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A B S T R A C T

The purpose of this paper is to consider the comparative statics of conjectural-variations oligopoly equilibria, following an industry-specific shift in costs, which could be due to a rise in the wage or in the cost of other inputs such as energy, a technological improvement, or taxation, of inputs or (physical) output. We establish very plausible conditions, necessary and sufficient for the model we consider, under which price overshifting (price rising by more than the excise tax) and profit overshifting (profit rising with the tax) obtain.

RESUMEN

En este trabajo estudiamos el comportamiento estático-comparativo de los equilibrios de variaciones-conjeturadas de un oligopolio que ha sido perturbado por un cambio en la estructura de costos. Entre ellos se encontrarían: un aumento en el salario, o en el costo de otros insumos como la energía, un cambio tecnológico o un impuesto ya sea sobre los insumos o sobre el producto. Encontramos condiciones necesarias y suficientes muy plausibles para que el precio del producto aumente más que el impuesto indirecto, y para que las ganancias aumenten con el impuesto (price overshifting y profit overshifting).

1. Introduction

The purpose of this paper is to consider the comparative statics of conjectural-variations oligopoly equilibria, following an industry-specific shift in costs, which could be due to a rise in the wage or in the cost of other inputs such as energy, a technological improvement, or taxation, of inputs or (physical) output. In so doing, my aim is two-fold. On the one hand, to contribute towards the bringing together questions from the public economics tradition, which has typically relied too heavily on assumptions of competitiveness, with models and concepts from industrial economics. On the other, to derive, analytically, two results on shifting of tax or cost rises, which are surprisingly general in their form and likely in their incidence. Namely we establish very plausible conditions, necessary and sufficient for the model we consider, under which price overshifting (price rising by more than the excise tax) and profit overshifting (profit rising with the tax) obtain.

The idea that profits may rise with costs, to which our main specific result below refers, is not altogether new in informal discussions on oligopoly. It has been widely discussed in the traditional literature on shifting, although to the best of my knowledge analyses have tended to be somewhat empiricist, establishing relations between observed data but not telling a good story to explain them. Similarly, during the oil crisis following the formation of OPEC in late 1973, the fact was widely noted that the profits of the large multinational oil companies increased as the cost of their crude input rose. But this tended to be explained through an "in adversity unite" effect of the cost rise on oligopolists' behaviour, which somehow awoke the sense of comradeship of the oil giants and made them more collusive. There may be something in that, but the underlying story is not convincing, or still needs to be told. A more precise, although still conjectural reference to the kinds of reasons why oligopoly may result in "surprising" results such as costs raising profits, is a brief recognition by Salop (1981; fn. 64 in p. 38)[1/] of the possibility that cost-rises may raise profits on account of repercussions on the oligopolistic equilibrium. Salop's distinctive intuition at play, but without exploring the point formally.

Closer to the spirit of this paper, analyzing the effects of policy (or other) changes on fully specified oligopoly models, are de Meza (1982), Stern (1982), and a very recent paper by Katz and Rosen (1983). David de Meza's interesting article is centered on the effects of entry on (i.e. the SR vs. LR comparison of) derived demand elasticities, but, along the way, some results related to ours below are obtained, on price and profit "overshifting" of cost rises, for the case of isoelastic demand and identical cost functions. Stern analyzes a variety of oligopoly models to look at policy questions largely separate from our interests here, but obtains one result in common with ours, again on price "overshifting" under isoelastic demands and symmetry. Lastly, Katz and Rosen formulate questions closer to ours, using the very same model we use below (for the symmetric case), and derive interesting results on the profit- and welfare-effects of taxation by numerical simulation. Their purpose is primarily to illustrate possibilities rather than to characterize outcomes, however, hence they rely on some specific examples of demand and cost functions only.

"Paradoxical" results are pervasive in oligopoly; exceptions to "normal" behaviour of these models can be found all over, so that their interest cannot be but commensurate to their robustness, which needs to be studied. Surprises can be important, aberrations much less so: it all depends on whether the former are seen to occur for a large and central set of circumstances or not. Unfortunately, a complete characterization of outcomes is usually hard to ascertain in oligopoly, on account of the algebraic barrier these problems can present. But then relying on special examples can be misleading: the generality of their behaviour remains open to question. We shall give in this paper a fairly full analytic characterization of the effects cost rises have on profit margins (price shifting) and on profits, for the general conjectural-variations oligopolistic equilibrium. It permeates that conventional wisdom is most unreliable in this problem. Price and profit "overshifting" are likely indeed.

Section 2 recalls the nature of the model, and the various conditions we shall use: first-order, second-order, and those for stability of equilibrium. Section 2 analyzes, for the symmetric-industry case, the effect of tax or cost rises on firms' output, price, and profits, and Section 4 extends the analysis to the asymmetric case. Section 5 contains some concluding remarks.

2. Framework

The model I shall be concerned with is the conjectural-variations or quasi-Cournot model of oligopolistic equilibrium, initially under conditions of industry-wide symmetry as studied in Seade (1980a) although this will later be relaxed to look at the general homogeneous-output case as in Seade (1980b).

