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INTRODUCTION AND BENEFIT OF TECHNOLOGICAL CHANGE UNDER OLIGOPOLY

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Introduction and Benefit of Technological Change under Oligopoly.

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A B S T R A C T

In this paper we analyze the comparative static effects of the introduction of cost reducing technologies on the output and profits of a non-collusive oligopoly, as well as those of its members. We show that even though each firm tries to maximize its profits by decreasing costs and increasing its market share these actions may bring about a decrease in everyone's profit rate. We also show that size matters, that in a world of heterogeneous firms dominant producers take better advantage of the new technology. Unevenness is heightened by the neutral diffusion of cost reducing technological change. Finally we prove that profit margins (average or marginal) are a bad indicator of what happens to profit volumes and rates: the former can rise while the latter could fall.

R E S U M E N

En este trabajo analizamos los efectos estático-comparativos de la introducción de tecnologías eficientes sobre la producción y las ganancias tanto de una industria oligopólica como de sus miembros. Probamos que aunque cada empresa trata de maximizar sus ganancias reduciendo costos y aumentando su penetración en el mercado, estos intentos pueden provocar una caída en la tasa de ganancia de todos los productores. Asimismo, probamos que el tamaño de una empresa es una variable crucial: en un mundo de empresas heterogéneas, las más grandes aprovechan mejor la introducción de nuevas tecnologías. La difusión neutral (instantánea y gratuita) de cambios tecnológicos que reducen los costos de todas las empresas en la misma cantidad aumenta la desigualdad industrial. También mostramos que los márgenes de ganancia son un muy mal indicador de lo que le sucede a los volúmenes y las tasas de ganancia: aquellos pueden aumentar aunque estos caigan.

In this paper, we analyze the comparative static effects of the introduction of cost reducing technologies on the output and profits[1] of a non-collusive oligopoly, as well as those of its individual members. In the case of a competitive producer who sells all it wants, the only motivation for such an introduction has to be the direct cost reductions that accrue to him. Similarly, a monopolist is only interested in the productivity gains of the new investment, since its sales are directly bound by the size of the market.

In an oligopolistic environment, however, the reaction of market prices to output changes depends on the behaviour of other producers. Hence firms can be enticed by the possibility of further market penetration. The first firms that lowers its asking price could increase its share of total sales to the detriment of its competitors, thereby increasing profits. The other firms would then have to take defensive actions, triggering a process only bound by the elasticity of the demand curve and the interaction amongst the players. The outcome is far from obvious. We show that it is possible for everyone to be worse off in the new equilibrium[2]. In that case, a fallacy of composition has appeared: each firm attempts to maximize its profits and by so doing brings about a lower rate of profit for the industry as a whole.

We also show that size matters. In a world of heterogeneous firms, the initial distribution of market shares, corrected by the relative responsiveness of all firms to the actions of each individual one, determines the degree to which the new technology heightens unevenness. Specifically we prove that the neutral (cost free and instantaneous) diffusion of cost reducing technological change can lead to a fall in the sales of small firms which are taken up by large ones. Furthermore, what happens to the industry as a whole is no guide to what happens to each specific producer. In a shrinking (expanding) market, big (small) firms can fare better (worse) than average: their output, profits and rates of profit may increase (decrease) in absolute terms.

Section I describes the basic one sector model[3] used in the exercise. It is a specific application of Seade's work (1983). Section II analyzes the effects of the introduction of profitable technologies in an industry composed of identical firms. Section III generalizes the results to a heterogeneous industry in which firms differ by their size, their production function, and their market importance. Section IV presents a numerical illustration of the results of the previous section and Section V offers some conclusions.

I The Model

We use Seade's conjectural variations oligopoly model (1983) to analyze the introduction of new technologies. Consider a firm (i) which tries to maximize its profits (π_i) by manipulating its output level (q_i). As is well known, if the firm sells its output in a competitive market, it is a price taker and its sales decisions have no bearing on the price at which it can sell its output. If, on the contrary, it is a monopolist, any attempt to sell more will bring about a price decrease which will chip into revenues. In an oligipolistic market, however, the reaction function of market prices to output decisions depends on the behavior of other producers. Assume there are n firms in this market, and call

$$Q \equiv \sum_{i=1}^{n} q_i \tag{1}$$

the total output produced and sold. We will label

$$Q^c = \Gamma(\underline{q}) \tag{2}$$

its conjectured value which is a function of the behaviour of all firms in the market. In general Γ will depend on the structure of the industry (represented by the vector of outputs $\underline{q}$) and the process of expectations formation[4]. In this simple setting we assume that

$$\gamma_i \equiv \frac{dQ^c}{dq_i} \tag{3}$$

can be treated parametrically[5]. i's cost function is represented by:

$$C_i = C\,(q_i\,,T) \qquad C_q \equiv \frac{dC}{dq_i} > 0 \tag{4}$$

where T is a shift parameter that reflects the state of technical knowledge and we assume there is no fixed capital. The market's inverse demand function is:

$$p = p\,(Q) \qquad p' \equiv \frac{dp}{dQ} < 0 \tag{5}$$

The firm will try to:

$$\max_{q_i} \quad \pi_i \equiv p\,(Q)\,q_i - C^i\,(q_i\,,T) \tag{6}$$

The first and second order conditions for a maximum of (6) are, respectively[6]:

$$p'\,\gamma_i q_i + p - C^i_q = 0 \tag{7}$$

and

$$p''\,\gamma_i q_i + 2p' - C^i_{qq} < 0 \tag{8}$$

For equilibrium to be stable we need, furthermore[7]:

$$p''\,\gamma_i n q_i + p'(n + \gamma_i) - C^i_{qq} < 0 \tag{9}$$

and

$$\gamma_i p' - C^i_{qq} < 0 \tag{10}$$

We assume equation (7) - (10) to be fulfilled for all firms.

II The Symmetrical Case

Suppose, to begin with, that all firms are identical: $\gamma_i = \gamma$, and $q_i = \frac{Q}{n} \equiv q, \forall i$. We prove in Appendix I that profitable technologies will:

A) make it profitable for each firm to produce more. (P1)

B) cause market prices to fall. (P2)

Those results are hardly surprising: since we have assumed the new technology to be cost reducing, total output should increase and its price decrease as we move along the market demand curve. These well known results in a competitive or monopoly setting extend to the case of identical oligopolistic firms, because market shares remain constant.

C) cause marginal profit margins <u>to decrease</u> if the elasticity of the slope of the inverse demand function (E) is bigger than 1 and if we posit constant returns to scale. (P3)

In the case of isoelastic demand curves, <u>prices will always fall</u> more than costs, an unexpected result if one were to extrapolate from the competitive case when they change in unison. On the contrary, it is very much in keeping with what happens to a monopolistic producer where:

$$p = MR \cdot \frac{\varepsilon}{\varepsilon - 1}$$

for ε the price elasticity of the demand for output. If we assume profit maximization:

$$MR = MC$$

and

$$\frac{dp}{dT} = \frac{\varepsilon}{\varepsilon-1} \cdot \frac{dMR}{dT}$$

where $\frac{\varepsilon}{\varepsilon-1}$ has to be greater than 1 if marginal revenues is to be positive.

