Do Real Balance Effects Invalidate the Taylor Principle in Closed and Open Economies?

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Abstract
This paper examines the implications for equilibrium determinacy of forward-looking monetary policy rules in a Neo-Wicksellian model that incorporates real balance effects. We show that in closed economies the presence of small, empirically plausible real balance effects significantly restricts the ability of the Taylor principle to prevent indeterminacy of the rational expectations equilibrium. This problem is further exacerbated in open economies, particularly if the monetary policy rule reacts to consumer-price, rather than domestic-price, inflation. These findings still hold even when output and the real exchange rate are introduced into the policy rule, thereby suggesting that the widespread neglect of real balance effects in the literature is ill-advised.

JEL Classification: E41; E52; F41

Keywords: Equilibrium Determinacy; Real Balance Effects; Trade Openness; Forward-looking Inflation Targeting; Taylor Principle.

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1 Introduction

The importance of forward-looking monetary policy has long been emphasized by researchers. The need to conduct monetary policy in a forward-looking, or preemptive manner, arises primarily because of the widely documented long and variable time lag after which a monetary policy action takes effect in the economy (Friedman, 1968). But it has also been theoretically rationalized on the grounds of central bank credibility, in order to anchor and manage private-sector expectations (see, e.g., Svensson, 1997; Batini and Haldane, 1999). Such benefits have not been overlooked by policymakers. Empirical evidence suggests that many central banks set the nominal interest rate in response to expected future inflation (see, e.g., Clarida et al., 1998, 2000; Orphanides, 2001, 2004; Mihailov, 2006). Indeed the popularity of conducting monetary policy in this way is such that now “forward-looking inflation targeting has become a defining characteristic of monetary policymaking worldwide” (Huang et al., 2009, p.409).

A key issue in the design of such forward-looking monetary policy then, when operationalized via simple feedback rules, is that the particular interest-rate feedback rule adopted by a central bank should ensure a determinate equilibrium.¹ That is, monetary policy should be designed to avoid generating real indeterminacy which can destabilize the economy through the emergence of sunspot equilibria and self-fulfilling expectations that result in large reductions in the welfare of the economy.² It has been well established in the New Keynesian (or Neo-Wickesllian) literature that under the Taylor principle, i.e. a policy that adjusts the nominal interest rate by proportionally more than the increase in inflation, a central bank can easily prevent the emergence of indeterminacy, provided it is not overly aggressive in its response to expected future inflation; or alternatively, by also including contemporaneous output into the feedback rule (see, e.g., Bernanke and Woodford, 1997; Clarida et al., 2000; Woodford, 2003). Recent studies have considered whether such policies are also consistent with equilibrium determinacy in open economies.³ Among other things, ¹We focus on simple feedback rules and not on optimal targeting rules. For a discussion on the benefits of considering simple feedback rules see, e.g., Taylor (1993), Batini and Haldane (1999), Woodford (2001), Svensson (2003). For a study of optimal targeting rules in a two-country model of a similar kind to ours, see, e.g., Benigno and Benigno (2006). ²By real indeterminacy we mean that there exists a continuum of equilibrium paths, starting from the same initial conditions, which converge to the steady state. Our attention rests solely with the consideration of local (real) determinacy as opposed to global determinacy. For further discussion of these issues see Clarida et al. (2000), Carlstrom and Fuerst (2001), Benhabib et al. (2002), Woodford (2003), Cochrane (2011). ³See, e.g., Zanna (2003), Batini et al. (2004), De Fiore and Liu (2005), Linnemann and Schabert (2006),
this literature has found that the Taylor principle may not be as effective in preventing indeterminacy in open economies if the central bank reacts to expected future consumer-price inflation, rather than expected future domestic-price inflation (see, e.g., Linnemann and Schabert, 2006; Llosa and Tuesta, 2008; Leith and Wren-Lewis, 2009; McKnight, 2011b).

By reacting to consumer-price inflation, the Taylor principle now becomes constrained by the economy’s degree of openness to international trade.

However, a general criticism of the above literature is the notable absence of monetary aggregates from the determinacy analysis.\(^4\) When the nominal interest rate is the monetary policy instrument, money demand plays no role for equilibrium determination. Therefore, the transmission mechanism of monetary policy operates entirely through an aggregate demand channel: changes in the nominal interest rate affect output, via changes in the real interest rate, which results in a change in inflation via a New Keynesian version of the Phillips curve.\(^5\) Yet, there are strong intuitive reasons for considering the real balance effects of money and their role in the monetary transmission mechanism. Since the classical works of Von Haberler (1937), de Scitovszky (1941), Pigou (1941, 1943) and Patinkin (1949, 1956), it has long been suggested that changes in real money balances can affect consumption and output, through changes in individual wealth. Another real balance effect of money arises from facilitating transactions services. As stressed by Woodford (2003), if money is considered to provide transaction services then the benefits of this service should be related to the individual’s volume of transactions. Empirical estimates suggest that such effects, while small, are found to exist in the data.\(^6\)

This paper considers the robustness of the Taylor principle under forward-looking inflation feedback rules when real balance effects of transactions services are explicitly modeled. Following Woodford (2003) and Kurozumi (2006), these effects are introduced via a money-in-the-utility-function (MIUF) set-up where consumption and real money balances enter non-separably.\(^7\) Now there is an additional monetary transmission mechanism where

\(^4\)These studies either assume a cashless economy or adopt a money-in-the-utility function model with separable preferences.

\(^5\)In the open economy there is an additional monetary transmission channel that arises through changes in the terms of trade. Now changes in the nominal interest rate also affect output (and thus inflation) via an expenditure switching effect towards/away from foreign goods.


\(^7\)Alternatively, Andrés et al. (2009) generate real balance effects through the introduction of portfolio
changes in the nominal interest rate result in changes in the demand for money, which affects the output and pricing decisions of firms, via changes in the real marginal cost of production. The analysis examines whether empirically realistic real balance effects can have important implications for determinacy of the rational expectations equilibrium in both closed and open economies. We begin by showing that, for closed economies, the presence of real balance effects significantly increases the severity of indeterminacy under the Taylor principle. This is first demonstrated for interest rate feedback rules that set the nominal interest rate in response solely to expected future inflation, and then shown to be robust when future, or contemporaneous, output is also incorporated into the feedback rule. Next, we investigate the determinacy implications of real balance effects for open economies, where the feedback rule can respond to either domestic-price or consumer-price inflation. Consistent with the empirical studies of Clarida et al. (1998, 2000), Orphanides (2004) and Mihailov (2006), we focus our attention on a feedback rule that reacts to expected future inflation and contemporaneous output. In general we find that in the presence of real balance effects the problem of indeterminacy is more severe for open economies than closed economies. When the indicator of inflation used in the policy rule is domestic-price inflation we find that the range of indeterminacy can increase as the degree of trade openness decreases. However, by reacting to consumer-price inflation, not only does the range of indeterminacy increase sizeably, relative to domestic-price inflation feedback rules, but the range of indeterminacy is now increasing with respect to the degree of trade openness. In contrast to the existing literature (e.g. Linnemann and Schabert, 2006; McKnight, 2011a) it is further shown that reacting to movements in the real exchange rate does little to mitigate the range of indeterminacy induced.

Overall our analysis suggests two key policy implications. First, real balance effects exert a destabilizing effect on the rational expectations equilibrium when monetary policy is governed by a forward-looking interest rate rule. Since the prevention of indeterminacy is an important issue, our analysis suggests that central banks have a difficult task of neither being too cautious, nor too aggressive in changing the nominal interest rate in response to expected changes in future inflation and output. Secondly, our analysis provides further adjustment costs (in terms of real money balances).

\footnote{In the open economy, changes in money demand have an additional effect, since changes in real marginal cost result in changes in the terms of trade, via the expenditure switching effect.}
evidence that central banks should use domestic-price inflation rather than consumer-price inflation in the conduct of monetary policy. Our analysis supports a number of recent studies that have argued against the current choice of the consumer-price index as the indicator of inflation used in the feedback rule. While reacting to domestic-price inflation cannot eliminate the indeterminacy problem in the presence of real balance effects, it does help to reduce its potential severity.

Our paper contributes to a small literature that has been studying the implications for equilibrium determinacy when the real balance effects of transactions services are introduced through the assumption of non-separability of the utility function (between consumption and real money balances). Among other things, Benhabib et al. (2001) show that non-separability has no implications for determinacy using a continuous-time MIUF model. Using a discrete-time MIUF model, Schabert and Stoltenberg (2005) find that in general the determinacy conditions are independent of the magnitude of real balance effects, and this result is robust regardless of whether prices are assumed to be flexible or sticky or monetary policy is conducted using an interest rate rule or a (constant) money growth policy rule. However, Kurozumi (2006) shows that real balance effects can be destabilizing if the policy rule responds, in addition to current inflation, also to current output.9 For a similar feedback rule Piergallini (2006) finds that when consumers are assumed to be finite-lived then real balance effects could actually have a stabilizing effect on the rational expectations equilibrium. While our approach has many similarities with this literature, there are two key differences. First of all, the existing literature examines real balance effects under contemporaneous interest-rate rules. McCallum (1999) among others has questioned whether such rules are even implementable in practice. This paper therefore addresses an important gap in the literature. We examine the determinacy implications of real balance effects under implementable, forward-looking, feedback rules, which are empirically motivated and thus critical to study. Secondly, we also consider the determinacy implications of real balance effects for the open economy. For many central banks, this is a highly significant issue in the design of interest-rate policy, given the large and increasing

9Kurozumi (2006) also finds that transaction frictions can play an important role in inducing indeterminacy. By extending the analysis of Carlstrom and Fuerst (2001) to allow for real balance effects, he shows that different timing assumptions on how money balances enter the utility function can alter the conditions for determinacy. A similar point is also made by Schabert and Stoltenberg (2005).
trade share of many inflation-targeting countries (De Fiore and Liu, 2005).\footnote{De Fiore and Liu (2005) study the determinacy implications of feedback rules for small open economies in the presence of transactions frictions represented by a cash-in-advance constraint. However in stark contrast to this analysis, they do not consider the role of transactions services, the importance of domestic-price versus consumer-price inflation, nor the role of output response in the feedback rule.}

The remainder of the paper is organized as follows. Section 2 outlines the model and Section 3 derives the linearized equilibrium system. The determinacy analysis for closed economies is addressed in Section 4. Section 5 derives the conditions for determinacy in an open-economy context. Finally, Section 6 concludes.