Faced with an inverse demand function for aggregate output $p(Y)$, a cost function for own output $c^f(y_f, \xi)$, where ξ is a shift parameter, and immersed in an industry consisting of n firms described by their cost functions $c^j(y_j,\xi)$, firm f choses output to maximize profits:

$$\max_{y_f} \quad \Pi^f = y_f p(Y) - c^f(y_f,\xi) \tag{1}$$

given, in general, a conjectured functional dependence of responses of aggregate output $Y \equiv \sum_j y_j$ to changes in own production. More generally this function could also depend on the entire position of the industry as described by the vector (y_j). Given only the existence (and, for simplicity, differentiability) of such a function for each f, one could postulate and study the existence and properties of equilibrium. Little is lost, however, if, for local analysis, the derivatives $dY^c/dy_f \equiv \lambda_f$ (c = conjectured) are treated parametrically. More is lost, of course, if symmetry across the λ's is assumed: even firms of similar size (symmetry of structure as such) can have different styles of management, whose outlook on their industrial environment as captured primarily by λ will also differ. Since my purpose in this paper is to derive general conditions under which certain results obtain rather than merely to establish them as possibilities, it seems important to allow for behavioural (and other) asymmetries.

The first and second order conditions for a maximum of (1) are, respectively,

$$p + \lambda_f y_f p' - c^f_y = 0 \qquad (2)$$

and $\lambda_f^2 y_f p'' + 2\lambda_f p' - c^f_{yy} < 0 \qquad (3)$

where suffixes of $c^f(\cdot)$ denote partial derivatives and all arguments in functions have been omitted. I shall also rely heavily on the following condition 2/ for stability:

$$(n + \lambda_f)p' + n\lambda_f y_f p'' - c^f_{yy} < 0 \qquad (4)$$

for all f, whose more common stronger version $p' + \lambda_f y_f p'' < 0$ would be weak enough for our purposes below.

3. Displacing symmetric equilibrium

3.1 Output

Let us first consider, to simplify the exposition, the symmetric case with all f's deleted. Equation (2) reduces to

$$\lambda y p'(ny) + p(ny) - c_y(y,\xi) = 0, \qquad (5)$$

noting that $Y = ny$ under symmetry. Let us now introduce a shift in the parameter ξ of the cost function: this could for instance reflect an input-price increase, a specific tax on output, or some technological shift in the production function of all producers; any industry-wide change, but not economy-wide, given our partial equilibrium framework. To be specific, I assume that, for all y, $c_{y\xi} > 0$.

Totally differentiating (5) and solving for $dy/d\xi$, we get:

$$\frac{dy}{d\xi} = \frac{c_{y\xi}}{(n+\lambda)p' + n\lambda y p'' - c_{yy}} \qquad (6)$$

which, from (4), is unambiguously positive. Output always falls as marginal costs increase at the margin, at any rate under symmetry. Conventional wisdom, derived from the simple competitive case when c_y itself is the supply function, is proved correct on this point, for a wide class of cases given the flexibility of interpretation the conjectural variations model lends itself to. The two usual limit cases of oligopoly can easily be obtained as special cases: monopoly setting $n = \lambda = 1$, and price-taking behaviour setting $\lambda = 0$.

The above of course assumes constant structure following the increase in costs: no bankruptcies and exit, for example. Were some firms to leave the market, following an adverse development like supposedly a cost-increase is, others might end up producing more than under constant structure — or even less, for the effects of entry and exit on firm-level output can be tricky for plausible cases in these models. This point depends directly on what happens to profits in the first place, in the new short-run no-entry equilibrium, to which I turn below.

3.2 Price

With output falling price will clearly rise: some shifting will occur (always, except under limit assumptions on demand or marginal-cost elasticities). But the interesting question is whether price may rise by more than marginal cost, i.e. whether the increase in the latter may be shifted on to consumers by more than 100%. Conventional wisdom, based on the competitive case, for which price-shifting to consumers is always between 0 and 100%, gives a strong negative answer to this question.3/

Differentiating $p = p(ny)$,

$$\frac{dp}{d\xi} = p'n\frac{dy}{d\xi} = \frac{p'n\ c_{y\xi}}{(n+\lambda)p' + n\lambda yp'' - c_{yy}} \qquad (7)$$

which is, as was argued for (6), unambiguously positive. Now define the shifting coefficient $S \equiv (dp/d\xi)/c_{y\xi}$, whose being $\gtrless 1$ is equivalent to shifting being $\gtrless$ 100%. For example, in a uniform excise-tax interpretation of $\xi \equiv t$, the before-tax cost function $b(y)$ becomes the after-tax function $c(y) \equiv b(y) + ty$, so that S reduces to dp/dt. From (7),

$$S = \frac{p'n}{(n+\lambda)p' + n\lambda yp'' - c_{yy}} \qquad (8)$$

so that $S > 1$ can easily arise: it will if $(\lambda/n)p' + \lambda yp'' - c_{yy}/n$ is a positive number, which can very well be the case, not under competition $(\lambda = 0)$, but in other structures. For example, if $c_{yy} = 0$, we get, from (8),

