INSULATING PROPERTIES OF DUAL EXCHANGE RATES:
A NEW-CLASSICAL MODEL

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Abstract

In this paper we offer a new approach to analyzing dual exchange rates that highlights the interactions of the real and financial sectors of the economy. We model the links between flows of foreign exchange, the availability of working capital, and domestic production. Furthermore, we identify an imperfection that justifies the imposition of dual exchange rates. We show that dual exchange rates reduce the impact of financial disturbances on domestic production, even when leaks between the official and financial markets exist. We also compare dual and flexible exchange rates. We show that the impact effect of a temporary financial disturbance is lower under the dual rates. However, persistence is greater. Finally, we analyze the time inconsistency of dual exchange rate systems.

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I. Introduction.

Dual exchange rate systems have been widely adopted as a means to limit the extent of financial flows.[1] Yet there is no consensus among economists about the benefits of such a system. Proponents argue that by inhibiting speculative capital flows, dual rates insulate the economy from temporary financial disturbances, and facilitate the implementation of stabilization programs.[2] Critics of dual exchange rates, however, see them as welfare reducing distortions.[3] Furthermore, some say that it is impossible to perfectly separate the financial and official foreign exchange markets. That is, the premium of the financial over the official exchange rate generates leaks between the two markets, such as the underinvoicing of exports, which undermine the system.

If there is a case for dual exchange rates, it must rest on the existence of a market imperfection. And the link between the dampening of financial flows and the insulation of production it must be made clear. Yet, much of the previous literature has ignored this essential link. Output has been considered as given or as demand-determined, narrowly focusing on the behavior of the money market and the financial exchange rate.[4] One exception is Flood and Marion (1982) who model production explicitly. However, they

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[1] Under dual exchange rate systems commercial and financial transactions take place in separate markets. Commercial transactions use an official exchange rate set by the central bank, while financial transactions occur at a freely determined exchange rate. In practice, some commercial transactions are allowed to take place at the financial rate, or multiple exchange rates exist. For an analysis of dual exchange rate systems see: Dornbusch (1986), Fleming (1971, 1974) and Lanyi (1975).

[2] According to Dornbusch (1985): "...running the world to the tune of assets markets is at best questionable. Hence the interest in institutional arrangements that delink the assets markets and free policies for what are perceived to be the 'true' priorities."

[3] According to Adams and Greenwood (1985): "Dual Exchange rates have been evaluated on arbitrary criteria such as their ability to insulate an economy from various economic disturbances, say a change in world interest rates. It is not always clear, however, why insulating an economy is a laudable goal and how it corresponds to maximizing an economy's welfare". Furthermore, "...a 'small' open economy should adopt a flexible exchange rate system and follow the optimum quantity of money rule in order to maximize its welfare".

[4] For example: Aizenman (1983), Cumby (1984), Flood (1978).

assume that the problem is one of nominal rigidities not of financial flows.[5] This leads

them to conclude that fixed and dual rates have the same insulating properties. In contrast,

Tobin (1978) stresses the central importance of financial flows:

> "...the basic problem today is not the exchange rate regime, whether fixed or floating. Debate on the regime evades and obscures the essential problem. That is the excessive international mobility of private financial capital. ...my proposal is to throw some sand in the wheels of our excessively efficient international money markets."

In this paper we offer a new approach to analyzing dual exchange rates where the dampening of financial flows plays a key role in the insulation of production from transitory financial disturbances. We also show that the existence of leakages between the official and financial markets does not undermine the system. In fact, leakages are one of the channels by which dual exchange rates insulate the economy. The distinctive features of the model are the following:

- **We highlight the links between flows of financial foreign exchange, the availability of working capital and the level of domestic production.** We model these links by recognizing that investors allocate their wealth between foreign assets and working capital. This working capital is invested in a nontradeable input, which is transformed after one period into an exportable good. Any increase in the foreign interest rate increases overseas investment and simultaneously reduces the availability of working capital and thus domestic production.

- **We identify an externality in the economy that justifies the imposition of dual exchange rates.** The externality stems from the fact that the population is not homogeneous, but composed of two groups: investors, who produce the exportable

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[5] They consider a one good economy with nominal rigidities, in which output variations occur only as a consequence of price prediction errors. In this instance, dual and fixed exchange rates fully insulate domestic production from foreign disturbances because agents know that any domestic price variation can be caused only by a foreign price variation --PPP holds and the official exchange rate is fixed. Therefore, the economy is fully indexed. Under flexible exchange rates, variations in domestic prices do not fully reflect variation in foreign prices. Consequently, foreign disturbances affect domestic output. Note that in this argument, the importance of financial flows has been neglected. The fact that dual exchange rates inhibit financial flows has no role in explaining why they insulate the economy.

good; and workers, who supply the nontradeable input --labor. Investors have access to the international capital market, while workers have no access to any capital market at all, and cannot migrate. By choosing their portfolio, investors indirectly determine the income of the workers. During periods of high foreign rates of return, investors shift their wealth from working capital to foreign assets, thereby reducing workers' income and thus consumption.

• We explicitly model production and "leakages" as optimal decisions made by investors who maximize their lifetime utility.

The main point of this paper is that dual exchange rates act as automatic insulators of the economy from transitory financial disturbances. Furthermore, this is robust to the introduction of leaks between the official and the financial markets. To illustrate the above we consider an unexpected and transitory increase in the foreign interest rate. To restore portfolio equilibrium, the investors shift their wealth overseas, causing a reduction in the availability of working capital, and thus a reduction in the demand for the nontradeable input. As a result, the marginal product of the nontradeable input is higher and the profitability of domestic investment increases. The process continues until marginal returns are equalized. We show that under dual exchange rates the shift to FFE is smaller than under uniform rates. This is because the transitory increase in the foreign interest rate generates an immediate depreciation and an expected appreciation of the financial exchange rate. Under dual rates this creates expected capital losses on the holdings of FFE. Meanwhile, the profitability of domestic investment remains unchanged because under dual rates the prices of imports and exports are delinked from the financial exchange rate. As a result, the net increase of the relative profitability of overseas investment is lower than the increase in the foreign interest rate. Under uniform rates, the expected appreciation does not reduce the profitability of overseas investment relative to domestic investment. Therefore the impact of the financial disturbance is not dampened. The reason for this is that the official and the financial markets are not segregated, thus the price of the input (imported)

increases with the depreciation of the exchange rate, and the prices of the output (exported) decreases with the future appreciation. Summing up, the underlying cause for the better insulation under dual exchange rates is that the financial exchange rate is able to alter the profitability of overseas investment relative to domestic investment, while under uniform rates it cannot.

We introduce leakages between the official and the financial markets by assuming that the authorities cannot monitor all transactions made in the economy. Therefore, when the premium of the financial over the official exchange rate is positive, investors find it profitable to underinvoice a share of their official exports and sell them through the financial market. Conversely, when the premium is negative, investors illegally sell a share of their holdings of FFE in the official market. We show in this paper that the introduction of leaks does not undermine the insulating properties of dual exchange rates. Even if investors can sell part of their holdings of FFE in the official market once the disturbance has ended, some capital losses on the holdings of FFE will still be generated. Therefore, the relative profitability of overseas investment relative to domestic investment is still reduced.

The model we present is also useful in identifying a trade-off between dual and uniform exchange rates. Namely, a temporary financial disturbance has a greater immediate impact on the availability of working capital under uniform rates, but more persistence under dual exchange rates. The impact is higher under uniform rates because, as mentioned earlier, the expected appreciation does not reduce the profitability of overseas investment relative to domestic investment as it does under dual rates. Persistence is higher under dual rates because, as mention earlier, investors cannot legally sell their excess stock of FFE in the official market once the financial disturbance is over. Therefore, they sell it gradually through illegal channels. For the market of FFE to clear while investors are trying to reduce their holdings of FFE, expected depreciations of the financial exchange rate must occur in order to increase the relative profitability of overseas investment. As a result, the

availability of working capital remains depressed and the fall in demand for the nontradeable input will persist.

The last issue we consider is whether the stated policy of prohibiting the sale of FFE in the official market in the future is credible. This problem is known in the literature as "time inconsistency." The argument is as follows: once the financial disturbance has ended, the authorities will find no opposition from the workers or the investors to the authorities' allowing FFE to be sold in the official market. By so doing, the authorities could eliminate persistence and restore domestic production. If the authorities cannot credibly precommit to avoiding this action in the future, then at the time of the shock, the extent of the shift from domestic investment to FFE would be higher. This is because investors foresee that the authorities will not allow a negative premium of the financial over the official exchange rate to occur in the future. Therefore, we conclude that dual exchange rates are time inconsistent.

In section II we describe in an intuitive way how dual exchange rates insulate domestic production. We analyze the investment and export underinvoicing decisions of the agents, and discuss the dynamics of the financial exchange rate following a temporary financial disturbance. In section III we present a formal treatment and embed the investment and underinvoicing problems in a lifetime utility maximization framework. We compare dual and flexible exchange rates in section IV. In section V we discuss the time inconsistency of dual exchange rates. Finally, in section VI we present the conclusions.

