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RESUELVE: A GAUSS PROGRAM TO SOLVE APPLIED EQUILIBRIUM AND DISEQUILIBRIUM MODELS*

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ABSTRACT

This paper contains a *GAUSS* program that can be used, among other things, to find equilibria for applied general equilibrium models, and fix-price equilibria for general non-Walrasian models. Simple applications of these two cases, as well as to linear and quadratic programming, are also provided.

*This work represents the first stage of research in progress, kindly supported by México's Secretaría de Comercio y Fomento Industrial, on the effects of changes in tariffs for economies with price rigidities. The program contained in this paper can be freely used for teaching, research, or any other noncommercial purpose.

1. INTRODUCTION

This paper provides a computer program, written by the author in *GAUSS*, that solves linear and nonlinear complementarity problems. It can be used, in particular, to find equilibria for applied general equilibrium models, as well as fix-price equilibria for general disequilibrium models. Simple applications of these two cases, as well as to linear and quadratic programming, are also provided.

The plan of the paper is as follows: Section 2 introduces the linear complementarity problem, an algorithm for solving it, and two applications. Section 3, on the other hand, presents the nonlinear complementarity problem, both in its classical and generalized versions, and an algorithm to solve it. Section 4 then shows how to cast general equilibrium models in a nonlinear complementarity framework, and also solves a particular example. Section 5 does the same for the case of general fix-price models, and solves a simple model à la Barro-Grossman-Malinvaud. Finally, Appendix A presents the computer code of the program, Appendix B presents the results for the model in Section 4, and Appendix C presents the ones for the model in Section 5.

A final point before starting: Our program was purportedly written to be used for problems in general form (Walrasian and non-Walrasian), and also to be as compact as possible (the computer code contains only 128 lines). For some particular models, however, the reader will surely be able to enhance the performance of the program by modifying it in an *ad-hoc* fashion. For instance, in the case of general equilibrium models, Mathiesen (1985a,b) shows how to take advantage of the sparsity of a matrix that plays a key role in the algorithm. To conclude, the reader is encouraged to modify the program when computer time is a constraint.

2. AN ALGORITHM TO SOLVE LINEAR COMPLEMENTARITY PROBLEMS

This section presents a description o ..e linear complementarity problem, together with Lemke's (1965) algorithm for its solution. It is worth · ..ting out, however, that Lemke's method is not the only procedure available (see, e.g., Murty's, 1974, and the references in Murty, 1988). We have chosen it simply because of its widespread popularity and easiness of implementation. In the rest of the section we illustrate the use of the program by applying it to linear programming and quadratic programming. For more applications of the linear complementarity problem, see the delightful book by Murty (1988).

2.1 The Linear Complementarity Problem

Consider the following problem:

(LCP) Given an n×1 vector q and an n×n matrix $\mathbf{M}$ find $w \geq 0$ and $z \geq 0$ such that $w = \mathbf{M}z + q$ and $w'z = 0$.

Because of its linear structure and the fact that the entries of w and z have to be complementary in the sense that $w_i z_i = 0$ $(i = 1,\ldots,n)$, the problem given above is called in the literature the linear complementarity problem (LCP). Given that optimization problems as common as linear programming, quadratic programming, and two-person nonzero sum games can be casted as linear complementarity problems, Cottle and Dantzig (1968, p. 103) refer to (LCP) as the "fundamental problem".

In order to solve (LCP), the program *RESUELVE* contains a procedure based on Lemke's algorithm of solution (Lemke, 1965, and Cottle and Dantzig, 1968). Briefly, and borrowing freely from the terminology used in the simplex algorithm

(see, e.g., Intriligator, 1971), the computational steps involved in Lemke's method of solution can be described as follows: Let e be the n×1 vector of ones and consider the related problem

$$\text{(RP)} \qquad w - Mz - ez_0 = q, \quad w \geq 0, \; z \geq 0, \; z_0 \geq 0, \; w'z = 0.$$

where z_0 is a new variable added to the problem. Obviously any solution with $z_0 = 0$ to this new problem is a solution to the original one.

Set now $z = 0$ in (RP). If $q \geq 0$, the solution is clearly given by $w = q$; otherwise, let q_r be the smallest negative component of q and set $z_0 = -q_r$. The initial basis is then given by z_0 and all the entries of the corresponding $w \geq 0$ except for w_r (which is obviously zero). Denoting by B the current basis matrix, the rest of the procedure can be described, quoting Tomlin (1978), as follows:

(1) If w_r (respectively z_r) has just become non-basic, z_r (resp. w_r) will enter the basis.

(2) If the candidate to become basic is w_r, let $\alpha = B^{-1}e$. And if it is z_r, let $\alpha = -B^{-1}m_r$ where m_r is the rth column of M.