$$S^{-1} - 1 = \frac{\lambda p' + n\lambda yp''}{np'}$$

$$= \frac{\lambda}{n}\left[1 + \frac{nyp''}{p'}\right]$$

$$= \frac{\lambda}{n}(1 - E), \qquad (9)$$

where

$$E \equiv -Yp''/p', \qquad (10)$$

the <u>elasticity of the slope</u> of inverse demand, whose value turned out to be the central parameter in determining the qualitative effects of entry in Seade (1980a), and which again will be very useful in what follows.[4/] The result in (9) says that, quite generally (given only constant marginal cost, which I adopt for simplicity) over-shifting will occur if and only if the electricity of the slope of demand is greater than 1.

The acceptability of this range of values of E needs, of course, to be checked against the second order and the stability conditions for the problem. In terms of E, for the $c_{yy} = 0$ case, these conditions [(3) and (4) resp.] become:

Second order: $E < 2n/\lambda,$ (10)

Stability: $E < (n/\lambda) + 1 .$ (11)

But in most cases of interest n/λ will be a number considerably larger than 1, unless tacit collusion is very high. For the iso-elastic case, with demand given by $p = AY^{-1/\varepsilon}$, the value of E is $1 + (1/\varepsilon)$, so that <u>any</u> isoelastic demand function consistent with stable market equilibrium will result in overshifting,[5/] whatever the market structure (including monopoly) and behavioural parameters, n and λ! Other demand functions can of course render this result less likely (or even rule it out, as will linear demand or more generally any concave demand curve), but E and hence the sign of $E - 1$ are all that matters (given $c_{yy} = 0$), which is useful.

This result obtains rather directly, but I was surprised to note it, for at least my conventional wisdom was very firmly of the view that $S > 1$ was an unlikely perverse outcome. But that the result should not have surprised me is clear from the following familiar formula for monopoly profit maximization:

$$p(1 - \frac{1}{\varepsilon}) = c_y, \tag{11}$$

so that whatever the value of ε may be, if it is constant, an upwards shift in c_y will be amplified by $(1 - \frac{1}{\varepsilon}) < 1$ into a larger change in p. The extension to general conjectural-variations equilibria is direct, for (11) again holds if we replace ε by the firms' <u>perceived</u> demand elasticity, namely $\varepsilon n/\lambda$. This formula can be modified to get an interesting extension of the above result on overshifting under isoelastic demand, to the case with $c_{yy} \neq 0$.[6/] For this, notice that marginal and average cost are related by $c_y = (1+e)(c/y)$, where e is the elasticity of average cost with respect to output. Hence (11) becomes

$$p(1 - \frac{1}{\varepsilon}) = (1 + e)(c/y). \tag{12}$$

Hence if e and ε are constant (the linear-cost example being a special case for the former), a rise in <u>average</u> cost is more than fully shifted iff $e > -1/\varepsilon$.

3.3 Profit

Overshifting of price, as a likely analytic possibility, may be of some interest, but more as a curious result than for its implications. Producers' (net) price may well rise upon taxation, but since output falls, their profits need not rise - they will probably fall, one would guess. Whether the price rise is larger or less than the change in marginal cost will not have particular qualitative signifance for market- or economy-wide structure or for the nature of the welfare effects (signs of components) of tax or pay rises or of technological or other changes. In contrast, if profits were an increasing function of marginal cost in a given industry, the invisible hand would be giving the socially wrong signal to investors following a sectoral efficiency loss, for example, and excise taxation would have quite different redistributive implications from what one might in principle assume. So I now turn to the effect of changes in ξ on firms' profits, again for the symmetric case.

Differentiating $\Pi \equiv yp(ny) - c(y,\xi)$, we get

$$\frac{d\Pi}{d\xi} = p\frac{dy}{d\xi} + yp'n\,\frac{dy}{d\xi} - c_y\,\frac{dy}{d\xi} - c_\xi$$

$$= \frac{(p+nyp'-c_y)c_{y\xi}}{(n+\lambda)p' + n\lambda yp''-c_{yy}} - c_\xi$$

$$= \frac{(n-\lambda)yp'c_{y\xi}}{(n+\lambda)p' + n\lambda yp'' - c_{yy}} - c_{\xi} \qquad (13)$$

using the first-order condition (2). Since n is a natural upper bound for λ in these models, and using the stability condition (4), the signs of the two terms in (13) are the signs of $c_{y\xi}$ and of $-c_{\xi}$ respectively. It seems reasonable to assume that a shift in input-price or production conditions that increases marginal cost, increases total cost too, so that the two effects in (13) will pull in opposite directions. This is what one would expect: the term c_{ξ} is the direct profit-loss suffered by producers in the absence of equilibrium effects, whilst the first term in (13) measures the beneficial effect of reduced output, and we saw above that output will fall in all stable cases, given that competition amongst producers makes them product too much relative to their Pareto profit-frontier. The cost-increase imposes upon them the collusion they themselves had been unable to achieve.