2 Model

The model is a two-country extension of the Neo-Wicksellian MIUF model employed by Woodford (2003) and Kurozumi (2006) for the closed economy. Within each country there exists a representative infinitely-lived household, a representative final-good producer, a continuum of intermediate-goods producing firms, and a monetary authority. Real balance effects are introduced by assuming that the utility function of the representative household is non-separable between consumption and real money balances. The representative final-good producer is a competitive firm that bundles domestic and imported intermediate goods into non-tradeable final goods. Intermediate-goods firms operate under monopolistic competition and set prices in a staggered fashion according to Calvo (1983). Monetary policy is governed by a Taylor rule where the nominal interest rate reacts to expected future inflation. In line with the recent literature, we assume that the law of one price holds, financial markets are complete and that the degree of trade openness is proxied by the inverse of home bias in preferences for traded inputs. Preferences and technologies are symmetric across the two countries. We present the features of the model for the home country on the understanding that the foreign case can be analogously derived, where an asterisk denotes foreign variables.
2.1 Final-Goods Sector

The home final good \((Z)\) is produced by a competitive firm that uses domestic \((Z_H)\) and imported \((Z_F)\) intermediate goods as inputs according to the aggregation technology index:

\[
Z_t = \left[ a^{\frac{1}{\theta}} Z_{H,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} Z_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},
\]

(1)

\[
Z_{H,t} = \left[ \int_0^1 z_{H,t}(i)^{\frac{1}{\varphi}} di \right]^{\frac{1}{1-\varphi}}, \quad Z_{F,t} = \left[ \int_0^1 z_{F,t}(j)^{\frac{1}{\varphi}} dj \right]^{\frac{1}{1-\varphi}},
\]

(2)

where \(z_H(i)\) and \(z_F(j)\) are the respective quantities of the domestic and imported type \(i\) and \(j\) intermediate goods. The parameter \(\theta > 0\) represents the constant elasticity of substitution between aggregate home and foreign intermediate goods, \(0.5 < a < 1\) captures the degree of home bias towards domestic intermediate goods and \(\varphi > 1\) is the elasticity of substitution across individual home (foreign) intermediate goods.

Let \(p_H(i)\) and \(p_F(j)\) represent the respective prices of \(z_H(i)\) and \(z_F(j)\) in home currency. Cost minimization in final good production yields the aggregate demand conditions for home and foreign goods:

\[
Z_{H,t} = a \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} Z_t, \quad Z_{F,t} = (1-a) \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} Z_t,
\]

(3)

where the demand for individual goods is given by

\[
z_{H,t}(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varphi} Z_{H,t}, \quad z_{F,t}(j) = \left( \frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varphi} Z_{F,t}.
\]

(4)

Since the final good producer is competitive, its price is equal to marginal cost:

\[
P_t = \left[ a^{\frac{1-\theta}{\theta}} P_{H,t}^{1-\theta} + (1-a)^{\frac{1-\theta}{\theta}} P_{F,t}^{1-\theta} \right]^{\frac{\theta}{1-\theta}},
\]

(5)

where \(P\) is the consumer price index and \(P_H\) and \(P_F\) are the respective price indices of home and foreign intermediate goods, all denominated in home currency

\[
P_{H,t} \equiv \left[ \int_0^1 p_{H,t}(i)^{1-\varphi} di \right]^{\frac{1}{1-\varphi}}, \quad P_{F,t} \equiv \left[ \int_0^1 p_{F,t}(j)^{1-\varphi} dj \right]^{\frac{1}{1-\varphi}}.
\]

(6)
We assume that there are no costs to trade between the two countries and the law of one price holds, which implies that

\[ P_{H,t} = S_t P^*_{H,t}, \quad P^*_{F,t} = \frac{P_{F,t}}{S_t}, \]  

(7)

where \( S \) is the nominal exchange rate. Letting \( Q \equiv \frac{S P^*}{P} \) denote the real exchange rate, under the law of one price the CPI index (5) and its foreign equivalent imply:

\[ \left( \frac{1}{Q_t} \right)^{1-\theta} = \left( \frac{P_t}{S_t P^*_t} \right)^{1-\theta} = \frac{a P^1_{H,t} - (1-a) (S_t P^*_{F,t})^{1-\theta}}{a \left( S_t P^*_{F,t} \right)^{1-\theta} + (1-a) P^1_{H,t}} \]  

(8)

Hence, with home bias between intermediate goods (i.e. \( a > 0.5 \)), the purchasing power parity (PPP) condition fails to hold. The relative price of foreign goods in terms of home goods, or the (home) terms of trade \( T \), is defined as \( T \equiv \frac{S P^*_F}{P_H} \).

2.2 Intermediate-Goods Sector

Intermediate-sector firms hire labor \( h \) to produce output given a real wage rate \( w_t \). A firm of type \( i \) has a linear production technology

\[ y_t(i) = h_t(i), \]  

(9)

and given competitive prices of labor, cost minimization yields

\[ m_{ct} = w_t \frac{P_t}{P_{H,t}}, \]  

(10)

where \( m_{ct} \equiv \frac{MC_t}{P_{ct}} \) is real marginal cost. Intermediate-sector firms set prices according to Calvo (1983), where in each period there is a constant probability \( 1 - \psi \) that a firm will be randomly selected to adjust its price, which is drawn independently of past history. A domestic firm \( i \), faced with changing its price at time \( t \), has to choose \( p_{H,t}(i) \) to maximize its expected discounted value of profits, taking as given the indexes \( P, P_H, P_F, Z \) and \( Z^* \):

\[ \max_{p_{H,t}(i)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \psi)^s X_{t,t+s} \left\{ [p_{H,t}(i) - MC_t+\delta(i)] [z_{H,t+s}(i) + z^*_{H,t+s}(i)] \right\}, \]  

(11)
where
\[ z_{H,t+s}(i) + z_{H,t+s}^*(i) \equiv (p_{H,t}(i))^{-\phi} [Z_{H,t+s} + Z_{H,t+s}^*], \]
and the firm’s stochastic discount factor used to value random date \( t+s \) payoffs is \( \beta^s X_{t,t+s} = \frac{U_C(C_{t+s}, m_{t+s})}{U_C(C_t, m_t)}(P_t/P_{t+s}) \).

Firms that are given the opportunity to change their price, at a particular time, all behave in an identical manner. The optimization condition to the firm’s maximization problem yields
\[ P_{H,t} = \frac{\phi}{\phi - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta^s)^s X_{t,t+s} P_{H,t+s}^s (Z_{H,t+s} + Z_{H,t+s}^*) MC_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta^s)^s X_{t,t+s} P_{H,t+s}^s (Z_{H,t+s} + Z_{H,t+s}^*)}. \] (12)

The optimal price set is a mark-up \( \frac{\phi}{\phi - 1} \) over a weighted average of future nominal marginal costs.

### 2.3 Representative Household

The representative household chooses real consumption \( C \), domestic real money balances \( m \equiv M/P \), and labor \( h \) to maximize expected discounted utility:
\[ \max E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t, \frac{M_t}{P_t}, h_t \right), \]
where the discount factor is \( 0 < \beta < 1 \), subject to the period budget constraint
\[ E_t \{ \Gamma_{t,t+1} B_{t+1} + M_t + P_t C_t \leq B_t + M_{t-1} + P_t w_t h_t + \int_0^1 \Pi_t(i) d(i) + \Upsilon_t \}. \] (13)

The household carries \( M_{t-1} \) units of money and \( B_t \) units of nominal bonds into period \( t \). Before proceeding to the goods market, the household visits the financial market where a state-contingent nominal bond \( B_{t+1} \) can be purchased that pays one unit of domestic currency in period \( t+1 \) if a specific state is realized at a period \( t \) price \( \Gamma_{t,t+1} \). During period \( t \) the household supplies labor to the intermediate-sector firms receiving real income from wages \( w_t \), nominal profits from the ownership of domestic intermediate-sector firms

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11. The assumption that all firms are owned by the representative household implies that the firm’s stochastic discount factor is equivalent to the household’s intertemporal marginal rate of substitution.
12. To facilitate comparison with the vast majority of the existing literature, we adopt the traditional convention that end-of-period real money balances enter the utility function. Assuming an alternative timing-assumption on money could have important consequences for equilibrium determinacy, as discussed by Carlstrom and Fuerst (2001), Kurozumi (2006), and McKnight (2011b).
Π_t and a lump-sum (net) nominal transfer Υ_t from the monetary authority. The household then uses these resources to purchase the final good.

The period utility function is assumed to be non-separable between consumption and real money balances but additively separable with respect to labor:\(^\text{13}\)

\[
U(C_t, m_t, h_t) \equiv u(C_t, m_t) - v(h_t).
\] (14)

The first-order conditions from the home household’s maximization problem yield:

\[
\beta E_t \left\{ \frac{u_c(C_{t+1}, m_{t+1})}{P_{t+1}} \right\} = \frac{u_c(C_t, m_t)}{P_t} \frac{1}{R_t}
\] (15)

\[
\frac{u_m(C_t, m_t)}{u_c(C_t, m_t)} = \frac{R_t - 1}{R_t}
\] (16)

\[
\frac{v_h(h_t)}{u_c(C_t, m_t)} = w_t
\] (17)

where \(R_t\) denotes the gross nominal yield on a one-period discount bond defined as \(R_t^{-1} \equiv E_t \{ \Gamma_{t,t+1} \}.\) Equation (15) is the consumption Euler equation, (16) defines the money demand function, and (17) determines labor supply. Optimizing behavior further implies that the budget constraint (13) holds with equality in each period and the appropriate transversality condition is satisfied. Analogous conditions apply to the foreign household.

Letting \(\Gamma_{t,t+1}^*\) denote the price of the foreign country’s state-contingent bonds then no-arbitrage implies

\[
R_t = R_t^* E_t \left\{ \frac{S_{t+1}}{S_t} \right\},
\] (18)

where \((R_t^*)^{-1} \equiv E_t \{ \Gamma_{t,t+1}^* \}.\) Equation (18) is the standard uncovered interest rate parity (UIP) condition. Equation (15), its foreign equivalent and the UIP condition (18) consequently imply

\[
Q_t = q_0 \frac{u_c^*(C_t^*, m_{t}^*)}{u_c^*(C_0^*, m_0^*)} \frac{u_c(C_t, m_t)}{u_c^*(C_t^*, m_{t}^*)},
\] (19)

which follows from the assumption of complete asset markets, where the constant \(q_0 \equiv Q_0 \left[ \frac{u_c(C_0, m_0)}{u_c^*(C_0^*, m_0^*)} \right].\)

\(^{13}\)As is standard, we assume that \(u(C, m)\) is concave and strictly increasing in each argument and both consumption and real money balances are normal goods. It is further assumed that \(v(h),\) the disutility of labor supply, is an increasing, convex function.
2.4 Monetary Authority

Motivated by the empirical studies of Clarida et al. (1998, 2000), Orphanides (2004) and Mihailov (2006), monetary policy is specified as a Taylor-type rule in which the nominal interest rate is a function of expected future inflation and current or expected future output. The monetary authority can adjust the (gross) nominal interest rate in response to changes in domestic-price inflation (PPI) $\pi^H_t$ or to changes in consumer-price inflation (CPI) $\pi_t$, according to the rules:

$$ R_t = \overline{R} \left( \frac{E_t(\pi^H_{t+1})}{\pi^H_t} \right)^{\mu_\pi} \left( \frac{E_t(Y_{t+k})}{\overline{Y}} \right)^{\mu_y}, \quad R_t = \overline{R} \left( \frac{E_t(\pi_{t+1})}{\pi} \right)^{\mu_\pi} \left( \frac{E_t(Y_{t+k})}{\overline{Y}} \right)^{\mu_y}, $$

where $\overline{R}$ and $\overline{Y}$ respectively denote the steady state nominal interest rate and steady state output, and where $\mu_\pi > 0$, $\mu_y \geq 0$ and $k = 0, 1$.

