POTENTIAL GAINS FROM MARKET INTEGRATION WITH INDIVIDUAL NON-CONVEXITIES

Jaime Sempere

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JAIME SEMPERE
C.F.E., El Colegio de México
Camino al Ajusco 20, México 01000 D.F., México

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Abstract

The theorems concerning gains from trade and from market integration in customs unions are revisited when the consumption sets or preferences are not necessarily convex. The main idea involved in showing Pareto gains is the use of dispersed compensation. This allows us to consider situations in which consumers could migrate to other countries forming the union, and even to analyze the case where survival of the whole population is not ensured. Two kinds of compensation mechanism are used. The first uses lump sum transfers. The second assumes that the government can freeze prices and dividends and use a poll subsidy.
1 Introduction

The gains from trade theorems as first shown by Samuelson (1939 and 1962) and Kemp (1962) claim that when a group of countries frees trade among them, improvements in efficiency are possible and these make possible a Pareto gain if the adequate compensation is implemented. Historically two kinds of compensation mechanism which give rise to a Pareto gain are used. The first, associated with classical welfare economics, assumed that governments had enough information available about people's characteristics to use lump-sum compensation. The second, forming part of the new welfare economics, takes into account the informational restrictions faced by governments and uses only incentive compatible compensation. Then two compensatory mechanisms have been used. The first is the one devised by Dixit and Norman (1980, 1986) based on the Diamond and Mirrlees (1971) justification of productive efficiency and uses movements in commodity taxes to implement the Pareto gain. The second is the one devised by Hammond and Sempere (1992) which extends the latter to avoid the assumption of free disposability and reduces the informational requirements. This compensation mechanism involves changes in commodity taxes and also a uniform subsidy.

All these works use the assumption of convexity. We claim that this is very restrictive in international trade models where certain indivisibilities are more evident. A very important example to consider is the case of migration. If we allow people to migrate, assuming convexity in consumer feasible sets means that all goods and services are internationally traded and they are also divisible. In fact, in most of the work on the gains from trade theorems labour services are like any other Arrow-Debreu commodity. If no distinction is made between tradeable and non-tradeable commodities, these models implicitly allow the analysis of gains from free trade of labour services. Each national agent could, after the trade liberalization, offer freely his labour services in all the rest of countries. Even in several of them simultaneously. It is assumed that it is costless to migrate from one country to another and that labour is perfectly divisible. In real economies it is hard to supply labour at several different places simultaneously. In our model we assume that labour is included in the set of non-tradeable commodities of a country. Then we will allow the consumer to offer his labour supply in other countries but this will be costly for him. These facts would explain differences in wages
between different countries, even after freeing trade and allowing migration, because we consider labour supplied in each different country as a different commodity. On the other hand, it shows how restrictive is the convexity of preferences when the same goods, but consumed in different countries, are mixed. For these reasons, in this paper we are not going to assume convexity in consumers' characteristics.

This will allow us to consider also the case of economies without full survival, as discussed by Coles and Hammond (1991), and thus also discuss the possibility of Pareto gains in cases where part of the population faces chances of starvation when they do not have enough income. With this complication a problem arises at the time of defining a Pareto gain in our static economy. We could make this concept depend on the assumption of local non-satiation of non-survivors (even in the situation where they starve). This would imply that non-survivors are better off if they have more food before dying. But we could also consider that the situation of a person who would starve in absence of the reform would be improved only in the case in which he is made to survive. This will not always be feasible. So a Pareto improvement in the economy without full survival will be a situation where all survivors are improved, some non-survivors are made survivors and made better off, and no survivor is made a non-survivor.

The methodology we will use to tackle non-convexities makes use of a model with a continuum of traders as originated in the work of Aumann (1964) and follows the methods of Hildenbrand (1974), as developed in Yamazaki (1981). These are based on the smoothing effects of aggregation when the distribution of agents' characteristics is dispersed in a certain way. In particular, at the time of designing the compensatory mechanism, we will require the additional property of dispersed compensation.

The paper is organized as follows: Section 2 sets up a general equilibrium model of an international economy with a continuum of traders. Section 3 defines our concept of potential Pareto gains. Section 4 analyzes the possibility of getting Pareto gains from creating customs unions when migration is allowed, and when using lump-sum transfers is feasible for each government. Section 5 proves the possibility of Pareto gains under the weaker assumption that only incentive compatible compensation is feasible. Section 6 analyzes the case when the assumption of full survival of the population is removed, and Section 7 concludes the paper with some final remarks.
2 An International Economy with a Continuum of traders

There are $K$ countries indexed by $k$. There is a continuum of consumers $I$ indexed by $i$. $I_k$ is the subset of consumers originally living in country $k$. We also assume that there is a finite set of commodities, denoted by $L$. This set is partitioned into $L = \bigcup_{k \in K} N_k \cup T$, where $N_k$ is the set of goods specific to country $k$ and $T$ is the set of internationally traded goods.