II. The Economy.

The economy we consider is populated by two groups of agents. One group, the investors, has access to the international capital market. The other group, the workers, produces a nontradeable good (labor) and has no access to capital markets whatsoever. During each period, investors can allocate their wealth in two different ways: (1)by buying foreign exchange in the financial market in order to invest overseas, and (2)by investing

domestically in working capital, i.e. by buying the nontradeable input from the workers and then transforming it after one period into an exportable good.[6,7] Given that workers have no access to capital markets, they cannot save. Therefore, any reduction in the investors' demand for the nontradeable input (NTI) reduces their income and thus their consumption. This is the externality that justifies the imposition of dual exchange rates (DER). If the entire population had access to capital markets and no nominal rigidities or other imperfections existed, then the insulation of production would be meaningless, since production would be effected at the optimum level.

Both groups consume a perishable imported good which can be imported only by the authorities and is sold for domestic money only. This assumption is critical for three reasons: (1) If workers consumed only a nontradeable good, then the decisions of investors would not affect the workers' welfare, and there would be no justification for dual rates.[8] (2) If the consumption good could be bought using foreign exchange, then there would be no demand for domestic money, and the exchange rate would be undefined. (3) By assuming that the consumption basket does not contain nontradeables we are assuming away wealth effects of financial disturbances on the demand for domestic production.

In order to focus exclusively on how DER insulate domestic production by dampening financial flows, we assume full nominal price flexibility in this economy. Consideration of price rigidities is important when comparing fixed vs. flexible exchange rates, but not when comparing uniform rates and DER.

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[6] In a more realistic setup we would distinguish between investors and producers. Investors choose their portfolio of foreign and domestic currencies. Then, the banking system transforms this financial liquidity into working capital by lending to producers. In this paper we abstract from the banking system.

[7] Note that there exists import content in the production of the exportable good. This is because the workers of the nontradeable sector consume an imported good.

[8] More generally, if no imported input would be used directly or indirectly by the nontradeable sector, then this sector would be autarkic. In this situation the only way that the investors' demand for the nontradeable input could affect the nontradeable sector's welfare is if nominal price rigidities existed in this sector. Throughout this paper we will assume full nominal price flexibility.

II.1 Financial Flows, Working Capital and Domestic Production.

During each period the representative investor can invest his holdings of domestic money in two ways:

i. By purchasing FFE in the financial market at a price f_t, investing overseas, obtaining a return $1+i^*$, and selling it during the next period at a price f_{t+1}.[9]

ii. By purchasing the NTI at a price P_{nt}, transforming it after one period into an exportable good (T) and selling it at a price P_{xt+1}.

In either case, the investor will use the proceeds to purchase the consumption good and to reinvest in the future.

The technology of the exportable good is given by:

$$T_{t+1} = T(N_t), \qquad T'>0, \quad T''<0 \qquad (1)$$

The nontradeable good is produced using only labor as an input. The technology is given by: $N_t=n_t/a$. There exists an excess supply of labor. However, the real wage in terms of the consumption good is fixed as a result of institutional rigidities. The wage rigidity gives rise to income and employment fluctuations. The behavior of workers' income will be the same if we instead assume real wage flexibility and full employment.

Under DER exports of the tradeable good and imports of the consumption good take place at a fixed rate set by the authorities which we will call the official rate "e." The trade of FFE takes place at a freely determined rate "f." If no leaks exist between the financial and official markets, then P_x and P_n are delinked from f. In the model we present: $P_x=P_n=e$.[10] Formally, the investment problem is:[11]

$$\underset{n_t, F_{dt}}{Max} \quad P_{x,t+1}T(N_t) + f_{t+1} \cdot [1+i^*]F_{dt} \qquad (2)$$

$$s.t. \quad M_t \geq P_{nt}N_t + f_t F_{dt} \qquad (3)$$

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[9] We assume that there does not exist a forward market for FFE. Given the illegal character of the activities that generally take place in this market, this assumption seems appropriate.

[10] We assume that the real wage is equal to 1/a units of the consumption good, and that $P_c^*=P_x^*=1$. The asterisk denotes foreign prices. Thus $P_n = P_x = e$.

[11] In section III we consider the lifetime maximization problem of the investor, and we analyze the evolution of monetary balances.

M_t represents the domestic money holdings of the investor; F_{dt} represents the demand for FFE. At the optimum, the marginal returns on the NTI and FFE are equalized:

$$[P_{x,t+1}/P_{nt}] \cdot T'(N_t) = [f_{t+1}/f_t] \cdot [1+i^*] \qquad (4)$$

By reducing his demand for the NTI by one unit at time t, the investor can buy P_{nt}/f_t units of FFE, earn a return of $[1+i^*]$, and sell them at time t+1 for domestic money at a price of f_{t+1}. Therefore, the marginal return on reducing the demand for the NTI is:

$[f_{t+1}/f_t] \cdot [1+i^*] \cdot P_{nt}$. The marginal cost is the sacrifice in future export revenues:

$P_{x,t+1} \cdot T'(N_t)$.

The transitory financial disturbance we consider is an unexpected increase in the foreign interest rate, which is known to last for one period only.[12]

That is, $\qquad i^*_1 = i^* + \Delta i^*, \qquad i^*_0 = i^*_2 = i^*_3 = ... = i^*$

This shock induces investors to shift their wealth overseas at t=1, causing a reduction in the availability of working capital. The scarcity of working capital in turn reduces the level of production and thus the demand for the NTI. As a result, the marginal product of the NTI is higher and therefore the profitability of domestic investment increases. The process continues until marginal returns are equalized and the arbitrage condition (4) is satisfied.

We will show that DER reduce the extent of the shift from the NTI into FFE. The argument is is in two parts:

i. The transitory increase in i* generates an immediate depreciation and an expected future appreciation of f. We will prove this in sections II.3 and III. If no leaks between the financial and official markets exist, then the fall in f_{t+1}/f_t generates expected capital losses on the holdings of FFE. Meanwhile, the profitability of domestic investment -- $[P_{x,t+1}/P_{nt}] \cdot T'(N_t)$-- remains unchanged because P_x and P_n are delinked from f. As a

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[12] The increase in i* can be interpreted as any shock which reduces the profitability of domestic investment relative to overseas investment.
In this paper we consider only transitory disturbances. If a disturbance would be permanent, then the insulating mechanisms of dual exchange rates would not exist.

result of these capital losses, the net increase in the relative attractiveness of overseas investment is lower than the increase in i*. Clearly, the reduction in the demand for the NTI required to restore portfolio equilibrium is lower than under uniform exchange rates.

ii. If we allow for the existence of leaks, then it is no longer clear whether the reduction in f_{t+1}/f_t is able to generate expected capital losses on the holdings of FFE and reduce the profitability of overseas investment relative to domestic investment. In section II.2 we take as given that f_{t+1}/f_t falls and show that, even if leaks exist, the above is unambiguously true.

The existence of leakages between the official and the financial markets is a very important real world phenomenon.[13] A natural way to introduce leaks into the model is by assuming that the authorities cannot monitor all the transactions made. In particular, the authorities can only discover and confiscate a proportion $[1-\varphi]$ of the exports underinvoiced and a proportion $[1-\pi]$ of the FFE illegally sold in the official market. This implies that when the premium is positive ($f>e$) and no trade taxes are existent[14] investors find it profitable to underinvoice a share Θ of their official exports and sell them through the financial market. Conversely, when the premium is negative, investors illegally sell a share Ω of their holdings of FFE in the official market.[15] As the underinvoicing rate Θ is increased the probability of being discovered increases: $\varphi'(\Theta)<0$, and at the limit, if the entire production is sold through the financial market, then confiscation is certain, i.e. $\varphi(1)=0$. The possibility of underinvoicing exports implies that the effective price of exports is not P_x, but rather a linear combination of P_x and f. Formally, the effective price of exports $-\hat{P}_x-$ is an optimal value function defined by the following problem:[16]

$$\hat{P}_x(f)= \quad \underset{\Theta}{Max} \quad \{P_x[1-\Theta] + f\varphi\Theta\} \qquad (5a)$$

$$s.t. \quad \varphi(0)=1, \quad \varphi(1)=0, \quad \varphi'(\Theta)<0, \quad \varphi''(\Theta)<0$$
$$\Theta \in [0,1]$$

The solution to this problem is:[17]

$$\Theta(f) = \begin{cases} [[e/f - \varphi]/\varphi'>0 & f>e \\ \\ 0 & f\leq e \end{cases} \qquad (6)$$

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[13] In many developing countries, underinvoicing of exports is in fact an important channel for capital flight. Gulati (1985) finds that according to partner country data, during the period 1977-83 underinvoicing of exports as a percentage of official exports was: 34% for Mexico, 13% for Brazil, 20% for Argentina.

[14] For the effect of trade taxes on the misreporting of exports and imports see Bhagwati (1974) and Dornbusch (1987). For analyses of illegal activities in international trade see also Bhandari and Decaluwe (1987) and De Macedo (1982).

[15] We do not consider the overinvoicing of imports because investors are not importers. The import content of production is embeded in the nontradeable input. This is because workers consume an imported good.

[16] Problem (5) assumes that at this stage investors are risk neutral.