2.5 Market Clearing and Equilibrium

Market clearing for the home intermediate-goods market requires

$$ Z_{H,t} + Z^*_{H,t} = Y_t. $$

(21)

Total home demand must equal the supply of the final good,

$$ Z_t = C_t, $$

(22)

and the labor market, the money market

$$ \Upsilon_t = M_t - M_{t-1}, $$

(23)

and the bond market all clear

$$ B_t + B^*_t = 0. $$

(24)

**Definition 1** (Rational-Expectations Equilibrium): Given an initial allocation of $B_{t_0}$, $B^*_{t_0}$, and $M_{t_0-1}$, $M^*_{t_0-1}$, a rational-expectations equilibrium is a set of sequences \{\(C_t, C^*_t, M_t, M^*_t, h_t, h^*_t, B_t, B^*_t, R_t, R^*_t, \Gamma_t, \Gamma^*_t, MC_t, MC^*_t, w_t, w^*_t, Y_t, Y^*_t, S_t, Q_t, P_t, P^*_t, P_{H,t}\),
for all \( t \geq t_0 \) characterized by: (i) the optimality conditions of the representative household, (15) to (17), the budget constraint (13) is satisfied and the transversality condition holds; (ii) cost-minimization (10), and price-setting behavior of intermediate-sector firms (12), and the aggregate version of the production function (9); (iii) the final good producer’s optimality conditions, (3) and (5); (iv) all markets clear, (21) to (24); (v) the monetary policy rule is satisfied (20); along with the foreign counterparts for (i)-(v) and conditions (7), (8), (18) and (19).

3 Local Equilibrium Dynamics

3.1 Linearized Model

As is common in the literature, the model is log-linearized around a zero-inflation symmetric steady state. In what follows, all hatted variables denote percentage deviations from the steady state. Linearizing (15) yields the IS equation for the home country:

\[
\hat{C}_t = E_t \hat{C}_{t+1} - \sigma \left[ \hat{R}_t - E_t \hat{\pi}_{t+1} + \chi (E_t \hat{m}_{t+1} - \hat{m}_t) \right]
\]

(25)

where \( \sigma \equiv -u_c/u_{cc} > 0 \) is the intertemporal elasticity of substitution in consumption, and \( \chi \equiv \bar{m} u_{cm}/u_c \) is the degree of non-separability between consumption and real money balances. For analytical tractability the ensuing analysis follows Kurozumi (2006) in imposing:

**Assumption 1** \( 0 \leq \chi < (\eta_c \sigma)^{-1} \iff 0 \leq 1 - \eta_c \sigma \chi \equiv \Omega < 1, \)

which as discussed in Section 3.2 below is of most empirical relevance.

Linearizing the price-setting equation (12) yields the AS equation for the home country:

\[
\hat{\pi}_t^H = \beta E_t \hat{\pi}_{t+1}^H + \lambda \hat{m}_t
\]

(26)

where \( \lambda \equiv \frac{(1-\psi)(1-\beta \psi)}{\psi} > 0 \) is the real marginal cost elasticity of inflation and real marginal cost is given by:

\[
\hat{m}_t = \omega \hat{Y}_t + \frac{1}{\sigma} \hat{C}_t - \chi \hat{m}_t + (1-a) \hat{T}_t
\]

(27)
after combining the linearized versions of (5), (9), (10) and (17), where \( \omega \equiv \frac{hv_{hh}}{v_{h}} > 0 \) is the output elasticity of real marginal cost. Domestic output follows from the linearized versions of (3), (5), their foreign equivalents and the market clearing conditions (21) and (22):

\[
\hat{Y}_t = 2a\theta(1 - a)\hat{T}_t + a\hat{C}_t + (1 - a)\hat{C}_t^*.
\]  

(28)

Linearizing equation (16) yields the LM equation

\[
\hat{m}_t = \eta_c \hat{C}_t - \eta_R \hat{R}_t
\]  

(29)

where \( \eta_c, \eta_R > 0 \) are the income elasticity and interest rate semi-elasticity of money demand, which are defined as follows:

\[
\eta_c \equiv \frac{\sigma - 1 + \vartheta}{\chi + \sigma_m^{-1}}; \quad \eta_R \equiv \left( \frac{\beta}{1 - \beta} \right) \left( \frac{1}{\chi + \sigma_m^{-1}} \right),
\]

where \( \sigma_m^{-1} = -\bar{m}u_{mm}/u_m, \vartheta = \bar{C}u_{mc}/u_m \) and \( \chi = s_m \vartheta, \) where \( s_m = \bar{m}u_m/u_cC \), with all partial derivatives evaluated at the steady state.

Linearizing equations (7), (8), (18) and (19) yields expressions for the uncovered interest parity condition, the terms of trade and the consumer-price inflation differential:

\[
\hat{R}_t - \hat{R}_t^* = E_t \Delta \hat{S}_{t+1}
\]  

(30)

\[
(2a - 1)\hat{T}_t = \sigma^{-1} \left( \hat{C}_t - \hat{C}_t^* \right) - \chi (\hat{m}_t - \hat{m}_t^*)
\]  

(31)

\[
\hat{\pi}_t - \hat{\pi}_t^* = (2a - 1) \left( \hat{\pi}_t^H - \hat{\pi}_t^F \right) + 2(1 - a)\Delta \hat{S}_t
\]  

(32)

where \( E_t \Delta \hat{S}_{t+1} \equiv E_t \hat{S}_{t+1} - \hat{S}_t \) is the expected depreciation of the home currency from \( t \) to \( t + 1 \). Note that an important consequence of assuming non-separability of the utility function is that real money balances enter the IS equation (25), the AS equation (26) and the terms of trade condition (29).
The linearized equilibrium system is given by equations (25)-(29), their foreign equivalents and equations (30)-(32), along with linearized versions of the interest rate rule (20):

\[ \hat{R}_t = \mu_\pi E_t \hat{\pi}^H_{t+1} + \mu_y E_t \hat{Y}_{t+k}; \quad \text{or} \quad \hat{R}_t = \mu_\pi E_t \hat{\pi}_{t+1} + \mu_y E_t \hat{Y}_{t+k}; \] (33)

where \( \mu_\pi > 0 \) is the inflation response coefficient, \( \mu_y \geq 0 \) is the output response coefficient and \( k = 0, 1 \).

It is important to stress that in the above system there are three channels of monetary policy. Using (27) and (28) to eliminate \( \hat{m}_c_t \) and \( \hat{Y}_t \) from (26) and combining this AS equation with its foreign equivalent and the terms of trade condition (31) generates the following expression for the domestic-price inflation differential (in deviations from the steady state):

\[ \hat{\pi}^H_t - \hat{\pi}^F_t = \beta E_t \left( \hat{\pi}^H_{t+1} - \hat{\pi}^F_{t+1} \right) + [\kappa_C + \kappa_T \zeta_C] \left( \hat{C}_t - \hat{C}_t^* \right) - [\kappa_\mu + \kappa_T \zeta_\mu] (\hat{m}_t - \hat{m}_t^*) \] (34)

where \( \kappa_T \equiv 2\lambda(1-a)[1 + 2a\omega\theta] > 0 \), \( \kappa_C \equiv \lambda[\omega(2a - 1) + 1/\sigma] > 0 \), \( \zeta_C \equiv [\sigma(2a - 1)]^{-1} > 0 \), \( \kappa_\mu \equiv \lambda \chi > 0 \) and \( \zeta_\mu \equiv \lambda \chi/(2a - 1) > 0 \). There is the conventional aggregate demand channel, where a relative increase in the home country’s interest rate lowers home consumption and reduces the domestic-price inflation differential, the sensitivity of which depends on the coefficient \( \kappa_C \). A second channel of monetary policy is the role that the demand for money plays in affecting the cost of production of intermediate-sector firms. With real balance effects the differential demand for money enters into (34) as a negative cost-push shock. Here an increase in the relative interest rate generates a relative reduction in the demand for domestic money, which results in an increase in real marginal cost and, given the coefficient \( \kappa_\mu \), an increase in the domestic-price inflation differential. Finally, there is a terms of trade channel, where a relative increase in the interest rate leads to an improvement in the terms of trade which has two separate effects on the domestic-price inflation differential. On the one hand an improvement in the terms of trade, via an expenditure switching effect to foreign intermediate-sector goods, reduces the domestic-price inflation differential to an extent determined by \( \kappa_T \zeta_C \), whereas on the other hand it is increased (because of real balance effects), through relative changes in real marginal cost depending on \( \kappa_T \zeta_\mu \).14

\[ \text{14Clearly in a closed economy both these terms of trade effects are absent since } \kappa_T = 0 \text{ as } a = 1. \]
Table 1: Linearized system of equations

\begin{align*}
\textbf{Aggregate System} \\
E_t \hat{C}_{t+1}^W &= \hat{C}_t^W + \sigma \hat{R}_t^W - \sigma E_t \hat{\pi}_{t+1}^W + \sigma \chi E_t \hat{m}_t^W - \sigma \chi \hat{m}_t^W & \text{IS}^W \\
\hat{m}_t^W &= \eta \hat{C}_t - \eta R_t^W & \text{LM}^W \\
\hat{\pi}_t^W &= \beta E_t \hat{\pi}_{t+1}^W + \lambda \omega \hat{Y}_t + \hat{\sigma} \hat{C}_t - \lambda \chi \hat{m}_t^W & \text{AS}^W \\
\hat{Y}_t^W &= \hat{C}_t^W & \text{Output}^W \\
\hat{R}_t^W &= \mu \pi E_t \hat{\pi}_{t+1}^W + \mu \eta E_t \hat{Y}_t^W & \text{Taylor rule}^W \\
\end{align*}

\begin{align*}
\textbf{Difference System} \\
E_t \hat{C}_{t+1}^R &= \hat{C}_t^R + \sigma \hat{R}_t^R - \sigma E_t \hat{\pi}_{t+1}^R + \sigma \chi E_t \hat{m}_t^R - \sigma \chi \hat{m}_t^R & \text{IS}^R \\
\hat{m}_t^R &= \eta \hat{C}_t - \eta R_t^R & \text{LM}^R \\
\hat{\pi}_t^R &= \beta E_t \hat{\pi}_{t+1}^R + 2 \lambda (1-a) \hat{T}_t + \frac{2 \hat{\sigma}}{\sigma} \hat{C}_t^R - \lambda \chi \hat{m}_t^R + \lambda \omega \hat{Y}_t^R & \text{AS}^R \\
\hat{Y}_t^R &= 4 \alpha \theta (1-a) \hat{T}_t + (2a-1) \hat{C}_t^R & \text{Output}^R \\
\hat{R}_t^R &= \Delta E_t \hat{S}_{t+1}^R & \text{UIP} \\
(2a-1) \hat{T}_t &= \frac{1}{2} \hat{C}_t^R - \chi \hat{m}_t^R & \text{ToT} \\
\hat{\pi}_t^R &= (2a-1) \hat{\pi}_{t+1}^R + 2(1-a) \Delta \hat{S}_t & \text{Inflation}^R \\
\hat{R}_t^R &= \mu \pi E_t \hat{\pi}_{t+1}^R + \mu \eta E_t \hat{Y}_t^R & \text{Taylor rule}^R: \text{PPI} \\
\hat{R}_t^R &= \mu \pi E_t \hat{\pi}_{t+1}^R + \mu \eta E_t \hat{Y}_t^R & \text{Taylor rule}^R: \text{CPI} \\
\end{align*}

Notes: The index \( W \) refers to world aggregates where \( \pi^W = \frac{\pi + \pi^*}{2} = \frac{\pi^H + \pi^*}{2} \).