Each individual is characterized by a preference relation $\geq_i$ which satisfies transitivity, completeness, continuity and local non-satiation. The set of preference orderings, as represented by their graphs, will be endowed with the closed convergence topology. Each individual also has a consumption set $X^i \subset \mathbb{R}^L$ which is assumed to be closed but non necessarily convex. Examples of non-convex consumption sets are the following:

EXAMPLE 1: Migration: Suppose that individuals have the possibility of migrating. This is like giving them the chance of both choosing between alternative sets of non-traded goods and buying traded goods at different prices (because of different distortionary taxation) from those in their original countries by expending a given amount on travelling, removal costs, learning the language, etc.... In the set of non-traded goods of one nation there would hypothetically be local public production, most labour services, and also other non-tradeables. The literature on local public goods analyses the case in which each agent decides where to live on the basis of public goods and taxes. This would be easily adapted here to the case in which public production is included as an additional firm producing non-traded goods and commodity taxes would fix different consumer prices in each of the countries. Each consumer would have a preference relation on the existing commodities and, on the basis of national prices and non-traded goods, migration costs, income and preferences, he would choose where to live.

If the individual lives in one of the countries, it is not feasible for him to consume non-traded goods of other countries unless he bears the migration costs in a minimal amount. Denote by $c_{kk'}^i$, a minimal amount of net trades necessary to arrange migration from $k$ to $k'$. Let $X_{kk'}^i$ be the set of feasible consumptions for consumer $i$ when he bears the amount needed $c_{kk'}^i$ of the cost of migration from $k$ to $k'$. It is easy to notice that if we define the
consumption set as $X^i \equiv \bigcup_{k' \in K} X^i_{kk'}$, there could be discontinuous preferences at points on the lower boundary of some $X^i_{kk'}$. These discontinuities would be due to the fact that a migrant to $k'$ consuming $x^i \in X^i_{kk'}$ might prefer points $\tilde{x}^i \in X^i_{kk'}$ quite a bit worse than $x^i$ to points $\tilde{x}^i$ which are close to $x^i$, because living in $k'$ could be valued for its own sake. To avoid this problem we will introduce consumption of location as a non-marketed good in the definition of the consumption set.

Let $\{k\}$ be here the variable representing consumption of location at country $k$. Thus consumption of location or, in other words, migration will be introduced as a good in its own right, even though not marketed. Now consumer $i$'s feasible set would be $X^i \equiv \bigcup_{k' \in K} \{k\} \times X^i_{kk'}$, where if $l \in N_{k'}$ and $x^i < c^i_{kk'}$, it implies that $x^l_i = 0$ if $x^i \in X^i$ and $i \in I_k$, $k \neq k'$. And also if $x^i \geq c^i_{kk'}$ and $x^l_i \neq 0$ for some $l \in N_{k'}$, then $x^l_i = 0$ if $l \in N_k$ if $x^i \in X^i$ and $i \in I_k$. In other words, if the individual consumes one of the non-traded goods set it must bear the migration costs to preserve feasibility. We will restrict our analysis to the case in which the consumer can migrate to only one of the other countries, so that if $x^l_i \neq 0$ for some $h \in N_{k'}$ then $x^l_i = 0$ for all $l \in N_{k''}$ whenever $k' \neq k''$. Obviously, $c^i_{kk} = 0$ because consumption in $i$'s original country is possible without purchasing some migration services.

We will assume also that $X^i_{kk'}$ for $k, k' \in K$ are convex and closed sets. Clearly the latter does not imply that the sets $X^i$ are convex. The appearance of indivisible costs of consumption of non-traded goods and the fact that $c^i_{kk'}$ could be considered as indivisible composite commodities imply non-convexities in consumption sets.

**Example 2: Non-survival economies (Coles and Hammond (1991)):** An individual survives if his budget set intersects the set of consumptions which he needs to survive. If this intersection is empty the individual will starve. Each agent has a survival consumption set $X^S$ and a non-survival consumption set $X^N$. Each one is assumed to be convex and closed. Then the consumer feasible set would be $X := X^S \cup X^N$. The problem with this consumption set is that preferences defined on it are typically discontinuous. This is sorted out by adding to the commodity space an extra good, representing survival. This is indicated by means of a variable $i \in \{-1, 0\}$ representing either survival or non-survival. We could relate this to the migration example by assuming that the household lives in the land of the dead and he would have to consume a minimal amount of commodities to migrate to the land of survivors.
Preferences can still be monotone in the extended commodity space and the price of the extra good is always zero. Then the consumption set is

\[ X = (\{0\} \times X^S) \cup (\{-1\} \times X^N) \]

This set is closed but clearly non-convex.