[17] The FOC is: $-e +f\varphi + f\varphi'\Theta = 0$. $f\varphi'\Theta<0$ implies that $e<f\varphi$. It follows that $\Theta>0$.
$\Theta(1) = [1 - \varphi(\Theta(1))]/\varphi' = 0$ because $\varphi(0)=1$ and $\varphi'(\Theta)>0$.

Θ equals zero when there is no premium ($e=f$). As the premium rises, the proportion of underinvoiced exports increases monotonically:

$$\Theta'(f \geq e) = -e/[2\varphi' + \Theta\varphi'']f^2 > 0 \qquad (7)$$

When the premium is negative ($f<e$) exports are not underinvoiced. Thus, the effective price of exports is P_x not $\hat{P}_x$. In this case, holders of FFE have an incentive to sell a share Ω of their holdings of FFE in the official market. Therefore, when the premium is negative the effective price of FFE ($\hat{f}$) is:

$$\hat{f}(f) = \underset{\Omega}{Max} \quad \{e\pi\Omega + f[1-\Omega]\} \cdot F \qquad (5b)$$

$$s.t. \quad \pi(i^*/1+i^*)=1^{18}, \ \pi(1)=0, \ \pi'(\Omega \geq i^*/1+i^*)<0, \ \pi''(\Omega)<0$$

The solution for Ω is analogous to Θ. It is zero when there is a positive premium, it is equal to $i^*/1+i^*$ when the premium is zero, and it is positive when $f<e$.

Using (5a) and (5b), we can rewrite the arbitrage condition (4) as:

$$T'(N_t) \cdot \hat{P}_{xt+1}/P_{nt} = [1+i^*] \cdot \hat{f}_{t+1}/f_t \qquad (4')$$

$$where: \hat{P}_{xt+1} = P_{xt+1}[1-\Theta] + f_{t+1}\varphi\Theta = e[1-\Theta] + f_{t+1}\varphi\Theta$$

$$\hat{f}_{t+1} = f_{t+1}[1-\Omega] + e\pi$$

Note, if $f<e$ then $\hat{P}_{xt+1} = P_{xt+1}$. And if $f>e$ then $\hat{f}_{t+1} = f_{t+1}$.

Now we are ready to prove the following proposition:

Proposition I.

Under dual exchange rates, the profitability of overseas investment relative to domestic investment is an increasing function of the ratio of future to present financial exchange rates (f_{t+1}/f_t). The existence of leakages between the official and financial markets does not break this dependency.

Proof:

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[18] This condition is necessary for the long run viability of the dual exchange rate system: it implies that in steady state, investors will sell their interest earnings on financial foreign exchange to the central bank. Some authors assume that interest payments are channelled through the official market, see Flood and Marion (1982).

From (4´) it follows that we need to prove that $[\hat{P}_{xt+1}/P_{nt}]/[\hat{f}_{t+1}/f_t]$ is increasing in

f_{t+1}/f_t. Note that since that P_{nt} is delinked from f_t and f_{t+1}, it is sufficient to show that

$\partial[\hat{P}_{xt+1}/\hat{f}_{t+1}]/\partial f_{t+1}<0$. Consider two cases:

Case i: $f_{t+1}>e$.

Given that $\hat{P}_{xt+1}$ is an optimal value function, we can use the envelope theorem to get

$\partial \hat{P}_{xt+1}/\partial f_{t+1} = \varphi\Theta$. Since the premium is positive, no FFE is sold in the official market

$(\Omega=0)$; thus $\partial \hat{f}_{t+1}/\partial f_{t+1}=1$. It follows from $\varphi\Theta<1$ that $\partial[\hat{P}_{xt+1}/\hat{f}_{t+1}]/\partial[f_{t+1}/f_t]|f_{t+1}>e<0$.

Case ii: $f_{t+1}<e$.

Using the envelope theorem, $\partial \hat{f}_{t+1}/\partial f_{t+1} = 1-\Omega$. Given that the premium is negative,

exports are not underinvoiced; thus $\partial \hat{P}_{xt+1}/\partial f_{t+1}=0$. It follows from $1-\Omega>0$ that

$\partial[\hat{P}_{xt+1}/\hat{f}_{t+1}]/\partial[f_{t+1}/f_t]|f_{t+1}<e<0$.   QED.

II.3 Dynamics of the financial exchange rate.

In this subsection we will show with the use of a phase diagram that a temporary

increase in i* generates an immediate depreciation and a future appreciation of f. The phase

diagram should be considered as merely suggestive, since our model is in discrete time. In

section III we present a formal treatment and derive from utility maximization the behavior

of consumption and money balances underlying the dynamics of f[19].

Let us first consider the supply of FFE. It has two components: a stock and a flow

component. The stock component is determined by last period's supply. The flow

component, Z, is determined by the premium of the financial over the official rate:

$$F_{st} = [1+i^*] \cdot F_{st-1} + Z_t \qquad\qquad (9)$$

$$Z_t = \begin{cases} \varphi\Theta(f_t)\cdot T(N_{t-1}) > 0 & \text{if } f>e \\ -\Omega(f_t)\cdot[1+i^*]F_{st-1} \leq 0 & \text{if } f\leq e \end{cases} \qquad (10)$$

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[19] For an analysis of exchange rate dynamics in black markets see Dornbusch (1983) and de Macedo (1982).

If the premium is positive, i.e. $f_t > e$, then exports are underinvoiced and a positive flow supply is generated. This flow is smaller than the amount of exports underinvoiced because of the confiscation made by the authorities ($\varphi > 0$). If the premium is negative, then exports are not underinvoiced. In this case FFE is channelled illegally to the official market. In steady state the premium is zero. Therefore, $Z_t = -i^* F_{st-1}$ and $F_{st} = F_{st-1}$. In figure 1, the locus along which the stock supply of FFE is constant ($\Delta F_s = F_{st} - F_{st-1} = 0$) is represented by the horizontal line $f = e$. For points above this locus $\Delta F_s > 0$ and for points bellow, $\Delta F_s < 0$.

Let us now consider the demand for FFE. It is determined simultaneously with the demand for the NTI. The marginal returns on the NTI and FFE must be equalized at each point in time as shown in (4´). The return on NTI has two components: a productivity component and a terms of trade component, i.e. $T'(N_t) \cdot \hat{P}_{xt+1} / P_{nt}$. Similarly, the return in terms of domestic currency on FFE has two components: the foreign return and capital gains or losses, i.e. $[1+i^*] \cdot \hat{f}_{t+1} / f_t$. Note that:

- The demand for FFE is decreasing in f_t. First, there is a negative wealth effect because real balances are reduced. Second, a depreciation reduces the relative profitability of overseas investment relative to domestic investment. This is because the return on FFE is reduced by the full amount of the depreciation, while the return on the NTI remains unchanged.

- The demand for FFE is increasing in f_{t+1}. Consider two cases: (1) $f_{t+1} < e$, in this case the effective price of exports ($\hat{P}_{xt+1}$) is not affected by the future appreciation and the result is straightforward. (2) $f_{t+1} \geq e$, in this case there exists underinvoicing of exports. Thus the effective price of exports increases, but by less than the future appreciation ($\Delta \hat{P}_{xt+1} = \Theta \Delta f_{t+1} < \Delta f_{t+1}$). Hence the relative profitability of FFE increases.

Portfolio equilibrium is represented in figure 1 by the $\Delta f / f = 0$ locus. This locus depicts combinations (F, f) that correspond to a steady state. Along this locus: (1) no capital gains or losses on holdings of FFE exist, and (2) the marginal returns of FFE and the NTI

~ are equalized. The $\Delta f/f=0$ locus is negatively sloped.[20] To see why, consider a depreciation of f_t while keeping f_{t+1}/f_t constant. Given level of F_{dt} the demand for the NTI is reduced, and thus the profitability of domestic investment increased. In order to restore portfolio equilibrium, the investor shifts part of his resources from FFE to the NTI. This reduces the marginal product of the NTI and equalizes marginal returns. Points to the right of $\Delta f/f=0$ represent excess supply of FFE. In order to clear the market, an expected depreciation must occur ($\Delta f>0$). The expected capital gains generated increase the demand for FFE. Points to the left of $\Delta f/f$ represent excess supply of FFE, thus $\Delta f<0$. Combining (4´) and (3) we obtain $\Delta f/f$:

$$\frac{f_{t+1} - f_t}{f_t} = \frac{T'(M_t - f_t F_t) \cdot \hat{P}_{xt+1}/P_{nt}}{1 + i^*} - 1 \qquad for \ f \geq e \qquad (11)$$

$$= \frac{T'(M_t - f_t F_t) \cdot e/P_{nt}[1-\Omega]}{1 + i^*} + e\pi\Omega/[1-\Omega]f_t - 1 \qquad for \ f \leq e$$

Note that when $f=e$ both terms in the rhs of (11) are identical. This is because if $f_{t+1}=e$, then $\hat{P}_{xt+1}=e$ and $\Omega=0$.

The system possesses saddle path stability: there exists a unique path which converges to the steady state equilibrium. Any other path violates the transversality condition: $\lim_{t\to\infty}\delta U'(C_t)M_t=0$, which characterizes the optimal consumption path of the representative investor (this is shown in section III). Hence, the saddle path is the only path consistent with optimality.