(3) If p is such that $\beta_p/\alpha_p = \text{Min } (\beta_i/\alpha_i, \text{ for } \alpha_i>0 \text{ and } i=1,\ldots,n)$, where $\beta = B^{-1}q$, then the pth entry in the current basis becomes non-basic.

(4) Stop if z_0 has become non-basic or $z_0 < \delta$, for some small δ. Otherwise update the representation of B^{-1} and go to (1).

The above procedure is simple enough, but care has to be taken in the updating of B^{-1}, the choice of the pivot, and other issues reviewed by Tomlin (1978). Due to these reasons, we have chosen to write our *GAUSS* program along

the slightly different approach taken by Ravindran (1972) and Proll (1974). Although their algorithm is not as fast and robust as Tomlin's, its simplicity convinced us to use it as a blueprint (with some minor changes) for the *GAUSS* procedure given in Appendix A under the name *LEMKE*.

Another warning before turning to applications: Lemke's procedure is not guaranteed to process a solution for all matrices **M**. It always computes a solution, however, in the case of a copositive matrix (i.e., $x'\mathbf{M}x \geq 0$ for all $x \geq 0$), as well as in other common cases (see, e.g., Murty, 1988, Chap. 2).

2.2 Application to Linear Programming

As the first example of a model that can be casted into a linear complementarity framework, consider the classical linear programming problem (already written in canonical form):

$$\text{Minimize } c'x$$

$$\text{s.t. } \mathbf{A}x \geq b,\ x \geq 0,$$

where x, the primal vector, and c are of order n×1, **A** is of order m×n, and b is of order m×1.

In order to solve this problem via the *LEMKE* procedure, one first has to express it as a linear complementarity problem. This can be accomplished as follows: If y denotes the dual vector, and u and v denote the primal and dual slack vectors, then defining

$$w = \begin{pmatrix} v \\ u \end{pmatrix},\quad M = \begin{pmatrix} 0 & -A' \\ A & 0 \end{pmatrix},\quad z = \begin{pmatrix} x \\ y \end{pmatrix},\quad q = \begin{pmatrix} c \\ -b \end{pmatrix},$$

it follows that the linear programming problem can be written as a linear

complementarity problem of the form (LCP). This is so because, by definition of the slack vectors, w has to be equal to $Mz+q$, and also because the complementary slackness conditions of linear programming (see, e.g., Intriligator, 1971, p. 83) imply that w and z are orthogonal.

The following *GAUSS* program solves a particular linear programming problem using *LEMKE*:

```
new;
let a = {3  6 -1  2,
        -4  2  1  5};
let b = {4,2};
let c = {6,20,3,20};
m = (zeros(4,4)~-a')|(a~zeros(2,2));
q = c|-b;
{z,w} = lemke(m,q);
```

The answer, which took seven iterations and less than a second to be computed,[1] is: $z^* = (0,.6154,0,.1538,2.3077,3.0769)$ and $w^* = (11.3846,0,2.2308,0,0,0)$.

2.3 Application to Quadratic Programming

As a second application, consider a more general model, the quadratic programming problem

$$\text{Minimize } c'x + \tfrac{1}{2}x'Px$$

$$\text{s.t. } Ax \geq b,\ x \geq 0,$$

where x and c are of order $n\times 1$, A is of order $m\times n$, b is of order $m\times 1$, and P is an $n\times n$ symmetric positive semidefinite matrix.

In order to cast the problem into a linear complementarity framework, we just have to note that if x^* solves the quadratic problem, then, as shown for

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[1] All the computations reported in this paper were done in a computer with a 80387SX chip and running at 16 MHz.

instance in Murty (1988, p. 24), it also solves the linear programming problem

$$\text{Minimize } (c' + x^{*\prime}P)x$$

$$\text{s.t. } Ax \geq b,\ x \geq 0.$$

Thus, using now the trick employed in the last subsection, we end up with a linear complementarity problem after defining:

$$w = \begin{pmatrix} v \\ u \end{pmatrix},\ M = \begin{pmatrix} P & -A' \\ A & 0 \end{pmatrix},\ z = \begin{pmatrix} x \\ y \end{pmatrix},\ q = \begin{pmatrix} c \\ -b \end{pmatrix}.$$

The following *GAUSS* program states a particular, somewhat pathological, quadratic programming problem, and then calls the procedure *LEMKE* to solve it:

```
new;
let a = {2  3  1  0,
        -2 -3 -1  0,
         1  4  0  1,
        -1 -4  0 -1};
let b = {6,-6,5,-5};
let c = {-1,-2,0,0};
let v = {1,1,0,0};
p = (eye(2)~zeros(2,2))|zeros(2,4);
m = (p~~a')|(a~zeros(4,4));
q = c|-b;
{z,w} = lemke(m,q);
```

The answer, which took 6 iterations and less than a second to be computed, is: z^* = (.7647,1.0588,1.2941,0,0,0,0,.2353) and w^* = (0,0,0,.2353,0,0,0,0).