This expression can easily be shown to take one or other sign under a variety of cost and demand conditions. However, no neat classificatory results seem to emerge for the general-cost-function case. To concentrate on the role of the demand side, which seems again to be the central player, I shall revert to the linear-costs assumption or example: $c(y,\xi) = a + m(\xi)y$; $c_y = m$; $c_{y\xi} = m_{\xi}$; $c_{yy} = 0$; $c_{\xi} = m_{\xi}y$. Substituting these in (13),

$$\frac{d\Pi}{d\xi} = \frac{(n-\lambda)yp'm_\xi}{(n+\lambda)p' + n\lambda yp''} - m_\xi y$$

$$= \frac{m_\xi y\left[(n-\lambda)p' - (n+\lambda)p' - n\lambda yp''\right]}{(n+\lambda)p' + n\lambda yp''}$$

$$= \frac{\lambda m_\xi y\left[-2\ p' - nyp''\right]}{(n+\lambda)p' + n\lambda yp''}$$

$$= \frac{\lambda m_\xi y\ (E - 2)}{p'\left[(n+\lambda)p' + n\lambda yp''\right]} \qquad (14)$$

where E is as defined in (10). The denominator of this expression is positive for stable equilibria.

We thus have a rather intriguing result, of general applicability to symmetric conjectural variations equilibria: cost-rises (linear costs case) result in profit rises, if and only if the elasticity of the slope of inverse demand exceeds the magic number 2, without reference to other aspects of demand or cost conditions, or indeed to the structural and behavioural parameters n and λ. Since constant elasticity ε makes $E = 1 + (1/\varepsilon)$, the result, for iso-elastic cases, says that <u>whenever ordinary demand elasticity is less than unity, profits increase with costs</u>.

This seems rather a perverse result, if by that we man counter-intuitive, but is not at all unlikely. Under monopoly, of course, elasticity will not be below 1 in the optimum. But more generally, the only requirement we can impose is (10), namely $E < (n/\lambda) + 1$, which for the isoelastic case reduces to $\varepsilon > \lambda/n$, and λ/n will usually be a number much smaller than 1, at any rate if tacit collusion is not too high.

Some examples of profit-raising cost-rises are given in Table 1 below, where permissible values of the elasticity of demand (for the isoelastic case) are indicated. Also shown, for the sake of comparison, are the ranges of E (or ε) under which entry into the industry results in expansion of output by individual firms, as found in Seade (1980a). Notice how this pathology, the one that was not ruled out by recourse to stability conditions in the said paper, becomes very unlikely to occur except for very concentrated industries, whilst the range over which profit increases with costs is always large, and actually widens with industry size. Casual inspection of the table suggests that, far from being a pathology which one can expect not to arise, profit overshifting may be about as likely to occur as not to, indeed if isoelasticity rather than constant gradient is the best "idealization" of demands as is commonly assumed by industrial economists. And even if the best assumption is linear demand, for which the result no longer obtains (not under linear costs) it is important to realize how strongly the comparative statics of oligopoly depend on the <u>curvature</u> of demand around the prevailing equilibrium's observed point.

TABLE 1.

Restrictions on the values of the elasticity of the slope of inverse demand (and, in brackets, on ordinary demand elasticity for isoelastic case) to meet the requirements or obtain the results indicated in rows 2-5. Relations shown are for linear cost and Cournot behaviour (for other λ's replace n by n/λ throughout).

n (number of firms, symmetric oligopoly)	2	3	5	10
Second-order condition: $E<2n$	$E<4$ $(\varepsilon>\frac{1}{3})$	$E<6$ $(\varepsilon>\frac{1}{5})$	$E<10$ $(\varepsilon>\frac{1}{9})$	$E<20$ $(\varepsilon>\frac{1}{19})$
Stability condition, necessary and sufficient under symmetry: $E<n+1$	$E<3$ $(\varepsilon>\frac{1}{2})$	$E<4$ $(\varepsilon>\frac{1}{3})$	$E<6$ $(\varepsilon>\frac{1}{5})$	$E<11$ $(\varepsilon>\frac{1}{10})$
Individual output rising with entry: $E>n$	$E>2$ $(\varepsilon<1)$	$E>3$ $(\varepsilon<\frac{1}{2})$	$E>5$ $(\varepsilon<\frac{1}{4})$	$E>10$ $(\varepsilon<\frac{1}{9})$
Profit overshifting, i.e. an increasing of marginal cost: $E>2$	$E>2$ $(\varepsilon<1)$	$E>2$ $(\varepsilon<1)$	$E>2$ $(\varepsilon<1)$	$E>2$ $(\varepsilon<1)$

An example from the 'real world' is of course given by the experience of the 'Seven Sisters" (the seven dominant oil concerns) in the post-OPEC years: readers will remember the surprise that was widely expressed when it was realized that their profits had increased sharply following the equally sharp rise they had to pay for their crude. Popular explanations tended to imply that their oligopolistic behaviour (tacit collusion, λ) had changed, which is a difficult explanation to substantiate or accept (they had all the same incentives and opportunity to collude in earlier years, or again in more recent years). But our result, simple as it is in not taking into account distinctive features of the oil industry, notably inventories, can explain quite well this outcome. The short-run elasticity of demand for final oil products was low indeed in the years in question, 73-75. And in the medium run, with a much larger (in fact, as it was, surprisingly large) relevant elasticity, profits have again fallen from their abnormally high levels of the early OPEC years. This all seems clear enough, for the sake of illustration rather than as a serious attempt to analyze the oil industry. But some more precise figures than my casual empiricist's views, or conversations with people better informed than I on these affairs, would do my illustration no harm: I hope to improve my reference to this (or other[7/]) example(s) in a future revision of this paper.