Since we are interested in obtaining analytical conditions for determinacy under both the closed and open economy dimensions of the model, it will be convenient to use the method of Aoki (1981) to split the linearized equilibrium system into two decoupled dynamic systems: the aggregate system that captures the properties of the closed world economy and the difference system that portrays the open-economy dimension. This decomposition of the linearized model into worldwide aggregates \( X^W \equiv \frac{\hat{X}}{2} + \frac{\hat{X}^*}{2} \) and cross-country differences \( X^R \equiv \hat{X} - \hat{X}^* \) is summarized in Table 1. For the closed economy version of the model, its determinacy properties are fully characterized by the aggregate system.\(^\text{15}\) However, for the equilibrium to be determinate in the open-economy it must be the case that there is a

\(^{15}\)Note that the measure of inflation targeted in the interest-rate feedback rule is irrelevant in the aggregate system, since in a closed economy domestic and consumer-price inflation are identical concepts. i.e. \( \pi^W = \frac{\pi + \pi^*}{2} = \frac{\pi^H + \pi^*}{2} \).
Table 2: Benchmark parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Intertemporal elasticity of substitution in consumption</td>
<td>6.4</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Output elasticity of (real) marginal cost</td>
<td>0.47</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Degree of price stickiness</td>
<td>0.75</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Real marginal cost elasticity of inflation</td>
<td>0.08</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Degree of non-separability of utility function</td>
<td>$0 \leq \chi \leq 0.03$</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>Output elasticity of money demand</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_R$</td>
<td>Interest-rate semi-elasticity of money demand</td>
<td>28</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution between <em>home</em> and <em>foreign</em> goods</td>
<td>1</td>
</tr>
<tr>
<td>$1 - a$</td>
<td>Degree of trade openness</td>
<td>0.15, 0.3, or 0.4</td>
</tr>
</tbody>
</table>

unique solution for both cross-country differences and world aggregates.\(^{16}\)

### 3.2 Parameterization

It will be useful to illustrate our results using the benchmark values for the parameters specified in Table 2. Parameter $\beta$ is standard in the literature and $\omega$ is taken from Woodford (2003). Following Woodford (2003) and Kurozumi (2006) we set the output elasticity of money demand $\eta_c = 1$, the interest-rate semi-elasticity of money demand $\eta_R = 28$ and the intertemporal elasticity of substitution in consumption $\sigma = 6.4$. The latter is consistent with Rotemberg and Woodford (1997) estimate of $\sigma = 6.37$ for the US economy and implies a value of the risk aversion coefficient of $1/\sigma \approx 0.16$.\(^{17}\) Consistent with recent empirical estimates by Woodford (2003), Ireland (2004) and Andrés *et al.* (2006) we only consider values for the degree of non-separability $\chi = [0, 0.03]$.\(^{18}\) As noted by Benhabib and Eusepi (2005), empirical estimates of nominal rigidity find $\psi$ to be between 0.66 and 0.83. Following Taylor (1999) we set $\psi = 0.75$ which constitutes an average price duration of one year and implies that the real marginal cost elasticity of inflation $\lambda = 0.08$. However, the robustness of the numerical results is examined for variations in $\psi$. Consistent with Bergin (2006), we set $\theta = 1$. Finally for illustrative purposes, three alternative values for the degree of trade openness $a = 0.15, 0.3, or 0.4$.

\(^{16}\)Determinacy of the aggregate and difference systems implies determinacy at the individual country level since $\hat{X} = X^W + \frac{X^R}{2}$ and $\hat{X}^* = X^W - \frac{X^R}{2}$.

\(^{17}\)Woodford (2003) argues that a low risk aversion coefficient is justified on the grounds that the intertemporal substitution elasticity of consumption is significantly higher once investment in capital and consumer durables are considered. The sensitivity analysis suggests that while lower values of $\sigma$ generate different quantitative results, the qualitative conclusions are the same.

\(^{18}\)These values for $\chi$ satisfy Assumption 1.
openness are also chosen, which are roughly consistent with the ratio of imports to GDP of the USA \((a = 0.85)\), UK \((a = 0.7)\) and Canada \((a = 0.6)\).

4 Determinacy Analysis for Closed Economies

This section considers the issue of local determinacy of the rational expectations equilibrium for the closed economy.

4.1 Policy Response to Future Expected Inflation

We first consider the determinacy implications for an interest-rate feedback rule that responds only to expected future inflation (i.e. \(\mu_y = 0\) in (33)).

**Proposition 1** If the policy rule reacts only to expected future inflation, then given Assumption 1, the necessary and sufficient conditions for local equilibrium determinacy in a closed economy are:

\[
1 < \mu_\pi < \min\{\Gamma_1^1, \Gamma_1^3\} \quad \text{or} \quad \max\{1, \Gamma_1^1\} < \mu_\pi < \min\{\Gamma_2^1, \Gamma_3^1\}
\]

(35)

where

\[
\Gamma_1^1 = \frac{\beta \Omega}{\eta R \lambda \omega \chi}; \quad \Gamma_1^2 = \frac{\Omega (1 + \beta)}{\eta R \lambda \omega \chi}; \quad \Gamma_3^1 = \frac{2 \Omega (1 + \beta) + \lambda (\Omega + \sigma \omega)}{\lambda (\Omega + \sigma \omega (1 + 2 \eta R \chi))}
\]

**Proof.** See Appendix A.

Note that with separability of the utility function \((\chi = 0)\) the bounds \(\Gamma_1^1\) and \(\Gamma_2^1\) no longer apply while \(\Gamma_3^1\) reduces to \(\Gamma_3^1_{\chi = 0} = 1 + \frac{2(1 + \beta)}{\lambda (1 + \sigma \omega)}\). Hence the determinacy conditions summarized in Proposition 1 collapse to:

\[
1 < \mu_\pi < 1 + \frac{2(1 + \beta)}{\lambda (1 + \sigma \omega)}.
\]

(36)

It is clear from (35) and (36) that while the Taylor principle \((\mu_\pi > 1)\) is a necessary condition for equilibrium determinacy, it is not sufficient. The numerical analysis suggests that, with very small values for \(\chi\), real balance effects play a significant role in reducing the upper bound on the inflation response coefficient \((\mu_\pi)\). Using the baseline parameter values
Figure 1: Regions of indeterminacy under a forward-looking inflation feedback rule

summarized in Table 2, Figure 1 illustrates the regions of determinacy and indeterminacy for combinations of $\mu_\pi$ and the degree of price rigidity ($\psi$) for alternative values of $\chi = 0, 0.1, 0.2, 0.3$. The top, left-hand corner of Fig. 1 shows the determinacy properties of the model under separability of the utility function. In this case, the Taylor principle easily induces determinacy of equilibrium. While indeterminacy can emerge under our baseline parameter values, the upper bound on $\mu_\pi$ needed is of a size to be unlikely to bind. The other three panels of Fig. 1 show the indeterminacy implications of real balance effects as the extent of non-separability of the utility function ($\chi$) is increased. It is evident from Fig. 1 that real balance effects exert a destabilizing effect on the rational expectations equilibrium. By inspection, the upper bound on $\mu_\pi$ is tighter, the higher the degree of non-separability, and the lower the degree of price stickiness. Consequently, the Taylor principle is now significantly weakened in its effectiveness in preventing indeterminacy. For example, if $\psi = 0.75$, the upper bound on the inflation response coefficient is $\mu_\pi < 12.57$ when $\chi = 0$ and only $\mu_\pi < 4.65$ when $\chi = 0.03$. This finding is in stark contrast to the existing literature on real balance effects (e.g. Kurozumi, 2006), where non-separability has no implications for determinacy when the Taylor-type feedback rule responds only to
current-looking inflation.

4.2 Policy Response to Output

We now consider the determinacy implications for the closed-economy dimension of the model if output is also included in the interest-rate feedback rule. Proposition 2 derives the determinacy conditions when the feedback rule responds to expected future output (i.e. \( k = 1 \) in (33)) and Proposition 3 when contemporaneous output enters the interest rate rule (i.e. \( k = 0 \) in (33)).

4.2.1 Policy Response to Expected Future Inflation and Expected Future Output

**Proposition 2** If the policy rule reacts to expected future inflation and expected future output, then given Assumption 1, the necessary and sufficient conditions for local equilibrium determinacy in a closed economy are:

\[
\max\{0, \Gamma_1^2\} < \mu_y < \min\{\Gamma_2^2, \Gamma_3^2\}
\]

(37)

where

\[
\Gamma_1^2 \equiv 1 - \frac{\sigma \mu_y (1 - \beta - \lambda \eta_R \chi)}{\lambda (\Omega + \sigma \omega)};
\]

\[
\Gamma_2^2 \equiv \frac{2 \Omega (1 + \beta)}{\lambda [\Omega + \sigma \omega(1 + 2 \eta_R \chi)]} + \frac{(\Omega + \sigma \omega)}{[\Omega + \sigma \omega(1 + 2 \eta_R \chi)]} - \frac{\sigma \mu_y [(1 + \beta)(1 + 2 \eta_R \chi) + \lambda \eta_R \chi]}{\lambda [\Omega + \sigma \omega(1 + 2 \eta_R \chi)]};
\]

\[
\Gamma_3^2 \equiv \beta \lambda \eta_R \chi \sigma \mu_y \left[ \frac{\Omega (1 + \frac{\sigma \omega}{\beta \mu_y}) + \sigma \omega(1 + \eta_R \chi)}{\Omega (1 + \frac{\sigma \omega}{\beta \mu_y}) + \sigma \omega(1 + \eta_R \chi)} \right] - \frac{\sigma \omega}{\Omega (1 + \frac{\sigma \omega}{\beta \mu_y}) + \sigma \omega(1 + \eta_R \chi)} - \frac{\mu_y \sigma [1 + \eta_R \chi (1 + \beta)]}{\lambda [\Omega (1 + \frac{\sigma \omega}{\beta \mu_y}) + \sigma \omega(1 + \eta_R \chi)]}.
\]

**Proof.** See Appendix B.
Figure 2: Regions of indeterminacy under a forward-looking inflation and forward-looking output feedback rule

Note that with $\chi = 0$, the bound $\Gamma_2^2$ no longer applies, and the determinacy conditions summarized in Proposition 2 collapse to:

$$\max \left\{ 0, 1 - \frac{\mu_y \sigma (1 - \beta)}{\lambda (1 + \sigma \omega)} \right\} < \mu_\pi < 1 + \frac{2(1 + \beta)}{\lambda (1 + \sigma \omega)} - \frac{\mu_y \sigma (1 + \beta)}{\lambda (1 + \sigma \omega)}.$$  \hspace{1cm} (38)

A policy rule that also reacts to output greatly increases the range of indeterminacy under the Taylor principle. Now the lower and upper bounds on the inflation response coefficient ($\mu_\pi$) given by (37) and (38) are a function of the policy response to future output ($\mu_y$). Using the baseline parameter values summarized in Table 2, Figure 2 illustrates the regions of determinacy and indeterminacy for combinations of $\mu_\pi$ and $\mu_y$ for alternative values of $\chi = 0.1, 0.2, 0.3$. The top, left-hand corner of Fig. 2 shows the determinacy properties of the model under separability of the utility function, and the other three panels show the implications for determinacy as the degree of non-separability of the utility function ($\chi$) is increased. By inspection of Fig. 2, indeterminacy is generated if the monetary authority is overly aggressive in its setting of either one of these policy coefficients. Indeed for high enough values of the output response coefficient equilibrium determinacy is impossible re-
gardless of the value of $\mu_\pi$. As Fig. 2 shows, not only is the upper bound on $\mu_\pi$ decreasing with respect to both $\chi$ and $\mu_y$, but in the presence of real balance effects it is of such a small magnitude to render the equilibrium indeterminant. For example, with $\chi = 0.03$ determinacy is impossible for all $\mu_\pi$ if $\mu_y \geq 0.08$.