We will now show that an unexpected increase in i*, which is known to last for only one period generates an immediate depreciation and a future appreciation of f. At time t=1, when i* increases, investors shift their wealth from the NTI to FFE. In figure 2 the $\Delta f_0=0$ locus shifts to $\Delta f_1=0$. However, under DER the stock supply of FFE is fixed at

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[20] $df/dF = P_{xt+1}f_tT'''/[\Theta\varphi T' - P_{xt+1}F_tT'']$. If $T'(N)>0$ then this expression is positive. However, for sufficiently high n (equivalently, sufficiently low fF), T' might become negative and the slope positive. We will rule out the latter possibility on the grounds that investors will never operate in the region of negative marginal product of the nontradeable input.
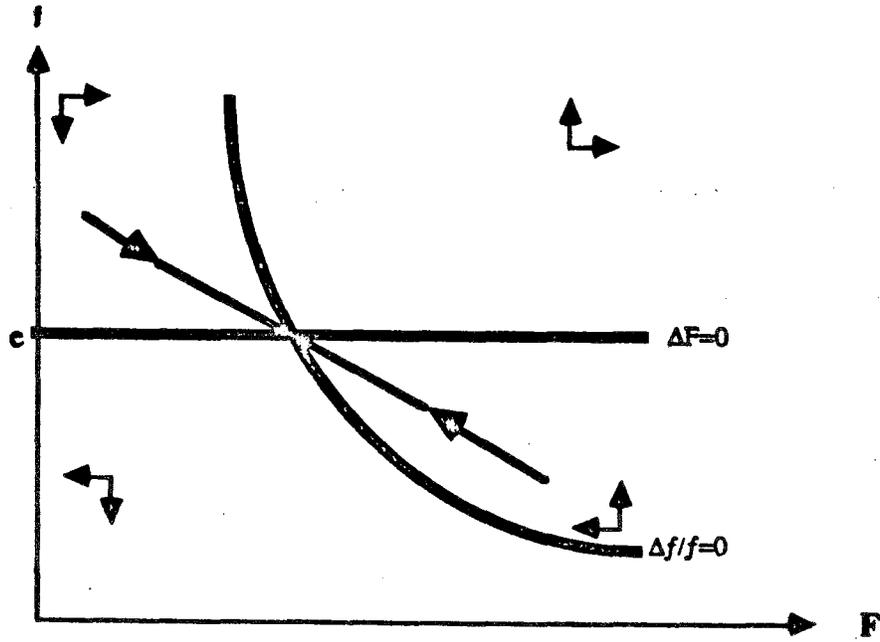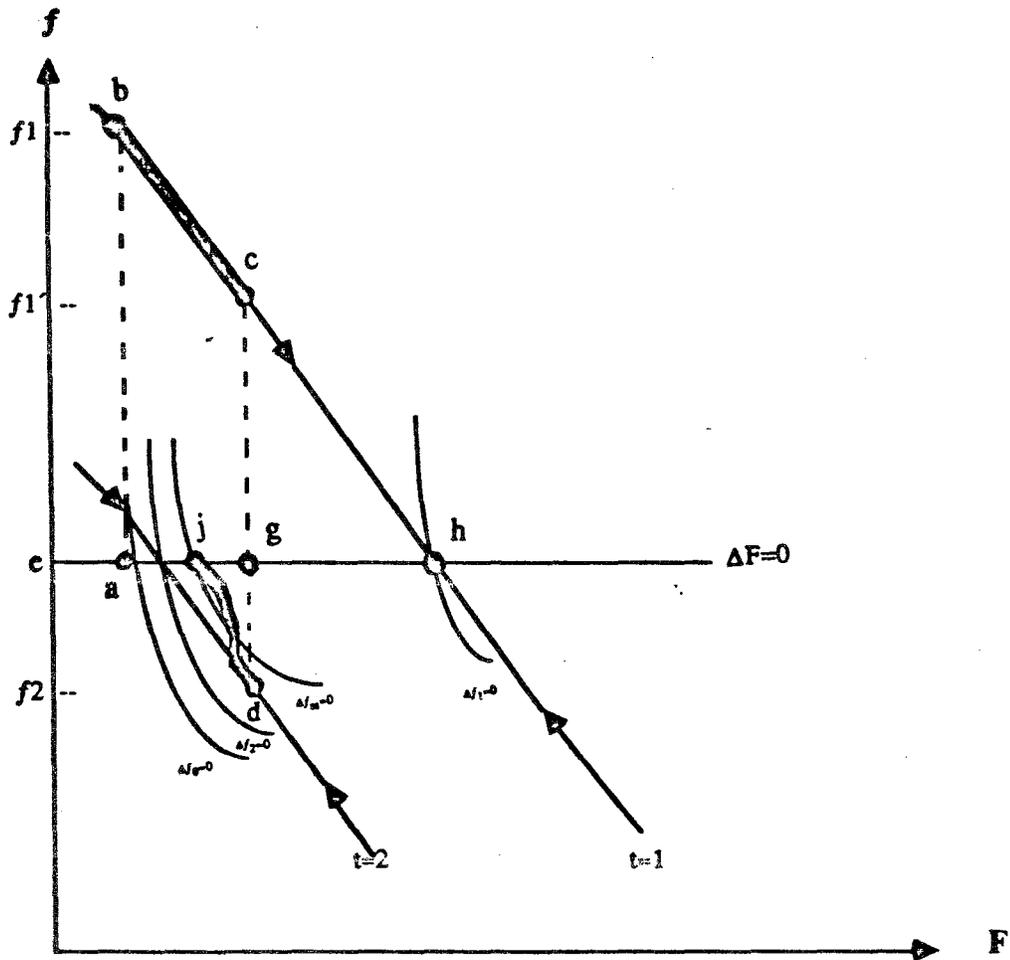
Figure 1

Figure 2

any point in time. In order to clear the excess demand of FFE at t=1, f depreciates from e to f_1 (from a to b on the saddle path corresponding to $\Delta f_1=0$). On the one side the positive premium of the financial over the official rate induces agents to underinvoice exports, generating a positive flow supply of FFE. On the other side, the demand for FFE is reduced. This is because the higher f reduces real money balances and the return on overseas investment. As excess demand is reduced, f appreciates from f_1 to f'_1[21].

At t=2 i* will return to its original level (the curve $\Delta f=0$ will shift back to $\Delta f_2=0$). This will reduce the profitability of overseas investment relative to domestic investment, for each level of i*, generating an excess supply of FFE at $f_2=f_1$. However, under DER, FFE cannot be sold legally in the official market. In order to clear the market, an expected depreciation will have to occur at t=2, i.e. $f_2<f_3$. This will create expected capital gains on the holdings of FFE and thus increase the demand for FFE at t=2.

The claim that DER insulate production hinghes upon $f_1>e>f_2$. However, we have only shown that $f_1>e$ and $f_3>f_2$, and this does not imply $f_1>e>f_2$. In principle the last inequality can be reversed. This is because the investors' monetary balances at t=2 are higher than in steady state due to their illegal sale of FFE in the official market at t=1. Therefore, ceteris paribus, their demand for FFE at t=2 will be higher and it might be possible that no excess supply of FFE will arise at $f_2 \geq f_1$. We can see this in figure 2. At t=2 the locus $\Delta f_2=0$ does not revert to its original steady state position, $\Delta f_0=0$. There are three possibilities:

   i. The increase in M is sufficiently small so that $\Delta f_2=0$ lies to the left of point g. In this case there is an appreciation and undershooting of f at t=2.

   ii. $\Delta f_2=0$ lies between g and h. In this case there is an appreciation of f at t=2, but the premium of the financial over the official exchange rate is positive.

   iii. $\Delta f_2=0$ lies to the right of h. In this case there is a depreciation of f at t=2.

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[21] The continuous-time interval between f_1 and f_1' in figure 2 corresponds to period t=1 in our discrete-time model.

If case iii occurs, then $f_1 < f_2$ and DER would not insulate production. Moreover DER would amplify the negative impact of the financial disturbance. Proposition II below establishes the impossibility of cases (ii) and (iii). Hence, at t=2, f appreciates and undershoots its steady state level, as shown in figure 2.

The segregation of the official and financial markets implies that the depreciation at t=1 and appreciation at t=2 reduce the marginal return on FFE ($\hat{f}_2/f_1[1+i^*]$) relative to the marginal return on the NTI ($\hat{P}_{x2}/P_{n1}T'(N_1)$). Therefore, under DER the marginal product of nontradeables does not need to increase to the same extent as i^*. Hence, the impact of a temporary financial disturbance on the demand for NTI is dampened. Proposition I established that this dampening effect is robust to the existence of leakages between the official and financial markets.

The insulation of the economy is achieved at a cost in terms of foreign exchange lost by the central bank. Reserves are not lost through a capital account deficit; they are lost indirectly through a current account deficit generated by the underinvoicing of exports. The system is, however, viable. If the shock is temporary, then the central bank will gradually build-up reserves, as the holders of FFE swap it illegally for domestic money. This can be seen in figure 2: at time t=2 the premium of f over e is negative. Consequently, exports are not underinvoiced. Moreover, a share Ω of FFE is illegally sold in the official market, increasing the reserves of the central bank. The remaining excess supply of FFE is cleared by an expected depreciation, i.e. $f_2 < f_3$. During the following periods the system evolves through the path "di", with a negative and decreasing premium until the new steady state is reached. During the process FFE is gradually sold in the official market. Note that in the new steady state the curve $\Delta f_{ss}=0$ lies to the right of the original one. This is because the financial disturbance increased the investors' wealth.