4. Displacing non-symmetric equilibria

The assumption of symmetry can be unduly strong or very acceptable depending on the context in which it is imposed, and the questions being asked. In the present case, it seems reasonable to conjecture that, whether a firm will gain or not from an industry-wide change in costs, may well depend on the position of that firm relative to the rest of the industry. That is, under asymmetry (and given a behavioural pattern for firms), some firms may end up cutting their output much more than others, thus handing the latter a beneficial market "externality". Under such conditions, some might see their profits rise while others' fall, following a development affecting all of them as considered in the previous section. Hence the robustness of our results to the introduction of asymmetry needs to be examined. All the more so, given that a special striking feature of those results, both on price-overshifting and on profit-overshifting, is their total independence from the structual and behavioural parameters of the symmetric case. Does this "magic" of the number 2, when compared with E, in being the determinant of profit overshifting, extend to the non-symmetric case ? The purpose of this section is to provide an affirmative answer to this question. After somewhat laborious computations, the same earlier simplicity will emerge.

This last statement needs to be qualified somewhat, in relation to two quite distinct meanings the word "asymmetry" can take in the present case. On the one hand there is the usual asymmetry of firm size, style of management or techniques, hence of cost functions, behavioural parameters and production levels. This I will allow unrestrictedly. But a different dimension of asymmetry arises when conducting a cost-shift exercise, like the one we are dealing with.
Namely, asymmetry of the direct <u>incidence</u> of the cost-increase on different firms, independently of equilibrium repercussions. That is, our arbitrary shift. parameter ξ may formally enter all the cost functions in the industry, even under symmetry of the first kind, while in fact affecting only one of them: say a fire broke out at a plant whose insurance policy had just lapsed. Obviously we cannot expect a direct, full generalization of our results to cover such cases, making all firms winners or losers depending only on demand conditions. I shall thus allow

for industrial asymmetry in the usual sense, but require effects on their marginal costs (which can themselves differ) to be the same: for each firm f of section 2 above, $c^f_{y\xi} = \gamma$ ($\forall f$). This will be true, for example, in the important special case of excise taxation (at rate ξ), when c^f is $b^f(y) + \xi y$. Similarly, for industry-wide wage rises under a common homothetic technology (with asymmetry in market shares and responses still being possible through differences in firm's outlooks on their environment and hence in their behaviour, as captured by the λ_f's).

Let us now proceed as before and displace the first-order condition for equilibrium, which need now be given by (2) instead of (5). Differentiating (2), and writing $x_f \equiv dy_f/d\xi$, we obtain :

$$p'\sum_i x_i + \lambda_f y_f p''\sum_i x_i + \lambda_f p' x_f - c^f_{yy} x_f - c^f_{y\xi} = 0 . \qquad (15)$$

Simplifying, through the assumption of linear costs,

$$c^f(y,\xi) = a^f + m^f(\xi) y_f, \qquad (16)$$

and defining

$$\alpha_f \equiv p' + \lambda_f y_f p'' \ ; \quad \beta_f \equiv \lambda_f p' \ ; \quad \gamma_f \equiv c^f_{y\xi} \ , \qquad (17)$$

(15) can be rewritten as

$$\alpha_f \sum_i x_i + \beta_f x_f = \gamma_f \ , \quad \forall f \ , \qquad (18)$$

or, in matrix form,

$$A\underline{x} = \underline{\gamma} \ , \qquad (18')$$

where

$$A \equiv \begin{bmatrix} \alpha_1+\beta_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_n \\ \alpha_1 & \alpha_2+\beta_2 & \alpha_3 & \cdots & \alpha_n \\ \alpha_1 & \alpha_2 & \alpha_3+\beta_3 & \cdots & \alpha_n \\ \cdots\cdots \\ \alpha_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_n+\beta_n \end{bmatrix}, \tag{19}$$

with $\underline{x}$ and $\underline{\gamma}$ being the column vectors (x_i), (γ_i).

Thus our first task is to invert matrix (19), a matrix pattern that, with different specific entries, occurs recurrently in comparative-statics and stability analyses of conjectural-variations equilibria (see Seade (1980b), appendices 1 and 2).

