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ON THE INVARIANCE OF MARKET INNOVATION TO THE NUMBER OF FIRMS. THE ROLE OF THE TIMING OF INNOVATION

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The paper studies a stochastic patent race in which firms can undertake several projects aimed to the same innovation. The paper shows that the invariance of the market portfolio of projects does not hold when we increase the number of firms in the market from one to two or more firms. This result has also implications for the efficiency of the market outcome. If firms can perfectly price discriminate so that the private prize for innovating is equal to the social prize, the paper shows that the market undertakes more projects than the socially desirable number. However, each project is undertaken at an efficient level of effort which is independent of the size of the prize for innovating.

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1 INTRODUCTION

The issue of how firms invest in R and D in a strategic setting have received a great deal of attention from the economics profession over the last ten years¹. Raaj K. Sah and Joseph E. Stiglitz (1987) wrote a very interesting article on the invariance of market innovation to the number of firms. The authors assume an economy in which firms can undertake several parallel projects aimed to the same innovation². The probability for any project to get the innovation is a function of the firm's investment in that project. There is no time in their model and the probability that two or more projects are successful is nonegligible. Thus, their model is not a game of timing. Firms compete in a market characterized by Bertrand competition. Hence, if two or more firms innovate, Bertrand competition will take away all the benefits for "This assumption gives ..-their-.. model a winnerinnovation. takes-all feature similar to that in the patent race literature" (Sah and Stiglitz (1987) p.98). The main result of their paper is:

i. "The market portfolio of projects -the number of projects undertaken as well as the expenditures on each of them- is unaffected by the number of firms" (Sah and Stiglitz (1987) p.99).

With uncorrelated projects and symmetric equilibria they were able to show that:

ii. The level of effort at which each project is undertaken, is independent of the size of the prize granted to the firm if she is the only winner.

iii. If social benefits are equal to private benefits -if firms can perfectly price discriminate- then the market portfolio of outcome is socially efficient in the sense that both the level of effort and the number of projects undertaken by the market are equal to the level chosen by a social planner³.

It turns out that conclusions (i) and (iii) are sensitive to the assumptions of their model. In particular, the fact that the model is not a game of timing and the fact that two or more projects can

¹ The reader may consult the excellent survey by Reinganum (1989).

² The authors do not have to assume that the projects are uncorrelated to prove their main result, however to make welfare analysis they do assume that the projects are uncorrelated.

³ Sah and Stiglitz do not talk explicitly about this result, rather they argue that private benefits are smaller than social benefits because the innovator cannot appropriate all the consumer surplus, therefore for them the number of projects that the market undertakes is smaller than those undertaken by the social planner. be successful. This paper studies the implications of a specification in which the timing of innovation matters. I study a model in which firms can undertake several projects aimed to the same innovation in the context of a stochastic patent race.

2 THE MODEL

Consider a model á la Loury (1979) with lump sum costs of R and D, in which firms can undertake several parallel projects aimed to the same innovation. The projects are statistically independent. As in Loury, the timing of innovation t' of any single project is distributed according to an exponential distribution function with parameter λ :

$Prob(t' \le t) = 1 - e^{-\lambda t}$

To make things comparable between the social planner solution and the market outcome⁴, assume that the parameter λ is a function of c_{ij} , the investment by firm *i* on project *j*. Assume also that the function $\lambda(\cdot)$ satisfies the following conditions: $\lambda(0)=0$, $\lim_{c_{ij} \leftarrow c_{ij} \leftarrow c_{ij$

finally $\lambda''(c_{ii}) < 0$ if $c_{ii} > \overline{c}$, with $\overline{c} > 0$.

The firms compete for a prize R, the loosers do not get anything. The reader may notice that the assumption of lump sum costs makes the cost expenditures independent of the duration of the race. We know this is trivially true for an economy without time. On the other hand, the assumption that the winner takes all is better justified in this case by assuming that a patent right gives the winner the privilege of exploitation of the innovation.

Under the above assumptions the probability that the rivals of firm *i* have not innovated by time t is given by:

$$\prod_{j\neq i}\prod_{k=1}^{k_j}e^{-\lambda(c_{jk})t}$$

where k_j represents the number of projects undertaken by firm j. Given the symmetry of the technology, any firm will choose the same level of effort for all projects. Hence, the last condition can be simplified to:

e^{-µt}

where $\mu = \sum_{j \neq i} k_j \lambda(c_j)$. On the other hand the probability that firm *i* innovates before time t is given by:

⁴ See Kamien and Schwartz (1982) p. 192.