### 4.2.2 Policy Response to Expected Future Inflation and Contemporaneous Output

We next examine the policy response to contemporaneous output. This is a particularly important specification of the interest-rate feedback rule, as there is evidence to suggest that it represents the actual monetary behavior of central banks, where contemporaneous output is typically used to help forecast expected future inflation (e.g. Clarida et al., 1998, 2000; Orphanides, 2004; Mihailov, 2006).

**Proposition 3** If the policy rule reacts to expected future inflation and contemporaneous output, then given Assumption 1, the necessary and sufficient conditions for local equilibrium determinacy in a closed economy are:

$$\max\{0, \Gamma_3^1\} < \mu_\pi < \min\{\Gamma_3^2, \Gamma_3^3\} \quad \text{or} \quad \max\{0, \Gamma_3^4, \Gamma_3^1\} < \mu_\pi < \min\{\Gamma_3^2, \Gamma_3^4\}$$

(39)

where

$$\Gamma_3^1 = \frac{\beta \Omega}{\lambda \sigma \omega \eta R \chi} + \frac{\beta \mu_y}{\lambda \omega}, \quad \Gamma_3^2 = \frac{(1 + \beta) \Omega}{\lambda \sigma \omega \eta R \chi} - \frac{\sigma \mu_y [1 + \eta R \chi (1 + \beta)]}{\lambda \sigma \omega \eta R \chi}, \quad \Gamma_3^3 = 1 - \frac{\sigma \mu_y (1 - \beta - \lambda \eta R \chi)}{\lambda (1 + \sigma \omega)};$$

$$\Gamma_3^3 = \frac{2 \Omega (1 + \beta)}{\lambda [\Omega + \sigma \omega (1 + 2 \eta R \chi)]} + \frac{(\Omega + \sigma \omega)}{[\Omega + \sigma \omega (1 + 2 \eta R \chi)]} + \frac{\sigma \mu_y [(1 + \beta) (1 + 2 \eta R \chi) + \lambda \eta R \chi]}{\lambda [\Omega + \sigma \omega (1 + 2 \eta R \chi)]}.$$

**Proof.** See Appendix C.

Note that with separability of the utility function ($\chi = 0$), the bounds $\Gamma_3^1$ and $\Gamma_3^2$ no longer apply, and the determinacy conditions summarized in Proposition 3 collapse to:

$$\max\left\{0, 1 - \frac{\mu_y \sigma (1 - \beta)}{\lambda (1 + \sigma \omega)}\right\} < \mu_\pi < 1 + \frac{2(1 + \beta)}{\lambda (1 + \sigma \omega)} + \frac{\mu_y \sigma (1 + \beta)}{\lambda (1 + \sigma \omega)}.$$  

(40)

---

19 The sensitivity analysis suggests that variations in the degree of price stickiness have little significant impact on the values of $\mu_\pi$ needed to rule out determinacy.
Figure 3: Regions of indeterminacy under a forward-looking inflation and contemporaneous output feedback rule ($\psi = 0.75$)

Figure 4: Regions of indeterminacy under a forward-looking inflation and contemporaneous output feedback rule ($\psi = 0.66$)
As before, the lower and upper bounds on the inflation response coefficient ($\mu_\pi$) given in (39) and (40) are a function of the output response coefficient ($\mu_y$). However, under a policy of reacting to contemporaneous output these bounds are now less likely to bind, and consequently the range of determinacy is greater relative to forward-looking output. For the baseline parameter values, Figure 3 illustrates the regions of determinacy and indeterminacy for combinations of $\mu_\pi$ and $\mu_y$ when $\psi = 0.75$, whereas Figure 4 considers the indeterminacy implications when a lower value for the degree of price stickiness is chosen, $\psi = 0.66$.

The top, left-hand corners of Figs. 3 and 4 show the determinacy properties of the model under separability of the utility function. When $\chi = 0$, the Taylor principle easily induces determinacy of equilibrium. However, by responding to current output, the lower bound on $\mu_\pi$ is reduced below unity and consequently determinacy is also attainable under a passive monetary policy ($\mu_\pi < 1$). The other three panels of Figs. 3 and 4 show the indeterminacy implications of real balance effects as the extent of non-separability of the utility function is increased. The profound impact real balance effects have on the determinacy implications of the Taylor principle is evident: as $\chi$ increases, the lower bound on $\mu_\pi$ given in (39) pivots clockwise around the $\mu_\pi = 1$ point, whereas the upper bound on $\mu_\pi$ given in (39) shifts anti-clockwise. The combined effect is a significant reduction in the regions of determinacy.

In addition, the analysis suggests that the lower and upper bounds on $\mu_\pi$ are also sensitive to the degree of price stickiness. Consequently for a given value of $\chi$, the more flexible are prices, the lower are the regions of determinacy.

Overall, we can conclude that for closed economies, Taylor-type feedback rules that respond to expected future inflation are significantly more likely to induce indeterminacy of equilibrium in the presence of small, empirically plausible real balance effects. These conclusions are in stark contrast to feedback rules that react to contemporaneous inflation. For example, Kurozumi (2006) finds that for such policy rule specifications, real balance effects only have a destabilizing effect on the rational expectations equilibrium when current output also enters into the feedback rule. Perhaps, more importantly, our analysis also raises concerns relating to the observed monetary policy conduct of central banks: setting the nominal interest rate in response to movements in contemporaneous output and expected future inflation. By ignoring the role the demand for money plays in the monetary transmission mechanism, the previous literature has suggested that such a feedback rule is desirable...
in preventing indeterminacy. In stark contrast, we have shown that the implementation of such a Taylor-type rule can actually be destabilizing in the presence of real balance effects.

5 Determinacy Analysis for Open Economies

This section considers the issue of local determinacy of the rational expectations equilibrium for the open economy. We wish to answer the following question: Do the determinacy conditions for open economies differ significantly from the conditions for closed economies in the presence of real balance effects? We restrict our focus to feedback rules that react to expected future (domestic or consumer-price) inflation and current output \((k = 0\) in (33)), since this has most empirical relevance.

5.1 Reacting to Domestic-Price Inflation

Let us first consider the determinacy conditions for the open economy when the feedback rule reacts to expected future domestic-price inflation.

**Proposition 4** If the policy rule reacts to expected future domestic-price inflation and contemporaneous output, then given Assumption 1, the necessary and sufficient conditions for local equilibrium determinacy in an open economy are:

\[
\max\{0, \Gamma_3^1, \Gamma_4^4\} < \mu_\pi < \min\{\Gamma_3^3, \Gamma_4^3, \Gamma_4^4\} \quad \text{or} \quad \max\{0, \Gamma_3^3, \Gamma_4^4\} < \mu_\pi < \min\{\Gamma_3^3, \Gamma_4^3, \Gamma_4^4\}
\]

(41)

where \(\Gamma_3^3, \Gamma_4^3, \Gamma_4^4\) are defined in Proposition 3 and:

\[
\Gamma_2^4 \equiv \frac{\Omega(1 + \beta)}{\lambda \sigma \eta_\chi (2a - 1)} + \mu_\pi \frac{\sigma (2a - 1)^2 + \Omega 4a\theta(1 - a) + (2a - 1)\sigma \eta_\chi (1 + \beta)}{(2a - 1)\lambda \sigma \eta_\chi}.
\]

\[
\Gamma_3^4 \equiv 1 - \mu_\pi \frac{[1 - \beta] \sigma (2a - 1)^2 + \Omega 4a\theta(1 - a) - \lambda \sigma \eta_\chi (2a - 1)]}{\lambda \sigma (2a - 1)^2 + \Omega [1 + 4a\theta \omega (1 - a)]}.
\]

\[
\Gamma_4^4 \equiv \frac{2\Omega(1 + \beta) + \lambda \sigma \omega (2a - 1)^2 + \lambda \Omega [1 + 4a\theta \omega (1 - a)]}{\lambda \Omega [1 + 4a\theta \omega (1 - a)] + \lambda \sigma \omega (2a - 1)[2a - 1 + 2\eta_\chi]}
\]

\[
+ \mu_\pi \frac{[1 + \beta] \sigma (2a - 1)^2 + \Omega 4a\theta(1 - a) + \sigma (2a - 1)^2 + \sigma \eta_\chi (2a - 1)[\lambda + 2(1 + \beta)]}{\lambda \Omega [1 + 4a\theta \omega (1 - a)] + \lambda \sigma \omega (2a - 1)[2a - 1 + 2\eta_\chi]}.
\]

**Proof.** See Appendix D.
Table 3: Reacting to expected domestic-price inflation: lower bounds on the inflation response coefficient ($\mu_{\pi}$) for determinacy when $\mu_y = 4$

<table>
<thead>
<tr>
<th>$\chi$ = 0</th>
<th>$\chi$ = 0.01</th>
<th>$\chi$ = 0.02</th>
<th>$\chi$ = 0.03</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed economy:</td>
<td>$\mu_{\pi} &gt; 2.6$</td>
<td>$\mu_{\pi} &gt; 2.06$</td>
<td>$\mu_{\pi} &gt; 3.93$</td>
</tr>
<tr>
<td>Open economy:</td>
<td>$\alpha = 0.85$</td>
<td>$\mu_{\pi} &gt; 0.37$</td>
<td>$\mu_{\pi} &gt; 2.27$</td>
</tr>
<tr>
<td>$\alpha = 0.70$</td>
<td>$\mu_{\pi} &gt; 0.54$</td>
<td>$\mu_{\pi} &gt; 2.13$</td>
<td>$\mu_{\pi} &gt; 3.93$</td>
</tr>
<tr>
<td>$\alpha = 0.60$</td>
<td>$\mu_{\pi} &gt; 0.64$</td>
<td>$\mu_{\pi} &gt; 2.06$</td>
<td>$\mu_{\pi} &gt; 3.93$</td>
</tr>
</tbody>
</table>