Under uniform exchange rates, the impact of a temporary financial disturbance on the demand for nontradeables is not dampened by the movements of the exchange rate; regardless of the investment made at t=1, the resulting foreign exchange proceeds will be

sold during t=2 at the same price. Thus, the present depreciation and expected appreciation do not affect the relative marginal returns of the two activities as they do under DER.

III. Microfoundations.[22]

In section II we showed that under DER the negative effect of a financial disturbance on the demand for the NTI is dampened by an immediate depreciation and an expected appreciation of f. We noted in section II that a necessary condition for an expected appreciation of f is that an excess supply of FFE arise once the disturbance has ended at (t=2). However, it is not immediately obvious why this will happen. At t=2 the stock supply of FFE is higher because of the underinvoicing of exports and the higher i* during t=1; yet, the demand for FFE is also higher because investors are wealthier. In this section we will derive the consumption decision from utility maximization and show that the evolution of wealth conjectured in section II is indeed correct, i.e. an excess supply of FFE arises at t=2.[23] We will also show that the choice of the saddle path as the unique path along which the system evolves is optimal and that DER are viable in the long run in the absence of permanent disturbances.

Variations in f affect holders of domestic money and holders of the exportable good differently. For the former group, a depreciation of f reduces real domestic money balances. For the latter group, a depreciation of f increases the earnings from underinvoiced exports. To reflect these differences we will assume that there exist two types of investors differing only in the timing of their investment decisions. When one type of investor receives the proceeds of his past investments, the other type invests. For expositional purposes we will call these types of investors "Odd" and "Even".[24] "Odd

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[22] This section can be skipped without any loss of continuity.

[23] In terms of figure 2 it is conjectured that the increase in M at t=2 is sufficiently small so that at t=2, the $\Delta f/f=0$ curve shifts leftwards enough to guarantee that f_2 undershoots its steady state level.

[24] This structure is similar to an overlaping generations model. Note however that in the OLG model the "period" is meant to represent a real time interval of a lifetime, whereas the phenomenon we are analyzing is short-run in nature.

investors" consume during odd periods and invest during even periods and, conversely, "Even investors" consume during even periods and invest during odd periods. The timing is as follows: At time t=1 Even enters the period with domestic money carried over from t=0 (M_{e1}, e stands for Even). Odd enters the period with a stock of FFE and his production of the exportable good. Let us describe the actions of each type of representative investor:

Odd: Since at t=1 i* is above its steady state level, $f_1 > e$; therefore Odd sells to Even his FFE and a share Θ_1 of his exports for domestic money. He sells the remaining exports in the official market. Odd spends part of the domestic money he just received (M_{o1}) on the importable consumption good; he carries over the rest to period t=2 ($M_{o2} = M_{o1} - P_{n1}C_1$).

Even: At t=1 he spends the domestic money with which he entered the period, buying FFE from Odd (F_{d1}) and buying the NTI (N_1).

During t=2 the roles are reversed. Since i* returns to its original level, $f_2 < e$; therefore Even sells all the exportable good he has produced and a share Ω_2 of his stock of FFE in the official market. He sells the remaining FFE to Odd. Even spends part of the domestic money just received ($M_{e2} = P_{t2}T_2 + \hat{f}_2 F_{d1}[1+i^*]$) on the consumption good, and he carries over the rest of the money to period t=3 ($M_{e3} = M_{e2} - P_{n2}C_2$). Odd spends all his money buying the NTI and FFE from Even ($M_{o2} = P_{n2}N_2 + f_2 F_{d2}$). In the periods following, this process is repeated.

During each period, all the domestic money received by the nontradeable sector is used to consume the imported consumption good which is imported by the government and sold at the official exchange rate ($P_c = eP_c^* = e$). Any imbalance in the official market is met with variations in the central bank's reserves.

Next, we will formalize the decisions of the representative investor. We can think of the decision interval of the representative investor as composed of two calendar periods. During each interval, the representative investor makes five decisions[25].

Even chooses the sequences:

$$\{\Theta_{2j}, \Omega_{2j}, C_{2j}, n_{2j+1}, F_{d,2j+1}\} \qquad \text{to Maximize} \quad \Sigma_{j=\tau} \delta^{j-\tau} U(C_{2j}) \tag{12a}$$

Odd chooses the sequences:

$$\{ n_{2j}, F_{d,2j}, \Theta_{2j+1}, \Omega_{2j+1}, C_{2j+1}\} \text{ to Maximize} \quad \Sigma_{j=\tau} \delta^{j-\tau} U(C_{2j+1}) \tag{12b}$$

$$\tau=...,-1,0,1,...; \quad j=\tau, \tau+1, ...$$

In section II we analyzed the choice of Θ and Ω for a given level of M. Let us now consider the consumption decision. This decision determines the evolution of M. The solution to (12) can be characterized using the dynamic programming approach. We can write (12) as:

$$J(M_t) = \underset{C_t}{Max}\, U(C_t) \; + \; \delta J(M_{t+2}) \tag{13}$$

$$M_{t+1} = M_t - P_{n,t}C_t$$
$$M_{t+2} = \underset{\alpha_{2t}}{Max}\, \hat{P}_{x,t+2} \cdot T(M_{t+1}\alpha_{t+1}/P_{n,t+1}) \; + \; [\hat{f}_{t+2}/f_{t+1}] \cdot M_{t+1}\alpha_{t+1}[1+i^*]$$

$$\alpha_t = P_{nt}n_t/M_t$$
$$\hat{P}_{xt} = f_t\Theta_t\varphi + e[1-\Theta_t] \tag{5a}$$
$$f_t = f_t[1-\Omega_t] + e\pi\Omega_t \tag{5b}$$

By choosing C_t, the control variable, next period's money balances (M_{t+1}) are determined. Given M_{t+1}, the investor chooses α_{t+1} and Θ_{t+2} optimally, as shown by equations (4)-(6). These choices in turn determine M_{t+2} which is the state variable during the next two-period interval. At time t+2, the investor chooses C_{t+2}, and the same decision cycle is repeated.

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[25] We assume that investors do not discount time within a two period interval and that $U'(0)=\infty$ & $U'(\infty)=0$.

The conditions for an optimum are[26]:

$$U'(C_t) \cdot f_{t+1} = U'(C_{t+2}) \cdot \hat{f}_{t+2} \qquad (14)$$

$$\lim_{i \to \infty} \delta^i U'(C_i) \cdot M_i = 0 \qquad (15)$$

By sacrificing one unit of consumption at time t, the agent's savings increase by $P_{c,t}$. Thus, at t+1 he will be able to increase his investment by $P_{c,t}/f_{t+1}$, which would result in an increase in t+2's income equal to: $P_{c,t} \cdot [1+i^*] \cdot \hat{f}_{t+2}/f_{t+1}$. This increase in income would allow consumption to be augmented by: $P_{c,t}/P_{c,t+2} \cdot [1+i] \cdot \hat{f}_{t+2}/f_{t+1}$. Hence, in present value terms, utility will increase by $U'(C_{t+2})\hat{f}_{t+2}/f_{t+1}$. The price of the consumption good does not influence the agent's intertemporal decision because under DER the offi[27]cial exchange rate is fixed. In this model: $P_{c,t}=P_{c,t+2}=e$.

The Euler condition (14) determines only the slope of the optimal consumption path. The transversality condition (15) allows us to fully characterize the consumption path. According to (15), the present value of money holdings must approach zero as t tends to infinity. $U'(C)$ can be interpreted as the shadow price of money.

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[26] Note that: $U'(C_t) - \delta J'(M_{t+2}) \cdot \partial M_{t+2}/\partial M_{t+1} \cdot P_{n,t} = 0$ (14')

Given that M_{t+2} is an optimal value function we can apply the envelope theorem to get:

$$\partial M_{t+2}/\partial M_{t+1} = \alpha_{t+1}\hat{P}_{x,t+2}/P_{n,t+1}T' + [1-\alpha_{t+1}][1+i^*]\hat{f}_{t+2}/f_{t+1}$$
$$= [1+i^*]f_{t+2}/f_{t+1}$$

The last equality follows from (4). Substituting it in (14'), and assuming that $\delta=1/[1+i^*]$, we get:

$$U'(C_t) = J'(M_{t+2}) \cdot \hat{f}_{t+2}/f_{t+1} \qquad (14'')$$

Note also that $J(M_{t+2})$ is a optimal value function, thus using the envelope theorem, it follows from (13) that:

$$J'(M_{t+2}) = \delta J'(M_{t+4}) \cdot \partial M_{t+4}/\partial M_{t+3} = J'(M_{t+4}) \cdot f_{t+4}/f_{t+3}$$
$$= U'(C_{t+2})$$

The last equality follows from (14'). By substituting it in (14'') we get (14).