D) cause the volume of profits and the average profit margin to <u>decrease</u>, if E ~~i~~s bigger than 2 and the technologies considered only decrease variable costs. (P4)

Profit volumes, average margins and rates will be constant if the elasticity of demand (ε) is 1. If it is smaller than 1, profits and average margins <u>will fall</u>. We know that no monopolist would ever produce in such a range because marginal revenues are negative. But it precisely the competition amongst non-collusive oligopolists for bigger market shares what pushes them into it[8]. While the marginal revenue for the market as a whole is clearly negative, each one of them could increase its revenue by $\Delta q.p^*$ (where p^* is the new market price) <u>if</u> it could sell cheaper. That provides an incentive for each producer to introduce the new technology as long as it is cost reducing (Okishio profitable).

As they all do the same, though, the marginal revenue of the market as a whole is negative, which brings total profits down. Cost reducing technological change has incited each of them to be more voracious and that has brought about their own demise. Their joint actions have moved them further away from the collusive solution and closer to the one that would prevail under perfect competition, thereby chipping into their profits.

E) sometimes bring about a <u>fall</u> in the rate of profit. (P

A decrease in profits is not enough to bring about a fall in the rate of profit since production costs could decrease still faster. The result shown in Appendix I indicates that the profit rate will fall in the isoleastic case as $\varepsilon \to 0$, as well as when $\varepsilon \to \infty$. Indeed, there can be no oligopolistic solution in either of those two cases because the marginal revenue curve is indistinguishable from the demand curve. Hence, the closer ε is to 0 (or to ∞), the more slippery the road that leads from the collusive solution to the competitive one. Therefore the more likely it is that profit rates will fall as oligopolistic "super profits" disappear.

It is also shown in Appendix I that by giving some of the parameters their appropiate values we obtain the usual results for the cases of perfect competition and monopoly.

III The Non Symmetrical Case

These unexpected results could be attributed to the "artificially imposed homogeneity" which makes all firms produce the same amount of output and react the same way. It could be argued that in a world in which firms react differently, some will pray on others and that these changes in market shares may significantly modify the propositions previously offered. We try, in this section, to extend our results to a situation in which firms differ in their behavioral responses, their production function and the amount of output they produce. We assume, however, that the new technology decreases marginal costs by the same amount across all firms:

$$(C_{qT})_i = \xi \qquad \forall i \tag{11}$$

We show in Appendix II that profitable technologies will cause:

A) the total output produced by all firms to increase. (P

B) market prices to fall. (P7)

As before, these two results are not surprising.

C) industry-wide marginal profit margins to decrease if we assume, as before, constant returns to scale and $E > 1$. (P8)

Once more the homogenous diffusion of technological change in a world of heterogenous firms will cause prices to fall more than costs, if the demand for output is isoelastic.

D) the output of some of the smaller and/or less important firms to decrease. (P9)

In other words, size and respectability matter. Condition (A31) defines the compensated market share of a firm as:

$$\delta_i = \frac{q_i}{Q} \sum_j \frac{\gamma_i}{\gamma_j}$$

Firm i will produce less output if:

$$\delta_i < \frac{1}{1+\varepsilon}$$

Why would technological change, that decreases all marginal costs by the same amount and is freely and instantaneously available to all firms, favor the bigger and/or more dominant ones? Consider two firms (I and II) with the same γ's but different market shares, and calculate their marginal costs of production. From equation (7) we see that:

$$C_q^{I} = p'\gamma q_I + p > 0$$

$$C_q^{II} = p'\gamma q_{II} + p > 0 \qquad \text{for} \quad \gamma_I = \gamma_{II} \equiv \gamma$$

Since $p' < 0$ and

$$q_{II} > q_I \text{ , then}$$

$$C_q^{II} < C_q^{I}$$

The firm that produces more <u>has to have</u> a smaller marginal cost of production. Indeed if both I an II were considering increasing their production by one unit, the conjectured market output would grow by γ units and the price would fall by the same amount in both cases. But firm II would incur a bigger loss of total revenue because it produces more. Hence, to be in marginal equilibrium, II must have lower production costs. That is why size matters. Notice though that it has nothing to do with differential access to technology, but is a consequence of the first order conditions for maximization. When the new technology is introduced, the bigger firms can undercut the smaller ones which will see their level of output decrease (and hence their market share).

E) the profit levels of the bigger and/or more dominant firms to increase even if industry wide profits decrease. (P10)

Given Proposition 9, Proposition 4 has to be amended to incorporate the effect of the size of the firm. The profits of the growing firms can rise even though the industry's may be falling if their market shares increase enough. On the other hand, the firms that loose the most market ($\delta_i << \frac{1}{1+\varepsilon}$) can see their profits shrink while the industry's are rising. In a heterogenous world, initial conditions are crucial: once an equilibrium is perturbed by technological change, big firms pray on small ones thereby increasing the degree of concentration.

F) the average profit margins of the bigger and/or more dominant firms to decrease if $E > 1$. (P11)

This proposition should not be surprising in view of P4, P9 and P10. It shows that average profit margins can be very unreliable. Indeed, if $E > 2$ the bigger firms could see them decrease while their profit volume rises. Conversely, for $1 < E < 2$, the smaller firm's profits could fall while their margins increase: their output decreases so much that bigger margins do not help.

G) industry wide profits will decrease if $E > 2$. (P12)

This is the equivalent of P4 and can be interpreted the same way: as oligopolists try to capture each other's market, they push one another into a region that is clearly unprofitable for them as a group. The bigger firms can still profit from such a change but the industry as a whole will loose (the losses of the small firms are bigger than the gains of the big ones).

H) the industry wide rate of profit will fall if the price elasticity of demand (ε) differs enough from 1. (P13)

This is the equivalent of P5.

I) the firm's rate of profit need not change as the industry's: it depends on the firm's market position. (P14)

We cannot establish the existence of firms that go against the current, because that depends on the actual distribution of outputs and "market influence". The more numerous and diverse firms are however, the more likely it is such firm(s) will exist. Hence heterogeneity can lead to higher levels of concentration in an industry because external shocks will favor (disfavor) the bigger (smaller) firms. If, to begin with, the industry was in long run equilibrium (equal rates of profit across firms), technological change will bring about differential changes in profit rates depending on each firm's market position.

Therefore the results of Section II are robust to the heterogeneity we have introduced with the covenant that a firm's market position (represented by s_i) makes a lot of difference.

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IV A Numerical Illustration

Consider an industry with ten firms (I, II, ... X) which produce, in total, 100 units. The first one produces 55 units while the other nine produce 5 units each. We assume an isoelastic demand curve, constant returns to scale and that the conjectured market reactions are the same for all firms ($\gamma_i = \gamma \ \forall_i$). Suppose our initial parameter values are:

$$\varepsilon = 2, \ \gamma = 2, \ p = 2 \ \text{ and } \ \xi = -10\%$$

In the new equilibrium, total output will increase 16.67%, and the market price will decrease by 33.34% as can be seen from (A26) and (A27). The big firm will produce 24.16 more units (obtained from A30) while the smallones will produce .83 fewer ones. Their profit rates will increase 17.54% in the former case and only 2.9% in the latter: production costs rise 14.16% = 24.16% - 10.00% but revenues grow 34.86% for firm I, cost decrease 26.6% = 16.6% - 10.0% and revenues fall by 23.7% for the other nine. Finally we may want to notice that total output produced, marginal costs of production (and their difference) depend directly on γ.