Let \((I, \mathcal{F}, \nu)\) denote an atomless measure space of economic agents, where \(\mathcal{F}\) is the \(\sigma\)-algebra of measurable subsets and \(\nu\) the distribution of agents characteristics. A continuum economy, as defined in Aumann (1964), is a measurable mapping

\[ \varepsilon : I \rightarrow \Theta \]

from the measure space into the space of agents characteristics. We also consider production, as Hildenbrand (1974), in the form of a coalition production economy. That is, a production set correspondence

\[ Y : \mathcal{F} \rightarrow \mathbb{R}^L \]

will be specified which will give a production set to each measurable subset of agents. For each coalition of consumers \(S\), the set \(Y(S) \subset \mathbb{R}^L\) satisfies \(0 \in Y(S)\). Each coalition chooses a vector of production \(y^S\). Then the production set of the agents of country \(k\) is \(Y(I_k)\) (denoted \(Y_k\) from now on so its elements will be written as \(y_k\)). It is also assumed that if \(l\) does not belong to \(N_k \cup T\), then \(y^S_l = 0\) whenever \(S \subset I_k\) (before the reform) and \(y^S \in Y(S)\). This assumption implies that even though owners of the firms can migrate and offer labour in other countries, they cannot transport their production activities with them. Each production set is assumed to be closed and convex. They are also assumed to satisfy the requirement that for each aggregate lower bound \(y\) the constrained set of international production allocations

\[ Y^K(y) = \{ y^K \in Y^K = \prod_{k \in K} Y_k \mid \sum_{k \in K} y_k \geq y \} \]

is bounded. This means that in the international economy, bounded inputs only allow bounded outputs in each separate country.

\[ ^1\text{As shown in Hildenbrand (1974), p. 227, problem 3, these economies have a Private Ownership Economy as a special case.} \]
Denote by $q_k$ the vector of consumer prices of country $k$, which could be different from producer prices $p$ because of commodity taxation. Let $q = \{q_k\}_{k \in K}$. Also denote by $m^i_k$ the net transfers consumer $i$ gets from production profits and government transfers if he lives in $k$. Let $m^i = \{m^i_k\}_{k \in K}$.

For each consumer we can define the budget set correspondence if he lives in country $k$,

$$B^i_k(q_k, m^i_k) = \{x^i \in X^i \mid q_k x^i \leq m^i_k\}$$

where $m^i_k$ is the net transfer received if $i$ lives in $k$. The budget set correspondence if he can migrate to any $k \in K$ is given by

$$B^i(q, m^i) = \bigcup_{k \in K} B^i_k(q_k, m^i_k)$$

The compensated demand correspondence is

$$\xi_c^i(q, m^i) = \{x^i \in B^i(q, m^i) \mid \hat{x}^i \succsim_i x^i \Rightarrow q_k \hat{x}^i \geq m^i \forall k\}$$

and the Walrasian demand correspondence,

$$\xi_w^i(q, m^i) = \{x^i \in B^i(q, m^i) \mid \hat{x}^i \succ_i x^i \Rightarrow q_k \hat{x}^i > m^i \forall k\}$$

These compensated demands will be non-empty if the budget correspondences are non-empty and will have a closed graph provided that the set of preference orderings is endowed with the closed convergence topology. Walrasian demands will be non-empty for $(q, m^i)$ such that the budget correspondence is non-empty and the correspondence will also have a closed graph whenever there is a cheaper point $\hat{x}^i \in X^i$ with $q_k \hat{x}^i < m^i_k$ in the corresponding “local” consumption set (conditioned to the decision of location).

With our definitions of demands, we have given each consumer the possibility of choosing between the different budget sets which he would face in each different country. Given that we have incorporated the migration costs in the consumption sets, his choice represents a choice of country. Compensated demands are demand that minimize cost, when the individual can choose the country where he consumes or supplies. In a similar way we can define Walrasian demands as those which maximize utility subject to a budget constraint that could be different for different countries where the consumer can choose between different sets of non-traded goods and different commodity taxes. This definition generalizes the one used in Bewley (1981) in the context of local public goods theory.
We want to find an allocation that is Pareto superior to a prespecified status quo in the customs union \((\bar{x}, \bar{y})\). In this paper we assume that the pre-reform allocation satisfies a balance of trade constraint, which could be expressed in terms of averages \(w\bar{z}_k = \bar{b}_k\) for each country \(k\), where \(w\) is the world price vector for traded goods, \(\bar{b}_k\) is a maximum allowable average deficit, and \(\bar{z}_k\) is the net mean import vector which could be defined by

\[
\bar{z}_k = \int_{I_k} \bar{x}^i d\nu - \bar{y}(I_k) = \bar{x}_k - \bar{y}_k
\]