Under domestic-price inflation targeting, whether indeterminacy is a relatively more serious problem in open economies than closed economies depends on the magnitude of the lower and upper bounds on the inflation response coefficient ($\mu_{\pi}$) given in (41). For the baseline parameterization, the numerical analysis suggests that $\Gamma_3^3 < \Gamma_4^3$ and $\Gamma_3^4 < \Gamma_4^4$, implying that there is no change in relation to the upper bounds on $\mu_{\pi}$. Therefore indeterminacy can only be greater in the open economy if the additional lower bound $\Gamma_4^3 > \Gamma_3^3$. As an example, Table 3 summarizes the lower bound computations on the inflation response coefficient if $\mu_y = 4$ for variations in the degree of trade openness ($1 - \alpha$) and the extent of non-separability of the utility function ($\chi$). It is evident from Table 3 that if the feedback rule reacts to domestic-price inflation, there are no serious additional consequences for indeterminacy. However, while the indeterminacy implications of opening up the economy are small, there are some interesting differences between the separable vs. non-separability cases. When $\chi = 0$, increases in the degree of trade openness increase the lower bound on $\mu_{\pi}$, thereby increasing the range of indeterminacy. This corresponds with the standard conclusion in the open-economy literature: as the degree of openness to international trade increases, the range of indeterminacy is typically increased (e.g. De Fiore and Liu, 2005, and Llosa and Tuesta, 2008). However, in the presence of real balance effects, the lower bound on $\mu_{\pi}$ is now decreasing with respect to the degree of trade openness. Consequently, in this case indeterminacy can actually be reduced as the degree of trade openness increases. This follows from inspection of equation (34). In the open economy, the degree of trade openness affects the weight of influence exerted on the domestic-price inflation differential by both the aggregate demand channel and the cost channel of monetary policy. Our results suggest that a higher degree of trade openness (i.e. ↓ $\alpha$) typically reduces the relative strength of
this cost channel.

5.2 Reacting to Consumer-Price Inflation

We now consider the determinacy implications when the feedback rule reacts to expected future consumer-price inflation. There is evidence to suggest that this is the actual indicator of inflation used by central banks in the setting of monetary policy (see, e.g. De Fiore and Liu, 2005).

Proposition 5 If the policy rule reacts to expected future consumer price inflation and contemporaneous output, then given Assumption 1, the necessary and sufficient conditions for local equilibrium determinacy in an open economy are:

\[
\max\{0, \Gamma_3^1, \Gamma_3^2, \Gamma_3^3, \Gamma_3^4\} < \mu_x < \min\{\Gamma_3^2, \Gamma_3^3, \Gamma_3^4\} \quad \text{or}
\]

\[
\max\{0, \Gamma_3^3, \Gamma_3^4\} < \mu_x < \min\{\Gamma_3^3, \Gamma_3^4\}
\]

(42)

where \(\Gamma_3^1, \Gamma_3^2, \Gamma_3^3, \Gamma_3^4\) are defined in Proposition 3 and:

\[
\Gamma_3^5 = \frac{\beta \Omega + \beta \mu_y \sigma \eta R \chi (2a - 1)}{2 \beta \Omega (1 - a) + \lambda \sigma \omega \eta R \chi (2a - 1)^2};
\]

\[
\Gamma_3^2 = \frac{\Omega (1 + \beta) + \mu_y \sigma \eta R \chi (2a - 1)(1 + \beta) + \mu_y \left[\sigma (2a - 1)^2 + 4a \theta \Omega (1 - a)\right]}{2 \Omega (1 - a)(1 + \beta) + \lambda \sigma \omega \eta R \chi (2a - 1)^2};
\]

\[
\Gamma_3^3 = 1 - \frac{\mu_y \left[(1 - \beta) \left[\sigma (2a - 1)^2 + \Omega 4a \theta (1 - a)\right] - \lambda \sigma \omega \eta R \chi (2a - 1)\right]}{\lambda \sigma \omega (2a - 1)^2 + \Omega \lambda [1 + 4a \theta \omega (1 - a)]};
\]

\[
\Gamma_3^4 = \frac{2 \Omega (1 + \beta) + \lambda \sigma \omega (2a - 1)^2 + \lambda \Omega [1 + 4a \theta \omega (1 - a)]}{4 \Omega (1 - a)(1 + \beta) + \lambda \Omega [1 + 4a \theta \omega (1 - a)] + \lambda \sigma \omega (2a - 1)^2 (1 + 2 \eta R \chi)}
\]

+ \frac{\mu_y \left[(1 + \beta) \left[\Omega 4a \theta (1 - a) + \sigma (2a - 1)^2\right] + \lambda \sigma \omega \eta R \chi (2a - 1)\left[\lambda + 2 (1 + \beta)\right]\right]}{4 \Omega (1 - a)(1 + \beta) + \lambda \Omega [1 + 4a \theta \omega (1 - a)] + \lambda \sigma \omega (2a - 1)^2 (1 + 2 \eta R \chi)}.

Proof. See Appendix E.

In order to gain some further insight, we illustrate condition (42) using the baseline parameter values summarized in Table 2. Figure 5 shows the regions of determinacy and indeterminacy for combinations of \(\mu_x\) and \(\mu_y\) for alternative values of \(\chi = 0.1, 0.2, 0.3\) when \(a = 0.85\), whereas Figure 6 considers the indeterminacy implications under a higher degree of trade openness, \(a = 0.60\). The top, left-hand corner of Figs. 5 and 6 show the
Figure 5: Regions of indeterminacy under a forward-looking consumer-price inflation and contemporaneous output feedback rule with low trade openness ($\alpha = 0.85$)

Figure 6: Regions of indeterminacy under a forward-looking consumer-price inflation and contemporaneous output feedback rule with high trade openness ($\alpha = 0.60$)
determinacy properties of the model under separability of the utility function. When \( \chi = 0 \), and in stark contrast to the closed economy, equilibrium determinacy is no longer guaranteed under the Taylor principle. Now the upper bound on \( \mu_\pi \) decreases as both the output response coefficient and the degree of trade openness are increased. The other three panels of Figs. 5 and 6 show the indeterminacy implications as the degree of non-separability is increased. By inspection, as \( \chi \) increases, the lower bound on the inflation response coefficient pivots clockwise around the \( \mu_\pi = 1 \) point, whereas the upper bound pivots in an anti-clockwise direction. Compared with the closed economy (see Fig. 3 of Section 4.2), the range of indeterminacy is clearly higher for open economies and the range of indeterminacy increases significantly as the degree of trade openness and non-separability of the utility function are both increased.\(^{20}\) Indeed, by inspection of the bottom, right-hand corner of Fig. 6, indeterminacy is very likely to emerge under the Taylor principle with \( a = 0.60 \) and \( \chi = 0.03 \).\(^{21}\)

To get some intuition behind this result, first note that in an open economy the consumer-price inflation rate differential depends on both the rate of domestic-price inflation differential and changes in the terms of trade:

\[
\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^* = \hat{\pi}_{t+1}^h - \hat{\pi}_{t+1}^f + 2(1 - a) \left( \tilde{T}_{t+1} - \tilde{T}_t \right).
\]

Under the Taylor principle, even when an increase in the nominal interest rate of the home country results in a relative reduction in \( \hat{\pi}_{t+1}^h - \hat{\pi}_{t+1}^f \), since this also results in a current improvement in the terms of trade \( (\tilde{T}_t \downarrow) \), indeterminacy is still possible provided the upward pressure on the consumer-price inflation differential generated by the adjustments in the terms of trade is sufficiently strong. As the degree of trade openness determines the weight of influence of the terms of trade on the consumer-price differential, the higher the degree of trade openness \( (\downarrow a) \), the more likely the consumer-price differential will increase despite a fall in the domestic-price inflation differential. In addition, our results suggest that the

---

\(^{20}\)The numerical analysis suggests that, for the baseline parameter values, the (empirically relevant) lower bound is higher in the open economy \( (\Gamma^3_3 < \Gamma^3_4) \) and the (empirically relevant) upper bound is lower \( (\Gamma^3_4 < \Gamma^3_3) \) when \( a = 0.85 \). For a higher degree of trade openness \( a = 0.60 \), the numerical analysis suggests that the lower bound on \( \mu_\pi \) is the same as in the closed economy \( (\Gamma^3_3 < \Gamma^3_4) \) but the upper bound is now significantly lower \( (\Gamma^3_4 < \Gamma^3_3) \).

\(^{21}\)The sensitivity analysis further suggests that the determinacy regions shrink even more as the degree of price stickiness is reduced.
greater the degree of non-separability ($\uparrow \chi$), the larger the improvement on the terms of trade ($\hat{T}_t \downarrow$), thereby implying self-fulfilling inflation expectations are now more likely.

Overall, we can conclude that under a feedback rule that reacts to expected inflation and current output, the Taylor principle can no longer ensure equilibrium determinacy in the presence of real balance effects. However, for open economies that use the consumer-price index as the indicator of inflation in the feedback rule, the indeterminacy implications of such a policy are even more severe. This analysis supports the majority of the existing literature that suggests that central banks should target domestic-price inflation in the feedback rule, rather than consumer-price inflation, since the later is more likely to destabilize the economy by generating self-fulfilling expectations (see e.g. Linnemann and Schabert, 2006; Llosa and Tuesta, 2008; Leith and Wren-Lewis, 2009).

5.2.1 Responding to the Real Exchange Rate

Can the indeterminacy problem, observed when the feedback rule reacts to expected future consumer-price inflation, be mitigated by incorporating the real exchange rate into the interest rate rule? There has been recent debates in the literature on whether central banks in open economies need to additionally respond to the real exchange rate (e.g. Taylor, 2001; Kirsanova et al., 2006; Bergin et al., 2007; Benigno and Benigno, 2008). Lubik and Schorfheide (2007) report evidence that both the Bank of Canada and the Bank of England include reacting to changes in the nominal (and hence) real exchange rate in empirically estimated feedback rules. For studies that ignore the demand for money, Linnemann and Schabert (2006) and McKnight (2011a) have shown that indeterminacy is significantly reduced if the central bank places a small weight on the real exchange rate coefficient. We now turn to the effectiveness of the real exchange rate in reducing indeterminacy in the presence of real balance effects.