3. AN ALGORITHM TO SOLVE NONLINEAR COMPLEMENTARITY PROBLEMS

The main component of the program *RESUELVE* is a procedure with the same name that solves a generalization of the nonlinear complementarity problem.

Before going through the most general case, however, it is helpful to consider first the classical nonlinear complementarity problem. Namely,

(NLCP) Given a differentiable vector function G: $R^n \longrightarrow R^n$, find an $n \times 1$ vector z, $z \geq 0$, such that $G(z)'z = 0$ and $G(z) \geq 0$.

As surveyed by Harker and Pang (1990), there are many applications in several sciences that lead to a framework like (NLCP). In the case of economics, it is to Mathiesen (1985a,b) credit to have realized that this nonlinear counterpart to (LCP) is also quite important, since applied general equilibrium models can be casted into this framework. In fact, Mathiesen's method of solution has become one of the most popular algorithms to solve AGE models, for, as noted by Preckel (1985), it is faster than other procedures currently in use (of the kind described in Scarf, 1985).

Once one has an algorithm to solve the linear complementarity problem of last section, a procedure to solve (NLCP) can be easily implemented by transforming, using for instance Newton's method, the nonlinear complementarity problem to a sequence of linear complementarity problems.[2] More precisely, an algorithm to solve (NLCP) can be described as follows:

(1) To initialize set $k = 0$ and specify an initial guess $z = z^0$.

(2) Set $k = k+1$ and linearize $G(z)$ around z^{k-1}:

$L(G, z^{k-1}) = q^k + M^k z$, where M^k is the Jacobian matrix of G evaluated at z^{k-1}, and $q^k = G(z^{k-1}) - M^k z^{k-1}$.

(3) Using Lemke's method of solution, find z^k and w that solves:

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[2] For a different, and very promising, approach to solve the nonlinear complementarity problem, see the recent paper by Harker and Xiao (1990).

$w = M^k z^k + q^k$, $w'z^k = 0$, $w \geq 0$, and $z^k \geq 0$.

(4) Stop if $\Sigma_{i=1}^{n} |G^i(z^k)| < \delta$, for some small δ. Otherwise go to (2).

The next section shows how applied general equilibrium models can be casted into this framework. For these models, Mathiesen (1985a,b, and 1987) and Rutherford (1989) provide refinements to the algorithm given above.

It should be noted, however, that although it is quite easy to implement in *GAUSS* the procedure, we do not do so in our program. The reason is because, as first noted by Lensberg (1983), there is a more general version of the nonlinear complementarity problem that can not only be used to study Walrasian equilibrium models, but also non-Walrasian ones. The generalized nonlinear complementarity problem can be stated as:

Given a differentiable vector function F: $R^n \times R^n \longrightarrow R^n$,

(NLCPG) find the $n \times 1$ vectors x and w, $x \geq 0$ and $w \geq 0$, such that

$F(x,w) = 0$ and $x'w = 0$.

Following Lensberg (1983) and Lensberg and Rasmussen (1986), an algorithm to solve (NLCPG) can be also implemented by transforming it to a sequence of linear complementarity problems. But before introducing the algorithm, some notation is in order. Let $N = (1,2,\ldots,2n)$, and let $v \in R^n \times R^n$ be defined as $v_i = x_i$ and $v_{i+n} = w_i$, $i = 1,\ldots,n$. For all $v \geq 0$ and such that $x'w = 0$, let $I(v)$ denote the set of n indices $i \in N$ for which $v_i > 0$ (if both x_i and w_i are zero then, say, let i be in $I(v)$). Thus, $I(v)$ contains the indices of the basic variables, while $N \backslash I(v)$ contains the indices of the non-basic ones. The main steps of the algorithm to solve (NLCPG) can then be described as follows:

(1*) To initialize set k = 0 and specify an initial guess $v = v^0$ (such that the corresponding x^0 and w^0 are orthogonal).

(2*) Set k = k+1 and linearize $F(v)$ around v^{k-1}: $L(F,v^{k-1}) = c + Mv$, where M is the Jacobian matrix of F evaluated at v^{k-1}, and $c = F(v^{k-1}) - Mv^{k-1}$.

(3*) Obtain the partitions $M = (M_I, M_{NI})$ and $v = (v_I, v_{NI})$, using the set of indices $I(v^{k-1})$.

(4*) Evaluate $L^k(F,v^{k-1}) = M_I^{-1}L(F,v^{k-1}) = c^k + v_I + M^k v_{NI}$, where $c^k = M_I^{-1}c$ and $M^k = M_I^{-1}M_{NI}$.

(5*) Using Lemke's method of solution, find v^k that solves: $v_I = -M^k v_{NI} - c^k$, $v_I' v_{NI} = 0$ and $v \geq 0$.

(6*) Stop if $\Sigma_{i=1}^{n} |F^i(v^k)| < \delta$, for some small δ. Otherwise go to (2*).