In order to leave the rest of the world unchanged when the customs union is formed, we assume, following most of the literature on the gains from union formation, that an external tariff can be set so that both world prices \(w\) and the average amount of trade per head

\[
\bar{z} = \int I \bar{x}^i d\nu - \bar{y}(I) = \bar{x} - \bar{y}
\]

of the union with the rest of the world, remain constant. This means that, even though the amount traded by any particular country could be different than before the reform, this difference is compensated by other offsetting changes so that the total is the same. It is important to realize that after the change, tariffs among the members disappear and therefore there will be a common vector of producer prices \(p\) for traded goods. Thus, in our model, the vector of external tariffs \(p - w\) and producer prices \(p\) will be endogenous, adjusting to clear commodity markets across the union while keeping \(\bar{z}\) and \(w\) constant.

We assume that coalitions of producers maximize profits after the reform. For every \(p \neq 0\), we define coalition \(S\)'s supply correspondence by

\[
\eta^S(p) = \arg \max \{py^S | y^S \in Y(S)\}
\]

which will be non-empty and have a closed graph for prices at which profits are bounded. We also can define the profit function as

\[
\pi^S(p) = \max \{py^S | y^S \in Y(S)\}
\]

Country \(k\)'s average supply function \(\eta_k(p) := \eta_{I_k}(p)\) and country \(k\)'s average profit function, \(\pi_k(p) := \pi_{I_k}(p)\) are defined in a similar fashion. Suppose too
we also assume that the production set correspondence is countably additive (so that $Y(\bigcup_{a=1}^{\infty} S_a) = \sum_{a=1}^{\infty} Y(S_a)$, for $S_a$ pairwise disjoint) and absolutely continuous with respect to $\nu$. Then by the Radon-Nikodym Theorem, there are measurable functions $\eta^i(p)$ and $\pi^i(p)$ such that

$$\eta^S(p) = \int_S \eta^i(p) d\nu$$
$$\pi^S(p) = \int_S \pi^i(p) d\nu$$

which represent unambiguous individual allocations of production and profits within each coalition. Now we assume that the reform improves production efficiency in the sense that for each country forming the customs union, the productive sector as a whole makes more profits, on average, by adjusting its plans than by staying where it is, so $\pi_k(p) > \bar{\pi}_k$. What is required here is that the pre-customs union allocation belongs to the interior of the aggregate production set of the union as a whole. This could be satisfied, even in the case where production was efficient before the reform in each of the different countries. We only need that the normals to the hyperplanes supporting the pre-reform production vectors were different for different countries before forming the customs union (as would happen because of the existence of distorting tariffs).

3 Potential Pareto Gains

Supply-side policies which improve production efficiency, even though they increase aggregate real income, do not ensure by themselves a Pareto improvement. A reform can make people better off (for instance, those owners of firms which benefit from the reform) and some others worse off (like those with ties to industries that are not competitive after the reform). In our examples of non-convex consumption sets the consequences of a reform could, dramatically, force some people to a situation of starvation. In the example with migration, it could force people to migrate in a very disadvantaged position.

\footnote{The Radon-Nikodym derivative is unique up to $\nu$-equivalence, assuming that the measure is $\sigma$-finite (each measurable set can be expressed as countable union of sets of finite measure) (see, for instance, Royden (1988)).}
Thus, getting a Pareto gain requires designing an appropriate redistribution mechanism. We will show two different kinds of mechanism depending on informational assumptions about the government. For both types we will specify some general assumptions. Let $m_i(p)$ be unearned income of consumer $i$ — that is, income coming from transfers from the public sector and dividends of firms. Let $q_k(p)$ be consumer prices of country $k$, its dependence on producer prices being due by commodity taxation. We assume that these functions are continuous and homogeneous of degree one in $p$. Later on, we will also make some important assumptions about the measure distribution of the transfers. Formally, finding a Pareto gain requires finding $\{p, [m_i(p)]_{i \in I}, [q_k(p)]_{k \in K}\}$ such that:

(i) $\hat{y}^i \in \eta^i(p) \ \forall i \in I \ \nu$-a.e.

(ii) $\hat{x}^i \in \xi^i_0(q(p), m_i(p)) \ \forall i \in I \ \nu$-a.e.

