V,0)-w^i]\end{aligned}$$

for some $h \in I_k$ and for each country k. This and the spanning condition on the poll subsidy is sufficient to ensure the existence of a positive present value of ϕ_k for every consumer in each country, so a Pareto gain. □

6 Final Remarks

Each of the reforms analysed in the preceding sections will favour some people and harm some others. If we want to appraise them while avoiding interpersonal comparison of different consumers' utilities, we have to look for the possibility of compensation of losers by the winners from the reform. In this sense, it is very important to consider the actual possibilities for compensation. These will depend clearly on the tax system and of crucial importance will be the information about consumers which the policy-maker in charge of the compensation has.

In order to get an unambiguous change in welfare derived from the integration of security and real markets, we had to assume a *spanning* condition. This was necessary to get an unanimous evaluation of the change in individual budget sets derived from the reform and corresponding compensatory policy. This assumption would be satisfied if the compensatory policies do not change the insurance opportunities provided by the financial markets before the reform. In other words, if these policies do not change the span of markets. The redistribution mechanism assumed that the government could know the transactions in asset markets that consumers would have made in absence of the reform. This was necessary to avoid people being harmed by capital losses. This, implicitly, assumes the knowledge of people's preferences and income and truthful revelation of this information would not be advantageous for individuals. The main advantage of this mechanism is that it does not assume that the government can transfer weath freely between different nodes. It is therefore *market structure compatible.*

It turns out that even though the policies analysed appear as a theoretical possibility, we have to be aware of the different constraints which place boundaries on the policy-maker's actions. It is actually a strong requirement to postulate the government being able to control all the parameters which determine each consumer's budget set. We are considering potential welfare gains. That is, in the spirit of the Kaldor-Hicks compensation test, we ask whether there exists a compensatory reply by the government which leads to a welfare gain for everybody. In this sense, we are supposing that the policy-maker acts optimally when in fact there are multiple constraints (such as bounded rationality of policy-maker or additional political constraints) which makes its performance differ substantially from an optimal one. Our conclusions are then pessimistic. Even in cases in which there are gains in production efficiency, it will be very difficult to achieve Pareto gains.

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Appendix: Existence proof with smooth demand and supply.

Assume that individual preferences can be represented by a utility function U^i which is assumed to satisfy:

I It is C^2.

II $h^\top D^2U^i(x)h < 0 \;\; \forall h \neq 0$ such that $DU^ih = 0 \;\; \forall x \in X^i$.

III $(U^i)^{-1}(c) \in \Re^N_{++} \;\; \forall c$.

IV U^i is strictly monotone.

About firms production sets we add to the assumptions of section 2 the following:

-The boundary of each Y^j is a C^2 manifold with strictly positive Gaussian curvature at each point.

PROPOSITION: *With our assumptions (regular economy) and assuming that the matrix,*

$$D_\phi(p\Box x)$$

has a positive dominant diagonal. Then a tuple (p, v) exists for the model of section 4 such that markets clear and every consumer is better off than before the reform.

The positiveness of the derivatives is interpreted as an aggregate normality condition. This means that each period-state aggregate demand is strictly increasing with the poll subsidy distributed in that period-state The dominant diagonal condition is the recognition that when markets are incomplete, people can transfer income through periods only imperfectly [5].

PROOF: We first eliminate asset prices from our system by requiring that they are related to spot prices in a way that excludes any arbitrage opportunities. That is, if v is an acceptable vector of asset prices, there must not be a portfolio θ with positive return ($W\theta \geq 0$). This implies that there exists $\beta \in \Re^{S+1}_{++}$ such that $v = \beta V$ (where we have normalized the β's so that $\beta_0 = 1$).

[5] In the extreme case of complete absence of security markets, this matrix would be diagonal.