The above algorithm is implemented as the procedure *RESUELVE* in our program provided in Appendix A. In section 5 is shown, and exemplified, how fix-price models can be stated neatly in a context such as (NLCPG); but before that, the next section deals with the case of Walrasian models.

4. SOLVING APPLIED GENERAL EQUILIBRIUM MODELS

We now illustrate how to solve applied general equilibrium models by first stating the procedure in an abstract setting, and then solving a particular example. In this section we assume that the reader is already familiar with the relevant microeconomic concepts (otherwise, see for instance Intriligator, 1971, or Shoven and Whalley, 1984).

Following Mathiesen (1985a,b), general equilibrium models can be casted as nonlinear complementarity problems by using an activity analysis framework *à la* Scarf. Thus, consider an economy in which there are m activities and n goods, and let

$y = (y_1,\ldots,y_m)'$ be the vector of activity levels,

$p = (p_1,\ldots,p_n)'$ be the vector of good prices,

$b = (b_1,\ldots,b_n)'$ be the vector of initial endowments,

$d(p) = (d_1(p),\ldots,d_n(p))'$ be the vector of demand for goods, and

$A(p) = [\ a_{ij}(p)\ ]$ be the input-output matrix of order m×n.

Note that the entries of the input-output matrix depend in general on prices, except for Leontieff technologies, and that positive (negative) entries in the matrix denote outputs (inputs).

In this framework, a competitive equilibrium is made of a vector of prices p^* and a vector of activity levels y^* such that:

(i) $-A(p^*)p^* \geq 0$, no activity earns positive profits;

(ii) $b + A(p^*)'y^* - d(p^*) \geq 0$, no commodity is in excess demand;

(iii) $p^* \geq 0$, $y^* \geq 0$, no prices or activity levels are negative;

(iv) $[-A(p^*)p^*]'y^* = 0$, no activity with negative profits is run;

(v) $[b + A(p^*)'y^* - d(p^*)]'p^* = 0$, Walras law, except for free goods.

If one defines the vector $z = (y_1,\ldots,y_m,p_1,\ldots,p_n)$, and the vector function $G(z)$ as $(G^y(z)',G^p(z)')'$, where

$$G^y(z) = -A(p)p,$$

$$G^p(z) = b + A(p)'y - d(p),$$

it is then clear that the conditions for a competitive equilibrium stated above can be rephrased as: $z^* = (y^{*\prime}, p^{*\prime})'$ solves the nonlinear complementarity problem

$$G(z)'z = 0 \text{ with } z \geq 0 \text{ and } G(z) \geq 0.$$

This is the departure point for Mathiesen's algorithm, which is a refined version of the one given last section to solve (NLCP). Note that, since only relative prices matter in general equilibrium models, then one has to choose a numeraire (and drop an equation) before attempting to solve the model.[3] This will be illustrated below.

As an alternative to the above transformation, let us now recast the general equilibrium model into the generalized nonlinear complementarity framework (NLCPG) defined last section. Let w denote the vector $(w_1', w_2')'$, where w_1 and w_2 are nonnegative vectors of order $m \times 1$ and $n \times 1$, respectively. Let x denote, on the other hand, the vector $(y', p')'$. Finally, define the vector function $F(x,w)$ as $(F^1(x,w)', F^2(x,w)')'$, where

$$F^1(x,w) = w_1 + \mathbf{A}(p)p,$$
$$F^2(x,z) = w_2 - b - \mathbf{A}(p)'y + d(p).$$

Then, it is clear that any competitive equilibrium vector of activity levels and prices, x^*, has to solve the generalized nonlinear complementarity problem

$$F(x,w) = 0, \text{ with } x \geq 0, \; w \geq 0, \text{ and } x'w = 0.$$

This last way of recasting a general equilibrium model is now exemplified by solving a very simple general equilibrium model presented in Shoven and

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[3] The choice of the numeraire can also influence the speed of convergence (or lack of it). See Mathiesen (1985b, and 1987) and Rutherford (1989).

Whalley (1984). In order to avoid lengthy expressions, we refer to their paper and to Appendix B below for the detailed equations. Here we simply point out how to rephrase their model in terms of the concepts discussed above.

On the production side, their model contains two final goods, 1 and 2, produced using labor and capital, and by means of CES production functions. Denote by $L_i(p,Q_i)$ and $K_i(p,Q_i)$ the conditional factor demands, by $y = (y_1,y_2)'$ the level of activity in sector i, and by $p = (p_1,p_2,p_L,p_K)'$ the vector of prices; then, the input-output matrix $A(p)$ in this model can be expressed as

$$A(p) = \begin{pmatrix} 1 & 0 & -L_1(p,1) & -K_1(p,1) \\ 0 & 1 & -L_2(p,1) & -K_2(p,1) \end{pmatrix}.$$

On the other hand, Shoven and Whalley's model has two consumers (1 and 2) with CES utility functions depending only on the two final goods. The first ("rich") consumer owns all the capital endowment, while the other ("poor") owns all labor. Using the value of the parameters given in Table 1 of Shoven and Whalley (1984), the generalized nonlinear complementarity framework given above, and the price of labor as numeraire, Appendix B presents the computer code and the output obtained using *RESUELVE*. As can be seen there, the program is able to replicate the results in Shoven and Whalley's Table 2.