(iii) $\int_I \hat{x}^i d\nu - \int_I \hat{y}^i d\nu = \bar{z}$ and

(iv) $\hat{x}^i \succ_i \hat{y}^i \ \forall i \in I \ \nu$-a.e.

4 Gains with Migration: Lump Sum Compensation

Here the government has available enough information about people's characteristics to use lump sum transfers. Now no commodity taxation is used so $q_k(p) = p$. Thus the choice of country only depends on non-internationally-traded goods, such as job opportunities and housing, and not on different consumer prices for tradeables.

Each country joining the union has to reform the tariff system. This will have positive effects (in terms of revenue) for some countries and negative effects for some others. These tariff revenue effects could impede the implementation of the Pareto gain. To avoid this possibility, we follow the literature (see, for instance, Grinols (1981)) and postulate intergovernmental transfers which compensate for the loss of tariff revenue. We assume that the union-wide revenue from the common external tariff forms a community fund. This gets divided between the member countries following the pre-reform pattern of trade. Specifically, when producer prices are $p$, each
member country gets a transfer of \((p - w) \tilde{z}_k\) even if its net import vector differs from \(\tilde{z}_k\). We also assume that the amount each country is allowed to distribute, on average, does not exceed its national mean income plus mean external borrowing and mean transfers derived from the external tariff, so

\[
\int_{l_k} m_i(p) d\nu \leq b_k \leq (p - w) \tilde{z}_k + \pi_k(p) = p\tilde{z}_k + \pi_k(p)
\]

The transfers must be constructed to assure each consumer more than enough income to purchase his pre-reform allocation at the new prices, so that \(m^i(p) > p\bar{x}^i\). With our assumption of increased production efficiency, this would be obviously feasible in average, given that

\[
\int_{l_k} m_k(p) d\nu = p\tilde{z}_k + \pi_k(p) > p\tilde{z}_k + p\bar{y}_k
\]

for each \(k \in K\). Under the assumptions of section 2, there exists a compensated equilibrium \(\{p, [\hat{x}^i_{i \in I}, [\hat{y}^i]_{i \in I}]\} in which:

(i) \(\hat{y}^i \in \eta^i(p) \ \forall i \in I \ \nu\text{-a.e.}\)

(ii) \(\hat{x}^i \in \xi^i_{i \in I}(p, m^i(p)) \ \forall i \in I_k\) and \(k \in K \ \nu\text{-a.e.}\)

(iii) \(\int_I \hat{x}^i d\nu - \int_I \hat{y}^i d\nu = \bar{z}\)

The construction of the transfers gives each consumer more than enough income to buy its pre-reform allocation. This ensures the existence of a cheaper point in the consumption set of each consumer, avoiding Arrow's exceptional case. In a model with convex consumption sets, compensated demand would coincide with Walrasian demand and our equilibrium would be a competitive one. In a model with non-convexities this is not true any more. Taking the case of migration, this is not enough to prevent the existence of a non-null set of individuals from country \(k\) who can more than afford their pre-reform consumption, but have now migrated and are stuck at the lower boundary of one of the sets \(X^i_{kk'}\ (k \in K)\). This would happen when migration is clearly better, even though the migrant is at a cheapest point of his conditionally feasible set, given the decision to migrate. This difficulty could prevent the existence of competitive equilibrium.

To avoid this problem we need an additional assumption, which is motivated by Yamazaki's (1981) dispersed endowments and Coles and Hammond's
(1991) dispersed needs assumption. To state this assumption formally, define first the sets, for \( i \in I_k \) and \( k \in K \),

\[
P^i = \{ p \mid \exists k'; \exists \hat{x}^i \in \xi^i(p, m^i(p)) \cap X^i_{kk'}:
\forall \hat{x}^i \in X^i_{kk'}: p\hat{x}^i \geq m^i(p) \}
\]

and,

\[
I(p) = \{ i \in I \mid p \in P^i \}
\]

Now assume:

**DISPERSED COMPENSATION 1:** The lump-sum transfers satisfy \( \nu(I(p)) = \frac{0}{0} \forall p \neq 0 \).

What we are requiring is that the measure distribution of the transfers conditioned to the consumption set is atomless at the points of difficulty, in the sense that it gives null measure to the set of agents receiving the same transfer and having the same feasible set, for those agents who would decide to migrate and their consumption allocation would be a cheapest point of the conditionally consumption set given their decision as to which country to live in. Then the possible discontinuities of individual demand are dispersed so mean demand preserves continuity. A very simple example of a transfer system satisfying our conditions is the following:

**EXAMPLE:** Assuming that all consumers in country \( k \) have the same consumption set and pre-reform allocation, the transfer system will give each \( i \in I_k \), for each \( k \),

\[
m^i(p) = p\hat{x}^i + \theta^i_k[\pi_k(p) - p\hat{y}_k]
\]

where \( \theta^i_k \) is uniformly distributed on \((0, \bar{\theta})\), where \( \bar{\theta} \) is fixed in order to satisfy \( \int_{I_k} \theta^i_k(d\nu) = 1 \).