Consider the following log-linearized feedback rule:

$$
\hat{R}_t = \mu_\pi \hat{\pi}_{t+1} + \mu_y \hat{Y}_t + \mu_q \hat{Q}_t,
$$

(43)

where $\mu_q \geq 0$.\textsuperscript{22} Rather than derive the analytical conditions for determinacy, we will simply

\textsuperscript{22}For the foreign country’s rule, the response coefficient of the real exchange rate is the negative of that for the home country.
report some numerical results for the case of (43). Figure 7 plots the regions of determinacy and indeterminacy for \(a = 0.6\) and \(\chi = 0.03\) for four values of the real exchange rate coefficient \(\mu_q = 0, 0.3, 0.6, 1.0\). The top, left-hand corner of Fig. 7 shows the determinacy properties of the model when \(\mu_q = 0\). Fig. 7 illustrates that reacting positively to the real exchange rate has a very marginal impact on the regions of (in)determinacy.\(^{23}\) The numerical analysis suggests that there is only a minimal increase in the upper bound on \(\mu_\pi\) for determinacy (and no effect on the lower bound). Even when the interest rate reacts one for one to real exchange rate movements (\(\mu_q = 1\)), the large regions of indeterminacy robustly remain in the presence of real balance effects.

6 Conclusion

In the conduct of monetary policy, it is becoming increasingly popular for central banks to set the nominal interest rate in response to expected future inflation. In addition, there is empirical evidence to suggest that contemporaneous output is also included in the monetary

\(^{23}\)The sensitivity analysis indicates that this conclusion is robust to variations in \(\chi\) or \(a\).
Table 4: Indeterminacy under the Taylor principle: summary of our key results

<table>
<thead>
<tr>
<th></th>
<th>$\hat{R}<em>t = \mu</em>\pi \hat{\pi}_{t+1}$</th>
<th>$+\mu_y \hat{Y}_t$</th>
<th>$+\mu_y \hat{Y}_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi = 0$</td>
<td>$\chi = 0$</td>
<td>$\chi = 0$</td>
<td>$\chi = 0$</td>
</tr>
<tr>
<td>Closed economy:</td>
<td>unlikely</td>
<td>likely</td>
<td>never</td>
</tr>
<tr>
<td>Open economy:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DPI measure</td>
<td>unlikely</td>
<td>likely</td>
<td>never</td>
</tr>
<tr>
<td>CPI measure</td>
<td>likely</td>
<td>very likely</td>
<td>likely</td>
</tr>
<tr>
<td>CPI &amp; RER</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

policy rule, since it contains a useful prediction of future inflationary pressure. The closed-economy literature has shown that the adoption of such a feedback policy rule can easily prevent indeterminacy and self-fulfilling fluctuations by implementing the Taylor principle. Yet, these studies have surprisingly ignored the role of money demand in their analysis. We have shown that real balance effects generate an additional monetary transmission channel, whereby changes in money demand affect inflation via changes in real marginal cost. With forward-looking monetary policy, this weakens sizeably the ability of the Taylor principle to induce equilibrium determinacy.

Table 4 presents a qualitative summary of our key results in terms of the probability of the Taylor principle inducing indeterminacy for the three forward-looking policy rule specifications we considered: a rule that reacts only to expected future inflation ($\hat{R}_t = \mu_\pi \hat{\pi}_{t+1}$); a rule that also reacts to current output ($+\mu_y \hat{Y}_t$); and a rule that also reacts to expected future output ($+\mu_y \hat{Y}_{t+1}$). The characterization used for indeterminacy, in descending order, is: almost sure; very likely; highly likely; likely; unlikely; highly unlikely; very unlikely; and almost never. By inspection, for all three policy rule specifications, the likelihood of indeterminacy increases when the utility function is non-separable ($\chi = 0.03$) for both closed and open economies relative to the common separability assumption ($\chi = 0$). Furthermore, indeterminacy is also more likely for open economies if the policy rule reacts to consumer-price inflation (CPI) (and also the real exchange rate (RER)) compared to when domestic-price inflation (DPI) enters the policy rule.

While we do not formally outline the determinacy conditions of the open economy under a policy rule that also reacts to future expected output, the likelihood of indeterminacy is easily inferred from the closed-economy analysis presented in Section 4.2.1. The determinacy conditions for the separability case are taken from Llosa and Tuesta (2008).
Overall, our analysis suggests that to avoid indeterminacy central banks can neither be too cautious, nor too aggressive in adjusting the nominal interest rate in response to changes in both expected future inflation and current output. For open economies the indeterminacy problem can be even more severe. Our analysis raises some important concerns relating to the ability of central banks to avoid indeterminacy through the implementation of forward-looking inflation feedback rules. However, it does suggest that, for open economies, it would be highly beneficial for central banks to switch from the current widespread use of consumer-price inflation as the policy inflation indicator towards domestic-price inflation.

Acknowledgments
This is a substantial revision to a previous working paper by McKnight and Mihailov (2007). We are grateful to Yunus Aksoy, Philippe Bacchetta, Gianluca Benigno, Tatiana Damjanovic, Martin Ellison, Etienne Farvaque, Florence Huart, Thomas Lubik, Celine Poilly, Sergio Rossi and Kenneth West for helpful suggestions. Feedback from seminar participants at the Universities of Fribourg, Lausanne, Lille 1 and Reading, the Royal Economic Society Annual Conference 2010, the COOL 3 Macroeconomics Conference 2010 held at Birkbeck College, University of London, and the 2011 Meeting of the European Economic Association and the Econometric Society, is also acknowledged. The usual disclaimer applies.
A  Proof of Proposition 1

The aggregate system summarized in Table 1 can be reduced to the following two-dimensional system in \( \hat{\mathbf{m}}^W \mathbf{C}^W \), where the coefficient matrix is:

\[
\mathbf{A}_1 \equiv \left[ \begin{array}{cc}
\frac{\sigma(\lambda(\omega+\sigma^{-1})+\frac{\beta\sigma}{\eta R})}{\mu_R \eta R \sigma \chi_\omega \mu_y} & \frac{\eta R (1-\eta R \chi_\omega \sigma \mu_y)}{\eta R \sigma \chi_\omega} \\
\frac{\sigma(\lambda(\omega+\sigma^{-1})+\frac{\beta\sigma}{\eta R})}{\mu_R \eta R \sigma \chi_\omega \mu_y} & \frac{\eta R (1-\eta R \chi_\omega \sigma \mu_y)}{\eta R \sigma \chi_\omega} \\
\end{array} \right].
\]

Its determinant and trace are: \( \det \mathbf{A}_1 = \frac{\Omega}{\eta R \sigma \chi_\omega} \) and \( \text{tr} \mathbf{A}_1 = 1 + \frac{(\mu_R - 1)(\lambda(\eta R + \Omega) - \Omega)}{\eta R \sigma \chi_\omega} \). Since \( \hat{\mathbf{m}}^W \) and \( \hat{\mathbf{C}}^W \) are non-predetermined variables, determinacy requires that the two eigenvalues of \( \mathbf{A}_1 \) are outside the unit circle. According to the Schur-Cohn criterion (see, e.g., LaSalle, 1986) this requires that (i) \( |\det \mathbf{A}_1| > 1 \) and (ii) \( |\text{tr} \mathbf{A}_1| < 1 + |\det \mathbf{A}_1| \). First note that \( \det \mathbf{A}_1 > 1 \) provided \( \mu_R < \Gamma_1 \). In this case condition (ii) implies that \( 1 < \mu_R < \Gamma_2 \). Next note that \( \det \mathbf{A}_1 < -1 \) provided \( \Gamma_1 < \mu_R < \Gamma_2 \). Then \( |\text{tr} \mathbf{A}_1| < -1 - |\det \mathbf{A}_1| \) implies \( 1 < \mu_R < \Gamma_2 \). This completes the proof. \( \square \)

B  Proof of Proposition 2

The aggregate system summarized in Table 1 can be reduced to the following three-dimensional system in \( \hat{\mathbf{m}}^W \mathbf{C}^W \), where the coefficient matrix is:

\[
\mathbf{A}_2 \equiv \left[ \begin{array}{ccc}
1 + \frac{\lambda(\mu_R - \mu_y)}{\eta R \chi_\omega \mu_y} & \frac{1-\eta R \chi_\omega \mu_y}{\eta R \sigma \chi_\omega} & \frac{\eta R (1-\eta R \chi_\omega \sigma \mu_y)}{\eta R \sigma \chi_\omega} \\
\frac{\lambda(\mu_R - \mu_y)}{\eta R \chi_\omega \mu_y} & 1 - \eta R \chi_\omega \mu_y & \frac{\eta R (1-\eta R \chi_\omega \sigma \mu_y)}{\eta R \sigma \chi_\omega} \\
\frac{\lambda(\mu_R - \mu_y)}{\eta R \chi_\omega \mu_y} & \frac{\lambda(\mu_R - \mu_y)}{\eta R \chi_\omega \mu_y} & 1 - \eta R \chi_\omega \mu_y \\
\end{array} \right].
\]

The three eigenvalues of \( \mathbf{A}_2 \) are solutions to the cubic equation \( r^3 + a_2 r^2 + a_1 r + a_0 = 0 \), where

\[
\begin{align*}
    a_2 &= -1 - \frac{1}{\beta} - \frac{\lambda(\mu_R + \mu_y)}{\beta \eta R \chi_\omega \mu_y} - \frac{1}{\beta \eta R \chi_\omega \mu_y} + \frac{\Omega}{\beta \eta R \sigma \chi_\omega \mu_y} \\
    a_1 &= \frac{1}{\beta} + \frac{\lambda(\mu_R - 1)(\Omega + \sigma \omega)}{\beta \eta R \sigma \chi_\omega \mu_y} + \frac{1}{\beta \eta R \chi_\omega \mu_y} + \frac{\lambda(\mu_R - 1)(\Omega + \sigma \omega)}{\beta \eta R \sigma \chi_\omega \mu_y} - \frac{(1+\beta)\Omega}{\beta \eta R \sigma \chi_\omega \mu_y} \\
    a_0 &= \frac{\Omega}{\beta \eta R \sigma \chi_\omega \mu_y}.
\end{align*}
\]

As there are no predetermined variables, determinacy requires that all the eigenvalues are outside the unit circle. Since \( a_0 > 0 \), this is the case if and only if the following Schur-Cohn criterion is satisfied: (i) \( 1+a_2+a_1+a_0 > 0 \), (ii) \( -1+a_2-a_1+a_0 > 0 \), and (iii) \( a_2^2-1 > |a_0 a_2-a_1| \). Conditions (i) and (ii) simplify respectively to \( \lambda(\mu_R - 1)(\Omega + \sigma \omega) + \sigma \mu_y (1-\beta-\lambda(\sigma \chi_\omega \mu_y)) > 0 \) and, \( 2\Omega(1+\beta) + \lambda(\Omega + \sigma \omega) > \mu_R [\Omega + \sigma \omega(1+2\eta R \chi_\omega)] + \sigma \mu_y [1+\beta+\eta R \chi_\omega(\lambda+2(1+\beta))] \), which imply \( \Gamma_1^2 \) and \( \Gamma_2^2 \) of the main text. For condition (iii), the case \( 1 - a_0^2 < a_0 a_2 - a_1 \)
implies $\Gamma_3^2$. For the case $a_0^2 - 1 > a_0 a_2 - a_1$ this can be expressed as:

$$
\Omega \left[ \frac{1}{\eta_R \sigma \chi y} + (1 - \beta)(1 + \beta + \lambda) + \frac{\lambda \eta_R \sigma y}{\mu_y} + \frac{\beta}{\eta_R x} \right] + (1 - \beta)\eta_R \sigma \chi y + \lambda \eta_R \sigma \omega \chi \mu_\pi + \lambda \Omega
+
\beta \sigma \mu_y + \eta_R \lambda \sigma \chi y + [\lambda(\mu_\pi - 1)(\Omega + \sigma \omega) + \sigma \mu_y(1 - \beta - \lambda \eta_R \chi)] > 0,
$$

which, by inspection, is always satisfied if condition (i) holds. This completes the proof. □