γ	1/2	1	2	3
$\Sigma \frac{dq_j}{dT}$	11.11	12.5	16.67	25.0
C_q^{I}	1.725	1.45	.9	.35
C_q^{II-X}	1.975	1.95	1.90	1.85
$C_q^{I} - C_q^{II-X}$	-.250	-.50	-1.00	-1.50

V Conclusions

This paper has tried to analyze the impact of the introduction of new technologies in output and profits in an oligopolistic market. While working in a microeconomic framework (the analysis of the behavior of specific production units), we have obtained as intrinsically macroeconomic result: a fallacy of composition. Even though each firm introduces the new technology to increase its profits (decreasing costs and thereby penetration new markets) the result is often that industry wide profit volumes, margins and rates decrease. Since the price (output) that characterizes a fully monopolized market is bigger (smaller) than that of any non-collusive oligopoly, cost decreases have induced firms to produce more, breaking the stalemate their uncoordinated decisions had put them in and pushing them towards the competitive solution which entails lower prices, more output and smaller profits. Such a failure of the invisible hand provides strong incentives for the establishment of a policing body (legal or not, state supervised or not).

We have also shown that size matters, that big firms fare better than small ones sometimes praying on them. For that result to obtain, though, we had to assume U-shaped cost curves.

Indeed in a world of constant marginal cost curves it is difficult to conceive of a mechanism that determines the evolution of market shares as a new technology becomes available. The evolution of market shares, however, cannot be predicted without a fully dynamic model. While producers smaller than a given size may tend to shrink while big ones grow, we cannot determine a priori whether one firm will slowly take over the whole market or some stable multifirm structure will emerge. Finally, we found that profit margins are a bad indicator of what happens to profit volumes or profit rates. The former can rise while the latter may fall.

Footnotes

1 Volumes, margins and rates.

2 We assume of course that both equilibria are stable.

3 It is not easy to construct a two sector model which captures the effects of oligopolistic competition without specifying the relationship between profit rates across sectors. That could have been a distraction from the main argument of the paper.

4 See Seade (1980a, 1980b) for the conditions under which this model is stable and Iwai (1981) for an extensive discussion of expectations.

5 $\gamma = 1$ for the case of a monopoly and $\gamma = 0$ in the case of perfect competition. Notice also that if there are n identical oligopolistic firms in the market, n is an upper bound on γ.

6 We assume all functions to be twice differentiable. Notice that equation (7) implies that $p \geq C_q^i$ the difference being the oligopolist's rent.

7 See Seade (1980, b) p.24.

8 Notice that for a stable solution to exist when $E > 2$ we need:

$\gamma q p'' + 2p' < 0$ from equation (8)

$\gamma n q p'' + (n + \gamma) p' < 0$ from equation (9)

and $n q p'' + 2p' > 0$ from equation (A6)

Those restrictions are compatible with each other as can be seen from the following numerical example: $p' = -100$, $q = 10$, $n = 30$, $p'' = 1$, $\gamma = 10$. It is obvious however that in the case of a monopoly ($\gamma = n = 1$) these three conditions are incompatible. I am thankfull to Amit Bhaduri for raising this issue.

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Appendix I

The Symmetrical Case

This Appendix relies heavily on Seade's (1983) paper.

Proof of Proposition 1 (P1): "Output produced will increase".

Rewrite (7) as:

$$p'(nq)\gamma q + p(nq) - C_q(q,T) = 0$$

Total diferentiate to obtain

$$p'\gamma\, dq + \gamma q p'' n dq + p' n dq - C_{qq} dq - C_{qT} dT = 0$$

where $$C_{qq} \equiv \frac{\partial C_q}{\partial q} \quad \text{and} \quad C_{qT} \equiv \frac{\partial C_q}{\partial T} \qquad \text{(A1)}$$

Rearranging (10) obtain

$$\frac{dq}{dT} = \frac{C_{qT}}{p''\gamma nq + p'(n+\gamma) - C_{qq}} \qquad \text{(A2)}$$

If we interpret Okishio's profitability criterion to mean that the marginal cost of production will decrease with the new technology ($C_{qT} < 0$), this expression is positive since the denominator of equation (A2) is none other than equation (9).

q.e.d.

Proof of Proposition 2 (P2): "Market prices will fall".

Assuming no bankrupcies, no exits and no entries in the industry we use equation (5)

$$p = p(nq)$$

to obtain

$$\frac{dp}{dT} = p'n\frac{dq}{dT} < 0 \tag{A3}$$

q.e.d.

Proof of Proposition 3 (P3): "Marginal Profit margins will decrease if $E > 1$".

To analyze the behavior of the marginal profit margins we calculate the difference between the fall in prices $(\frac{dp}{dT})$ and the fall in the marginal cost of production (C_{qT}):

$$D \equiv \frac{dp}{dT} - C_{qT} = p'n\frac{dq}{dT} - C_{qT}$$

$$D = \frac{p'nC_{qT}}{p''\gamma nq + p'(n+\gamma) - C_{qq}} - C_{qT}$$

$$= C_{qT}\,\frac{[p'n - p''\gamma nq - p'n - p'\gamma + C_{qq}]}{p''\gamma nq + p'(n+\gamma) - C_{qq}}$$

$$= C_{qT}\,\frac{[C_{qq} - p''\gamma nq - p'\gamma]}{p''\gamma nq + p'(n+\gamma) - C_{qq}} \tag{A4}$$

Since $C_{qT} < 0$ and the denominator is also negative because of equation (9), D is $\lessgtr 0$ depending on

$$C_{qq} - p''\gamma nq - p'\gamma \text{ being } \lessgtr 0 \tag{A5}$$

The slope of the inverse demand function is $p' \equiv \frac{dp}{dQ}$ and its elasticity is:

$$E \equiv - \frac{(dp')}{dQ} \bigg| \frac{p'}{Q}$$

$$= - \frac{Qp''}{p'} = - \frac{nqp''}{p'} \tag{A6}$$

If $E > 1$

$$- nqp'' < p' \qquad \text{since} \quad p' < 0$$

As can be seen from equation (A5), if we assume Constant Returns to Scale ($C_{qq} = 0$), marginal profit margins will fall when $E > 1$.

q.e.d.

Proof of Proposition 4 (P4): "Profits and profit margins will decrease if $E > 2$".

Given the definition of profits:

$$\pi \equiv qp(nq) - C(q,T)$$

calculate

$$\frac{d\pi}{dT} = p\frac{dq}{dT} + qnp'\frac{dq}{dT} - C_q\frac{dq}{dT} - C_T$$

$$= \frac{dq}{dT}\left[p + qnp' - C_q\right] - C_T$$

Hence, technological change has two impacts on profits. The first one works through the change (increase) in output produced, and has three components:

- revenues increase (at a given market price for output)
- but the price charged decreases as we move along the market demand function.
- and production costs increase as production does (at a given marginal cost of producing).