As shown below, this is sufficient to have a cheaper point in the local feasible set, for almost every consumer and so to make our compensated equilibrium a competitive one.

**LEMMA:** Assuming that the transfers satisfy "Dispersed compensation 1", \( x^i \succ_i \hat{x}^i \Rightarrow px^i > p\hat{x}^i \ nu-a.e. \) at the compensated equilibrium.

**PROOF:** There are two possible cases to consider. The first happens when the individual does not migrate. In this case, because of the construction of the
transfers and budget exhaustion, \( p\hat{x}^i = m^i > p\tilde{x}^i \). The pre-reform allocation provides trivially a cheaper point. If, at the compensated equilibrium the consumer has migrated to country \( k' \) (\( \hat{x}^i \in X_{kk'} \)), suppose \( p\hat{x}^i = px^i \) and \( x^i \succ_i \hat{x}^i \). Then, by our assumption, for \( I \setminus I(p) \), there exists \( \hat{x}^i \in X_{kk'} \) such that \( p\hat{x}^i > p\tilde{x}^i \). By convexity of \( X_{kk'} \) and continuity of preferences, \( x^i(\epsilon) = (1 - \epsilon)x^i + \epsilon\hat{x}^i \in X_{kk'} \) and \( x^i(\epsilon) \succ_i \hat{x}^i \) for small \( \epsilon > 0 \), while \( px^i(\epsilon) < p\hat{x}^i \). This contradicts the hypothesis that \( \hat{x}^i \) was a compensated demand.

From the lemma and the discussion above we conclude:

**Proposition 1** With the assumptions of section 2 and assuming “dispersed compensation 1”, there exists a competitive equilibrium \( v.a.e. \) In this equilibrium, every consumer is better off than at the pre-reform allocation.

This transfer system depends on what each individual would have consumed in absence of the reform. This is a function of the preferences and incomes of consumers and these parameters are generally private information. Therefore the transfer system, following Hammond (1979), is generally not incentive compatible. A different problem concerning the economy with migration analysed in this section is that achieving the Pareto gain also requires the government of each \( k \) to be able to compensate all the individuals in \( I_k \), even though the set of consumers currently living in \( k \) could be different. That is, each government has to compensate all consumers who were living in its country before the reform, independently of where are they currently living. This happens because, even though each national production sector has profited with the reform, it does not mean that each country can afford the pre-reform consumption allocation of all its inhabitants (including immigrants) at new union prices. The first of these difficulties will be sorted out in the next section, where we use an incentive compatible mechanism.

## 5 Gains with Migration: Incentive Compatible Compensation

In this section, we use an incentive compatible mechanism to distribute the potential gains. Dixit and Norman (1980) devised an incentive compatible mechanism based on movements in commodity taxes. They assumed that
the government had enough tax instruments to freeze consumer prices and that firms did not have positive profits. Then they assumed the existence of a positive direction of reform in commodity taxes — i.e. the existence of a commodity either purchased or sold by every consumer. By movements on the price of this commodity they could get a Pareto gain. In order to get an equilibrium, they needed the government to buy the excesses in supply and to be able to dispose of them freely. The mechanism we are going to use is the one designed by Hammond and Sempere (1992). They extended Dixit and Norman (1980), assuming explicitly that the government could freeze consumer prices and dividends and using a poll subsidy to distribute the gains, without requiring the knowledge of a positive direction of commodity tax reform. They showed the existence of a competitive equilibrium without assuming any sort of free disposal.

Now we require the original consumers' allocation \( \bar{x}^i \) to be an equilibrium for the consumers at prices \( \tilde{q} = \{\tilde{q}_k\}_{k \in K} \) and unearned incomes \( \tilde{m}^i \) so that, for each consumer \( i \in I_k \) and each \( k \),

\[
\bar{x}^i \in \xi_w^i(\tilde{q}, \tilde{m}^i)
\]

We also assume that this set is single-valued at incomes \( \tilde{m}^i \) and prices \( \tilde{q} \) and that \( \bar{x}^i \) and \( \tilde{m}^i \) are integrable in \( I \).

