ON THE LIMITS TO INCOME REDISTRIBUTION WITH POLL
SUBSIDIES AND COMMODITY TAXATION

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Abstract

The limits to income redistribution are analyzed when the only feasible tax instruments are poll subsidies and commodity taxation. These instruments are incentive compatible and easily administered so they could be used by countries with less developed tax systems. We compare our results with those of Sah (1983) where the limits to redistribution in developing countries are set in terms of how much the government could distribute using commodity taxation.
1 Introduction

Perhaps one of the most accepted objectives of taxation is that of income redistribution. At the time of fixing income distribution policies, policymakers have to take into account the actual possibilities offered by the tax system. The latter includes a variety of factors such as tax laws, money spent in monitoring, tax honesty of citizens, the degree of sophistication of tax administration, other institutional constraints, etc.... These are variable among different countries. We can believe, for instance, that the possibilities for redistribution of income are a lot bigger in countries with a very sophisticated tax system, like Germany, than in countries with undeveloped tax systems. In the latter, efficient monitoring of income taxes is almost impossible and even commodity taxation finds difficulties given the easy appearance of black markets. Despite the problems that commodity taxation could encounter, it is commonly recognized that its management is a lot easier than the administration of the income tax. This makes Sah (1983) assert that the limits to redistribution in developing countries should be set in terms of how much the government can redistribute through commodity taxes. Additional tax instruments the government could use are poll subsidies and land taxation. However the last one could find problems in countries where land owners are able to form pressure groups. There is nothing comparable impeding the use of poll subsidies financed by (additional) commodity taxes.

In this note we reconsider Sah’s question where we allow the governments to use commodity taxation but also uniform subsidies in pursuing its objectives of income redistribution. It turns out that both tax instruments are incentive compatible and fairly simple to manage even in developing countries. In Sah’s analysis the obvious limits to redistribution appear from the structure of demand. Ours increases the number of possibilities. We could even use commodity taxation merely to get revenue while letting the uniform subsidy perform its redistributing role. It is shown however that, at the second best optimum, the possible redistribution is probably but not necessarily increased. This is because the amount that can be optimally redistributed through the poll subsidy depends on the structure of demand, even though the introduction of the income subsidy could change demand elasticities and commodity tax rules.

The comparison between redistribution mechanisms using commodity
taxes alone and those which also use poll subsidies becomes even clearer when we are away from a second best optimum in a general equilibrium model. The first type of mechanism is used, for instance, by Dixit and Norman (1980) to get Pareto gains from market integration. The idea is to freeze consumer prices through commodity taxes, assume that there exists a positive direction of commodity tax reform and no positive profits, and then move one of the consumer prices in the positive direction. They need an assumption of free disposal to achieve final equilibrium of the system. The second type of mechanism is used in Hammond and Sempere (1992) also to show Pareto gains from market integration. In that mechanism consumer prices are frozen through commodity taxation, and then a poll subsidy is used to produce a Pareto improvement. There is no need either for information about a positive direction of reform or for free disposal.

The paper is organized as follows: Section 2 introduces the welfare money metric; Section 3 computes optimal redistributive taxes and sets the welfare gain limits; Then Section 4 concludes the paper with some final remarks.

2 A Redistribution Measure

Our objective is to show limited possible gains in redistribution. In order to exaggerate the redistribution objectives we use, following Sah (1983), a Rawlsian welfare function. So the government has the objective of maximizing the utility of the worst off individual. In order to have a consistent welfare measure even for large changes which could reverse the classification of individuals in the characteristic poverty, it is easiest to assume that the poorest individual has always the same indirect utility function, no matter who this is. The money metric measure we use is the equivalent social variation per unit of income of the poorest individual. We use $p$ for untaxed producer prices which are supposed constant during the analysis, $q = p(1 + t)$ are consumer prices including the vector of commodity taxes $t$, $s$ is the poll subsidy, $E^1$ is the expenditure function of 1 (from now on, the poorest individual), $m^1$ is unearned income of consumer 1, and $v^1$ denotes the indirect utility function of 1. Then we define the welfare metric as

$$\frac{I^1}{m^1} = \frac{E^1[p, v^1(q, m^1 + s)]}{m^1} - 1$$

(1)
This would be the increase in money metric utility per unit of income due to commodity taxes and the poll subsidy.