The second impact raises profits directly by decreasing total costs ($-C_T$). To see which is stronger use equations (A2) and (7) to obtain:

$$\frac{d\pi}{dT} = \frac{C_{qT}\, p'q(n-\gamma)}{p''\gamma nq + p'(n+\gamma)} - C_T$$

Since n is a natural upper bound for γ the first term is negative while the second is positive ($-C_T$) but we do not know a priori which is bigger in absolute value. If we restrict ourselves to constant returns to scale production functions for which reductions in total costs are proportional to output (no change in fixed costs), we can advance some more. Suppose a cost function such that

$$C_T = C_{qT} \cdot q \tag{A7}$$

then we can write

$$\frac{d\pi}{dT} = \frac{C_{qT}p'q(n-\gamma) - C_{qT}q\,[p''\gamma nq + p'(n+\gamma)]}{p''\gamma nq + p'(n+\gamma)}$$

$$= \frac{C_{qT}\gamma q(-2p' - p''nq)}{p''\gamma nq + p'(n+\gamma)}$$

and, using equation (A6)

$$\frac{d\pi}{dT} = \frac{C_{qT}\gamma q\,(E-2)\,p'}{p''\gamma nq + p'(n+\gamma)} \tag{A8}$$

Hence, as long as $E > 2$ profits will fall.

Define the average profit margin (P_m) as

$$P_m \equiv \frac{\pi}{q}$$

Hence, using equations (A2) and (A8)

$$\frac{dP_m}{dT} = \frac{C_{qT}}{q^2\,[p''\gamma nq + p'(n+\gamma)]}\left[\gamma q^2(E-2)\,p' - \pi\right] \tag{A9}$$

which is < 0 if $E > 2$

q.e.d.

E being > 2 is a sufficient but not a necessary condition. Indeed, if we assume constant marginal cost $E > 1$ is enough. To prove that, replace equation (7) in $\gamma q^2 (E - 2) p' - \pi$ to obtain

$$\frac{dP_m}{dT} = - \frac{C_{qT} \qquad \pi}{q^2 \left[p''\gamma n q + p'(n + \gamma)\right]} \qquad (E - 1)$$

<u>Proof of Proposition 5 (P5): "Profit rates may very well fall"</u>.

Since we assumed that none of the technologies considered uses fixed capital and that the payments to the factors of production have to be done at the beginning of the production period, all capital is working capital and the profit rate (r) is defined as:

$$r = \frac{\pi}{C}$$

$$= \frac{pq}{C} - 1$$

If we maintain our previous assumptions ($C_{qq} = 0$, $C_T = C_{qT} \cdot q$, $\frac{d\varepsilon}{dQ} = 0$),

$$\frac{dr}{dT} = \frac{1}{C^2} \left[p \frac{dq}{dT} + Cq \frac{dp}{dT} - pq\, C_q \frac{dq}{dT} - pq\, C_T \right]$$

Using equations (7), (A3) and (A7) we obtain

$$\frac{dr}{dT} = \frac{1}{C^2} \left[\frac{dq}{dT} (Cp + Cqp'n - pq^2 p'\gamma - p^2 q) - pq^2\, C_{qT} \right]$$

Using equation (A2)

$$\frac{dr}{dT} = \frac{C_q T}{C^2 D}\left[Cp + Cqp'n - pq^2 p'\gamma - p^2 q - pq^2 p''\gamma n - pq^2 p'n - pq^2 p'\gamma\right] \quad \text{(A10)}$$

where $D \equiv p''\gamma nq + p'(n+\gamma) < 0$

Call the direct demand function:

$$Q = Q(p)$$

and define its elasticity to be

$$\varepsilon = -\frac{dQ}{dp}\frac{p}{Q}$$

Hence $\quad \dfrac{dQ}{dp} = -\dfrac{\varepsilon Q}{p}$

Since we assumed the demand function to be invertible

$$p' \equiv \frac{dp}{dQ} = -\frac{p}{\varepsilon Q} \quad \text{(A11)}$$

Hence $\quad \dfrac{d^2p}{dQ^2} = \dfrac{p}{\varepsilon^2 Q^2}\left[1 + \varepsilon + Q\dfrac{d\varepsilon}{dQ}\right]$

Replace these expressions for $\frac{dp}{dQ}$ and $\frac{d^2p}{dQ^2}$ in E's definition

$$E \equiv -Q\left[\frac{d^2p}{dQ^2} \Big| \frac{dp}{dQ}\right]$$

to obtain

$$E = 1 + \frac{1}{\varepsilon} + \frac{Q}{\varepsilon} \frac{d\varepsilon}{dQ} \tag{A12}$$

Use (A11) in (A10)

$$\frac{dr}{dt} = \frac{C_qT}{C^2D}\left[Cp - \frac{Cp}{\varepsilon} - 2pq^2p'\gamma + \frac{p^2q}{\varepsilon} - pq^3p''\gamma n - p^2q\right]$$

$$= \frac{C_qT}{C^2D}\left[\frac{Cp\varepsilon n}{\varepsilon n} - \frac{Cpn}{\varepsilon n} - \frac{p^2q\varepsilon n}{\varepsilon n} + \frac{p^2qn}{\varepsilon n} - pq^2\gamma\left[2p' + qp''n\right]\right]$$

Using equation (A6) and (A11):

$$2p' + qp''n = p'(2 - E)$$

$$= -\frac{p(2 - E)}{\varepsilon nq}$$

$$\frac{dr}{dT} = \frac{C_qT}{C^2D\varepsilon n}\left[Cp\varepsilon n - Cpn - p^2q\varepsilon n + p^2q\gamma(2 - E) + p^2qn\right]$$

$$\frac{dr}{dT} = \frac{C_qT}{C^2D\varepsilon n}\left[pn(C - pq)(\varepsilon - 1) + p^2p\gamma(2 - E)\right]$$

Since we only consider isoelastic demand curves

$$E = 1 + \frac{1}{\varepsilon} \quad \text{and} \quad 2 - E = \frac{1}{\varepsilon}(\varepsilon - 1)$$

$$\frac{dr}{dT} = \frac{C_qT^P}{C^2D\varepsilon n} \quad (\varepsilon - 1)\left[n(C - pq) + \frac{pq\gamma}{\varepsilon}\right]$$

Since $\pi = pq - C$

$$\frac{dr}{dT} = \frac{C_{qT}P}{C^2 D\varepsilon n} (\varepsilon - 1)\left(\frac{pq\gamma}{\varepsilon} - n\pi\right) \tag{A13}$$

and hence the behavior of profit rates depends on the sign of $\varepsilon - 1$ and $\frac{pq\gamma}{\varepsilon} - n\pi$ which are a function of ε.

Since:

$$\lim_{\varepsilon \to 0} \frac{pq\gamma}{\varepsilon} - n\pi > 0$$

$$\lim_{\varepsilon \to \infty} \frac{pq\gamma}{\varepsilon} - n\pi < 0$$

and $\frac{pq\gamma}{\varepsilon^*} - n\pi = 0$ when $\varepsilon^* = \frac{pq\gamma}{n\pi}$

the rate of profit will rise only when

$$1 > \varepsilon > \varepsilon^*$$

or $\varepsilon^* > \varepsilon > 1$

depending on the value of $\varepsilon^* - 1$ as can be seen from Table I

Table I

	0		1		$+\infty$
ε		< 1		> 1	
$\varepsilon - 1$		< 0	0	> 0	
E		> 2	2	< 2	
$\frac{qp\gamma}{\varepsilon}$		$> pq\gamma$		$< pq\gamma$	

If $\varepsilon^* < 1$

		ε^*			
$\frac{pq\gamma}{\varepsilon} - n\pi$	> 0	0	< 0		< 0
$\frac{dr}{dT}$	< 0	0	> 0	0	< 0

If $\varepsilon^* > 1$

				ε^*	
$\frac{pq\gamma}{\varepsilon} - n\pi$	> 0		> 0	0	< 0
$\frac{dr}{dT}$	< 0	0	> 0	0	< 0

q.e.d.