The amount of time taken by the program was around four and a half seconds, which seems to us to be quite fast. Granted, by casting the general equilibrium model into (NLCPG), rather than (NLCP), the dimension of the Jacobian matrix is increased (and hence the computer time for numerical differentiation). But that is the price that one has to pay to be able to have an algorithm that also solves other models, such as the general fix-price models to be reviewed next.

5. SOLVING APPLIED GENERAL DISEQUILIBRIUM MODELS

In this last section we discuss fix-price models. As noted by Lensberg (1983) [see also Hahn (1978) and Laroque (1981)], a general disequilibrium model can be usually given a *dual* formulation in terms of the shadow prices associated with the quantity constraints in the primal. For instance, if there is rationing in market i, then there are implicitly two shadow prices (in terms of money) on the binding constraint: the buyer's shadow price x_i, and the seller's shadow price y_i. But since at most only one side will be rationed, then it follows that x_i and y_i are complementary: $x_i y_i = 0$. Furthermore, if $\bar{p}$ denotes the fixed price in market i, the virtual price that would make excess demands to be zero in the case of the buyer (or seller) would be given by $\bar{p}+x_i$ (or $\bar{p}-y_i$). Of course, this discussion can be trivially extended to the case of models that only exhibit downwards price stickiness (or upwards for that matter).

In order to exemplify the above points, and also as an example of how a general disequilibrium model can be casted into a generalized nonlinear complementarity framework, we now consider a very simple fix-price model in the spirit of Barro and Grossman (1971) and Malinvaud (1977). It may be noted in passing that, curiously enough, there seems to be few published studies based on applied general non-Walrasian models. For a nice exception see Steigum (1987); see also Urzúa (1991) and references therein.

The first sector in the model involves a representative firm that uses labor to supply a single good (Q_S). Labor is inelastically supplied up to a fixed amount of $\bar{L}$ and the production function is assumed to be given by:

$$Q_S = L_D^{\theta}, \tag{5.1}$$

where L_D denotes the firm's demand for labor and θ is a positive exponent less than one.

In this fix-price model there is a rigid nominal price of labor, $\bar{W}$, and also a fixed output price, $\bar{P}$. Consequently, the firm in principle would be constrained in both the labor and the output markets. Nevertheless, if W_D and P_S denote the firm's shadow prices of labor and output, then, as noted earlier, the virtual prices $\bar{P}-P_S$ and $\bar{W}+W_D$ can be used to find the demand for labor and the supply of the good that maximize profits regardless of whether or not those constraints are binding:

$$(5.2) \qquad L_D = [\theta(\bar{P}-P_S)/(\bar{W}+W_D)]^{1/(1-\theta)},$$

$$(5.3) \qquad Q_S = [\theta(\bar{P}-P_S)/(\bar{W}+W_D)]^{1/(1-\theta)}.$$

The other sector of the model is made of a representative household that inelastically supplies labor, and demands goods (Q_D) together with money (M) according to the utility function:

$$(5.4) \qquad U(Q_D, M/\bar{P}) = \mu \log(Q_D) + (1-\mu)\log(M/\bar{P}),$$

where μ is the budget share for commodities. Given initial money holdings M_0, the household's budget constraint is given by

$$(5.5) \qquad \bar{P}Q_D + M \leq \bar{P}Q_S + M_0.$$

As in the case of the firm, the household could end up being rationed. Let P_D denote the household's shadow price corresponding to the constraint on the goods market, so that the virtual (imputed) price of the good is given by $\bar{P}+P_D$. Using this price, let I be the household's imputed income. Then the optimal demands for goods and money are given by:

(5.6) $$Q_D = \mu I/(\bar{P}+P_D),$$

(5.7) $$M = (1-\mu)I.$$

The model can now be casted into the nonlinear complementarity framework (NLCPG) as follows: Let ES, ED, and UI be, respectively, the excess supply of goods, the excess demand for labor, and the household's unused income. That is,

(5.8) $$ES = Q_S - Q_D = [\theta(\bar{P}-P_S)/(\bar{W}+W_D)]^{1/(1-\theta)} - \mu I/(\bar{P}+P_D),$$

(5.9) $$ED = L_D + U - \bar{L} = [\theta(\bar{P}-P_S)/(\bar{W}+W_D)]^{1/(1-\theta)} + U - \bar{L},$$

(5.10) $$UI = \bar{P}Q_S + M_0 - \bar{P}Q_D - M + S$$

$$= \bar{P}\{[\theta(\bar{P}-P_S)/(\bar{W}+W_D)]^{1/(1-\theta)} - \mu I/(\bar{P}+P_D)\} + M_0 - (1-\mu)I + S.$$

where U denotes unemployment and S is a slack variable that is necessarily equal to zero at the solution.

