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THE DYNAMICS OF REAL EXCHANGE RATE AND FINANCIAL ASSETS OF PRIVATELY FINANCED CURRENT ACCOUNT DEFICITS

Ángel Calderón Madrid

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THE DYNAMICS OF REAL EXCHANGE RATE AND FINANCIAL ASSETS OF PRIVATELY FINANCED CURRENT ACCOUNT DEFICITS.[1]

In the analysis of the last few years' developments in the balance of payments in countries such as Mexico and the U. K. a claim commonly found is that current account deficits when associated with private sector dissavings should not be a reason for concern. One of the arguments behind this claim is that, unlike public sector dissavings -i.e. fiscal deficits-, those corresponding to the private sector should be in the long-run self-correcting. The reasoning is that the agents of the private sector are not going to ignore their ex-ante intertemporal budget constraint. This implies that they will eventually want to bring spending into line with their underlying asset and debt position.

Claims along these lines are further substantiated by pointing out that, in some instances- e.g. Mexico, Denmark and Ireland- private dissavings are inducing current account deficits, even though they occur simultaneously with reductions in public sector deficits. These latter reductions, on their own, would lead to current account surplus. An explanation of private sector dissavings more than compensating public sector savings is attributed to the effect that fiscal policy has on the perceptions that the private sector has about the medium- and long-term developments in the economy. That is, since fiscal contractions are considered by the private sector to be sustainable, they are perceived to lead to long-run reductions in public indebtedness. - See Giavazzi and Pagano (1990) and Hellwig and Neumann (1987).

In this paper we deploy a model for understanding the behaviour of an open economy with an initial phase of its dynamic trayectory characterized by reductions in fiscal deficits accompanied by more than compensating reductions in private surplus, thereby exhibiting privately-financed current account deficits. This model - which follows the modern version of the Mundell-Flemming approach- enables us to consider the evolution of the real levels of the exchange rate, interest rates, and of the level and composition of private sector wealth in an open economy exhibiting these peculiarities.

The private sector in our model can hold money as well as domestic and foreign currency denominated non-monetary financial assets. In turn, its savings behaviour is interpreted as an adjustment of the stock of wealth to a long-run level desired by this sector. Non-zero savings by the private sector are linked to fiscal and current-account balances. In the model, the long-run equilibrium refers to a situation in which trade balance and interest revenue on foreign assets offset each other and consequently capital flows are zero. This implies,

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by construction, that its steady-state has two properties. First, as in the papers by Dornbusch and Fischer (1980) and Krugman (1988), the levels of real exchange rate and of net holdings of foreign assets by the private sector are inversely related across steady states. Second, it can capture a mechanism through which variations in the level of domestic public debt are related to changes in the real level of the exchange rate: as in the work by Sachs and Wyplosz (1984), we allow fiscal unbalances to change the level of domestic public indebtedness and consider its implications for exchange rate dynamics.

Due to these peculiarities of the model, we have, at the outset, the following two implications embedded in our analysis:

On the one hand, stating that a long-run decrease in the real level of the cumulated sum of the domesticaly-financed fiscal deficits- i.e. of the level of domestic public indebtedness- is perceived as feasible by the private sector is tantamount to stating that a long-run real exchange rate appreciation is expected to take place. On the other hand, if *ex-hypothesis* in the short run private sector dissaving more than compensates public sector savings, then an exchange rate appreciation must occur for the implied current account deficit to be registered. These two implications, in turn, generate the following question:

How can both a short and long-run real exchange rate appreciation be possible, given the constraint requiring that long-run variations in interest earnings on foreign assets, resulting from a different net foreign asset positon of the economy, be matched with net exports at a post-disturbance steady state?

The answer to this question, to be explored in this paper, is that the exchange rate must follow a dynamic trajectory such as to enable a sequence of current account deficits to be more than compensated by a sequence of current account surpluses. One possibility arises if there are no productivity gains in the tradable sector. This trajectory must imply that the convergence path of the real exchange rate to its new long-run equilibrium must have one characteristic: it cannot be monotonic. This will be so because the initial exchange rate apreciation must be followed by a temporary sequence of an exchange rate depreciated with respect to the pre-disturbance level.

A further possibility, which is not going to be explored in this paper is the following one. A non-monotonic behavior of the exchange rate -i.e. a depreciation alternating with an appreciation- would not be required if the model can capture productivity gains enabling, at the post-disturbance steady state, the export sector to have a more competitive performance, as well as a process leading the private sector to desire a lower long-run level of financial assets.

Most existing analytical models with forward-looking expectations of exchange rate behaviour share the limitations of our model to address the second answer.[2]

[2] Other models, for example the work by Kouri (1979) and (1982), Dournbusch (1988) and Serven (1990), do not include the issues raised in this paper, but address part of the second answer by modelling the relationship between investment and movements in the real level of the exchange rate.

In addition, due to the reduced size of their dynamic system, they are ill equipped to consider the first answer as well[3] and to explicitly consider the evolution of the size of the private sector financial portfolio.

Our paper analyses the first possibility in a analytically tractable model and, in the process, enables us to gain insight of the functioning and required extensions of the modern version of a stock-flow model within the Mundell-Flemming tradition.[4]

The remaining part of this paper is divided into three sections.

In section II we present the model, discuss its dynamic structure and the equations which determine the steady-state or long-run solution.

In section III we analyse the characteristics of the long-run equilibrium of the model and specify and interpret the conditions under which the long-run equilibrium of the model has the saddle-point properties required owing to the forward-looking nature of expectations about exchange rate movements.

On the bases of these characteristics we analyse, in section IV, the way in which a change in domestic public indebtedness is related to the real levels of the exchange rate and the net stock of privately held financial assets.

We finally examine the response that shocks to the system would have on the short-run or impact levels of exchange and interest rates. We discuss the characteristics of the dynamic trajectory to the new steady-state and explain why a non-monotonic convergence path would be registered with the problem addresed in this paper.

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[3]It is a common procedure to rely on a dynamic system with less than three dynamic equations. This procedure rules out, by construction, a non-monotonic behavior of the exchange rate. This follows because of the forward-looking assumption about exchange rate movements. Due to this assumption the models must exhibit saddle-point characteristics. That is, associated to a non-predetermined or 'jump' variable there must be one unstable root in the dynamic system. The influence of this root is neutralized - by means of a transversality condition- thereby constraining the system to lie on the stable manifold. (Cfr. Buiter (1984)).

[4]There is an additional problem, not explicitly addressed in this paper. A non-monotonic movement in exchange rate which, *ex-hypothesis*, is foreseen by the private sector could alter the country-risk since an exchange rate depreciation following an apppreciation is foreseen to happen.