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The characterization of the steady state will serve to show that DER are viable in the long run. It will also be useful in deriving the response of the system to a financial shock. In steady state, the excess demand for FFE is zero at $f = e$. Thus, the flow supply of FFE is zero and $f = e$. The fact that f is constant over time combined with the Euler condition (14) imply that the consumption of each type of investor is constant over time. Moreover, the transversality condition (15) requires that the monetary balances before consumption of each type of investor be constant over time. This implies that the steady state level of consumption must equal the total return on the agents' investments:

$$eC_t = \{ i^* \cdot [1 - \alpha_{t-1}] - \alpha_{t-1} \} M_{jt-1} + eT(n_{t-1}) \quad j = o \text{ for } t \text{ even, } e \text{ for } t \text{ odd} \quad (16)$$

If consumption were lower, then monetary balances would be growing at an increasing rate. Eventually they would grow faster than δ, in which case the transversality condition (15) would be violated. If consumption were higher, then monetary balances would erode and eventually they would become zero. This would cause a reduction in C to zero, making the marginal utility of the investor infinite,[28] violating (14) and (15). Next, note that the level of consumption and monetary balances of both types of investors must be identical. If not, then $F_{dt} \neq F_{dt-1}$. This would imply $e \neq f$, which cannot be true in steady state. Hence, in steady state:

$$M_{e0} = M_{o1} = M_{e2} = M_{o3} = \ldots \quad (17)$$

$$M_{o0} = M_{e1} = M_{o2} = M_{e3} = \ldots$$

Next, we will prove that DER are viable in the long run by showing that the net flow of the central bank's reserves is zero. Under DER all current account transactions are carried out at the fixed official exchange rate (e). The central bank must sell or buy any amount of foreign exchange derived from current account transactions. In this model, demand for reserves is derived from the consumption of the imported consumption good. The supply of reserves is derived from "official" exports, from confiscations of underinvoiced exports, and from illegal sales of FFE to the official market. We will show

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[28] We have assumed $U'(0) = \infty$.

that in steady state these flows sum up to zero. Let us first analyze the supply of FFE.

Consider an odd period, say $t=1$. Since in steady state $f=e$ and $\pi(i^*/1+i^*)=1$, Odd sells all his export proceeds and interest income in the official market. Therefore, the supply of reserves (R_{s1}) in terms of foreign currency is:

$$R_{s1} = T_1 + i^*F_{d0}$$
$$= T_1 + i^*[1-\alpha_0]M_{00}/e$$

The demand for reserves is determined by the demand for the consumption good by the Odd investor and by the workers of the nontradeable sector. Given that workers have no access to capital markets, their consumption expenditure is equal to their income. Thus:

$$R_{d1} = P_{n1}N_1/e + P_cC_1$$

Note that the workers' income is equal to Even's investment in the nontradeable input ($\alpha_1 M_{e1}$) and that (by (16)) Odd's consumption expenditures are equal to his investments' returns ($i^*[1-\alpha_0]M_{00} + eT_1 - \alpha_0M_{00}$). Furthermore, in steady state, α is constant and $M_{00}=M_{e1}$ (by 17). Therefore,

$$eC_1 + N_1P_{n1}/e = i^*[1-\alpha_0]M_{00} + eT_1, \text{ and}$$
$$R_{d1} = T_1 + i^*[1-\alpha_0]M_{00}/e$$

Thus $R_d=R_s$, i.e. the net flow of reserves in steady state is zero. Hence, the system is viable in the long run[29].

III.2 Temporary Financial Disturbance.

In this subsection we will derive the impact effect of a temporary increase in the foreign interest rate on the production of the nontradeable good. We will also show that the dynamics of the financial exchange rate derived in section II are indeed correct.

The extent of the shift from NTI to FFE (dN_1 and dF_{d1}) depends not only on df_1, but also on df_2, which is itself a function of dF_{d2}. Moreover, dF_{d2} in turn depends on df_3 and so on. We will first show that for an unchanged f_2: $dN_1<0$, $df_1>0$ and $dF_{d1}>0$.

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[29] Long run viability of dual exchange systems is discussed by Frenkel and Razin (1985).

Second, we will show that the optimality conditions (14) and (15) imply that f_2 must undershoot its long run value (e), i.e. $df_2 < 0$. Therefore, we will conclude that the impact effect of Δi^* (dN_1/di^*) is dampened. This is because $d^2N_1/df_1df_2 < 0$.

As in section II, let $i_1^* = i^* + \Delta i^*$, $i_0^* = i_2^* = ... = i^*$. At time t=1, at an unchanged f_2, the increase in i^* induces Even to demand more FFE (relative to the steady state) and less of the NTI. In order to generate a positive flow supply of FFE, f has to be depreciated $(df_1 > 0)$[30]. The positive premium induces Odd to sell part of his exports in the financial market, generating a flow supply of FFE equal to: $\Theta(f_1) \cdot \varphi \cdot T(N_0)$. From (3), (4´) and (9), it follows that for a given df_2[31]:

$$dN_1 = -\{ [1+i^*]d\widehat{f}_2 + \widehat{f}_2 di^* \} \cdot \{ F_{d1} + \Theta' f_1 T(N_0) \}/f_1 \zeta \qquad (18)$$

$$df_1 = \{ \widehat{f}_2/f_1 \} \cdot P_{n,1}/\zeta$$

$$dF_{d1} = \{ \widehat{f}_2/f_1 \} \cdot \Theta' P_{n,1} T(N_0)/\zeta$$

where: $\zeta = e[1+i^*] \cdot \widehat{f}_2/f_1 - \{F_{d1} + \Theta' f_1 T(N_0)\} \cdot T''(N_1) > 0$

$\Theta'(f_1) = -e/\{2\varphi' + \Theta\varphi''\} \cdot f_1^2 > 0$

$d\widehat{f}_2 = [1-\Omega]df_2$ *by the envelope theorem.*

We can relate (18) to the analysis of section II. If f_2 were equal to f_1, then no expected capital losses on FFE would be generated. Consequently: $dN_1 < 0$, $dF_{d1} > 0$ and $df_1 > 0$ (substitute $df_2 = df_1 > 0$ in (18)). As f_2 appreciates, the negative impact of the financial disturbance on production is dampened: $d^2N_1/df_1df_2 = -[1-\Omega] \cdot [1-i^*] \cdot [F_{d1} + \Theta' f_1 T(N_0)]/f_1\zeta < 0$

Proposition II below establishes that f appreciates at t=2. Moreover, it undershoots its long run level e. This is proved by showing that at f_2=e, there exists an excess supply of FFE conditional on $f_3 < e$. Proposition III proves that the Euler and transversality conditions (14) and (15) imply that f_3 must indeed be smaller than e.

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[30] If $F_{d0}[1+i^*] \geq F_{d1}$, then there is no movement in f_1 (at unchanged f_2).

[31] Consumption decisions affect n_1, f_1 and F_{d1} only through f_2.

Proposition II.

A temporary increase in the foreign interest rate at $t=1$ generates an immediate depreciation and an expected appreciation of the financial exchange rate. Moreover, at $t=2$, the premium of the financial over the official rate must be negative, i.e. $f_2 < e < f_1$.

Proof:

Throughout this proof we will assume that $f_3 \le e$. Proposition III shows that this is correct. We will first show that if $f_2 = f_1 > e$, then there exists an excess supply of FFE at $t=2$. Note that this is not obvious because both supply and demand for FFE increase.

i. The supply of FFE at $t=2$ is higher than in steady state because:

 • the increase in i* leads to a higher stock supply of FFE at $t=2$. This occurs because at $t=1$, i* and Even's demand for FFE were higher than their steady state levels.

 • The positive premium $(f_2 > e)$ generates a positive flow supply of FFE:

$$F_{s2}(f_2 \ge e) = F_{d1}[1 + i^* + \Delta i^*] + \varphi \Theta(f_2) T(N_1) \qquad (9')$$

ii. However, the demand for FFE at $t=2$ also increases $--F_{d2} > F_{d0}$ because at $t=2$ Odd is "wealthier" due to the higher price at which he sold his FFE during $t=1$.

$$F_{d2} = [eF_{d0} + \Delta M_{o1} - P_{c1}\Delta C_1 - P_{n2}\Delta N_2]/f_2 \qquad (8')$$

 where Δ represents the increase relative to the previous steady state level, at $t=0$.

At first glance it is not clear whether there is an excess supply or demand. Note, however, that under the assumption that $f_1 = f_2 \ge f_3$, F_{d2} is bounded above by:

$$[eF_{d0} + \Delta M_{o1}]/f_2 > F_{d2} \qquad (19)$$

The reason is that if $f_1 = f_2 \ge f_3$, then $\Delta C_1 \ge 0$ and $\Delta N_2 \ge 0$. The increase in Odd's consumption follows from the Euler condition (14): $f_2 U'(C_1) = f_3 U'(C_3)$ --substitution effect-- and the fact that $\Delta M_{o1} > 0$ --wealth effect. Odd's demand for the NTI increases because at $t=2$ he expects capital losses on his holdings of FFE, and i* is back to its original level.