By concavity in prices of the expenditure function, it is easy to check that \( I^1/m^1 \leq -T^1/m^1 \), where \(-T^1\) is the net subsidy or tax received by the worst-off individual in his total purchases plus the poll subsidy. This is shown with the following lemma.

**LEMMA:** \( I^1 + T^1 \leq 0 \).

**PROOF:** The net amount of taxes paid by 1 is

\[
T^1 = (q - p)E^1_q(q, v^1(q, m^1 + s)) + s
\]

From convexity of the expenditure function

\[
I^1 = E^1(p, v^1(q, m^1 + s)) - E^1(q, v^1(q, m^1 + s)) \leq (p - q)E^1_q(q, v^1(q, m + s))
\]

The result follows from \( E^1(q, v^1(q, m^1 + s)) = m^1 + s \). \( \square \)

We could also notice that \( I^1 + T^1 \) is the difference between the tax collected from the poorest individual and the welfare change produced. This could be a sort of personalized deadweight loss. The less distortionary the taxation that is used, the more accurate the index of welfare gain \(-T^1/m^1\) turns out to be.

### 3 Optimal Taxes

We set rules for commodity taxes and the poll subsidy in order to maximize the utility of the worst-off consumer subject to the government's revenue constraint. These will be the taxes maximizing the amount of redistribution. The government faces the problem of finding the maximizers of \( v^1(t, m^1 + s) \) subject to \( \sum_{i \in I} \sum_{j \in I} t_i x^j_i - Is \geq 0 \). The Lagrangean for this problem is

\[
L = v^1(q, m^1 + s) + \lambda \left( \sum_{i \in I} \sum_{j \in I} t_i x^j_i - Is \right)
\]

The first order conditions of this problem could be written as

\[
-\alpha^1 x^1_k + \lambda \left[ \sum_{i \in I} x^i_k \left( 1 - \sum_{i \in I} t_i \frac{\partial x^i}{\partial m^i} \right) + \sum_{i \in I} \sum_{j \in I} t_i S^i_{ik} \right] = 0 \quad \forall k \in L
\]
\[ \alpha^1 + \lambda \left[ \sum_{i \in L} \sum_{i \in i} t_i \frac{\partial x_i^1}{\partial m^1} - I \right] = 0 \]

where we have used the Slutsky equations and Roy’s identity. Here \( \alpha^1 \) is the marginal utility of income of individual 1 and \( S_{ik} \) is the \( lk \) term of the Slutsky matrix of consumer \( i \) while \( \lambda \) is the shadow price of government revenue. Write \( \partial T^i / \partial m^i \) for \( \sum_{i \in L} t_i \partial x_i^1 / \partial m^i \) i.e. consumer’s \( i \) income effect on government’s indirect tax revenue. Also let \( \bar{S}_{ik} \) be the average across agents of the \( lk \) term of the Slutsky matrix. Then, substituting \( \alpha^1 / \lambda \) from the second equation into the first, we get

\[
I \sum_{i \in L} t_i \bar{S}_{ik} = \sum_{i \in i} \left[ 1 - \frac{\partial T^i}{\partial m^i} \right] (x_k^1 - x_i^1) \quad \forall k \in L
\]