II.1 **Specification of the model.**

a) <u>Expectational Variables and Flow relationships:</u>

$$e^{e} = (de/dt)/e \tag{1}$$

$$Y = C + G + X - Z \tag{2}$$

$$X - Z = (X_q - Z_q)\,e + Z_y Y \qquad (X_q - Z_q) > 0,\ Z_y < 0 \tag{3}$$

$$C = C_y Y - C_T T + C_N N + C_v l_o (de/dt)/e + C_r r + C_w F + C^{a} \qquad C_y = C_T = C_N > 0;\ C_v, C_w > 0;\ C_r \leq 0 \tag{4}$$

$$N = r_o b + b_o r + r_o^{*}(l + l_o e) \tag{5}$$

$$G = -(C + X - Z) \tag{6}$$

$$T = r_o b + r b_o \tag{7}$$

c) <u>Financial Markets.</u>

$$F = h + b + (l + l_o e) \tag{8}$$

$$H_y Y + H_r r + H_w F = h \qquad H_y, H_w > 0;\ H_r < 0 \tag{9}$$

$$\rho = \rho_1 (l + l_o e) + \rho_2 F - \rho_2 h \qquad \rho_1 < 0;\ \ \rho_2 > 0;\ \ -\rho_1 > \rho_2 \tag{10}$$

$$r = r^{*} + \rho + e^{e} \tag{11}$$

d) <u>Dynamic Equations.</u>

$$dl/dt = (X_q - Z_q)\,e + r_o^{*} l + l_o [r_o^{*} e + (de/dt)/e] \tag{12}$$

$$dF/dt = (1 - C_N)\, r_o^{*}(l + l_o e) + [(1 - C_v)\, l_o - C_r]\,(de/dt)/e - C_r (r^{*} + \rho) - C_w F - C^{a} \tag{13}$$

$$(de/dt)/e = -\rho_1 l - [\rho_2 + (H_w/H_r)]\,F - l_o \rho_1 e + [(1/H_r) + \rho_2]\,h \tag{14}$$

$$\frac{dh}{dt} = \Omega\,(G + r_o b + r b_o - T) \tag{15}$$

Expectational Variables and Flow Relationships:

Our analysis is based on a portfolio model for a small open economy represented by these fifteen equations plus one constraint on the value of Q in (15), to be described below in our discussion about the composition of fiscal deficit financed with money. The consistency between stock-flow relationships is stressed and forward-looking expectations about exchange rate movements is assumed. All variables represent deviations from pre-disturbance situations, except those with subscript "$_0$" which denote pre-disturbance levels.

Expected movements in the real level of the exchange rate, e^e, are assumed to be rational, in the sense of being self-fulfilling, because of the assumption of perfect forseight. This is represented by (1). The term on the right-hand side of this equation stands for the actual rate of change in the real level of the exchange rate, represented by the variable e.

Equation (2) is the conventional goods market equilibrium condition stating that, in deviations from the pre-disturbance situation, the real level of GDP, Y, must be equal to the sum of desired private expenditure, C, public expenditure, G, and net exports, X - Z. In turn, the level of net exports, as represented by (3), is specified as a function of the real level of domestic GDP and of the real level of the exchange rate.

By means of equation (4) we postulate a linearised private expenditure function depending on the levels of disposable income and assume that the propensity to consume out of the three components of private disponible income - GDP, taxes, T, interest earnings, N, and capital gains - do not differ from each other. The real levels of financial wealth, F, and of the domestic interest rate, r, are also among its determinants. We have added the term C^a. This allows us to represent non-permanent autonomous changes in the level of private expenditure with respect to its pre-disturbance level.

The variable N, defined as the real level of interest revenue on non-monetary financial assets, which are either government bonds, b, or foreign assets, l, is represented by (5). In turn, r_0^*, which represents the pre-disturbance real level of the foreign interest rate, is assumed fixed. The financial assets are specified in real terms by deflating their nominal value (expressed in domestic currency) by the producer's price index. As explained below, domestic price changes are assumed away, hence capital gains and losses due to asset price changes are only those represented by $l_0(de/dt)/e$.

Our main concern is the dynamic adjustment path of an open economy with an initial phase of its trayectory characterized by a public sector having a surplus accompanied by a more than compensating reduction in private surplus, thereby exhibiting privately-financed current account deficits. We are particularly concerned with capturing the financial effects of fiscal policy when changes in the level of domestic-currency government bonds are explicitly considered, but not in a predetermined way. In order to simplify the dynamics of a system capable of capturing these effects, we concentrate on the case in which the economic authorities, in the event of a variation of private expenditure engage in an active use of public expenditure to ensure that the real level of aggregate demand remains constant. This assumption is stated by (6). It implies that the variable Y, which represents deviations of GDP from its pre-disturbance level, is zero because it is the composition, but not the level of aggregate demand which changes. It also implies that pressures on the domestic price level due to excess-demand for goods are ruled out by assumption.

Moreover, this simplification allows for a dynamic specification with a desired property: in contrast to the policy rule postulated by other authors,[5] ours implies that the size of the fiscal deficit along the trajectory to a new long-run equilibria is not exogenously specified.

On the other hand, since we are not interested on the potential source of instability associated with the domestic debt service component, we assume that the government increases its flow revenue by means of lump-sum taxes to pay for the additional interest payments. That is, T is a lump-sum tax which is determined according to (7).

Our set of assumptions is useful to simplify the algebra. It ensures that changes in private disposable income can be due only to two effects. The first is changes in the real flow of interest earnings on assets denominated in foreign currency and the other is variations in the real level of these assets due to movements in the real exchange rate.

To analyse the long-run and dynamic properties of the model we consider the case of a temporary change in the autonomously determined component of private expenditure. Viz. a non-permanent positive value of C^a. This component is determined at a value of zero except for a temporary period, at the initial phase of a dynamic trajectory, in which it acquires a positive value. Notice that, according to our equation (6) this change in private expenditure takes place simultaneusly with a fiscal surplus, by means of which public debt is withdrawn. When the autonomous component C^a returns to its initial value of zero, the fiscal budget and the current account remain unbalanced until the intrinsic dynamics of the system lead the economy to a new long-run equilibrium -provided that the required saddle-point properties are fulfilled.

Financial Markets:

In order to consider current account unbalances financed exclusively by the private sector, we assume that the public sector neither finances its deficit with foreign indebtedness nor holds foreign assets. If exogenously determined, foreign financing to the public sector can easily be incorporated. Their explicitly exlusion is analyticaly advantageous. It enables us to represent the discrepancy between the variation in the level of the net stock of assets to be held by the private sector and the change in the level of foreign assets as the counterpart of the financing of the fiscal deficit.

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[5] Sachs and Wyplosz (1984) and Giavazzi et. al. (1985) for example, assume that the government closes an exogenously given fiscal deficit at a given rate, bringing its debt to a previously determined target level. Their rule is specified, in our notation, as $db/dt = m(b^* - b)$. Where b^*, the target level, and m are exogenously determined parameters.

Equality (8), which states that financial wealth is equal to the sum of its components- namely money, h, bonds and foreign-currency denominated assets- holds at any point in time. The demand for money is represented as a function of GDP,[6] of the domestic interest rate and of the real level of wealth. The money market equilibrium condition is therefore given by (9). In our analysis two of the arguments of the demand for money change accros steady-states. These are the real levels of private sector wealth and of the domestic interest rate.

Risk-premium is represented in (10) by ρ. This equation (10) is deduced from a portfolio-balanced model specification following Sachs and Wyplosz (1984). The procedure, as described in the appendix of this paper, is based on a linearization of the equations of a portfolio balanced model, obtaining the risk premium as a function of the relative levels of the foreign and domestic-currency denominated non-monetary assets. F and h appear in (10) instead of b by means of (8).

The long-run levels of the domestic interest rate change when the levels of foreign interest rate and/or when the risk-premium change. During the transition to a new long-run equilibrium, the level of domestic interest rate is also determined by the expectations of exchange rate movements in such a way as to fulfill the covered interest rate parity condition, as specified by (11).

Dynamic Equations:

Variations in the level of foreign assets at a point in time are, as it is represented by (12), induced either by an unbalanced current account or by the revaluation of existing assets denominated in foreign currency.

We have explicitly incorporated a number of assumptions in order to identify variations in private disposable income with only two of its components, namely interest earnings on foreign assets and capital gains and losses due to exchange rate movements. Subtracting from the flow of private disposable income the flow of private expenditure we get an aggregate which corresponds to the change, at a point in time, of the demand for the net stock of privately held financial assets, dF/dt. This is represented by equation (13).[7]

In order to deduce the differential equation which specifies how are changes in the exchange rate determined, we use the money market equilibrium condition

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[6] Notice that it is explicitly assumed that the demand for money is not a function of the flow of interest revenue on non-monetary financial assets. Some authors -e.g. Frenkel and Razin (1987) have included this variable as one of the arguments of the demand for money, thereby adding a further mechanism through which a fiscal expansion affects the market excess demand for money.