Given values for the parameters $\bar{P}$, $\bar{W}$, θ, μ, $\bar{L}$ and M_0, a fix-price equilibrium (P_S,P_D,W_D,U,I,S) is obtained if (1) the three functions (5.8)-(5.10) evaluated at this point are equal to zero, and (2) the following complementarity conditions are fulfilled:

(5.11) $$P_S P_D = 0,$$

(5.12) $$W_D U = 0,$$

(5.13) $$IS = 0,$$

(5.14) $$P_S, P_D, W_D, U, I, S \geq 0.$$

Given the form of equations (5.8)-(5.11), it is clear that the problem of finding the non-Walrasian equilibrium is in fact a nonlinear complementarity problem. This is shown in Appendix C where the program *RESUELVE* is used to find the vector of fix-price equilibrium given the following parameters (taken from Lensberg and Rasmussen, 1986): $\bar{P} = 2$, $\bar{W} = 1$, $\theta = 0.45$, $\mu = 0.8$, $\bar{L} = 1$ and $M_0 = 0.5$. As noted in that appendix, the solution, which took about one and a half seconds to be computed, is given by: $P_S = 0$, $P_D = .18006$, $W_D = 0$, $U = .17433$, $I = 2.5$ and $S = 0$.

APPENDIX A: COMPUTER CODE FOR THE PROGRAM *RESUELVE*

The main component of the program, written by the author in *GAUSS* (version 2.0 or later), is made of the procedure *RESUELVE*. This procedure invokes in turn the procedure *LEMKE*, given also below. Note that the latter can be used independently in the case of linear complementarity problems. The procedure *RESUELVE*, as exemplified in the other appendices, requires an initial guess for vector "v", an initial basis ("basis"), a maximum number of iterations to be allowed ("imax"), and a procedure that defines the vector function ("f"). The procedure *LEMKE* only requires the matrix "m" and the vector "q".

Please note that since the program is in a preliminary stage, I do not guarantee that it is free of bugs. If the reader finds mistakes in it, I would appreciate hearing about them. The program contained in this paper can be freely used (or modified) for teaching, research, or any other noncommercial purpose.

```
proc resuelve(v,basis,imax);
    local d,g,gb,gbinv,gnb,i,k,m,n,nbasis,q,vb,vnb;
    n = rows(basis);
    i = 0;
    do while i < imax;
        i = i+1;
        print "ITERATION";; format /rd 4,0; i;
        format /rd 10,4; print "v' =";; v'; print "F(v)' =";; f(v)'; print;
        g = gradp(&f,v);
        gb = submat(g,0,basis);
        d = basis .> n*ones(n,1);
        nbasis = substute(basis+n*ones(n,1),d,basis-n*ones(n,1));
        gnb = submat(g,0,nbasis);
        gbinv = inv(gb);
        m = -gbinv*gnb;
        q = -gbinv*(f(v)-g*v);
        print "LEMKE'S PROCEDURE IS CALLED";
        {vnb,vb} = lemke(m,q);
        k = 0;
        do while k < n;
            k = k+1;
            if vnb[k] > 0;
                v[nbasis[k]] = vnb[k];
                basis[k] = nbasis[k];
```

```
            else;
                v[basis[k]] = vb[k];
            endif;
        endo;
        if sumc(abs(f(v))) <= 1E-10;
            print "SOLUTION FOUND";
            print "v' ="; v'; print "F(v)' ="; f(v)'; print;
            retp(v);
        endif;
    endo;
    print "NO SOLUTION AFTER"; format /rd 3,0; imax; print "ITERATIONS";
endp;
proc (2) = lemke(m,q);
    local alpha,b,basic,d,in,j,k,n,out,r,r0,w,z,z0;
    n = rows(m);
    /* Introduce the new variable z0 and get the basis */
    z0 = -minc(q);
      = minindc(q);
    z = zeros(n,1);
    if z0 < 0;
        w = q;
        print "TRIVIAL LINEAR COMPLEMENTARY SOLUTION: w = q, z = 0"; print;
        retp(z,w);
    endif;
    q = q+z0;
    q[r] = z0;
    b = eye(n);
    b[.,r] = -ones(n,1);
    basic = ones(n,1)|seqa(1,1,n);
    basic[r] = 0;
    basic[r+n] = 0;
    r0 = r;
    w = q;
    w[r] = 0;
    out = 1|r;
    j = 1;
    /* Find the new basis column to enter or detect solution */
d1: if out[1] == 1;
        in = 2|out[2];
        alpha = -b*m[.,in[
    elseif out[1] == 2;
        in = 1|out[2];
        alpha = b[.,in[2]];
    else;
d2:     print "LINEAR COMPLEMENTARY SOLUTION (AFTER";;
        format /rd 3,0; j; print "ITERATIONS)";
        d = q .<= 1E-10*ones(n,1);
        q = substute(q,d,0);
        w = zeros(n,1);
        z = zeros(n,1);
        k = 0;
        do while k < n;
```

```
            k = k+1;
            if basic[k] == 1;
                w[basic[k+n]] = q[k];
            elseif basic[k] == 2;
                z[basic[k+n]] = q[k];
            endif;
        endo;
        print "          z          w"; format /rd 10,4; z-w; print;
        retp(z,w);
    endif;
    /* Find the pivot row for next iteration */
    if q[r0] <= 1E-10;
        goto d2;
    endif;
    if alpha <= zeros(n,1);
        print "NO LINEAR COMPLEMENTARY SOLUTION"; print;
        retp(z,w);
    else;
        r = maxindc(alpha);
    endif;
    k = 0;
    do while k < n;
        k = k+1;
        if alpha[k] <= 0;
            continue;
        elseif q[k]/alpha[k] < q[r]/alpha[r];
            r = k;
        endif;
    endo;
    /* Make pivot operation and update the basis and q */
    b[r,.] = b[r,.]/alpha[r];
    q[r] = q[r]/alpha[r];
    k = 0;
    do while k < n;
        k = k+1;
        if k /= r;
            q[k] = q[k]-q[r]*alpha[k];
            b[k,.] = b[k,.]-b[r,.]*alpha[k];
        endif;
    endo;
    out[1] = basic[r];
    out[2] = basic[r+n];
    basic[r] = in[1];
    basic[r+n] = in[2];
    if j <= 99;
        j = j+1;
        goto d1;
    else;
        print "NO LINEAR COMPLEMENTARY SOLUTION AFTER 100 ITERATIONS"; print;
        retp(z,w);
    endif;
endp;
```