This trivially relates the marginal amount of each tax distortion (measured by the substitution terms of the Slutsky matrix) to the differences, for each commodity, net of marginal income effects on the public budget, between average consumption and the consumption of the poorest individual. Post-multiplying this system of equations by the tax vector \( t_i \), we have

\[
I \sum_{i \in L} t_i \bar{S}_{ik} t_k = \sum_{i \in i} \left[ 1 - \frac{\partial T^i}{\partial m^i} \right] (T^1 - T^i)
\]

rearranging, we get

\[
- \frac{T^i}{m^1} = - I \sum_{i \in L} t_i \bar{S}_{ik} t_k + \sum_{i \in i} \left[ 1 - \frac{\partial T^i}{\partial m^i} \right] T^i
\]

as an expression for our redistribution limit, where \( T^i \) is the total net tax paid by consumer \( i \). This expression relates the limit to the welfare gain with Harberger’s measure of deadweight loss and the sum of tax payments net of indirect income effects made by individuals. The limits to income redistribution come from the demand structure and from the income effects of the poll subsidy on tax revenue. Unless Engel curves are linear, income taxation changes the demand structure and this would change the rules for optimal redistributive commodity taxation.
The corresponding formula presented in Sah (1983), for the case where only commodity taxation is used, is

$$- \frac{T^1}{m^1} = \frac{-I \sum_{i \in L} t_i \tilde{S}_{ik} t_k + \sum_{i \in L} [1 - \partial T^i / \partial m^1] T^i}{\sum_{i \in L} m^i [1 - \partial T^i / \partial m^1]}$$ (4)

The main formal difference between (3) and (4) is that in the denominator of (3) only appears the income of the poorest individual whereas in (4) the denominator is $\sum_{i \in L} [1 - \partial T^i / \partial m^1] m^i$, i.e. the weighted summation of all individual incomes.

To see a clearer relation between the expressions, assume that Engel curves are parallel across individuals. Then $\partial x_i / \partial m^1 = \partial x_j / \partial m^2$ for all $i \neq j$ and all $l \in L$ so the second term of the numerator disappears because of the government revenue constraint. The expression for our limit is,

$$- \frac{T^1}{m^1} = \frac{- \sum_{i \in L} t_i \tilde{S}_{ik} t_k}{m^1 [1 - \partial T^i / \partial m^1]}$$

This is twice the average Harberger measure of deadweight loss per unit of income of the poorest individual net of indirect marginal income effects on commodity tax collection from individual 1. Provided that average national income is bigger than the income of the poorest, this limit at first looks greater than the one corresponding to Sah (1983). However, our tax rules are different and so lead to different consumer price vectors. The Slutsky matrix depends on final consumer prices so we do not have a fully consistent comparison between both formulae.

In the general case the most we can say is that, if we reduce our comparison to commodity taxes giving the same amount of deadweight loss, a positive poll subsidy would increase the possibilities of redistribution. This increase would depend on the differences between average income and income of the worst-off individual. In the example of Sah (1983 p.95), where as much as a half of any increase in income is paid in increased taxes (i.e., $\sum_{i \in L} [1 - dT^i / dm^1] = 1/2$) and deadweight loss is 20% of income, particularized to the case of average income equal to 300, when Engel curves are parallel, the worst-off individual's real income can be improved using commodity taxes by less than 80% independently of what this income is. Using commodity taxes and poll subsidies we could improve income of the worst-off
by less than 160% if his income is, for instance, 150 — or by less than 120% if his income is 200.

This comparison is very sensitive. It depends on the crucial assumption that the optimal deadweight loss is the same when one uses only commodity taxation and when one uses commodity taxation and also a poll subsidy. Without this assumption it is possible that the amount redistributed would continue to be low. We also have to take into account that what we are comparing are upper bounds on the actual gain. Differences in actual gains could be smaller.

An example where less redistribution at the optimum is got with only commodity taxes than with commodity taxes and the poll subsidy is the case of the Linear Expenditure System (LES) demand with consumers differentiated only by income. In this case Sah (1983) obtains small but positive possible redistribution. Suppose that we set up our maximization problem for LES preferences with indirect utility function

\[ v'(m^i + s, t) = \sum_{i \in L} \alpha_i \ln((m^i + s - m)(\frac{\alpha_i}{p_i(1 + t_i)}) \]

where \( m = \sum_{i \in L} \beta_i p_i(1 + t_i) \) is subsistence income. Then standard optimal taxation rules give