Next, note that the increase in Odd's money holdings at t=1 is equal to the increase in Even's expenditures in FFE relative to the original steady state. That is,

$$\Delta M_{o1} = F_{d0}[1+i^*] \cdot [f_1 - e] + \Theta_1 T(N_0)[\varphi f_1 - e], \text{ and}$$

$$F_{d1} = F_{d0}[1+i^*] + \varphi \Theta(f_1)T(n_0)$$

Substituting these expressions in (8´) and (19) we get:

$$F_{s2} - F_{d2}(f_2 \geq f_3) \geq \{F_0[1+i^*] + \varphi \Theta(f_1)T_1\}\{[1+i^*+\Delta i^*] - f_1/f_2\} + \varphi \Theta(f_2)T_2 + ieF_0/f_2$$

$$(20)$$

The last two terms in the rhs of (20) are positive. Therefore, there is an excess supply of FFE at t=2 if $f_1=f_2 \geq f_3$. Note that an appreciation of f_2 brings about a reduction in both the stock and the flow component of Even's supply of FFE (F_{s2}), and an increase in Odd's demand for FFE (F_{d2}). Hence, $f_2 < f_1$ --given that $f_3 \leq f_2$.

Let us now prove that $f_2 < e$ by showing that there exists an excess supply of FFE at $f_2 = e$ under the maintained assumption that $f_3 \leq f_2$. According to (20), a sufficient condition for there to be an excess supply of FFE at $e = f_2 \geq f_3$ is:

$$[1+i^*+\Delta i^*] \geq f_1/e$$

We will prove that this is in fact true using the arbitrage condition (4´) and noting that investors have perfect foresight. At t=1, under the assumption that $f_2 = e$, the following must hold:

$$1+i^*+\Delta i^* = [f_1/P_n] \cdot T'(N_1)$$

Note that $\quad T'(N_1) > T'(N_{ss}) \qquad$ *because $N_1 < N_{ss}$ by (18), and $T'' < 0$*

$$T'(N_{ss}) = [1+i^*]P_n/e > P_n/e \qquad f_t = f_{t+1} \text{ in steady state (ss)}$$

thus, $\quad 1+i^*+\Delta i^* \quad > f_1/e$

Hence, there is an excess supply of FFE at $f_2 = e$. Given that $\partial F_{s2}/\partial f_2 > 0$ and $\partial F_{d2}/\partial f_2 = -F_2/f_2 < 0$, we conclude that $f_2 < e$. $\qquad\qquad$ QED

Proposition III.

Following a temporary increase of the foreign interest rate at t=1, the premium of the financial over the official exchange rate at t=3 is nonpositive ($f_3 \leq e$). A positive premium would generate an ever-increasing premium which would lead to a violation of the transversality condition.

Proof:

Suppose it is not true. i.e. let the market for FFE during t=3 clear at $f_3 > e$. Assume for the moment that wealth remains fixed. The positive premium generates an inflow of FFE that will generate an excess supply of FFE at t=4. Therefore, an expected capital gain will have to occur at t=4 ($f_4 < f_5$) in order to induce Odd to hold the bigger stock of FFE. At t=5 the stock supply of FFE will be even higher than at t=4 because of the positive premium. Consequently, at t=5 higher expected capital gains would be needed in order to clear the market --$0 < f_5 - f_4 < f_6 - f_5$. It is clear that the sequence of financial exchange rates generated is not a Cauchy sequence. Thus, it will not converge. That is, the distance between f_t and f_{t+1} is nondecreasing. Hence, f_t would grow without bound. This violates the transversality condition (15) because: $M_t \rightarrow \infty$ as $f_t \rightarrow \infty$. Hence, f_3 cannot be higher than e.

Note that wealth does not remain constant. If $f_3 > e$, then Odd would underinvoice a share of his exports at t=3. Consequently, his wealth would be higher than if $f_3 < e$. The higher wealth would lead Odd to demand more FFE at t=4. If the increase in F_{d4} matched the increase in F_{s4}, then no expected capital gains would be needed to clear the market at t=4, and the sequence $e < f_3 = f_4 = f_5...$ would be feasible. Next, we will show that this is not possible.

Since $f_3 > e$, Odd sells all his FFE and a share of his exports in the financial market. That is:

$$F_{s3} = F_{d2}[1 + i^*] + \varphi \Theta_3 T_3$$

Thus, his money holdings before consumption are:

$$M_{o3} = f_3 F_{s3} + e[1 - \Theta_3] T_3$$

Part of this money is spent in consumption during t=3 and part is spent in the NTI at t=4. The rest is used to buy FFE at t=4. If $f_3=f_4=f_5=...$, then it follows from the optimality conditions (4), (14) and (15) that investment in the NTI and consumption are constant over time. Furthermore:

$$P_3C_3 = i^*f_3F_{d2} + [f_3T_3 - P_nN_2]$$

This implies that $F_{d4} = F_{d2}$. However, $F_{s4} = F_{s3}[1+i^*] + \varphi\Theta_4T_4$, which is bigger than F_{d4}. Thus, $e< f_3=f_4=f_5=...$ leads to a contradiction. Hence, either $f_3\leq e$ or $f_4<f_5$.

<div align="right">QED.</div>

In sum, we have shown that a temporary increase in i* generates an immediate depreciation and an expected future appreciation of f. These occur because when i* returns to its original level, the demand for FFE is reduced. An expected capital gain must be generated in order to absorb the excess supply of FFE, i.e. $f_2<f_3$. Given that financial exchange rates --for $t \geq 3$-- above the official rate are ruled out because they would lead to explosive behavior, f_2 must undershoot its steady state level "e." After t=3, f continues to depreciate until it converges to e. During the process, the stock of FFE accumulated at t=1 is gradually channelled to the official market until it attains its new steady state level. The process is gradual because there are convex costs associated with the illegal sale of FFE in the official market.

IV. Comparison of Dual and Uniform Flexible Exchange Rate Systems.

In this section we will show that if working capital is introduced, then a trade-off arises between dual and uniform exchange rate systems, even if nominal prices are fully flexible. The trade-off is that while the impact effect of a financial shock on domestic production is lower under DER, but the persistence of the shock is lower under flexible rates.

Under uniform flexible exchange rates (UFL) official and financial transactions are not separated. This means that: (1) investors can buy FFE at the same exchange rate at which the nontradeable input is priced[32], and (2) investors are able to sell their holdings of FFE at the same exchange rate at which they sell their production of exportables. This implies that the profitability of domestic investment relative to overseas investment is independent of the ratio f_{t+1}/f_t. In other words, the arbitrage equation which under DER is: $T'(n_t) \cdot \hat{P}_{xt+1}/P_{nt}=[1+i^*] \cdot \hat{f}_{t+1}/f_t$,[33] under UFL becomes:

$$T'(N_t) = [1 + i^*] \qquad (21)$$

Consider a temporary increase in i* at t=1. This increases the profitability of overseas investment relative to domestic investment. This induces investors to increase their demand for FFE, reducing the availability of working capital and thus the demand for the nontradeable input (NTI). As a result the marginal product of the NTI goes up, bringing (21) back to equilibrium. The reduction in the demand for the NTI at t=1 reduces workers' income. Given that they have no access to capital markets, their consumption and welfare are reduced.

Under DER, the temporary increase in i* generates an immediate depreciation and an expected future appreciation of f. This creates expected capital losses on the holdings of FFE that reduce the extent of the shift from the NTI to FFE at t=1. These insulating forces do not exist under UFL because the profitability of FFE relative to the NTI is not affected by the behavior of f_{t+1}/f_t. Consequently, all the adjustment has to come from an increase in the marginal product of the NTI.

Formally, under UFL the impact effect of a temporary financial shock on the production of the NTI is:

$$dN_1/_{UFL} = [1/T''] \cdot di^* < 0 \qquad (22)$$

Using (18) we can express $dn_1|_{DER}$ as:

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[32] In this case $P_{nt}=f_t=P_{ct}=f_t P_c^*$, $P_c^*=1$ and $P_{xt}=f_t=f_t P_x^*$.

[33] In this case $\hat{P}_{xt} = P_{xt} \cdot \{\Theta_t \varphi f_t + [1-\Theta_t]e\}$ and $\hat{f}_t = f_t[1-\Omega_t] + e\pi\Omega_t$.

$$dn_1/_{DER} \leq \frac{[\hat{f_2}/f_1]\cdot di^* + [1+i^*]df_2}{T'' - [e\cdot\hat{f_2}/f_1]\cdot[1+i^*]/[F_{d1}+\Theta'f_1T_1]} < 0$$

$$where, \hat{f_2} = e\pi\Omega + f_2[1-\Omega] \in (f_2, f_1)$$

We can conclude that the impact effect of a temporary increase in i^* on the demand for the NTI is lower under DER than under UFL ($|dN_1|_{UFL}| > |dN_1|_{DER}|$) because $f_2 < f_1$ and $df_2 < 0$. These two inequalities follow from the fact that $f_1 > e > f_2$, as proven in proposition II.