Non-singularity is easy to check. Divide each column i of A by β_i :

$$|A| = \Pi_i \beta_i \begin{vmatrix} 1+\frac{\alpha_1}{\beta_1} & \frac{\alpha_2}{\beta_2} & \cdots & \frac{\alpha_n}{\beta_n} \\ \vdots & & \ddots & \vdots \\ \frac{\alpha_1}{\beta_1} & \frac{\alpha_2}{\beta_2} & \cdots & 1+\frac{\alpha_n}{\beta_n} \end{vmatrix} \tag{20}$$

and substract row $i-1$ from row i, for each $i \geq 2$:

$$|A| = \Pi_i \beta_i \begin{vmatrix} 1+\frac{\alpha_1}{\beta_1} & \frac{\alpha_2}{\beta_2} & \frac{\alpha_3}{\beta_3} & \cdots\cdots & \frac{\alpha_n}{\beta_n} \\ -1 & 1 & 0 & \cdots\cdots & 0 \\ 0 & -1 & 1 & \cdots\cdots & 0 \\ \cdots\cdots & & & & \\ 0 & 0 & 0 & \cdots -1 & 1 \end{vmatrix} \qquad (21)$$

which is easy to compute, for the square submatrices associated to the terms of the first row are diagonal matrices with 1 or -1 in the diagonal, in such a way that the cofactors of first-row elements all reduce to 1. Hence,

$$|A| = \Pi_i \; \beta_i (1+\Sigma\frac{\alpha_i}{\beta_i}) \;, \qquad (22)$$

which can readily be checked to be non-vanishing under the stability condition (4) 8/.

Now to invert A, we use the following theorem in matrix algebra (see Maddala (1977, p. 446)), 9/ giving an often convenient decomposition of the inverse of a matrix M :

$$M^{-1} \equiv (N+qr')^{-1} = N^{-1} - (\frac{1}{1+r'N^{-1}q}) N^{-1} qr' N^{-1} \;, \qquad (23)$$

where M is an arbitrary non-singular square matrix, but one which can be decomposed in a useful way into the sum of an easy-to-invert non-singular matrix N, and the outer product of two vectors q and r. In the present

... that it can indeed be decomposed

as in (23), with N being a simple diagonal matrix :

$$A \equiv M = \begin{bmatrix} \beta_1 & o & \cdots & o \\ o & \beta_2 & \cdots & o \\ \cdots\cdots & & & \\ o & o & \cdots & \beta_n \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \cdots\cdots & & & \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{bmatrix} \tag{24}$$

$$= B + e\alpha' ,$$

where B is the diagonal matrix in (24), and e and α the row n-vectors with 1's and with the α_i's as elements. Applying (23) to (24), we obtain

$$A^{-1} = \begin{bmatrix} \frac{1-\delta_1}{\beta_1} & -\frac{\delta_2}{\beta_1} & -\frac{\delta_3}{\beta_1} & \cdots\cdots \\ -\frac{\delta_1}{\beta_2} & \frac{1-\delta_2}{\beta_2} & -\frac{\delta_3}{\beta_2} & \cdots\cdots \\ -\frac{\delta_1}{\beta_3} & -\frac{\delta_2}{\beta_3} & \frac{1-\delta_3}{\beta_3} & \cdots\cdots \\ \cdots\cdots & & & \end{bmatrix} \tag{25}$$

where $\delta_f \equiv [\alpha_f/\beta_f] \; / \; [1+\sum_i \alpha_i/\beta_i]$.

We thus get, solving (18') (and recalling the definition of x_f , namely $dy_f/d\xi$) ,

$$\frac{dy_f}{d\xi} = \frac{\gamma_f - \Sigma\gamma_i\delta_i}{\beta_f} , \tag{26}$$

describing the general form of the adjustment in each firm's output for the model, in response to any change on the cost side (under linear costs, which can easily be relaxed to come this far without them, but this would not be useful for what follows). It is at this stage convenient to specialize as indicated in the opening remarks of this section, and set $\gamma_i = \gamma \ \forall\ i$. This, as we noted, will be true when ξ stands for an excise tax rate, and is an interesting case to consider more generally. After some algebra, (26) transforms into

$$\frac{dy_f}{d\xi} = \frac{\gamma}{\beta_f\left[1 + \sum_i \frac{\alpha_i}{\beta_i}\right]} \tag{27}$$

or in turn into

$$\frac{dy_f}{d\xi} = \frac{\gamma/\lambda_f p'}{1 - E + \Sigma(1/\lambda_i)} \tag{27'}$$

(where γ, we recall, is the homogeneous shift in marginal cost). The expression in brackets in the denominator of (27) is unambiguously positive (see footnote 8, above), and thus our first result in the previous section is confirmed.

$$\frac{dy_f}{d\xi} < 0 \tag{28}$$

for each firm, in all stable equilibria.

Next let us look at the price. Differentiating $p = p(\Sigma y_i)$,

$$\frac{dp}{d\xi} = p'\Sigma\frac{dy_i}{d\xi}$$

$$= \frac{\gamma\Sigma\frac{1}{\lambda_i}}{\quad}$$

using (27'). To study, again, whether price increases faster than marginal cost or not, and hence whether overshifting occurs or not, let us reintroduce the shifting coefficient $S \equiv (dp/d\xi)/\gamma$. Signing $S-1$,

$$S^{-1} - 1 = \frac{1-E+\Sigma\frac{1}{\lambda_i}}{\Sigma\frac{1}{\lambda_i}} - 1$$

$$= \frac{1}{\Sigma\frac{1}{\lambda_i}}(1-E) \qquad (30)$$

Hence, the result on price overshifting is robust to asymmetry:

$S>1$ iff $E>1$.