The budget constraint of consumer i is

$$B^i(q,v,w^i,V,\phi) = \left(\begin{array}{c} x^i \in X^i,\ \theta^i \mid \\ q_0(x_0^i - w_0^i) = -v\theta^i + (v\theta^i - v\bar{\theta}^i) + \phi_0 \\ q_{0+}\square(x_{0+}^i - w_{0+}^i) = \theta^i V + \phi_{0+} \end{array} \right)$$

We can plug the non arbitrage condition in the contraint for the root node. Multiplying the second set of constraints by β and adding to the first one, we have

$$q'(x^i - w^i) = \beta V(\theta^i - \bar{\theta}^i) + \phi'$$

where $q' = \beta q$ and $\phi' = \beta\phi$ so we could normalize the budget constraint by β and it would be equivalent. That means that the budget constraint is independent of asset prices or that the asset market clearing condition is redundant once the non arbitrage condition is imposed. Now demands are $x^i(q,w^i,V,\phi)$. The *equilibrium of plans* equations are:

(i) $\sum_{i\in I}[x^i(q,w^i,V,\phi) - w^i] = \sum_{j\in J} y^j(p)$

(ii) $p\square\sum_{i\in I}[x^i(q,w^i,V,\phi) - w^i] = p\square\sum_{j\in J} y^j(p)$

where the second is a sequence of budget constraints faced by the economy. Given that V and q are fixed, we could treat them as a system of equations in p and ϕ provided that supply functions do not depend on ϕ the matrix derivatives of the system with respect ϕ are

$$D_\phi(p\square x)$$

by our assumption this matrix must have full rank so, applying the Implicit Function Theorem, there must be a C^1 $\phi(p)$ which satisfies the system of equations. Substituting this functions in our first condition, *equilibrium of plans* can be defined as a system of prices which solves

$$\sum_{i\in I}[x^i(p,w^i,V) - w^i] = \sum_{j\in J} y^j(p)$$

or, denoting by $\hat{Z}(p,w,V) : \Delta_{++}^N \times \Re_{++}^N \times G_J \longmapsto \Re^N$ the aggregate excess demand function

$$\hat{Z}(p,w,V) = 0$$

where excess demand has the following properties:

(P. I) $\hat{Z}$ *is* C^2

(P. II) $p\Box\hat{Z}(p,w,V) = 0 \;\; \forall\, p \in \triangle^N_{++}$

(P. III) $\hat{Z}$ is proper.

The first cames because supply functions are twice differentiable and ϕ was obtained as C^1 so demands are C^2. The second is a straightforward consequence of the construction of ϕ. The last is satisfied because, even though demand is affected by producer prices only through the poll subsidy, assuming closedness and convexity of aggregate production set, for any sequence of prices going to the boundary of the normal cone of the production set, the norm of the corresponding best responses goes to infinity (see, for instance Mas-Colell (1985)). With free disposal, for a sequence of prices going to the boundary of the simplex the norm of the corresponding best responses goes to infinity.

We conclude the proof by an homotopy argument. We use the linear homotopy with the same arguments as the one devised by Brown and DeMarzo (1991)

$$H(p,t) = t\hat{Z}(p) + (1-t)f_e(p)$$

where $f_e : \triangle^N_{++} \longmapsto \Re^N$ is defined by

$$f^l_e(p) = (\frac{p\,e}{N})\,\frac{1}{p^l} - e^l$$

f_e verifies all the necesary properties (smoothness, Walras' Law, properness, uniqueness of the solution of $f_e(p) = 0$ and Rank $D_e f_e(p) = N-1$). Also it can trivially be shown that Rank $D_w\hat{Z}(p) = N-1$.

The homotopy verifies Walras' Law (because f_e and $\hat{Z}$ do). For $t < 1$, $D_e H(p,t)$ has full rank $(N-1)$. For $t = 1$, $D_w H(p,t)$ has full rank. The full rank property means that the homotopy is transversal to zero. By the Transversality Theorem, this is true for almost all e. By the Regular Value Theorem, this means that $H^{-1}(0)$ is a differentiable manifold of dimension 1 ($dim\ \triangle^N_{++} \times (0,1) - (N-1)$). Given that f_e and $\hat{Z}$ are proper, the homotopy is boundary free. By the Homotopy Invariance Theorem, given that $f_e(p) = 0$ has an odd number of solutions, $\hat{Z}(p) = 0$ must have an odd number of solutions. $\Box$

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