[7] Notice that, the following relationships holds -and can be deduced from (12), (13) and (2):

$$dF/dt - [dl/dt + l_o (de/dt)/e] = G + r_o b_t + b_o r - T$$

Public deficits are financed by domestic-currency denominated bonds or by money.

(9) together with the covered interest ite parity condition (11) - *Ex-hypothesis* these two equations hold at any point in time along the transition between an initial point after a shock ocurred and a new long-run equilibrium. Using equation (1) to substitute for e^e in (11) and solving for r with the resulting equation, we can restate (9) by (14).

As will be discussed below, in this model the long-run money market e .lib-ium must be consistent with a level of interest rate determined by the inte. est rate parity condition represented by (11), when expected movements of the exchange rate are zero. For this requirement to be fulfilled, the assumed behavi..ur of the monetary authorities must accordingly be modelled, given that the domestic price level cannot take the rol of the a "sting variable. We therefore assume a behavior for the monetary authorities which ensures that long-run changes in the demand for money are accommodated.

That is, the long deviations of the supply of money with respect to a pre-disturbance level becomes one of the variables endogenously determined by the model. How can this property of be incorporated into our model?

To answer this question we first notice that, along the transition to a new long-run equilibrium, a fiscal defici: .s financed with money and government bonds. Assuming, as in equation (15), that the public sector finances a share, Ω, of its fiscal deficit with money, enables us to have, in the long-run, the required accomodating monetary policy. It will be shown that this is so, provided that this share is equal to the proportion of total change in money with respect to the cumulative sum of fiscal deficits resulting in the long-run. That is, equal in the long-run to the coefficient $h/(h+b)$- which results a magnitude determined by the parameters of the model.

It is worth pointing-out that this assumption does not preclude money market disequilibria neither during the transition to a ne long-run equilibrium nor at the moment that a shock to the system occurs. A change in the level of the supply of money does not necessarily coincide with a variation in the demand for money at the point in time in which the change in supply occurs. In addition, this assumption has the property of allowing us to consider, within an analytically tractable framework, those cases in which a constant price deflator of the money supply is consistent with a money market equilibrium condition.[8]

[8] We follc this procedure in order to avcid an issue highlighted by Branson and Buit (1983). Namely, when the nominal .evel of the money suppy is assumed constant, the long-run level of the domestic price level must change as a result of a policy action which induces a variation in one of the arguments of the m. ey demand function.

II.2 Steady-state representation of the model.

$$r = r^{*} + \rho_1 (l + l_o e) + \rho_2 F - \rho_2 h \tag{11a}$$

$$l = F - (h + b) - l_o e \tag{8a}$$

$$h = [\frac{H_r}{(1+\rho_2 H_r)}]\, \rho_1 (l + l_o e) + [\frac{H_r}{(1+\rho_2 H_r)}]\, [\rho_2 + \frac{H_w}{H_r}]\, F + [\frac{H_r}{(1+\rho_2 H_r)}]\, r^{*} \tag{14a}$$

$$e = -[\frac{r_o^{*} l}{(X_q - Z_q) - r_o^{*} l_o}] \tag{12a}$$

$$F = [\frac{(1-C_N)}{C_w}]\, r_o^{*} (l + l_o e) - (\frac{C_r}{C_w})\, r \tag{13a}$$

Across steady-states the domestic level of interest rate moves only if either the risk premium or the level of the foreign interest rate changes. This is given by (11a), and follows from the interest parity condition and from the postulate that, in steady-state, the level of exchange rate is expected to remain constant. In turn, the level of foreign-currency denominated financial assets must be consistent with the relationship determining the components of financial wealth, hence we have equation (8a).

In addition to these two relationships, we have three equations more for the steady-state solution of the model. These follow from considering (de/dt), (dF/dt), and (dl/dt) to be all equal to zero.

For a constant long-run level of the exchange rate, -i.e. de/dt equal zero- equation (14) provides us with the money market equilibrium condition (14a). In turn, for a constant level of foreign assets and of exchange rate, equation (12) becomes a relationship which states that the current account must be balanced, from this relationship we deduce (12a).

Finally, one of the characteristics of the steady state is that the net stock of privately held financial assets available in the system must coincide with the long-run desired level. This condition is represented by equation (13a) which follows from (13), for the case in which the flow of real savings is equal to zero.

The long-run equilibrium can be analysed on the basis of a system with five reduced form equations, for a given level of government bonds, b, foreign interest rate, r^{*}, and initial levels of foreign asset holdings, l_0. These five equations - (11a), (8a), (12a), (13a) and (14a) solve for the equilibrium values of the net stock of privately held financial assets, F, the real exchange rate, e, the level of assets denominated in foreign currency, *l*, the supply of money, h, and the level of the domestic interest rate, r.

III.1 Analysis of the Steady-state Characteristics of the Model.

Stock-flow Relationships.

The steady-state of the model has a number of characteristics orth mentioning. One of them is that capital gains and losses are zero the component C^a -the variations in the autonomous component of private exper. iture - is constrained to be zero. Hence, substituting for r in the consumption function (4) and using (13a) to solve for F, we get the following equation determining the variations in the level of consumption across steady-states:

$$C = r_o^* \, (l + l_o e)$$

This equation states that the long-run propensity to spend out of disposable income is one. That is, across steady-states changes in private expenditure must be equal to the changes in the level of private disposable income. *Ex-hypothesis* interest earnings on foreign assets are the only determinant of changes in disposable income. GDP is constant throughout the analysis and variations in interest earnings on government bonds are taxed away.

We specified the model in such a way that throughout the analysis, including the steady state, the level of GDP does not differ from its pre-disturbance level. Hence Y in (2) equals zero. This result can be considered to be the counterpart of the policy ru e for public expenditure stated by (6). This policy rule can also be interpreted as stating that the changes in the level of public expenditure across steady states will be zero if the changes in the level of consumption are of the same magnitude, but of opposite sign to the changes in net exports.

Consider now the function representing the long-run desired level of the net stock of privately held financial assets - i.e. equation (13a). The following implications can be deduced from this relationship:

First, due to the "disposable income effect", the private sector will demand a higher level of financial wealth in those cases in which interest earnings on the stock of foreign assets increase. Moreover, there is a given stock-flow relationship between the net stock of financial assets held by the private sector and its disposable income, for those cases in which the level of the domestic interest rate is not affected. This relationship is:

$$\frac{F}{C} = \frac{(1 - C_N)}{C_N}$$

Second, a *ceteris paribus* permanent decrease in the real level of interest rate -e.g. because of a lower risk-premium- induces a reduction in the long-run desired level of the net stock of privately held financial assets. That is, due to the "real interest rate effect" exclusively, the desired level of financial wealth will be lower.

An insightful way to analyse which one of these two effects dominates, as well as other characteristics of our model, is to consider the way in which a change in the cumulative sum of fiscal deficits -i.e. (b+h)- is related to the risk-premium and to other endogenous variables of the model. This analysis is presented next.

Here, we point-out an interesting characteristic of the model: in the general case in which the domestic level of interest rate changes in response to a diferent risk-premium, a **parameter** determines a stock-flow relationship between the net stock of financial assets held by the private sector and its disposable income. This relationship will be shown to be:

$$\frac{F}{C}=\frac{(1-C_W)}{C_W}+[\frac{C_r}{C_W H_r r_0^{*}}][\frac{(\beta H_W-\Omega)}{(\beta-1)}]$$

Where the *parameters* β and Ω will be deduced below.