Now, in order to avoid any consumer being harmed by a change in commodity prices or unearned income, assume that each country \( k \) uses indirect taxes to freeze consumer prices and income taxes to freeze any sort of unearned income. Given that across the union there is a unique producer price vector, that means that each country is fixing separately its own vector of commodity taxes. Assume also that if these policies create imbalances in each national public sector budget, these are given to the consumers in the form of a nationwide poll subsidy \( s_k \). From these assumptions, the budget constraint for \( i \) if he lives in \( k' \) after the reform is

\[
B_{k'}^i(s_k) = \{x^i \in X^i \mid \tilde{q}_k x^i \leq \tilde{m}^i + s_k\}
\]

so the budget constraint faced by consumer \( i \in I \) is

\[
B^i(s_k) = \bigcup_{k' \in K} B_{k'}^i(s_k)
\]

In order to show the possibility of Pareto gains we have to prove existence of equilibrium with frozen consumer prices and dividends, with free producer
prices and a poll subsidy distributing the potential gains. We do the proof in two steps. First we show the existence of compensated equilibrium. After that we set out conditions that make the compensated equilibrium a competitive one.

We start by showing the existence of a compensated equilibrium.

**Lemma:** With our assumptions, there exists a compensated equilibrium.

**Proof:** We will apply a fixed point argument to a correspondence suitably constructed inspired by the one used in Hammond and Sempere (1992).

Consider first the mean compensated demand correspondence for individuals who originated in country $k$, which is

$$\xi_c(s_k) = \int_{I_k} \xi^i_c(s_k) d\nu$$

Each $\xi^i_c(s_k)$ has a closed and measurable graph in the domain of characteristics of the measurable economy. Trivially, if we define $\bar{q} = \inf_{i \in L} \{q^i\}$, each budget set is bounded above by the vector $ [(s_k + \bar{m}^i)/\bar{q}] \bar{1}$ where $\bar{1}$ is the vector $(1, 1, \ldots, 1) \in \mathbb{R}^L$. For any given $s_k$ in a bounded set and any $\bar{q} > 0$, the function $\xi^i_c(s_k)$ is integrable in $I_k$. From the discussion of Yamazaki (1981), we conclude that mean demand is non-empty valued and it has a closed graph. Given that $\nu$ is non-atomic, mean demand is also convex valued. Consumption sets are not bounded. Given that the restricted set of possible mean international productions for the customs union is bounded (write $y^*$ for an upper bound) any feasible mean consumption must satisfy

$$x \leq y^* + \bar{z} - \int_I x^i d\nu$$

We define the truncated mean consumption sets

$$X_h = \{ x \in \int_I X^i d\nu \mid x \leq y^* + \bar{z} - \int_I x^i d\nu + h\bar{1} \}$$

for each consumer. Here $h$ is a strictly positive natural number and $\bar{1}$ is a vector of ones. The truncated mean consumption sets $X_h$ are obviously compact and convex sets.
Each aggregate supply correspondence is non-empty valued, defined in a suitable compact set \( Y \). As shown by Debreu (1959), an equilibrium relative to these sets will also be a equilibrium relative to the original \( Y(I) \), given that the constrained set of international productions is compact and that \( Y(I) \) is convex (see also Hammond and Sempere (1992)). It will have a closed graph. Similarly, the mean national supply correspondence \( \eta_k(p) \) will have non-empty, compact and convex values, a closed graph and it will take values in a compact, convex set.

Define \( x_k \in \xi_c(s_k) \), a compensated national average demand. From each \( x_k \) we get union average consumption \( x = \int_J x^i \, dx \). Write also \( y_k \in \eta_k(p) \) for each national average production. Given the additivity of the production set correspondence, these will be unambiguously related to the union average production \( y \). Write also \( z \) for \( x - y - \bar{z} \).

As the price adjustment correspondence, we use

\[
P(z) = \begin{cases} \frac{z}{\|z\|} & \text{if } \|z\| > 0 \\ B & \text{if } \|z\| = 0 \end{cases}
\]

where \( B = \{ p \in \mathbb{R}^k \mid \|p\| \leq 1 \} \). It consists of a single point on the boundary of \( B \) unless \( x - y - \bar{z} = 0 \), in which case it consists of the whole of \( B \). This correspondence will have non-empty, compact and convex values in a compact and convex set and also a closed graph, given that the value of the mean excess demand has a closed graph.