Finally it is easy to check that the "usual" results obtain for the cases of perfect competition and monopoly, by replacing some of the parameters by their appropiate values:

	Perfect Competition	Monopoly	
	$\gamma=0,\ p'=0,\ n=\infty$	$\gamma=1,\ n=1$	
Proposition 1 becomes	$\frac{dq}{dT}=\infty$	$\frac{dq}{dT}>0$	
Proposition 2 becomes	$\frac{dp}{dT}<0=C_{qT}$	$\frac{dq}{dT}<0$	if $C_{qq}=0$
Proposition 3 becomes	$D=0$	same as oligopoly	
Proposition 4 becomes	$\frac{d\pi}{dT}>0,\ \frac{dP_m}{dT}>0$	$\frac{d\pi}{dT}>0,\ \frac{dP_m}{dT}>0$	
Proposition 5 becomes	$\frac{dr}{dT}>0$	$\frac{dr}{dT}>0$	

Appendix II

The Non Symmetrical Case

Proof of Proposition 6 (P6): "Total output will increase".

To calculate the impact of technological innovation on output, differentiate equations (7) rewritten as:

$$p'(\sum_j q_j)\gamma_i q_i + p(\sum_j q_j) - C_q^i(q, T) = 0 \quad \text{for all } i\text{'s to}$$

obtain

$$p'\gamma_i \frac{dqi}{dT} + \gamma_i q_i p'' \sum_j \frac{dqj}{dT} + p' \sum_j \frac{dqj}{dT} - C_{qq}^i \frac{dqi}{dT} - C_{qT}^i = 0 \quad \text{for}$$

all i's.

Define $\frac{dqi}{dT} \equiv \chi_i$, and assume that the production functions exhibit Constant Return to Scale as before ($C_{qq}^i = 0, \forall i$). We then have

$$(p' + \gamma_i q_i p'') \sum_j \lambda_i + p'\gamma_i \chi_i - \xi = 0 \text{ for all } i\text{'s} \tag{A14}$$

Define:

$$a_i \equiv p' + \gamma_i q_i p'' \tag{A15}$$

$$A \equiv \begin{bmatrix} a_1 + p'\gamma_1 & a_1 & \cdots & a_1 \\ a_2 & a_2 + p'\gamma_2 & \cdots & a_2 \\ \vdots & \vdots & & \vdots \\ a_n & a_n & & a_n + p'\gamma n \end{bmatrix}$$

$$\underline{\chi} \equiv \begin{bmatrix} \chi_1 \\ \chi_2 \\ \vdots \\ \chi_n \end{bmatrix} \quad , \quad \underline{C} \equiv \begin{bmatrix} \xi \\ \xi \\ \vdots \\ \xi \end{bmatrix}$$

Equation (A14) can be written as:

$$A\underline{\chi} = \underline{C} \tag{A16}$$

and, since it is easy to show that A is non-singular

$$\underline{\chi} = A^{-1}\underline{C} \tag{A17}$$

To find the inverse of A, define the following diagonal matrix:

$$\Gamma \equiv p' \begin{bmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \dots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \dots & \gamma_n \end{bmatrix}$$

and the two vectors:

$$\underline{a}' \equiv (a_1 \; a_2 \dots \dots a_n)$$

$$\underline{1}' \equiv (1 \;\; 1 \dots \dots 1)$$

Rewrite A as:

$$A \equiv \Gamma + \underline{a} \cdot \underline{1}'$$

As Maddala (1977, p.446) has shown, the inverse of A can be written as:

$$\Gamma^{-1} - \frac{\Gamma^{-1}\underline{a}\,\underline{1}'\,\Gamma^{-1}}{1 + \underline{1}'\Gamma^{-1}\underline{a}}$$

In this case,

$$\Gamma^{-1} = \frac{1}{p'}\begin{bmatrix} \frac{1}{\gamma_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{\gamma_2} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \frac{1}{\gamma_n} \end{bmatrix}$$

$$1 + \underline{1}'\,\Gamma^{-1}\,\underline{a} = 1 + \frac{1}{p'}\sum_j \frac{a_j}{\gamma_j}$$

and

$$\Gamma^{-1}\underline{a}\,\underline{1}'\,\Gamma^{-1} = \frac{1}{p'^2}\begin{bmatrix} \frac{a_1}{\gamma_1^2} & \frac{a_1}{\gamma_1\gamma_2} & \cdots & \frac{a_1}{\gamma_1\gamma_n} \\ \frac{a_2}{\gamma_2\gamma_1} & \frac{a_2}{\gamma_2^2} & \cdots & \frac{a_2}{\gamma_2\gamma_n} \\ \vdots & \vdots & & \vdots \\ \frac{a_n}{\gamma_n\gamma_1} & \frac{a_n}{\gamma_n\gamma_2} & \cdots & \frac{a_n}{\gamma_n^2} \end{bmatrix}$$

Define:

$$\delta_i = \frac{a_i}{\gamma_i\left(p' + \sum_j a_j \gamma_j^{-1}\right)} \qquad \text{(A18)}$$

We can now rewrite the inverse of A as:

$$A^{-1} = \frac{1}{p'} \begin{bmatrix} \frac{1-\delta_1}{\gamma_1} & \frac{-\delta_1}{\gamma_2} & \cdots\cdots & \frac{-\delta_1}{\gamma_n} \\ \frac{-\delta_2}{\gamma_1} & \frac{1-\delta_2}{\gamma_2} & \cdots\cdots & \frac{-\delta_2}{\gamma_n} \\ \vdots & \vdots & & \vdots \\ \frac{-\delta_n}{\gamma_1} & \frac{-\delta_n}{\gamma_2} & \cdots\cdots & \frac{1-\delta_n}{\gamma_n} \end{bmatrix} \tag{A19}$$

Hence, given equation (A17)

$$\frac{dq_i}{dT} = \frac{\xi}{p'} \left(\frac{1}{\gamma_i} - \delta_i \sum_j \gamma_j^{-1}\right) \text{ for all } i\text{'s} \tag{A20}$$

The effect of the new technology on total output produced and sold in the market as a whole is:

$$\sum_j \frac{dq_j}{dT} = \frac{\xi \sum_j \gamma_j^{-1}}{p'} \left(1 - \sum_j \delta_j\right) \tag{A21}$$