Effects of Domestic Public Indebtedness on the Levels of the Net Stock of Privately Held Financial Assets and of the Exchange rate

Consider the money market equilibrium condition (14a), using (11a) we can deduce that this market clears when the variation in the long-run level of the domestic interest rate is given by:

$$r=-(\frac{H_W}{H_r})F+(\frac{1}{H_r})h \qquad (11a")$$

This relationship implies a compatibility requirement. Namely, that the level of interest rate at which the money market clears must not differ from the level determined by the interest rate parity condition represented by (11a). This consistency is achieved by means of a restriction to the share of the fiscal deficit which, on the transition to the steady state, is financed with money. That is, it is by means of a constraint on the value of Ω in equation (15) that the variable h, which represents the deviations of the supply of money with respect to a pre-disturbance level, becomes an endogenously determined variables which fulfills the mencioned compatibility requirement.[9] The constraint to the value of Ω is given by:[10]

$$\Omega=\alpha_1\beta-\alpha_2 \qquad (17)$$

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[9]Notice that the long-run solution allows us to have h determined by b in a direct form. With this relationship we can identify the share of money in the cummulative sum of fiscal deficit -i.e h/(h+b)- as a coefficient given by the parameters of the model.

[10]From the financing rule of the public sector (15) the long-run money supply function can be deduced to be $h=\Omega(b+h)$, in turn, given the demand for money and the money market equilibrium condition (14) it is possible to solve for Ω as a function of the parameters of the model.

where:[11]

$$\alpha_1 = \{[H_v + H_r(\rho_1 + \rho_2)] / (1 + H_r\rho_2)\} \qquad \alpha_2 = (H_r\rho_1) / (1 + H_r\rho_2) \qquad \alpha_1,\ \alpha_2 > 0$$

and:

$$\beta = -\left(\frac{(1 - C_N)\, r_o^* - C_r\rho_1 + (C_r\rho_2\alpha_2)}{(C_v + C_r\rho_2 - C_r\rho_2\alpha_1) - (1 - C_N)\, r_o^* + C_r\rho_1} \right) \tag{18}$$

With a flexible exchange-rate regime and current account deficits financed by the private sector exclusively, the variations in the level of the supply of money and of government bonds occur as a result of unbalanced fiscal budgets. This implies that the sum of changes in government bonds and money, (b + h), must necessarily correspond to the cumul ve sum of fiscal deficits along the transition between the pre-disturbance equilibrium situation and the new long-run equilibrium.[12]

We can restate our equations (8a), (11a), (12a), (13a), and (14a) as follows, in order to present our model in terms of effects attributed to the long-run implications of the total sequence of fiscal unbalances:

[11] We represent the parameters α_1 and α_2 with a positive value. For this to be the case, the following inequality must hold:

$$(1 + H_r\rho_2) > 0$$

This inequality is included because of its implications for the equation determining the expected rate of variation n the level of the exchange rate. It ensures that an initial increase in the supply of money resulting from an open market operation induce an instantaneous decline of the domestic interest rate and an expected apreciation of the exchange rate.

[12] By construction, changes in the real value of financial assets by means of inflation are eliminated.

$$r = -\left(\frac{H_w\beta - \Omega}{H_r}\right)(b+h) + r^* \qquad \beta \leq \frac{\Omega}{H_w} \tag{11b}$$

$$l = -(1-\beta)(b+h) - l_o e \tag{8b}$$

$$h = \Omega(b+h) \tag{14b}$$

$$e = -\left(\frac{r_o^*}{X_q - Z_q}\right)F + \left(\frac{r_o^*}{X_q - Z_q}\right)(b+h) \tag{12b}$$

$$F = \beta(b+h) \qquad \beta < 1 \tag{13b}$$

We concentrate on those cases in which the level of foreign interest rate is constant - i.e. r^* in (11b) is assumed to be zero. Hence, long-run reductions in the level of the domestic interest rate are atributted exclusively to the smaller risk-premium implied by a reduction in domestic public indebtedness, viz, reductions in (b+h).

Given the demand for money and the financing rule of the public sector -as it is represented by (15)- we can restate the money market equilibrium (14a) by (14b) provided Ω is given by the value stated by (17).- A constraint we have imposed to the financing behaviour of the public sector.

Equation (8a) indicates that the long-run level of the net stock of privately held financial assets must be equal to the sum of its components. Using this equation we can substitute for $(l + l_o e)$ in (12a) in order to restate it by (12b).

Equation (13b) follows from (13a) after a number of substitutions and for a value of beta, which will be shown to be equal to (18). With (13b) we can restate (8a) by (8b).

When stocks and flows -that is F and C- move in the same direction it is infered that the disposable income effect on the demand for net stock of financial assets is not outweighted by the real interest rate effect. Whether this will be the case depends on the value of β in (13b), to be discussed below.[13]

The value of β in equation (13b) is negative in those cases in which the above discussed disposable income effect dominates the interest rate effect on the demand for the net stock of privately held financial assets. This will be the case in which domestic and foreign non-monetary assets are close or very close

[13]Notice that stock-flow relationship between the net stock of financial assets held by the private sector and its disposable income can be represented by $\beta/r^*(\beta-1)$.

substitutes in private portfolio- A case captured, in this model with lower values of ρ in the equation determining risk premium, namely (10). On the other hand, in those cases in which a reduction in public indebtedness induces reduces the domestic interest rate to such an extent as to outweight disposable income effects, β will be positive.

To deduce the value of β in equation (13b), we first use equations (11a), (12a) and (14a) to solve for the levels disposable income and interest rate in equation (13a) -which determines the level of the net stock of privately held financial assets. From this substitution we get:

$$F = -\left(\frac{(1-C_y)\,r_o^{*}-C_r\rho_1}{C_w+C_r\rho_2-C_r\rho_2\alpha_1}\right)\left(\frac{X_q-Z_q}{r_o^{*}}\right)e-\left(\frac{(C_r\rho_2 H_r\rho_1)}{(C_w+C_r\rho_2-C_r\rho_2\alpha_1)\,(1+H_r\rho_2)}\right)(b+h) \qquad (13c)$$

The resulting equation together with the relationship determining a balanced current account- equation (12b)- constitute a semi-reduced form system of two equations determining the equilibrium values of e and F, for given values of the cumulative sum of fiscal deficits (b + h). In order to consider how are the variables F and (b+h) related, we use these two equations to get:

$$F = -\left(\frac{(1-C_N)\,r_o^{*}-C_r\rho_1+(C_r\rho_2\alpha_2)}{(C_w+C_r\rho_2-C_r\rho_2\alpha_1)-(1-C_N)\,r_o^{*}+C_r\rho_1}\right)(b+h) \qquad \textbf{(13d)}$$

Consider now equation (12b). From this equation and (13b) follows that, for values of β less than one, the effect of a reduction in long-run domestic public indebtedness, is a real exchange rate appreciation,-i.e. (b + h) is positively related with e. In turn, the condition ensuring a value of β less than one is:

$$(1-C_N)\,r^{*} < C_W \qquad (19)$$

In the appendix we identify this condition among the saddle-point requirements of the model. That is, among the conditions which determine that the model has only one unstable root, thereby ensuring the stability of the system.

Notice that when this condition holds, the denominator of β in (18) is positive. The sign of the numerator, in turn, is determined by the parameters representing the degree of asset substitutability -i.e the ρ. That is, value of β is negative for relative low values of ρ.

A Graphical Analysis

From our analysis follows that the effect of a reduction in long-run domestic public indebtedness is a real exchange rate appreciation. We now rely on a graphical analysis to consider its effects on the net stock of privately held financial assets.

Equation (13c) can be considered to be a relationship determining the combination of long-run equilibrium levels of e and F at which the available level of the net stock of privately held financial assets is equal to the long-run desired level by the private sector.- For given values of (b + h). In turn, re-arranging terms, equation (12b) can be represented by:

$$F = -[(X_q - Z_q)/r_o^*]e + (b+h) \qquad (12c)$$

By means of equation (12c) we represent the schedule EE. This schedule determines the combination of long-run equilibrium values of e and F for which the current account is balanced. -For given values of (b + h). The slope of the EE schedule is unambiguosly negative, as it is represented in Fig 1.