$1 - e^{-k^i \lambda(c_i) t}$

The probability that firm innovates in t:

 $k^{i}\lambda(c_{i})e^{-k^{i}\lambda(c_{i})t}$

Expected profits of firm *i*, are equal to the following expression:

$$V^{i}(\mu, R, r) = \underset{c_{i} \ k_{i}}{\max} \int_{0}^{\infty} e^{-\mu t} k^{i} \lambda(c_{i}) e^{-k^{i} \lambda(c_{i})} e^{-rt} R dt - k^{i} c_{i}$$

The firm will win the prize R, only if it innovates before all the other rivals do. After integrating the last expression, I get the following expression for expected benefits:

$$V^{i}(\mu, R, r) = Max \left[\frac{Rk^{i}\lambda(C_{i})}{\mu + k^{i}\lambda(C_{i}) + r} - k^{i}C_{i}\right]$$

3 RESULTS

I concentrate my analysis on symmetric Nash equilibria, the first order conditions are⁵:

(1)
$$\frac{\frac{((n^{m}-k^{i})\lambda(c_{i})+r)R\lambda(c_{i}))}{(n^{m}\lambda(c_{i})+r)^{2}}=c_{i}}{\frac{((n^{m}-k^{i})\lambda(c_{i})+r)Rk^{i}\lambda'(c_{i}))}{(n^{m}\lambda(c_{i})+r)^{2}}=k^{i}}$$

Where $n^{m}=N^{m}k_{i}$, with N^{m} equal to the number of firms. From the last two conditions it can be verified immediately that:

 $\frac{\lambda(C)}{C} = \lambda'(C)$

The last expression implies the following proposition:

Proposition 1: All firms choose the efficient level of effort, regardless of the size of the prize.

The last proposition confirms result ii of Sah and Stiglitz.

⁵ In the first order conditions for k_i , I assume for simplicity that k_i is a continuous variable. The analysis with k_i discrete does not change the results. Loury (1979), Lee and Wilde (1980) and Reinganum (1982) make similar assumptions with respect to discrete variables.

Notice that the conditions in (1) (the first order condition for a noncooperative Nash solution) imply also that n^{m} , the total number of projects undertaken by the market, is invariant to a change in the number of firms in a noncooperative Nash solution. If N^{m} increases the firms will respond by reducing the number of projects per firm (k^{1}) so that n^{m} remains constant. In the context of a stochastic patent race, Result i of Sah and Stiglitz holds if the solution is a noncooperative Nash solution (i.e. whenever we have two or more firms), the following proposition summarizes this finding:

Proposition 2: The invariance result is true for a noncooperative Nash solution (i.e. whenever there are two or more firms in the market).

Now let me compare the total number of projects undertaken by the market with the monopoly outcome. Since the monopoly does not care on which project succeeds, the monopolist is only concerned with $n^{s_{\lambda}}(c)$, the aggregate probability of innovation in the next instant of time. Expected benefits for the monopolist become:

$$V^{s}(R,r) = Max \left[\frac{n^{s}\lambda(C_{s})R}{n^{s}, C_{s}} - n^{s}C_{s} \right]$$

The first order conditions for n^s and c_s are:

(2)
$$C_{s} = \frac{Rr\lambda(C_{s})}{(n^{s}\lambda(C_{s})+r)^{2}} \qquad \frac{1}{\lambda'(C_{s})} = \frac{Rr}{(n^{s}\lambda(C_{s})+r)^{2}}$$

From the last conditions we notice immediately that $c_s=\overline{c}=c_i$. Let me compare the number of projects undertaken by the monopolist with those undertaken by the market. By combining (1) and (2):

 $\frac{(n^{m}-k)\lambda(c)+r}{(n^{m}\lambda(c)+r)^{2}} = \frac{r}{(n^{s}\lambda(c)+r)^{2}}$

From the last equation we get that $n^m > n^s$. If the number of firms that the monopolist controls is equal to the number of firms in the market, $k^i > k^s$, the representative firm undertakes a larger number of projects than in the cooperative solution. This result is not surprising, in a market environment firms care about their own success, whereas in the monopolist case it does not matter which firm succeeds. There is an externality in the market environment generated by the fact that each firm wants to be first in achieving the innovation.