Next, we will show that the persistence of a temporary increase in i^* is higher under DER than under UFL. Under DER, the existence of convex penalty costs $(1-\pi)$ for illegally selling FFE in the official market prevents the excess stock supply of FFE from being channelled to the official market immediately after i^* returns to its original level at $t=2$.[34] For the financial market to clear while investors are trying to reduce their holdings of FFE, the home currency must be expected to depreciate in order to increase the relative profitability of overseas investment. As a result, the availability of working capital remains depressed and the fall in demand for the NTI will persist until the excess stock supply of FFE (at $f=e$) is equal to zero. Under UFL, regardless of the pattern of f_{t+1}/f_t, there is no persistence. At $t=2$ investors can freely sell any excess FFE. This implies that investment decisions satisfy: $T'(N_t) = [1+i^*]$. Thus, at $t=2$ when i^* returns to its original level, the production of the NTI returns to its steady state level.

V. Time Inconsistency of Dual Exchange Rates Systems.

A necessary condition for DER to be effective is that in the aftermath of a financial shock authorities do not attempt to reduce the excess supply of FFE and that this future policy be fully believed by the investors. This requires that:

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[34] Convex penalty costs imply that it is more likely that an investor is discovered if he underinvoices once a share Θ of his official exports than if he underinvoices twice a share $\Theta/2$. We should expect this to happen in the real world because the higher Θ, the higher the "visibility" of the action.

- _ The central bank have enough reserves to sustain the current account deficits generated by the underinvoicing of exports during periods of negative financial shocks.

- The authorities do not allow the official exchange rate to move in response to temporary financial disturbances.

- The authorities will not allow investors to sell their undesired stock of FFE in the official market in the future. More importantly, the authorities must be able to credibly precommit themselves to follow this policy in the future.

Next, we will discuss why it is difficult for the authorities to precommit to this future policy. Suppose that the foreign interest rate increases during t=1 and returns to its original level at t=2. At time t=2 the authorities could eliminate the persistence of the financial disturbance by declaring "amnesty" and allowing investors to sell the excess stock supply of FFE in the official market. This would eliminate the need to generate expected capital gains on holdings of FFE ($f_2 < f_3$) in order to clear the market during t=2 and, therefore, would bring the demand for the NTI back to its steady state level[35]. Next, note that the declaration of amnesty would be Pareto welfare improving from a t=2 perspective. Workers in the nontradeable sector would benefit from a higher level of employment. Investors would also gain by being able to freely sell their excess stock of FFE in the official market. It is now clear that at t=2 the government will find no opposition from workers or investors to declaring amnesty.

Using a backwards-induction argument, at t=1 the investors know that at t=2, once the disturbance is over, the authorities will find it optimal to declare amnesty and eliminate the excess supply of FFE. Therefore, the investors know that at t=2 f will not appreciate as much. It will follow the path through point C rather than through point B in figure 3. Thus,

---

[35] The demand for the NTI is determined by: $P_{xt+1}/P_{nt} \cdot T'(N_t) = f_{t+1}/f_t[1+i^*]$. In steady state $f_{t+1} = f_t = e$.

Figure No. 3
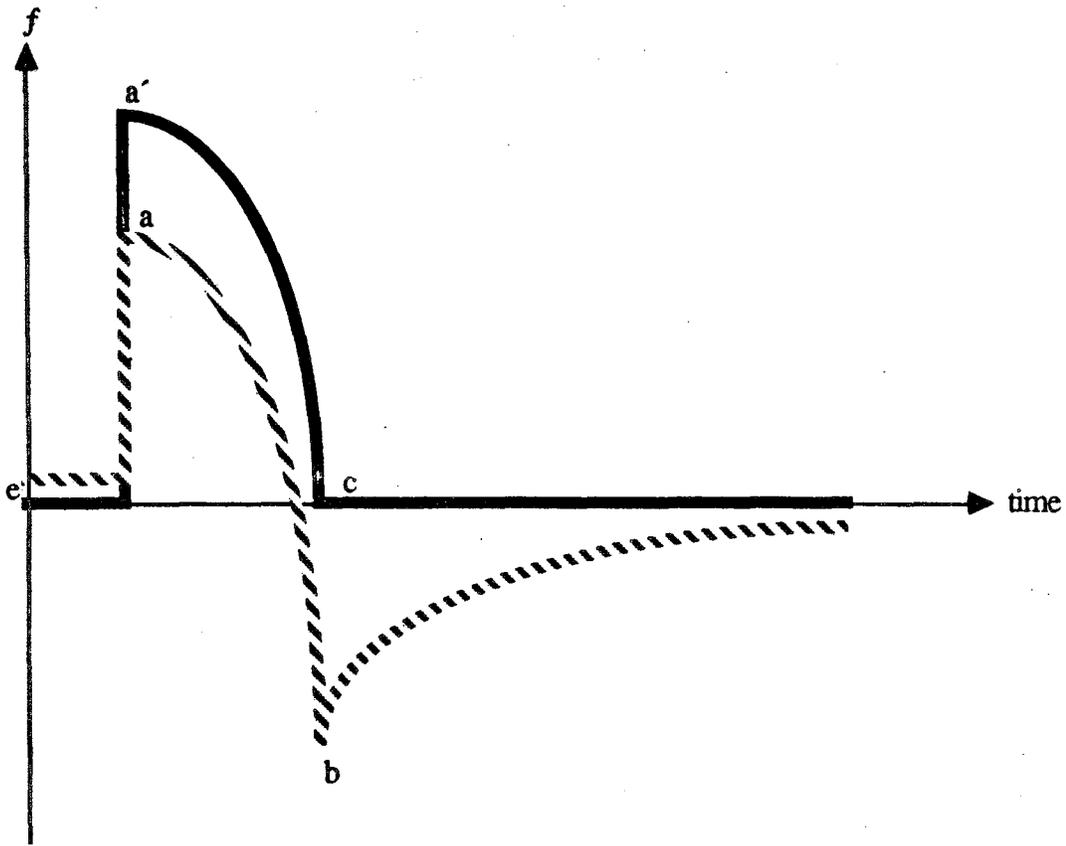
at t=1 the investors predict lower capital losses on their holdings of FFE and increase the

extent of the shift from the NTI to FFE.

We might conclude that an expected future attempt by the authorities to eliminate the

persistence of a shock results in a higher impact effect. The financial exchange rate will

follow not the path "eab" in figure 3 --which corresponds to the case where the government

can precommit itself not to change its plan in the future-- but the path "ea´c". Consequently,

the reduction in employment during t=1 would be greater.

Some authors have proposed a " neutral intervention policy," in which "the

monetary authority buys foreign exchange in the financial exchange market equal to the net

loss in official reserves arising from a current account deficit."[36] In the model we present,

such a policy undermines the insulating properties of DER because by reducing future

disequilibria in the financial market, a neutral intervention policy lowers the expected

appreciation of the financial exchange rate. This in turn amplifies the impact of a temporary

financial disturbance on domestic production.

VI. Conclusions.

In this paper we highlight the links between foreign exchange flows, the availability

of working capital, and the level of domestic production. We model these links by

recognizing that investors choose the allocation of their wealth between foreign assets and

working capital simultaneously. The working capital is then invested domestically in a

nontradeable input, which is transformed into an exportable good after one period.

Using this structure we analyze how, by limiting the extent of financial foreign

exchange flows, dual exchange rates insulate an economy from transitory financial

disturbances. We consider the case of a transitory increase in the foreign interest rate. This

shock induces investors to transfer their wealth abroad, reducing the availability of working

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[36] This quote is from Lanyi (1975). For an analysis of central bank intervention under DER see also
Fleming (1974) and Haaparanta (1986).

capital, and thus the demand for the nontradeable input. Under uniform exchange rates, an equilibrium is reached when the increase in the marginal productivity of the nontradeable input is equal to the size of the shock. Under dual rates there exists insulation because the expected future appreciation of the financial exchange rate creates expected capital losses on the holdings of financial foreign exchange. However, the profitability of domestic investment remains unchanged because the prices of exportables and nontradeables are delinked from the financial exchange rate. As a result, the net increase in the profitability of overseas investment relative to domestic investment is lower than under uniform exchange rates.

The insulation property of dual exchange rates is robust to the introduction of leaks between the official and financial markets, such as the underinvoicing of exports. Although the effective price of exports is linked to the financial exchange rate through the illegal sales in the financial market, it varies less than proportionally. Therefore, the expected appreciation caused by the shock still reduces the profitability of overseas relative to domestic investment.

We also show that a trade-off exists between uniform and dual exchange rates: the impact of a transitory disturbance on production is lower under dual rates, but persistence is higher. Persistence is higher under dual exchange rates because the excess stock of financial foreign exchange that is generated when the shock ends cannot be sold legally in the official market. It is optimally sold gradually through illegal channels. To clear the market over this period of time, expected depreciations must occur in order to increase the relative profitability of financial foreign exchange. This reduces the availability of working capital below its steady state level. Under uniform rates, the system attains its steady state immediately after the shock has ended.

Finally, we show that the stated policy of prohibiting the sale of financial foreign exchange in the official market during future dates is not credible, i.e. is time inconsistent. This is because once the financial disturbance has ended, the authorities will face no

opposition from the workers or from the investors should they choose to abandon this policy. By abandoning it, the authorities would eliminate persistence and domestic production would be immediately reestablished. If authorities cannot credibly precommit to avoid this action in the future, then the impact of the shock would be amplified since investors would anticipate the policy reversal.

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