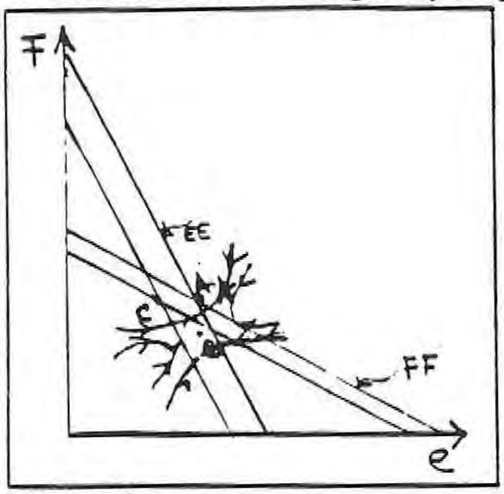

Figure 1

In figure 1 we also represent, using (13c), the FF schedule as a negatively sloped relation. We concentrate on the case in which the slope of the FF is negative. We follow these procedure because, as it is shown in the appendix, the sufficient conditions for the saddle-point stability conditions of the model are fulfilled. The two conditions determining a negatively sign slope represent cases in which either non-monetary assets are perfect substitutes or the degree of imperfect substitutability is moderate, these are:

$$\rho_2 < -\frac{C_w}{C_r} \qquad (20)$$

$$-\rho_1 < -[(1-C_N)/C_r]r_o^* \qquad (21)$$

These conditions establish a limit for the sensitivity of parameters which determine how the risk-premium on domestic assets changes when the composition of assets in the portfolio changes. Hence we define, based first on (20), cases with a low degree of imperfect asset substitutability as those in which the parameter representing the sensitivity of the risk-premium to the variations in the level of government bonds in the portfolio, ρ_2, is smaller than another parameter. This latter parameter is given by the inverse of the partial derivative of the long-run desired net stock of privately held financial assets with respect to the real level of the domestic interest rate, which is equal to $-C_w/C_r$.

In turn, condition (21) enables us to establish an upper limit for the absolute value of the sensitivity of the risk-premium to changes in the level of foreign assets, $-\rho_1$. This limit can be specified in terms of partial derivatives of the long-run desired level of the net stock of privately held financial assets and of the foreign level of interest rate.

The absolute value of slope of the FF schedule is smaller than corresponding one of the EE schedule -as it is represented in figure 1-, when the following inequality holds:

$$\frac{C_w + C_r\rho_2 - C_r\rho_2\alpha_1}{(1-C_N)r_o^* - C_r\rho_1} > 1 \qquad (22)$$

When (21) holds the denominator of inequality (22) is positive, hence this latter inequality can be restated as:

$$(C_r/C_w)\ (\rho_2+\rho_1) - (C_r\rho_2\alpha_1/C_w) > [(1-C_N)\,r_o^*/C_w] - 1 \qquad (22a)$$

The left-hand side of inequality (22a) has a positive value.[14] In turn, the right-hand side has a negative value, since a sufficient condition for this to e the case is (19), a stability requirement which we assume to hold.

ccording to equations (12c) and (13c) both schedules shift to the left when the ulative sum of fiscal deficits creases. Consider an initial point A in figure 1. For the size of the shift of the FF schedule to be larger in absolute magnitud than the size of the shift of the EE schedule, and hence for the new long-run equilibrium to be at point C in the figure, the following requirement must be fulfilled[15]:

$$-\left(\frac{(C_r\rho_2\alpha_2)}{(C_w+C_r\rho_2-C_r\rho_2\alpha_1)}\right) < 1 \qquad (32)$$

Substituting for α_1 and α_2 as stated in (17) and simplifing terms, these requirement can be represented by:

$$\rho_2 < -[(\frac{C_w}{C_r})]\ [\frac{(1+H_r\rho_2)}{(1-H_w)}] \qquad (32a)$$

By means of (20) an upper limit for the sensitivity of risk-premium to the changes in domestic currency denominated assets was specified. In addition of this condition, (32a) is satisfied, provided that the following inequality holds as well:

$$-(H_w/H_r) < (-C_w/C_r) \qquad (25)$$

This condition, as shown in the appendix, constitutes one of the requirements ensuring that the long-run equilibrium of the model has the required saddle-point characteristics.

In order to consider the sign of β in equation (18) we firstly point-out that the denominator of this parameter is positive in those cases in which the slope of the FF schedule is smaller in absolute value than the slope of the EE schedule. Hence, after re-arranging terms, we get that the combination of parameters which imply that the variables F and (b+h) are negatively related across steady-states is:

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[14]The parameter determining the sensitivity of the risk-premium to variations in the level of foreign assets, ρ_1, is negative and greater in absolute value than ρ_2 the parameter related to variations in the level of government bonds.

[15]When non-monetary assets are perfect substitutes it is only the FF schedule which shifts.

$$C_r\rho_1/(1+H_r\rho_2) < (1-C_N)\,r_o^* \qquad (24)$$

This inequality can be re-expressed by:

$$-\rho_1 < -[(1-C_N)/C_r]\,r_o^*\,(1+H_r\rho_2) \qquad (24a)$$

Comparing (21) with (24a) we can infer that the latter requires a lower sensitivity of risk-premium than the former one, since the combination of parameters $H_r\rho_2$ -although less than one in absolute terms- has a negative sign.

We conclude that the value of β in (18) is negative when non-monetary assets are not such imperfect substitutes so as to violate conditions (24a) or (20). In these cases changes in the real level of domestic public indebtedness and the variations in the real level of the net stock of privately held financial assets are negative related across steady states. That is, the "income effect" on the demand for financial wealth by the private sector dominates the "interest rate effect".

III.2 Impact exchange rate adjustments resulting from private dissavings cum-contractionary fiscal policies.

A current account deficit will be registered even if there is a fiscal surplus, provided that, as a result of the shock to the system, the level of expenditure of the private sector is greater than its disposable income and in addition private expansion outweights public contraction in aggregate demand.[16]That is, provided private dissaving outweights public sector saving.

When the exchange rate is flexible, an initial current account deficit must be the counterpart of an impact exchange rate appreciation and of a capital account surplus. In turn, the incipient capital inflows which lead the capital account of the balance of payments into a surplus must correspond to the financing of the private sector dissaving which does not have as a counterpart the fiscal surplus.

A point to be stressed is that these incipient capital inflows - which are due to the desire by the private sector to change the composition of its financial portfolio- are induced by the behaviour of forward-looking agents. That is, the resulting movements of the exchange rate are dependent of what agents foresee as the transitional and long-run effects of the shocks to the economy. Because of the forward-looking nature of this behavior, reflected in the financial markets, the movements in the level of the exchange rate are identified with those of the non-predetermined or "jumping" variable of the model. Hence, as it is ilustrated in figures 1 and 2, when an unforseen shock occurs, the level of exchange rate exhibits an initial discrete jump, from A to B, to a point on the saddle-path trayectory, thereafter moves continuously until the economy reaches its new long-run equilibrium. Moreover, because of the characteristics of the non-predetermined variables, the initial discrete jump of the exchange rate will take

[16]This follows from the goods market equilibrium condition.

place even in those cases in which the shocks have not taken place yet, but the "news" of their future occurrence arrive.

An exchange rate appreciation can happen together with either an increase or a fall in the level of domestic interest rate, depending on the expected direction of the movement of the exchange rate. This follows from the covered interest rate parity condition, as it is represented by (11). However, the movements in the domestic level of interest rates must also be consistent with the clearing process in the money market.