APPENDIX B: PROGRAM AND OUTPUT FOR THE EQUILIBRIUM MODEL IN SECTION 4

In order to solve Shoven and Whalley's model, we choose as numeraire the price of labor. Thus we can drop the equation representing the excess supply of labor (i.e., $b_L - L_1y_1 - L_2y_2$). Next define $v' = (y_1, y_2, p_1, p_2, p_K, w_1, w_2, w_3, w_4, w_5)$, where all the prices are now relative prices. The computer code that defines the corresponding function $F(v)$ is given in the procedure "proc f(v)" below. Finally, before calling *RESUELVE* an initial guess for the basis is required. In the case of general equilibrium models it is natural to guess that the basis is made of all variables except the w's, since these are greater than zero only if there is excess supply or positive profits. Furthermore, we chose as an initial point the one where all relative prices equal one and all activity levels equal 10. Thus the initial basis is $(y_1, y_2, p_1, p_2, p_K)$ with values (10,10,1,1,1).

```
new;
    let v = {10,10,1,1,1,0,0,0,0,0};
    let basis = {1,2,3,4,5};
    imax = 100;
    call resuelve(v,basis,imax);
end;

proc f(v);
    local k1,k2,l1,l2,x11,x12,x21,x22,u;
    u = zeros(5,1);
    k1 = 1/(1.5*(.9*v[5]+.4)^2);
    k2 = .15+(.0525/v[5])^.5;
    l1 = .24*v[5]^2/(.36*v[5]+.16)^2;
    l2 = .35+(.0525*v[5])^.5;
    x11 = 25*v[5]/(v[3]+(v[3]^1.5)/(v[4]^.5));
    x12 = 18/(.3*v[3]+.7*(v[3]^.75)*(v[4]^.25));
    x21 = 25*v[5]/(v[4]+(v[4]^1.5)/(v[3]^.5));
    x22 = 42/(.7*v[4]+.3*(v[4]^.75)*(v[3]^.25));
    u[1] = v[6]+v[3]-l1-v[5]*k1;
    u[2] = v[7]+v[4]-l2-v[5]*k2;
    u[3] = v[8]-v[1]+x11+x12;
    u[4] = v[9]-v[2]+x21+x22;
    u[5] = v[10]-25+v[1]*k1+v[2]*k2;
    retp(u);
endp;
```