From (A18)

$$1 - \sum_j \delta_j = 1 - \frac{\sum_j a_j \gamma_j^{-1}}{p' + \sum_j a_j \gamma_j^{-1}}$$

$$= \frac{p'}{p' + \sum_j a_j \gamma_j^{-1}} \tag{A22}$$

Furthermore using a_j's definition in equation (A15)

$$p' + \sum_j a_j \gamma_j^{-1} = p'(1 + 1\,\frac{1}{p'}\,\sum_j a_j\, \gamma_j^{-1}) \tag{A23}$$

$$= p'(1 + \sum_j \gamma_j^{-1} + \frac{p''}{p'}\,\sum_j q_j) \tag{A24}$$

Using equation (9):

$$p''\,\gamma_i\, nq_i + p'n + p'\gamma_i < 0$$

and dividing throughout by

$$\frac{1}{n\gamma_i p'} < 0$$

we obtain

$$\frac{p''}{p'}\,q_i + \frac{1}{\gamma_i} + \frac{1}{n} > 0 \tag{A25}$$

The sum of equations (A25) across all firms implies that

$$1 + \sum_j \gamma_j^{-1} + \frac{p''}{p'}\,\sum_j q_j > 0$$

Using (A22) and (A23) rewrite (A21) as:

$$\sum_j \frac{dq_j}{dT} = \frac{\xi \sum_j \gamma_j^{-1}}{p' + \sum_j a_j \gamma_j^{-1}} > 0 \tag{A26}$$

But since $p' < 0$, $\xi < 0$ and using (A24) and (A25)

$$\sum_j \frac{dq_j}{dT} > 0$$

q.e.d.

<u>Proof of Proposition 7 (P7): "Market prices will fall".</u>

Since

$$p = p(\sum_j q_j)$$

$$\frac{dp}{dT} = p' \sum_j \frac{dq_j}{dT}$$

and, using equation (A26)

$$\frac{dp}{dT} = \frac{p'\xi\sum_j \gamma_j^{-1}}{p' + \sum_j a_j \gamma_j^{-1}} \tag{A27}$$

$$< 0$$

q.e.d.

<u>Proof of Proposition 8 (P8): "Industry wide profit margins will fall i</u>

Calculate, as before:

$$D \equiv \frac{dp}{dT} - C_{qT}$$

Using equation (A23), (A24) and the definitions of E and Q

$$Q \equiv \sum_j q_j$$

$$E \equiv - \frac{p''Q}{p'}$$

$$1 + \frac{1}{p'} \sum_j a_j \gamma_j^{-1} = 1 + \sum_j \gamma_j^{-1} - E \tag{A28}$$

Hence,

$$D = \frac{-\xi}{1 + \frac{1}{p'} \sum_j a_j \gamma_j^{-1}} \quad (E - 1) \tag{A29}$$

q.e.d.

<u>Proof of Proposition 9 (P9): "The smaller and/or less dominant firms will produce less output".</u>

From equations (A20) and (A18) we obtain:

$$\frac{dq_i}{dT} = \frac{\xi}{p'\gamma_i} \left[1 - \frac{a_i \sum_j \gamma_j^{-1}}{p' + \sum_j a_j \gamma_j^{-1}} \right] \qquad \text{for all } i\text{'s}$$

$$= \frac{\xi (p' + \sum_j (a_j - a_i) \gamma_j^{-1}}{p' \gamma_i (p' + \sum_j a_j \gamma_j^{-1})} \qquad \text{for all } i\text{'s} \tag{A30}$$

where $a_i \equiv p' + p'' \gamma_i q_i$

Hence, firm i will produce less output iff:

$$p' + \sum_j (a_j - a_i)\gamma_j^{-1} > 0$$

Since,

$$a_j - a_i = p''(\gamma_j q_j - \gamma_i q_i)$$

i's output will decrease iff

$$p' + p'' Q - p'' \gamma_i q_i \sum_j \gamma_j^{-1} > 0$$

or

$$\delta_i \equiv \frac{q_i}{Q} \sum_j \frac{\gamma_i}{\gamma_j} < \frac{1}{1+\varepsilon}$$

Indeed, for any value of ε, it is more likely that condition (A31) will hold, the smaller a firm's share of the market $(\frac{q_i}{Q})$ and the less "dominant" it is relative to all others ($\gamma_i << \gamma_j, \forall j$ say). In the case of a small firm that has little impact on its competitors the latter will chew its market share up, bringing a fall in its production level. Conversely, big and/or dominator firms ($\delta_i > \frac{1}{1+e}$) will produce more.

q.e.d.

Proof of Proposition 10 (P10): "The profits of the bigger and/or more dominant firms will increase even if industry wide profits decrease (E > 2)".

From equation (6)

$$\pi_i \equiv q_i p(\sum_j q_j) - C^i (q_i, T)$$

and hence

$$\frac{d\pi_i}{dT} = p\ \chi_i + p' q_i \sum_j \chi_j - C^i_q \chi_i - C^i_T \tag{A32}$$

As before, technological change has two impacts on profits. The first one works through the change in output produced and has the three components previously mentioned. The difference now is that, since production may fall, the behavior of profits is also contingent on a firm's market position. As before, however, the second impact directly raises profits.

Rewrite (A32) as

$$\frac{d\pi_i}{dT} = (p - C^i_q)\ \chi_i + p' q_i \sum_j \chi_j - C^i_T$$

Use equations (7), (A7) and (11) to obtain

$$\frac{d\pi_i}{dT} = q_i (p' \sum_j \chi_j - p' \gamma_i \chi_i - \xi)$$

and (A30) and (A26) to find

$$\frac{d\pi_i}{dT} = q_i \frac{p'\xi\sum_j \gamma_j^{-1} - p'\xi - \xi\sum_j a_j\gamma_j^{-1} - \xi\left(p' + \sum_j (a_j - a_i)\gamma_j^{-1}\right)}{p' + \sum_j a_j\gamma_j^{-1}}$$

We get the value of $\sum_j a_j\gamma_j^{-1}$ from (A28) and hence

$$\frac{d\pi_i}{dT} = q_i \frac{\xi p'(E - 2) - \xi\sum_j (a_j - a_i)\gamma_j^{-1}}{p' + \sum_j a_j\gamma_j^{-1}} \qquad \text{(A33)}$$

For i's output to increase:

$$\sum_j (a_j - a_i)\gamma_j^{-1} < -p'$$

If we replace $\sum_j (a_j - a_i)\gamma_j^{-1}$ by $-p'(1 - k_i)$, $k_i > 0$

(A33) becomes

$$\frac{d\pi_i}{dT} = q_i \frac{\xi p'(E - 1 - k_i)}{p' + \sum_j a_j\gamma_j^{-1}} \qquad \text{(A34)}$$

We can obtain a similar expression if i's output decreases. In general we will write:

$$\frac{d\pi_i}{dT} = \frac{q_i\xi p'(E - 1 - k_i)}{p' + \sum_j a_j\gamma_j^{-1}} \qquad \text{(A35)}$$

where k_i

is negative if i's output decreases

is positive if i's output increases

and is 1 if all firm are identical

Profits of firm i^* could very well rise even if total industry profits fall ($E > 2$ as shown in P13) as long as its output increases enough ($k_i^* > E - 1$).

q.e.d.