Therefore, short-term movements in the level of the domestic interest rate, resulting from unexpected shocks, can be determined, in this model, on the basis of initial adjustment on the money market. In those cases in which, *ex-hypothesis*, neither the level of the nominal supply of money nor the level of its deflator are altered by an initial shock, the adjustment in this market must occur through compensating movements of the arguments of the demand for money. These arguments are the levels of the net stock of privately held financial assets and of the domestic interest rate.

Initial positive wealth effects in the demand for money, occurring because of a downward-jump in the exchange rate implies that this latter variable must be expected to move towards its pre-disturbance level. That is, an expected exchange rate depreciation must be the counterpart of the increase in domestic interest rate which clears the money market. On the other hand, an initial decline in the level of the domestic interest rate -associated with an expected exchange rate appreciation can occur along with an impact exchange appreciation. This case can happen when an increase of private expenditure is accompanied by an initial excess supply of money.

In our model the real level of the net stock of privately held financial assets at an initial point of the traverse to a new long-run equilibrium can differ from its pre-disturbance level. This is so because part of this stock is denominated in foreign-currency and the exchange rate exhibits an initial discrete movement.[17] This characteristic enables us to analyse the case in which an excess supply of money is associated with a decline in the real level of the net stock of privately held financial assets, because the private sector is a net debtor in foreign currency. We suggest, however, that the results can be generalized by relaxing our assumptions of no capital gains and losses on non-monetary assets due to variations in the domestic level of interest rate or when the price deflactor of the money supply is affected by exchange rate movements.

The effects of initial variations in the real level of the financial wealth of the private sector must be explicitly considered not only in the money market, but in the goods market as well. The effects on the goods market are associated

[17] The revaluation of the private debt commitments as a channel through which the exchange rate influences the initial level of GDP has been investigated by Frenkel and Razin (1987).

with wealth effects on private expenditure. As it is represented in (4), changes in the real level of wealth have two effects on private expenditure, one direct and the other indirect via induced movements in interest rates. In order to establish which of them dominates, we rely on one of the conditions for the long-run equilibrium of the model to have the required saddle-point characteristics. This condition, specified as (25) can be expressed as:

$$C_v - C_r(H_w/H_r) > 0 \qquad (25a)$$

As it has already been discussed, an initial increase in the real level of the net stock of privately held financial assets induces, via adjustments in the money market, an increase in the level of the domestic interest rate. We interpretate condition (25a) as implying that the direct expenditure-inducing wealth effect on private expenditure must not be outweighed by the expenditure-reducing interest rate effect.

An initial exchange rate appreciation can ocurr in our model even when the private sector is a net debtor. For this case to happen, the following situation must occur in the goods market: An autonomously determinied increase of private sector expenditure, C^a, must be of an amount which is greater than the sum of the absolute value of two expenditure reducing effects - so as to have a current account deficit: the induced negative net stock effect, which follows because condition (25a) is postulated to hold, and the initial fiscal contraction which follows from the assumed behaviour stated in (6). When the private sector is a net creditor in foreign currency the induced positive wealth effect will be positive, thereby requiring an autonomously determined private expenditure of a reduced magnitud to lead to this result.

III.3 A non-monotonic convergence path.

In the previous section we analised the characteristics of the long-run equilibrium of our model. We concluded that the long-run relationship between changes in the level of domestic public indebtedness, (b+h), and in the real levels of exchange rate, e, was negative.

In a model with forward-looking expectations like ours, this has the following implication: when agents foresee that, as a result of the future use of fiscal instruments, a long-run decrease in the level of domestic public indebtedness will occur, they expect, along with this result, that the long-run real exchange rate level will result appreciated with respect to its pre-disturbance level.

When this long-run implication is considered as part of case with a short-run case in which private sector dissaving more than compensates public sector savings, the following question arises, because of the initial exchange rate appreciation associated to the current account deficit which this short-run scenario implies:

How can a short and long-run real exchange rate appreciation be possible, given the constraint requiring long-run variations in interest earnings on foreign assets, resulting from a different net foreign asset positon of the economy- to be matched with net exports at a post-disturbance steady state?

Our model highlights the implications that this question has for the relationship between initial, transitional and steady-state · ations in the level and composition on the net stock of privately held fina... al assets, as well as for the trayectory of the exchange rate towards its new steady-state.

The answer to this question is that the trayectory of the exchange rate must be such as to enable the effects due to the initial sequence of current account deficits to be more than compensated by those of a sequence of current account surplus, which is going to be followed by final sequence of current account deficits. The dynamic characteristics of this trayectory rules-out a monotonic convergence path for the real exchange rate to its new long-run equilibria.

The reason for this cyclical or non-mo tonic behaviour is the llowing one:

On the one hand, we have that, whenever the real exchange rate is below its pre-disturbance level (i.e. appreciated), the current account is in deficit and, consequently, a reduction in the level of foreign assets held by the private sector takes place. This is illustrated in figure 3. On the other hand, since a long-run exchange rate appreciation is associated to a net level of foreign assets at the new steady-state resulting higher than at its pre-disturbance situation, there must necessarily be an e.. tual reversal of the movement in the level of foreign assets during the transit on to the long-run (point E in figure 3). This reversal must also occur in the eal level of the net stock of privately held financial assets when the value of β in (18) is negative. That is, when assets are close of perfect substitutes.

For these reversals to ocurr, the convergence path of the real exchange rate must be non-monotonic Thereby enabling changes in the level of foreign assets owing to a sequence of current account surplus to be larger than the absolute value of the change in assets owing to current account deficits. The non-monotonic behaviour of the exchange rate is illustrated in figures 2 and 3.

On impact, due to the forward-looking expectations, the exchange rate exhibits a discrete downward movement from point A to point B as a result of the shock. The domestic level of interest ajusts as well so as to clear the money market and enable ortfolio adjustments desired by agents which foresee the transitory and long-run effects of the shock.

With this initial level of (appreciated) exchange rate, there is a current account deficit reflecting short run private dissavings more than compensating public sector savings. Incipient capital inflows finance the dissavings of private sector which do not have as a counterpart savings of the public sector. This discrepancy corresponds to the current account deficit and to the initial capital account surplus.

It must be pointed-out that the initial shock to the system can be either an unforseen event or only the "news" of its future occurence.[18] In both cases the

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[18] In the case of "news" that the shocks will occur in the future, the initial jump of the exchange rate would not be to the saddle-path trajectory. it would only be at a point in this path when the shock actualy takes place. Cfr.

exchange rate exhibits a discrete movement.