\[ \frac{t_k}{p_k(1 + t_k)} = \sum_{i \in L} \alpha_i \frac{t_i}{p_i(1 + t_i)} - \frac{m^1 - m}{m - m}[1 - \sum_{i \in L} \alpha_i \frac{t_i}{p_i(1 + t_i)}] \tag{5} \]

for all \( k \), where \( m \) denotes average national income. This expression implies that all the commodities should be taxed at the same rate. This result is related to those of Atkinson (1977), Atkinson and Stiglitz (1976) and Deaton (1979) for more general social welfare functions. It would work whenever preferences are weakly separable between commodities and leisure and consumers have parallel linear Engel curves. This together with the government budget constraint \( \sum_{i \in L} \sum_{i \in L} t_i x_i - Is \geq 0 \) allows us to calculate the exact value of (1) for a set of commodity taxes consistent with (5) and a poll subsidy consistent with those commodity taxes and the budget constraint. We use, as did Sah (1983), the LES system estimated for the U.K. by Pollak and Wales (1978).\(^1\). Average income is 392.8. The results, obtained first for

\(^1\)The sample period is 1966-72 and income is measured in Shillings per week.
a uniform commodity tax of 20% and a corresponding poll subsidy of 78.56 and, second, for a uniform commodity tax of 50% and a corresponding poll subsidy of 196.4, appear in the following table where they are compared with Sah’s limits.

<table>
<thead>
<tr>
<th>Income of the poorest</th>
<th>150</th>
<th>200</th>
<th>300</th>
<th>392.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sah’s Welfare gain</td>
<td>0.134</td>
<td>0.056</td>
<td>0.008</td>
<td>0</td>
</tr>
<tr>
<td>Welfare gain ((t = 0.2, s = 78.56))</td>
<td>0.269</td>
<td>0.16</td>
<td>0.051</td>
<td>0</td>
</tr>
<tr>
<td>Welfare gain ((t = 0.5, s = 196.4))</td>
<td>0.539</td>
<td>0.321</td>
<td>0.103</td>
<td>0</td>
</tr>
</tbody>
</table>

Thus, for instance, the second column means that when the income of the worst off individual is 200, by using commodity taxes only his welfare could be increased by 5%; by using a uniform commodity tax of 20% with a corresponding poll subsidy of 78.56 his welfare could be increased by 16%; and by using a uniform commodity tax of 50% with a corresponding poll subsidy of 196.4, his welfare could be increased by 32%. The table shows that introducing the poll subsidy could amplify the increase in money metric utility. Commodity taxes are used to get revenue while the uniform subsidy performs its redistributing role.

## 4 Final Remarks

It appears possible that introducing additional incentive compatible redistributive tax instruments like poll subsidies could change the pessimistic results of Sah (1983). The maximum amount that we can distribute will depend anyway on demand structure, but now the government can use commodity taxes to get revenue which is distributed with the poll subsidy. In cases where consumers’ preferences are weakly separable between commodities and leisure and consumers have parallel linear Engel curves, the possible redistribution is increased with the introduction of poll subsidies into the optimal taxation problem.

A very important question is whether or not introducing time into our model has effects on the limits to redistribution. In this respect, Rogers (1987) shows in a two period economy how a government concerned with redistribution and efficiency has an incentive to reform announced savings taxes in a second period. The reason is that once individuals are committed to a particular level of savings, a tax on savings becomes a lump-sum tax...
and the government has an incentive to increase the use of these taxes and to reduce distortionary taxation. In the extension of our economy to two periods, the government is not allowed to use a saving tax but it could change savings decisions by distorting intertemporal consumer prices through use of different tax rates in different periods. The government could be interested in changing announced tax rates if it could increase public revenue and use this additional revenue to improve income distribution. In the case where the government only uses commodity taxation and poll subsidies with people having the same preferences in both periods, announced optimal taxes will coincide with time consistent ones. Demand structure sets what the optimal redistributive taxes ought to be and so the limits to redistribution. The first order conditions for the second period conditioned on first period consumer saving decisions would be included in the first order conditions of the two period optimal taxation problem.

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