<u>Proof of Proposition 11 (P11): "The profit margins of the bigger and/or more dominant firms will decrease if $E > 1$".</u>

By definition

$$P_m^i = \frac{\pi_i}{q_i}$$

Hence

$$\frac{dP_m^i}{dT} = \frac{q_i \dfrac{d\pi_i}{dT} - \pi_i \dfrac{d\pi_i}{dT}}{q_i^2}$$

Using equations (A30) and (A33)

$$\frac{dP_m^i}{dT} = \frac{\xi}{q_i^2 p' \gamma_i (p' + \sum_j \alpha_j \gamma_j^{-1})} \Big[p'(E-1)(q_i^2 p' \gamma_i) - (p' + \sum_j (a_j - a_i)\gamma_j^{-1}) (\pi_i + p' \gamma_i q_i^2) \Big] \tag{A36}$$

From equation (7) we obtain

$$p'\gamma_i q_i^2 = C_q^i q_i - pq_i$$

Since marginal cost is bigger or equal to average cost

$$C_q^i q_i \geq C_i$$

and

$$\pi_i + p'\gamma_i q_i^2 \geq 0$$

As shown in equation (A31) a firm will produce more output iff

$$p' + \sum_j (a_j - a_i)\gamma_j^{-1} < 0$$

Hence

$$\frac{dP_m^i}{dT} < 0 \qquad (A37)$$

if $E > 1$ even though we are considering a firm that produces more output.

q.e.d.

Proof of Proposition 12 (P12): "Industry wide profits will decrease if $E > 2$".

From equation (6) we calculate industry-wide profits:

$$\sum_j \pi_j = Q_p(Q) - \sum_j C^j(q_j, T)$$

Hence,

$$\frac{d\sum_j \pi_j}{d} = Qp' \frac{dQ}{dT} + p\frac{dQ}{dT} - \sum_j C_q^j \frac{dq_j}{dT} - \sum_j C_T^j$$

From equation (7) we obtain

$$p - C_q^j = -p'\gamma_j q_j$$

and hence

$$\frac{d\sum_j \pi_j}{dT} = Qp'\frac{dQ}{dT} - p'\sum_j \gamma_j q_j \frac{dq_j}{dT} - \sum_j C_T^j$$

Now use equations (11), (A7) and (A26) to get

$$\frac{d\sum_j \pi_j}{dT} = \frac{Qp'\xi\sum_j \gamma_j^{-1} - Qp'\xi - Q\xi\sum_j a_j\gamma_j^{-1}}{p' + \sum_j a_j\gamma_j^{-1}} - p'\sum_j \gamma_j q_j \frac{dq_j}{dT}$$

Using equation (A28)

$$\frac{d\sum_j \pi_j}{dT} = \frac{\xi Qp'(E-1)}{p' + \sum_j a_j\gamma_j^{-1}} - p'\sum_j \gamma_j q_j \frac{dq_j}{dT}$$

Hence

$$\frac{d\Sigma\pi_j}{dT} = \frac{\xi Q p'(E-2)}{p' + \sum_j a_j \gamma_j^{-1}}$$

q.e.d.

Proof of Proposition 13 (P13): "The industry-wide rate of profit may very well fall".

Define the industry-wide rate of profit to be

$$R \equiv \frac{\Sigma\pi_i}{\Sigma C_i} = \frac{pQ}{\Sigma C_i} - 1$$

and $K \equiv \Sigma C_i$

$$D \equiv (p' + \Sigma a_j \gamma_j^{-1}) < 0$$

$$\frac{dR}{dT} = \frac{1}{K^2}\left[Kp\,\frac{dQ}{dT} + KQ\,\frac{dp}{dT} - pQ\sum_i C_q^i\,\frac{dq_i}{dT} - pQ\sum_i C_{qT}\right]$$

Use equations (7) (11) (A7) (A26) (A27) and (A30) to obtain

$$\frac{dR}{dT} = \frac{1}{K^2}\left[\frac{Kp\xi\sum_j \gamma_j^{-1}}{D} + \frac{KQp'\xi\sum_j \gamma_j^{-1}}{D} - \frac{pQ^2\xi\,(p' + \sum_j a_j\gamma_j^{-1})}{D}\right]$$

$$- \frac{pQ\xi\sum_i (p + p'\gamma_i q_i)(p' + \sum_j (a_j - a_i)\gamma_j^{-1})}{p'\gamma_i D}$$

Now use equation (A28)

$$\frac{dR}{dT} = \frac{\xi}{K^2 D}\left[K\sum_j \gamma_j^{-1}(p + p'Q) - pp'Q^2 - pQ^2(p'\sum_j \gamma_j^{-1} - p'E)\right.$$
$$\left. - \frac{pQ}{p'}\sum_i \frac{(p + p'\gamma_i q_i)(p' + p''Q - p''\gamma_i q_i \sum_j \gamma_j^{-1})}{\gamma_i}\right]$$

because

$$(a_j - a_i)\gamma_j^{-1} = p''q_j - \frac{p''\gamma_i q_i}{\gamma_j}$$

Hence

$$\frac{dR}{dT} = \frac{\xi}{K^2 D}\left[K\sum_j \gamma_j^{-1}(p + p'Q) - pp'Q^2 - pp'Q^2\sum_j \gamma_j^{-1} + pp'Q^2 E - p^2 Q\sum_i \gamma_i^{-1}\right.$$
$$\left. - \frac{p^2Q^2p''\sum_j \gamma_j}{p'} + \frac{p^2Q^2p''\sum_j \gamma_j}{p'} - pp'Q^2 - pp''Q^3 + pp''Q\sum_i \gamma_i q_i^2 \sum_j \gamma_j^{-1}\right]$$

But

$$- pp''Q^3 + pp''Q\sum_i \gamma_i q_i^2 \sum_j \gamma_j^{-1} = - pp''Q\sum_i q_i\left[\sum_j q_j - \gamma_i q_i \sum_j \gamma_j^{1}\right]$$

$$= 0 \quad \text{by equation} \qquad \text{(A38)}$$

Hence

$$\frac{dR}{dT} = \frac{\xi}{K^2 D}\left[\sum_j \gamma_j^{-1}(Kp + Kp'Q - pp'Q^2 - p^2 Q) - 2pp'Q^2 + pp'Q^2 E\right]$$

But since $p'Q = -\frac{p}{\varepsilon}$

$$\frac{dR}{dT} = \frac{\xi}{\varepsilon K^2 D}\left[\sum_j \gamma_j^{-1}\left(Kp(\varepsilon - 1) - p^2 Q(\varepsilon - 1)\right) - p^2 Q(E - 2)\right]$$

$$= -\frac{\xi p}{\varepsilon K^2 D}\left[\sum_j \gamma_j^{-1}(\varepsilon - 1)\Pi + pQ\,(E - 2)\right]$$

because $\quad \Pi = pQ - K$

If we stick to isoelastic demand curves

$$\frac{dR}{dT} = \frac{\xi p}{\varepsilon K^2 D}(\varepsilon - 1)\left(\frac{pQ}{\varepsilon} - \Pi \sum_j \gamma_j^{-1}\right) \qquad \text{(A39)}$$

a result very similar to the one in equation (A13). The interpretation will be the same but we now have:

$$\varepsilon^* = \frac{pQ}{\Pi \sum_j \gamma_j^{-1}}$$

q.e.d.