C Proof of Proposition 3

The aggregate system summarized in Table 1 can be reduced to the following two-dimensional system in $\left[ \hat{m}_t^W \hat{C}_t^W \right]'$, where the coefficient matrix is:

$$
A_3 \equiv \left[ \begin{array}{cc}
\frac{\sigma_1 - \Delta_2}{\lambda_1} \left( \frac{\sigma(\mu_\pi - 1)}{\eta_R \sigma \chi y} + \sigma \chi \right) & \frac{\lambda_2 \Delta_2}{\lambda_1} \left( \frac{\sigma(\mu_\pi - 1)}{\eta_R \sigma \chi y} + \sigma \chi \right) \\
\frac{\sigma^2 \chi}{\lambda_1} - (\beta \sigma + \eta_R \sigma \mu_\pi) \left( \frac{\sigma(\mu_\pi - 1)}{\eta_R \sigma \chi y} + \sigma \chi \right) & \lambda_1(\beta \sigma + \eta_R \sigma \mu_\pi) - \sigma^2 \chi (\eta_R \sigma \mu_\pi) \cdot \frac{\lambda_1}{\lambda_1} \end{array} \right],
$$

with $\Lambda_1 \equiv \beta \sigma (\Omega + \eta_R \sigma \chi y) - \eta_R \sigma \omega \chi \mu_\pi$, $\Lambda_2 \equiv \beta \sigma (\eta_R \sigma \chi y) + \lambda \eta_R \sigma \chi y (1 + \sigma \omega)$, $\Lambda_3 \equiv 1 + \sigma \mu_y + (\mu_\pi - 1)(\eta_R \sigma \chi y) \frac{\sigma}{\eta_R \sigma \mu_\pi}$, $\sigma_3 = \frac{\Omega + \eta R \sigma \chi y}{\beta \sigma + \eta R \sigma \mu_\pi}$, and $trA_3 = 1 + \frac{\Omega - \lambda \mu_\pi - 1}{\beta \sigma + \eta R \sigma \mu_\pi} + \frac{\sigma}{\eta R \sigma \mu_\pi}$, $\beta \sigma + \eta R \sigma \mu_\pi$. Determinacy again requires that the two eigenvalues are outside the unit circle. Using the Schur-Cohn criteria, the det $A_3 > 1$ provided $\mu_\pi < \Gamma_1^3$ and in this case $|trA_3| < 1$ implies $A_3$ implies $\max \{0, \Gamma_3^3\} < \mu_\pi < \Gamma_3^3$. Next note that $det A_3 < -1$, $\Gamma_1^3 < \mu_\pi < \Gamma_2^3$. Then $|trA_3| < 1$, $\Gamma_2^3 < \mu_\pi < \Gamma_2^3$. This completes the proof. □

D Proof of Proposition 4

The difference system summarized in Table 1 can be reduced to the following two-dimensional system in $\left[ \hat{m}_t^R \hat{C}_t^R \right]'$, where the coefficient matrix is:

$$
B \equiv \left[ \begin{array}{cc}
\frac{\sigma \Lambda_1 \Lambda_2 + \Lambda_3}{\eta R \sigma \chi y (2a - 1)} & \frac{\Lambda_1 \Lambda_2 - \Lambda_3}{\eta R \sigma \chi y (2a - 1)} \\
\frac{\sigma \Lambda_1 \Lambda_2}{\sigma \Lambda_1 \Lambda_2 + \sigma \chi} & \frac{\sigma \chi}{\sigma \Lambda_1 \Lambda_2 + \sigma \chi} \\
\frac{\sigma \Lambda_1 \Lambda_2}{\eta R \sigma \chi y (2a - 1)} & \frac{\Lambda_1 \Lambda_2 - \Lambda_3}{\eta R \sigma \chi y (2a - 1)} \\
\frac{\sigma \Lambda_1 \Lambda_2}{\sigma \Lambda_1 \Lambda_2 + \sigma \chi} & \frac{\sigma \chi}{\sigma \Lambda_1 \Lambda_2 + \sigma \chi} \\
\end{array} \right],
$$

with $\Lambda_1 \equiv \beta \eta_R - \frac{\eta R \mu_\pi}{\sigma (2a - 1)} \left[ 4a \theta(1 - a) + \sigma (2a - 1)^2 + \frac{\lambda \eta R \mu_\pi}{\sigma (2a - 1)} (1 + \sigma \omega(2a - 1)^2 + 4a \theta(1 - a)) \right]$, $\Lambda_2 \equiv \chi + \frac{(2a - 1)(\mu_\pi - 1) + \frac{\mu_\pi}{\mu_\pi}}{\eta R \sigma \mu_\pi}$, $\Lambda_3 \equiv \frac{\eta R \mu_\pi}{\sigma (2a - 1)} \left[ 4a \theta(1 - a) + \sigma (2a - 1)^2 \right]$, $\Lambda_3 \equiv \chi + \frac{(2a - 1)(\mu_\pi - 1) + \frac{\mu_\pi}{\mu_\pi}}{\eta R \sigma \mu_\pi}$, $\Lambda_4 \equiv 1 + \frac{\eta R \mu_\pi (2a - 1)(\mu_\pi - 1)}{\sigma (2a - 1)} + \frac{\mu_\pi}{\mu_\pi} \frac{\lambda \eta R \mu_\pi}{\lambda \eta R \mu_\pi}$, det $B = \frac{\Omega + \eta R \sigma \chi y}{\beta \sigma + \eta R \sigma \mu_\pi} \frac{1 + \sigma \omega(2a - 1)^2 + 4a \theta(1 - a)}{\beta \sigma + \eta R \sigma \mu_\pi}$ and $trB = 1 + \frac{\Omega - \lambda \mu_\pi - 1}{\beta \sigma + \eta R \sigma \mu_\pi} \frac{\sigma \omega(2a - 1)^2 + 4a \theta(1 - a)}{\beta \sigma + \eta R \sigma \mu_\pi}$. Determinacy again requires that the two eigenvalues are outside the unit circle. Using the Schur-Cohn conditions, det $B > 1$ provided $\mu_\pi < \frac{\beta \sigma + \eta R \sigma \mu_\pi}{\eta R \sigma \omega \chi (2a - 1)} \equiv \Gamma_1^4$ and in this case $|trB| < 1$ implies $\max \{0, \Gamma_3^3\} < \mu_\pi < \Gamma_3^3$. Next note that $det B < -1$ provided $\Gamma_1^4 < \mu_\pi < \Gamma_2^4$. Then $|trB| < 1$, $\Gamma_2^4 < \mu_\pi < \Gamma_2^4$. Therefore
max\{0, \Gamma_3^2\} < \mu_\pi < \Gamma_3^2$, and either $\mu_\pi < \Gamma_1^4$ or $\Gamma_1^4 < \mu_\pi < \Gamma_2^4$, are the necessary and sufficient conditions for the difference system. Comparing these bounds on $\mu_\pi$ with the determinacy conditions obtained for the aggregate system given in Proposition 3, it is straightforward to verify that $\Gamma_1^4 < \Gamma_1^4$, $\Gamma_2^4 < \Gamma_2^4$, $\Gamma_3^3 < \Gamma_3^3$, and $\Gamma_4^4 < \Gamma_4^4$. This completes the proof.

### E Proof of Proposition 5

The difference system summarized in Table 1 can be reduced to the following two-dimensional system in $[\hat{m}_t^R \hat{C}_t^R]$, where the coefficient matrix is:

$$
C = \begin{bmatrix}
-\sigma_1 \Lambda_1 - \Lambda_3 & \beta \eta_1 \sigma_1 (2a-1) \lambda \omega_\mu_\pi (2a-1) - \beta \mu_y \\
-\sigma_2 \Lambda_2 - \sigma_\chi \Lambda_3 & \beta \eta_2 \sigma_1 (2a-1) \lambda \omega_\mu_\pi (2a-1) - \beta \mu_y \\
\beta \eta_1 \sigma_1 (2a-1) \lambda \omega_\mu_\pi (2a-1) - \beta \mu_y & \beta \eta_2 \sigma_1 (2a-1) \lambda \omega_\mu_\pi (2a-1) - \beta \mu_y
\end{bmatrix},
$$

with $\Lambda_1 \equiv \beta \eta_1 [1-2(1-a) \mu_\pi - \frac{\beta \eta_1 \mu_y}{\sigma_2 (2a-1)} \{4a\theta (1-a) + \sigma (2a-1)^2\} + \frac{\lambda \omega_\mu_\pi}{\sigma} \{1 + \sigma \omega (2a-1)^2 + 4a\theta \omega (1-a)\}]$, $\Lambda_2 \equiv \chi + \frac{\mu_y (1-1)}{\beta \eta_1} + \frac{\mu_y a \theta_1 (1-a)}{\mu_\pi (2a-1)}$, $\Lambda_3 \equiv \eta_1 \mu_y a \theta_1 (1-a) - [1 - 2(1-a) \mu_\pi]$, $\Lambda_4 \equiv 1 + \frac{\sigma \omega (1-1)}{\beta \eta_1}$, $\Lambda_5 \equiv \eta_1 [1 - 2(1-a) \mu_\pi] - \frac{\sigma \omega (2a-1)^2}{\sigma_2 (2a-1)}$ and $\Lambda_6 \equiv \beta [1 - 2(1-a) \mu_\pi] - \frac{\beta \eta_1 \mu_y a \theta_1 (1-a)}{2a-1} + \lambda \sigma_1 \eta_1 \omega_\mu_\pi \{1 + 4a\theta \omega (1-a)\}$.

Therefore, the Schur-Cohn conditions, $\det C > 1$ provided $\mu_\pi < \frac{\beta \eta_1 \mu_y a \theta_1 (1-a)}{2a-1} + \lambda \sigma_1 \eta_1 \omega_\mu_\pi \{1 + 4a\theta \omega (1-a)\}$ and in this case $|\operatorname{tr} C| < 1$ + $\det C$ implies $\max\{0, \Gamma_3^2\} < \mu_\pi < \Gamma_3^2$. Next note that $\det C < -1$ provided $\Gamma_3^2 < \mu_\pi < \Gamma_5^2$. Then $|\operatorname{tr} C| < -1 - \det C$ implies $\Gamma_3^2 < \mu_\pi < \Gamma_5^2$. Therefore max\{0, $\Gamma_3^2$\} < $\mu_\pi < \Gamma_5^2$, and either $\mu_\pi < \Gamma_1^4$ or $\Gamma_1^4 < \mu_\pi < \Gamma_2^4$, are the necessary and sufficient conditions for the difference system. This completes the proof.
References