The rest of this appendix reproduces the output obtained after running the program (using a format with four decimal points):

```
ITERATION   1
v' =   10.0000    10.0000     1.0000     1.0000     1.0000     0.0000
    0.0000     0.0000     0.0000     0.0000
F(v)' =   -0.2821     0.0417    20.5000    44.5000   -17.2639

LEMKE'S PROCEDURE IS CALLED
TRIVIAL LINEAR COMPLEMENTARY SOLUTION: w = q, z = 0

ITERATION   2
v' =   22.1599    52.3385     1.4961     1.1639     1.5425     0.0000
    0.0000     0.0000     0.0000     0.0000
F(v)' =    0.0584     0.0134     2.4872     0.6620    -2.8736

LEMKE'S PROCEDURE IS CALLED
TRIVIAL LINEAR COMPLEMENTARY SOLUTION: w = q, z = 0

ITERATION   3
v' =   24.6723    54.4272     1.3934     1.0795     1.3303     0.0000
    0.0000     0.0000     0.0000     0.0000
F(v)' =    0.0053     0.0014     0.0035     0.1094     0.4240

LEMKE'S PROCEDURE IS CALLED
TRIVIAL LINEAR COMPLEMENTARY SOLUTION: w = q, z = 0

ITERATION   4
v' =   24.9333    54.3804     1.3991     1.0927     1.3723     0.0000
    0.0000     0.0000     0.0000     0.0000
F(v)' =    0.0003     0.0001    -0.0003     0.0010     0.0115

LEMKE'S PROCEDURE IS CALLED
TRIVIAL LINEAR COMPLEMENTARY SOLUTION: w = q, z = 0

ITERATION   5
v' =   24.9425    54.3782     1.3991     1.0931     1.3735     0.0000
    0.0000     0.0000     0.0000     0.0000
F(v)' =    0.0000     0.0000     0.0000     0.0000     0.0000

LEMKE'S PROCEDURE IS CALLED
TRIVIAL LINEAR COMPLEMENTARY SOLUTION: w = q, z = 0

SOLUTION FOUND
v' =   24.9425    54.3782     1.3991     1.0931     1.3735     0.0000
    0.0000     0.0000     0.0000     0.0000
F(v)' =    0.0000     0.0000     0.0000    -0.0000     0.0000
```

APPENDIX C: PROGRAM AND OUTPUT FOR THE DISEQUILIBRIUM MODEL IN SECTION 5

In order to solve the system of equations (5.8)–(5.14), one has to recast it into a generalized nonlinear complementarity framework. Let $v' = (P_S, W_D, I, P_D, U, S)$, in such a way that the complementarity conditions are given by $v_i v_{i+3} = 0$ [$i \leq 3$], and let $F(v)' = (ES(v), ED(v), UI(v))$. The computer code that defines this function is given in the procedure "proc f(v)" below. Finally, before calling *RESUELVE* an initial guess for the basis is required. In our ex can be noted in the next lines, the guess was (P_S, I, U) with values $(0,0,2)$.

```
new;
    let v = (0,0,2,0,0,0);
    let basis = (1,3,5);
    imax = 100;
    call resuelve(v,basis,imax);
end;

proc f(v);
    local a,dg,dl,m0,r,pf,sg,sl,u,wf;
    u = zeros(3,1);
    a = 0.45;
    m0 = 0.5;
    pf = 2;
    r = 0.8;
    wf = 1;
    sl = 1;
    dl = (a*(pf-v[1])/(wf+v[2]))^(1/(1-a));
    sg = dl^a;
    dg = r*v[3]/(pf+v[4]);
    u[1] = sg-dg;
    u[2] = dl+v[5]-sl;
    u[3] = pf*sg+m0-pf*dg-(1-r)*v[3]+v[6];
    retp(u);
endp;
```

The rest of this appendix reproduces the output obtained after running the program (using a format with four decimal points):

```
ITERATION   1
v' =    0.0000     0.0000     2.0000     0.0000     0.0000     0.0000
F(v)' =    0.1174    -0.1743     0.3348

LEMKE'S PROCEDURE IS CALLED
LINEAR COMPLEMENTARY SOLUTION (AFTER  2 ITERATIONS)
         z          w

    0.2065     0.0000
    0.0000     0.1743
    0.0000     2.5000

ITERATION   2
v' =    0.0000     0.0000     2.5000     0.2065     0.1743     0.0000
F(v)' =    0.0110    -0.0000     0.0220

LEMKE'S PROCEDURE IS CALLED
TRIVIAL LINEAR COMPLEMENTARY SOLUTION: w = q, z = 0

ITERATION   3
v' =    0.0000     0.0000     2.5000     0.1797     0.1743     0.0000
F(v)' =   -0.0001     0.0000    -0.0003

LEMKE'S PROCEDURE IS CALLED
TRIVIAL LINEAR COMPLEMENTARY SOLUTION: w = q, z = 0

ITERATION   4
v' =    0.0000     0.0000     2.5000     0.1801     0.1743     0.0000
F(v)' =   -0.0000     0.0000    -0.0000

LEMKE'S PROCEDURE IS CALLED
TRIVIAL LINEAR COMPLEMENTARY SOLUTION: w = q, z = 0

SOLUTION FOUND
v' =    0.0000     0.0000     2.5000     0.1801     0.1743     0.0000
F(v)' =   -0.0000     0.0000    -0.0000
```

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