<u>Proof of Proposition 14 (P14): "The firm's rate of profit need not change as the industry's".</u>

Define i's profit rate to be

$$r_i \equiv \frac{\pi_i}{C_i}$$

Hence

$$\frac{dr_i}{dT} = \frac{1}{C_i^2}\left[\frac{C_i d\pi_i}{dT} - \pi_i C_q^i \frac{dq_i}{dT} - \pi_i C_{qT}\right]$$

Use equations (7) (A30) and (A33) to obtain

$$\frac{dr_i}{dT} = \frac{1}{C_i^2}\left[\frac{C_i q_i \xi(p'(E-2) - \sum_j (a_j - a_i)\gamma_j^{-1})}{D} - \frac{\xi\pi_i(p'\gamma_i q_i + p)(p' + \sum_j (a_j - a_i)\gamma_j^{-1}}{p'\gamma_i D}\right.$$

$$\left. - \frac{\pi_i \xi q_i p'\gamma_i(p' + \sum_j a_j\gamma_j^{-1})}{p'\gamma_i D}\right]$$

where $D \equiv p' + \sum_j a_j \gamma_j^{-1}$

$$= \frac{\xi}{C_i^2 D p'\gamma_i}\left[p'^2 q_i\gamma_i C_i(E-2) - p'\gamma_i q_i C_i \sum_j (a_j - a_i)\gamma_j^{-1} - 2p'^2 q_i\gamma_i\pi_i\right.$$

$$- p' q_i \gamma_i \pi_i \sum_j (a_j - a_i)\gamma_j^{-1} - pp'\pi_i - p\pi_i \sum_j (a_j - a_i)\gamma_j^{-1}$$

$$\left. - p' q_i \gamma_i \pi_i \sum_j a_j \gamma_j^{-1}\right]$$

$$= \frac{\xi}{C_i^2 D p'\gamma_i}\left[-(p' + \sum_j (a_j - a_i)\gamma_j^{-1})(p' q_i\gamma_i(C_i + \pi_i) + p\pi_i)\right.$$

$$\left. + p'^2 q_i \gamma_i C_i (E-1) - p' q_i \gamma_i \pi_i \sum_j a_j \gamma_j^{-1} - p'^2 q_i \gamma_i \pi_i\right]$$

Since

$$\sum_j (a_j - a_i)\gamma_j^{-1} = -p'E + \frac{p'E}{Q}\gamma_i q_i \sum_j \gamma_j^{-1}$$

and $$\sum_j a_j \gamma_j^{-1} = p'(\sum_j \gamma_j^{-1} -$$

$$\frac{dr_i}{dT} = \frac{\xi}{C_i^2 D p' \gamma_i}\Big[-(p' - p'E + \frac{p'E}{Q}\gamma_i \qquad q_i\gamma_i(C_i + \pi_i) + p\ \ _i) + p'^2 q_i\gamma$$

$$(E-1) - p'^2 q_i \gamma_i \pi_i \qquad - p'^2 q_i \gamma_i \pi_i \Big]$$

$$= \frac{\xi}{C_i^2 D p' \gamma_i}\Big[2p'^2 q_i \gamma_i (C_i + \pi_i)(E - 1 \qquad - 1)$$

$$- p' q_i \gamma_i \sum_j \gamma_j \Big(\frac{p' q_i \gamma_i I}{Q} \qquad + \frac{p \pi_i E}{Q} + p' \pi_i \Big]$$

Since

$$p' = -\frac{p}{\varepsilon Q}, \quad C_i + \pi_i = \qquad E - 1 = \frac{1}{\varepsilon} \text{ in the}$$

isoelastic case

$$\frac{dr_i}{dT} = \frac{\xi}{C_i^2 D \gamma_i}\Big[-\frac{2p^2 q_i^2 \gamma_i}{\varepsilon^2 Q} + \frac{p\pi_i}{\varepsilon} - q_i \qquad \frac{q_i^2 \gamma_i E}{\varepsilon Q^2} + \frac{p\pi_i E}{Q} - \frac{p\pi_i}{\varepsilon Q}\Big)\Big]$$

$$= \frac{\xi p}{\varepsilon Q \gamma_i C_i^2 D}\Big[-\frac{2pq_i^2\gamma_i}{\varepsilon} + \pi_i Q - q_i \qquad \frac{q_i^2 \gamma_i E}{Q} + \varepsilon \pi_i E - \pi_i\Big)\Big]$$

$$\frac{dr_i}{dT} = \frac{\xi p}{\varepsilon Q \gamma_i C_i^2 D}\left[p q_i^2 \gamma_i \left(\frac{q_i}{Q} \sum_j \frac{\gamma_i}{\gamma_j} (1 + \frac{1}{\varepsilon}) \frac{2}{\varepsilon}\right) - \pi_i Q \left(\frac{\varepsilon q_i}{Q} \sum_j \frac{\gamma_i}{\gamma_j} - 1\right)\right]$$

Using equation (A31) we obtain

$$\frac{dr_i}{dT} = \frac{\xi p}{\varepsilon Q \gamma_i C_i^2 (p' + \sum_j a_j \gamma_j^{-1})} \left[p q_i^2 \gamma_i (s_i (1 + \frac{1}{\varepsilon}) - \frac{2}{\varepsilon}) - \pi_i Q (s_i \varepsilon - 1)\right]$$

Hence the sign of $\frac{dr_i}{dT}$ is the sign of

$$p q_i^2 \gamma_i \left[s_i \frac{(1 + \varepsilon) - 2}{\varepsilon} \right] - \pi_i Q \left[s_i \varepsilon - 1 \right] \qquad (A40)$$

The sign of (A40) depends on the value of s_i as can be seen in the following table:

Table II

Suppose $\varepsilon > 1$

s_i	0		$\frac{1}{\varepsilon}$		$\frac{2}{1+\varepsilon}$		∞
$\frac{s_i(1+\varepsilon)-2}{\varepsilon}$	$-\frac{2}{\varepsilon}$	< 0		< 0	0	> 0	
$s_i\varepsilon - 1$	-1	< 0	0	> 0		> 0	
$\frac{dr_i}{dT}$		?		< 0		?	

and if $\varepsilon < 1$

s_i	0		$\frac{2}{1+\varepsilon}$		$\frac{1}{\varepsilon}$		∞
$\frac{s_i(1+\varepsilon)-2}{\varepsilon}$	$-\frac{2}{\varepsilon}$	< 0	0	> 0		> 0	
$s_i\varepsilon - 1$	-1	< 0		< 0	0	> 0	
$\frac{dr_i}{dT}$		?		> 0		?	

Notice that even though

$$\frac{q_i}{Q} < 1 \qquad \forall i$$

since

$$\sum_{j=1}^{n} \frac{\gamma_k}{\gamma_j} \text{ can be } > 1$$

δ_i can be bigger than 1. Actually, if q_i and γ_i are positively correlated, (the "rest of the market reacts more" when a big firm moves than when a small one does) there will always exist and $\delta_k > 1$ because in that case:

$$q_k > \frac{Q}{n} \doteq \bar{q}$$

and $$\sum_{j=1}^{n} \frac{\gamma_k}{\gamma_j} > n$$

Let us consider the following two cases:

a) $\varepsilon \leq \varepsilon^* < 1$

b) $1 < \varepsilon \leq \varepsilon^*$

From tables I and II we can see that in case a) the profit rate of all firms such that

$$\frac{2}{1+\varepsilon} < \delta_h < \frac{1}{\varepsilon}$$

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