Lastly, let us consider the effect of ξ on different firms' profits. Differentiating $\Pi^f = y_f p(Y) - c^f(y_f, \xi)$, and again writing $x_f \equiv dy_f/d\xi$, m^f for (constant) marginal cost c^f_y, and γ for its (uniform) shift $c^f_{y\xi}$, we get

$$\frac{d\Pi^f}{d\xi} = (p-m^f)x_f + y_f p'\sum_i x_i - \gamma y_f \qquad (31)$$

$$= \frac{(p-m^f)\gamma}{\beta_f \eta} + \frac{y_f p'\gamma}{\eta}\sum_i \frac{1}{\beta_i} - \gamma y_f$$

using (27) and writing $\eta \equiv \left[1+\Sigma\alpha_i/\beta_i\right]$. But, by the first order condition (2), $p-m^f = -\lambda_f y_f p'$, so that

$$\frac{d\Pi^f}{d\xi} = \gamma\{\frac{-\lambda_f y_f p'}{\eta\lambda_f p'} + \frac{y_f}{\eta}\sum_i\frac{1}{\lambda_i} - y_f\},$$

(using $\beta_f \equiv \lambda_f p'$)

$= \gamma\{\frac{y_f}{\ } \ 1 \ \ y_f \ \ldots$

$$= \frac{\gamma y_f}{\eta} \{ \Sigma \frac{1}{\lambda_i} - 1 - \eta \} . \tag{32}$$

But $\eta = 1 - E + \Sigma(1/\lambda_i)$, so that we finally obtain

$$\frac{d\Pi^f}{d\xi} = \gamma y_f (E-2)/\eta . \tag{33}$$

We already noted that $\eta > 0$. We therefore obtain that the result on profit-overshifting generalizes to the asymmetric, fully general (except for our assumptions on costs) conjectural variations equilibria: industry-homogeneous cost increases such as an excise tax, increase/decrease every firm's level of profits if and only if the elasticity of the slope of inverse demand is greater/less than 2, i.e., for the isoelastic case, if and only if demand elasticity is less/greater than unity.

Another interesting feature to be noted on this very simple expression is the fact that, despite the asymmetries being allowed for on size, efficiency and behaviour, and hence in profit margins and responses to the tax, the effect of excise taxes on profits, per unit of output, is the <u>same</u> for all firms.

5. Concluding Remarks

In this paper we have studied the comparative statics effects of changes in cost conditions, such as excise tax or a wage or technology-shift, in an oligopolistic industry selling homogeneous output to a market described by an arbitary demand function and whose non-homogeneous firms behave in a conjectural-variations fashion.

The main results that have obtained appear to me to be surprising, both for their generality or plausibility, being in some cases highly counter-intuitive results, and for their very simplicity. These are that, following a rise in excise taxation or some similar industry - homogeneous cost rise, and assuming for simplicity linear costs, (i) output of all firms will unambiguously fall in all stable equilibria, (ii) consumers' price will accordingly rise, but will do so to a greater extent than the shift in marginal cost, representing a more than 100% shift of excise tax (say) to consumers, if and only if the elasticity of the slope of inverse demand E is greater than 1 (its value for stable equilibria need only be less than $n + 1$, for n Cournot firms in the symmetric case), which for isoelastic demands means always, and (iii) the increase in price will be sufficient to more than offset the fall in volume of sales and the rise in costs, thereby raising the profits of all the firms in the industry, if and only if E is greater than 2, which in the isoelastic case means ordinary demand elasticity less than 1 (stability requiring it to be, in the isolestic case, greater than $1/n$, for the symmetric Cournot case with n firms).

Some instances of applications or situations where the above results may be of interest have already been mentioned, such as the analysis of tax-shifting both in the price and profit senses which are old topics,of constant interest particularly in the applied literature, or the interpretation of developments such as the example given in the text on the oil industry. In a foreign-trade context, the result would call for an output tax (or even better, to better avoid retaliation, an input tax) on an oligopolistic export sector facing inelastic world demand: their profits would rise, apart from yielding revenue. This is reminiscent of optimum-tariff arguments, but refers to intervention one or even two stages earlier than the export point in the process (taxing all output or its inputs), in a way that would not necessarily result in a net benefit, less so in a Paretian gain,to government and producers,if conventional wisdom were necessarily right in placing tax revenue and private profits (or more generally surplus) on the two sides of the scale in chosing tax.