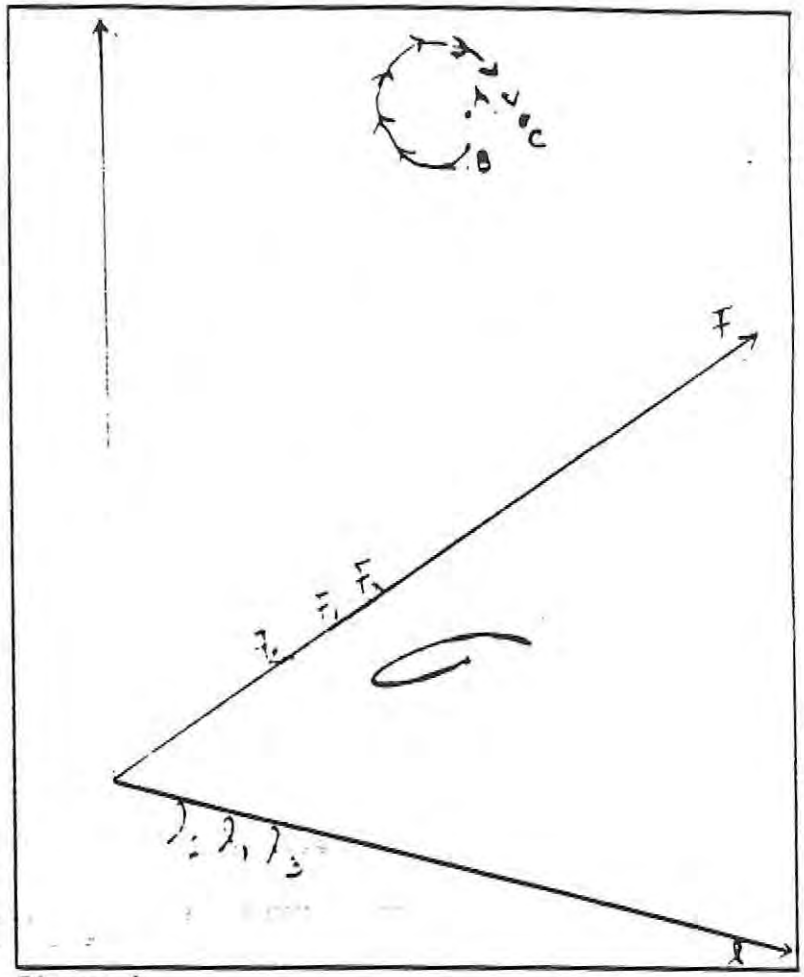
Figure 2

As time elapses and an initial series of current account deficits reduces the net foreign asset position of the private sector from l_1 to l_2, the total level of private sector financial wealth is diminished from F_1 to F_2. This private decumulation is not only caused by current account deficits, but also by the fiscal surplus which occurrs simultaneusly with the private sector dissaving.

Along with the reduction in the level of the financial wealth of the private sector its disposable income diminishes too. At some point, its expenditure is adjusted downward in line with the lower levels of disposable income and of financial wealth. This induced behaviour implies that the private sector eventualy becomes a net saver. This behaviour can also be seen as the counterpart of a desire by the private sector to increase the level of its financial wealth in order to achieve a targeted stock-flow ratio.

For this increase in privately held financial wealth to be possible, the current account deficit must turn into a surplus.

At some point of the trajectory the level of foreign assets stops falling and starts to increase. Here the exchange rate has not only return to its pre-disturbance level, e_0. but, is already above it. This is so because a depreciation with respect to the pre-disturbance level is required in order to induce a current account surplus.

With a current account surplus the net stock of foreign assets held by the private sector increases. These surplus are required not only to make the higher levels of wealth by the private sector possible. There is an additional reason as well. Namely, to make the long-run exchange rate appreciation possible. This

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Wilson (1979).

appreciation cannot ocurr -as stated by our equation (12a), unless the flow of interest earnings from foreign assets is higher than its pre-disturbance equilibrium level. Notice, that this will happen only if the effects of initial sequence of current account deficits of the dynamic trajectory is more than compensated by those of the sequence of current account surpluses. When this happens, the new steady-state level of foreign assets, l- in figure 2, would be to the right of the initial level.

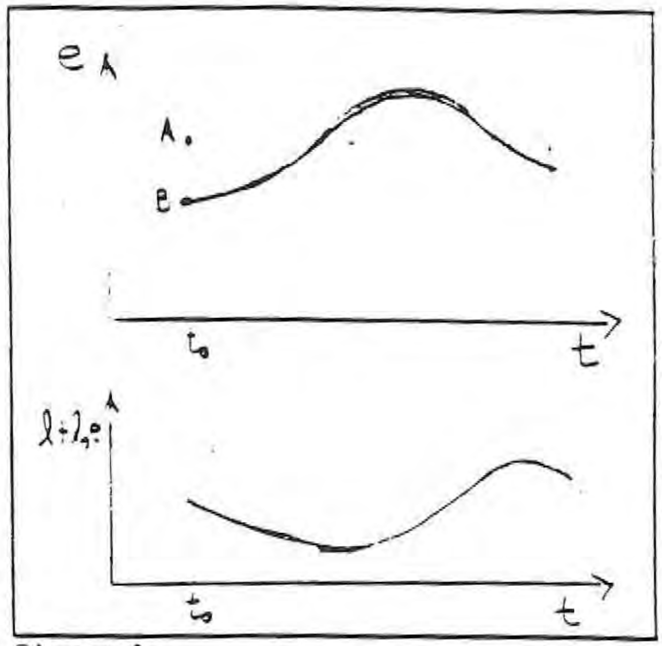

Figure 3

At the new steady-state, point C in the diagram, the exchange rate achieves a level which implies an appreciation with respect to its pre-disturbance level - point A - and, in this case, a depreciation with respect to point B - the initial point of the traverse to a new long-run equilibrium. That is, there is an initial overshooting of the permanent exchange rate appreciation induced by a the short run private expansion and fiscal actions.

A point we wish to emphasize is that exchange rate models with forward-looking characteristics, such as the one deployed in this paper, must exhibit saddle-point stability. This implies that one root of the dynamic system must be unstable, being the other ones the relevant roots determining the dynamics of the system when the shocks have ocurred. Hence for a non-monotonic behaviour of the exhange rate - such as the one represented in figure 2- not to be unadvertentedly excluded from a model, the following is a necessary condition:
It must have at least three dynamic equations. Thereby enabling the capturing of cyclical movements of the exchange rate by means of the two stable roots which determine the dynamic trajectory of the system. The work presented here helps to make progress in this direction, since most analytical models addresing the problem of exchange rate behaviour do not fulfill this condition.

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APPENDIX

Analysis of the Dynamic Properties of the System Matrix.

The dynamic structure of our model can be represented by:

$$\begin{bmatrix} dl/dt \\ dF/dt \\ de/dt \end{bmatrix} = \mathbf{A} \begin{bmatrix} l \\ F \\ e \end{bmatrix} + \mathbf{B} \begin{bmatrix} r^{*} \\ C^{a} \end{bmatrix}$$

Where:

$$\mathbf{A} = \begin{bmatrix} r_o^{*} - l_o(\rho_1+\delta) & -l_o[\rho_2+(H_w/H_r)-\delta] & (X_q-Z_q)+l_o[r_o^{*}-l_o(\rho_1+\delta) \\ a_{21} & a_{22} & a_{23} \\ -\rho_1-\delta & -[\rho_2+H_w/H_r)]+\delta & -l_o(\rho_1+\delta) \end{bmatrix}$$

and:

$$\delta = [\rho_2 + (1/H_r)]\Omega$$
$$a_{21} = (1-C_N) r_o^{*} - l_o(1-C_v)(\rho_1-\delta) - (C_r\Omega/H_r)$$
$$a_{22} = C_r(H_w/H_r) - C_w - l_o(1-C_v)[\rho_2+(H_w/H_r)+\delta] + (C_r\Omega/H_r)$$
$$a_{23} = l_o[(1-C_N) r_o^{*} - l_o(1-C_v)(\rho_1+\delta)] - l_o(C_r\Omega/H_r)$$

The first differential equation of the system is given by the balance of payments relationship.- i.e. (12). Using (14) to substitute for (de/dt)/e we get:

$$\frac{dl}{dt} = (r_o^{*} - l_o\rho_1)l - l_o[\rho_2+(H_w/H_r)]F + \{(X_q-Z_q) + l_o[r_o^{*}-l_o\rho_1]\}e + l_o[(1/H_r)+\rho_2]h \quad (27)$$

A second differential equation is the one determining the variations in the real level of the net stock of privately held financial assets. This is our equation (13), which after substituting for (de/dt)/e, e^{e} and ρ by means of (14), (1) and (10)- becomes:

$$\frac{dF}{dt} = [(1-C_N) r_o^{*} - l_o(1-C_v)\rho_1]l + \{C_r(H_w/H_r) - C_w - [l_o(1-C_v)]([\rho_2+(H_w/H_r)\}F + l_o[(1-C_N) r_o^{*} - l_o(1-C_v)\rho_1]e - \{l_o(1-C_v)[\rho_2+(1/H_r)] - (C_r/H_r)\}h \quad (28)$$

Our third differential equation is given by equation (14) in the text, which determines the behaviour of the exchange rate on the basis of the clearing of the money market.