Define s as the social prize of innovation. If the monopolist can perfectly price discriminate, s=R. The social planner solution is identical to the monopolist solution, the latter reasoning applies unmodified to the social planner solution. Therefore, the market undertakes a larger number of projects than the socially desirable level. The following proposition summarizes these latter reasonings:

Proposition 3: i) In a stochastic patent race, the number of projects undertaken by the market is not invariant to the number of firms, in particular, when we compare the number of projects undertaken by one firm (the monopolist) with the number of projects undertaken by two or more firms we notice that the number of projects undertaken by the latter market structure is higher than the one for the single monopolist.

ii) Whenever the monopolist can price discriminate and therefore R=S, the market outcome yields an excessive number of projects as compared with the social planner solution.

Proposition 1 and 3 illustrate the restrictiveness of the specification of the Loury (1979) model. The externality present in the market outcome was translated into too many firms allocating suboptimal levels to R and D. When firms are not restricted to choose only one project, the inefficiency is translated into too many projects working at the optimal level.

4 CONCLUDING REMARKS

The driving force in Sah and Stiglitz results is the assumption of an economy without time. This assumption implies that the probability that two or more projects innovate is nonnegligible. A firm will undertake an additional project by comparing its expected benefits with the costs of undertaking it. "This project yields a benefit only if the other projects undertaken by the firm fail, as well as if all of the other projects undertaken by other firms fail. The marginal decisions are thus influenced by the total number of projects undertaken in the market and not by how these projects are partitioned between the firms" (Sah and Stiglitz (1987) p. 101).

In contrast, in this setting, the firm that innovates wins the A firm increases its number of projects with the aim of race. reducing the expected day of discovery. The probability that two projects innovate at the same time is zero, therefore the probability that two or more projects succeed is zero. A firm will undertake an additional project only if the reduction on the expected date of innovation is such that the increase in expected benefits overcompensates the additional costs of undertaken the project. Due to the fact that in a strategic environment (two or more firms), each firm wants to be first, the expected date of innovation is lower than the expected date of innovation for the cooperative solution (the monopolist case). The way noncooperative firms reduce the expected date of discovery is by undertaking more projects each. In Sah and Stiglitz, the fact that other projects undertaken by a firm have a positive probability of being successful, undermines the incentive to undertake the marginal project. This project will yield a benefit only if the other projects in the market (including those undertaken by the firm)

fail.

This paper highlights the role of timing in the innovation process. The fact that each firm wants to be first yields an inefficient outcome. When we relax the restriction that forces each firm to undertake a single project, we eliminate one inefficiency present in the work by Loury (1979), the fact that each firm works at a suboptimal level. However, we still get to much effort allocated to R and D when we compare the market outcome with the social planner solution.

Besides, in a noncooperative environment as in Sah and Stiglitz, firms do take into account in their marginal decisions the number of projects undertaken by the whole market, in such a way that the number of firms is irrelevant in the marginal decisions. However, the invariance theorem is not true when we increase the number of firms from one to two or more.

Sah and Stiglitz have a contest model in which several projects may be successful. This paper modifies the conclusions reached by Sah and Stiglitz in the context of a stochastic patent race in which firms can undertake several projects.

The paper has policy implications, we know from proposition 1 that each firm will undertake each project at the most efficient level. The objective of policy in an economy in which the timing of innovation matters is to grant patent rights for a duration of time that can reduce R and therefore expected profits to a level that commits the firms to undertake the same number of projects that the social planner choose⁶.

Finally, it is interesting to contrast the invariance theorem with the previous belief about the relation between market structure and innovation. The previous belief considered the number of firms as an important variable that affected the incentives to innovate. Moreover, the expectations of monopoly rents were considered as crucial for undertaking R and D. Whenever we have a noncooperative solution, the result of this model undermines the structure of the market as an important variable for determining the market structure of projects for innovation. But the possibility of getting monopoly rents is still important.

⁶ Remember that we are assuming that the social benefits are equal to private benefits.

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