The motivation I offer for the paper, however, is also, and to a large extent, theoretical. Not that the actual results obtained are of much interest from that point of view, but more generally for the fact that it was found to be feasible to manipulate and study the conventional-variations model analytically, and in so doing to raise questions of fiscal policy under the richer industrial structures that are commonplace in industrial, but not in public, economics. Indeed, much too often or even as a rule, studies of the effects of taxes on firms' pricing and output restrict attention to the polar forms of monopoly and perfect competition, and immediately shift attention from standard neoclassical tools to mark-up models if oligopoly is to be

Of course the main questions to be asked in this connection lie ahead or elsewhere, and not in this paper, notably the welfare effects (and design) of taxation. Some interesting results have successfully been derived for special cases, by Stern [1982] and Katz and Rosen [1983]. But imperfect competition is by and large still to make its full entry into public economics. One general difficulty is that, in the latter field, one tends to shy away these days from partial-equilibrium analysis, which is pretty much necessary to study many problems in industrial economics. One should perhaps be more open-minded about such things.

This takes me to the question of limitations or extensions of the analysis. One that I think is not a limitation, firstly, at least in connection with the specific results obtained as opposed to the modelling done, is the assumption of partial equilibrium. In a general equilibrium context it is clear that one can have effects such as a firms' profits rising if the wage rate goes up, for example on account of the increased income of customers-workers. It is in a partial-equilibrium context that the possibility of profit-raising cost rises is relatively striking.

Similarly, the no-entry structure we have adopted is not restrictive as far as our main results, on profits, are concerned. In a long-run, free-entry equilibrium context, the conditions under which short-run profits increase/decrease with cost rises, translate into entry/exit from the industry until equilibrium is restored. Fortunately it will then be restored, for as was shown in Seade [1980a] individual profits will unambigiously fall/rise with entry/exit. The result on price-overshifting, however, may be affected in the long-run, the reason being that it holds in a larger class of cases than the profit-rises result: if profits rise with costs and entry is induced, aggregate output will unambiguously rise (again see the paper cited), and hence price will fall from its short-run increased level. I have not explored carefully conditions under which this effect may or may not overturn the initial overshifting, but I guess both possibilities will be there. On the other hand, in cases with profits falling but overshifting still occurring ($1 < E < 2$, or $\varepsilon > 1$ for isoelastic cases), exit will further contract output and hence result in a further degree of price overshifting in the long run.

Two extensions that I have only looked at briefly but which could take some interesting turn, still within the framework of the model of this paper, would be to relax the linear-costs assumption, and to analyse shifts on the demand side. Doing the former does not seem to lead to interpretable expressions or to yield any insight, although special cases can be analyzed or results obtained, such as the result on price-overshifting noted in connection with equation (12) above, or the analysis of the quadratic-costs case in Katz and Rosen [1983]. Shifting the demand instead of the cost function, does not seem to yield anything interesting or useful. The linear-costs isoelastic-demand case ($p = A(\xi)Y^{-1/\varepsilon}$) turns out to validate conventional wisdom inevitably: profits rise with expansions of demand if and only if equilibrium is stable. This is what one would expect, since the two opposing effects of shifting costs, the direct damage of the cost rise and the indirect benefit of the induced contraction, are not both present under a demand shift. But ambiguities should surely be expected for arbitrary demands and forms of expansion of demand.

More interesting, finally, would be to alter, in certain useful ways, the structure of the problem formulated. Firstly, introducing product-differentiation while striving to keep the analysis tractable, for both simplicity and product-diversity will be essential to bring a model like the present one into any form of general equilibrium framework. Secondly, adopting alternative solutions to the oligopoly game.

Footnotes

1/ I am grateful to Bobby Willig for bringing this reference to my attention in this connection.

2/ A sufficient condition, if paired with the assumption $\lambda_f p^1 - c^f_{yy} < 0$ which I also make, and a necessary condition too relative to the class of cases where either (4) or its opposite-sign expression hold for all firms ("weakly homogeneous industry" in Seade [1980b, p.18]).

3/ Thus, none less than L. Johansen (1971, p.280), referring to what for us is equation (8) for the case of monopoly, states that "it is theoretically possible (though this would hardly occur with any frequency in practice) that the price p will increase by more than the increase in the excise duty rate ...".

4/ Defined here as minus the E of the said paper, for convenience, to make it positive in central cases of interest. Its relation to ordinary demand elasticity ε, in general, is given by

$$E = 1 + \frac{1}{\varepsilon} + \eta_{\varepsilon Y}, \qquad (10')$$

where $\eta_{\varepsilon Y} \equiv Y(d\varepsilon/dY)/\varepsilon$, so that in the isoelastic case $E = 1 + 1/\varepsilon$.

5/ This result, for the isoelastic demand case under symmetry, has also been noted by D. de Meza (1982) and by N. Stern (1982).

6/ I am grateful to Norman Ireland for this suggestion.

7/ Another example seems to have been the soft-drinks industry, after the sharp increase of the price of sugar in the mid-70's.

8/ The expression in brackets is the sum of the left-hand sides of (4) across i, each divided by $n\lambda_i p'$. This expression reduces to $1 - E + \Sigma(1/\lambda_i)$.

9/ I am grateful to Peter Burridge and Ken Wallis for bringing the result to my attention.

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