We linearise (de/dt)/e by (de/dt)/e_0 and for convenience postulate the pre-disturbance level of the real exchange rate, e_0, equal to one. In addition, the following relationship - which holds at any point in time because of the postulated financing behaviour of the public sector, as represented by (15):[19] - is being used:

Requirements for the steady-state equilibrium to be a saddle-point

[19] Notice that the corresponding time subscripts of h, b, l and e, to indicate points of the trajectory, have been ommited for simplicity.

$$h=\Omega[(F-(1-l_o e)]\qquad(29)$$

Due to rational expectations about the exchange rate movements, the long-run equilibrium of the model must have saddle-point properties to ensure that, after an initial jump, the system will converge to a new long-run equilibrium. Therefore, this three differential-equations system requires two stable roots for its long-run equilibrium to have saddle-point properties. That is, It must have only one unstable root.

A positive value for the determinant of matrix **A** is one of the necessary conditions for the system to have only one unstable root.[20] A positive sign for the determinant of **A** is not sufficient to rule-out the case of three unstable (positive) roots. We must therefore consider complementary procedures based on a negative sign of the trace of **A** and on the sign of the sum of the determinants of its three 2x2 minors.

Conditions under which the steady-state equilibrium of the system has saddle-point properties.

It can be demonstrated that the following is a necessary and sufficient condition to ensure that the determinant of matrix **A** has a positive sign:

$$(1-C_N)r_o^*\rho_2+C_v\rho_1+(H_w/H_r)[(1-C_N)r_o^*-C_r\rho_1]<0\qquad(30)$$

Arranging terms this inequality can be re-expressed by:

$$-(H_w+H_r\rho_2)<[H_r\rho_1/(1-C_N)r_o^*)][C_v-C_r(H_w/H_r)]\qquad(30a)$$

Condition (19) presented in section III.2 of the text ensures that the saddle-point stability condition (30a) holds, for those cases in which imperfect assets mobility is not strong, as it is represented by (21) in section III.3.

If the sign of the determinant of matrix **A** is positive, then a sufficient but not necessary condition for one unstable root is its trace having a negative sign. The condition for the trace of **A** to have a negative sign is given by:

$$r_o^*-2l_o\rho_1-l_o[(1-C_v)[\rho_2+(H_w/H_r)]+C_r(H_w/H_r)-C_w<0\qquad(31)$$

This requirement can alternatively be represented by:

$$r_o^*+C_r(H_w/H_r)-C_w<l_o(\rho_1+\rho_2)+l_o(\rho_1-C_v\rho_2)+l_o(1-C_v)(H_w/H_r)\qquad(31a)$$

Hence, when the initial value of assets denominated in foreign currency, l_o, is zero the trace will have a negative sign if the following condition holds: Since the value of $-\rho_1$ has a larger value than ρ_2, we can infer infer two

[20] This statement follows because the product of the roots of the system has the same sign of the determinant of this matrix.

$$C_w - C_r(H_w/H_r) > r_o^{\cdot} \tag{32}$$

additional propositions: a) when the initial value of assets denominated in foreign currency has a negative value, (32) is a sufficient, but not necessary condition for the trace to have a negative sign. b) with this criterion the possibility of three unstable roots cannot be rejected in those cases in which the initial level of foreign assets is positive.

Notice that a negative trace is a sufficient but not necessary requirement for only one unstable root. Hence, if it is not fulfilled it does not preclude the use of other complementary conditions. Therefore, we can turn to the third criterion mentioned above.

This last criterion, based on the sum of the determinants of the 2x2 principal minors of matrix **A**, allows us to discard the possibility of three unstable roots in those cases in which the parameter C_w is greater than the product of $C_r(H_w/H_r)$. That is, in those cases in which wealth effects dominate over interest rate effects, in both consumption and demand for money, viz. $C_w/H_w > C_r/H_r$.

A negative value of the determinant of **A** is a necessary condition for the long-run of the model to be a saddle-point. As it can be seen from the condition which determines whether this would be the case - (30a), when a *ceteris paribus* increase in privately held financial assets induces a reduction in the demand for money -i.e. when $(H_w + H_r\rho_2) < 0$ - the determinant of **A** will not have the required sign unless the following condition holds:

$$C_w - C_r(H_w/H_r) > 0 \tag{33}$$

As just mentioned, the fulfilment of this inequality is also required to guarantee the sufficient condition for only one unstable root when the combination of parameters given by $(H_w + H_r\rho_2)$ is either positive or zero and the initial level of foreign assets is either zero or negative, but small.

Interpretation of the Saddle-point Stability Conditions

A priori restrictions based on theoretically sound arguments can be used to establish that side conditions -such as our inequality (33)- must hold.[21] The explicit inclusion of the money market as one of the components of the model, together with its consistent stock-flow equilibrium in the long-run, enable us to posit that a parameter C_w smaller than the absolute value of $C_r(H_w/H_r)$ is incompatible with the following requirement:

$$-H_w(C_r/C_w) + H_r < 0 \tag{C.3a}$$

This inequality implies that, if a shock to the <u>long-run</u> equilibrium of the

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[21] Notice that (C.1) indicates that the trace can have a negative sign even if C_w - $C_r(H_w/H_r)$ has negative value, provided that the pre-disturbance level of foreign assets is large and negative. In turn, as stated by (30a), when the combination of parameters given by the sum of H_w and $H_r\rho_2$ is positive the determinant of **A** could still have a positive sign even if (33) does not hold.

system produces a *ceteris paribus* increase in the level of the domestic interest rate, this increase must induce a decline in the demand for money.

In order to interpret the requirement for saddle-point stability represented by (33) in terms of its implications for the flow relationships of the model, two issues must be considered.

On the one hand, the explicit inclusion of the money market equilibrium condition in our model implies that the equilibrium changes in the level of the domestic interest rate must be compatible with the clearing process in this market. Hence, a *ceteris paribus* change in the level of government bonds, b, induces a variation in the level of the interest rate which is determined by $-(H_w/H_r)b$.

On the other hand, in this model a *ceteris paribus* increase in the level of government bonds increases the net wealth of the private sector. This implies that an increase in the level of government bonds induces a positive response of the level of consumption due to a wealth effect determined by the parameter C_w.

The variation in the level of consumption due to a change in the level of government bonds will be determined by two effects. One due to a change in the level of wealth, the other to a change in the level of the interest rate. Therefore, (33) can be interpreted as a condition which states that as the level of government bonds increases, the level of private consumption must increase.

In general, the requirement represented by (33) can be considered to imply that, if due to an initial increase in the real level of the net stock of privately held financial assets the domestic interest rate increases, the level of private expenditure must increase as well because the expenditure-inducing wealth effect must not be outweighed by the expenditure-reducing interest rate effect.

When this requirement is not fulfilled and the combination of parameters given by $(H_w + H_r\rho_2)$ is negative, the model is completely unstable. A shock to the system -or news of its future occurrence- would imply an explosive path for the endogenous variables.

On the other hand, (33) has the following implication. A fiscal deficit occurring simultaneously with a current account surplus increases the level of the net stock of privately held financial assets. This increase in financial wealth induces a higher level of private expenditure. However, if (33) does not hold, a fall in private expenditure could be induced. This fall in private expenditure could lead to a larger size of the fiscal deficit and of current account surplus which would cause an even larger increase in the level of government bonds and of foreign assets. This, in turn, would induce a further fall in private expenditure. This process could go on in an unending explosive process.

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