Write \( \bar{m}_k \) for \( \int_{\mathcal{L}_k} \bar{m} \, dv \). We also define the poll subsidy adjustment correspondence,

\[
\sigma_k(x_k, y_k, p) = \max \{ 0, p(\bar{z}_k + y_k - x_k) + \bar{q}_k x_k - \bar{m}_k + 1 - \|p\| \}
\]

This correspondence will have non-empty and convex values. Its range \( S_k \) is convex and compact (because it is u.h.c. in a compact domain). We can now apply Kakutani’s fixed point theorem to the product of the correspondences

\[
\prod_{k \in K} [\xi^i_k(s_k) \times \eta_k(p) \times \sigma_k(x_k, y_k, p)] \times P(x, y)
\]

So there are infinite sequences of fixed points \( \{ [\hat{x}_{kh}, \hat{y}_{kh}, \hat{s}_{kh}]_{k \in K}, p_k \} \) and of integrably bounded measurable mappings \( \{ \hat{x}^i_{kh}, \hat{y}^i_{kh} \} \) such that: \( \hat{x}^i_{kh} \in \xi^i_k(s_{kh}) \)
a.e. in $I_k$; $\hat{x}_{kh} = \int_{I_k} \tilde{x}^i_{kh}(du)$ and $\hat{y}_{kh} = \int_{I_k} \eta^i(p_h)$ a.e. in $I_k$; $\hat{y}_{kh} = \int_{I_k} y^i_{kh}(du)$. The following lemma shows that for each $h$, $\|p_h\| = 1$ and $\|\hat{x}_h - \hat{y}_h - \tilde{z}\| = \|z_h\| = 0$.

**Lemma:** At the fixed point for each $h$ one has $\|p_h\| = 1$ and $\|\hat{x}_h - \hat{y}_h - \tilde{z}\| = \|z_h\| = 0$.

**Proof:** Suppose instead that $\|\hat{x}_h - \hat{y}_h - \tilde{z}\| > 0$. From the definition of $P(z)$, $p_h = z_h/\|z_h\|$ so

$$\|z_h\| [p_h z_h] = \|z_h\| \frac{z_h}{\|z_h\|} z_h = \|z_h\|^2 > 0$$

This implies that $p_h z_h > 0$ and so there exists $k$ for which $p_h \hat{z}_{kh} = p_h (\hat{x}_{kh} - \hat{y}_{kh} - \tilde{z}_k) > 0$. Then, since $\hat{z}_{kh}$ is a fixed point, from the definition of $\sigma$, it follows that

$$\hat{z}_{kh} = \max\{0, p_h (\zeta_k + \hat{y}_k - \hat{x}_{kh}) + \tilde{q}_k \hat{x}_{kh} - \hat{m}_k + 1 - \|p_h\|\}$$

If $\hat{z}_{kh} = 0$, given the assumption that the pre-reform individual demands were single valued, it follows that $\bar{x}_k = \int_{I_k} \xi^i(0) du$. By definition of maximum,

$$0 < p_h (\bar{x}_k - \hat{y}_k - \tilde{z}_k) \leq \tilde{q}_k \bar{x}_k - \hat{m}_k + 1 - \|p_h\|$$

We also assumed that at the pre-reform allocation consumers were satisfying their budget sets, so $\tilde{q}_k \bar{x}_k = \hat{m}_k$ and hence $\tilde{q}_k \bar{x}_k = \hat{m}_k$. This implies that $\|p_h\| < 1$ since there is at least one $k$ for which $p_h \hat{z}_{kh} > 0$.

If $\hat{z}_{kh} > 0$ then the fixed point satisfies

$$p_h (\hat{x}_{kh} - \hat{y}_{kh} - \tilde{z}_k) = \tilde{q}_k \hat{x}_{kh} - \hat{m}_k - \hat{z}_{kh} + 1 - \|p_h\|$$

Consumer budget balance implies that also $\|p_h\| < 1$ if $p_h \hat{z}_{kh} > 0$. From the above discussion, we conclude that $p_h \hat{z}_h > 0$ implies $\|p_h\| < 1$. By definition of $P(z)$, $\|\hat{z}_h\| > 0$ means that $\|p_h\| = \| [z_h/\|z_h\|] \| = 1$. This contradiction implies that $\|p_h\| = 1$ and $\|z_h\| = 0$. □

Given these results, the sequence of fixed points takes values satisfying

$$\hat{x}_h \in \{x_h \in \mathbb{R}^L \mid \int_I \bar{x}^i(du) \leq x_h = y_h + \tilde{z} \leq y^* + \tilde{z}\}$$
\[ \delta_{kh} \in \{ s_{kh} \in \mathbb{R} \mid 0 \leq s_{kh} \leq p_h (\bar{z}_k + y_{kh} - x_{kh}) + \bar{q}_k x_{kh} - \bar{m}_k \} \]

and \( p_h \in \bar{B} = \{ p_h \in \mathbb{R}^L \mid \|p_h\| = 1 \} \). The lower bound of the poll subsidy\(^3\) gives obvious lower bounds for mean demands and upper bounds on mean supplies. The upper bound of the restricted production set gives upper bounds for mean demand and the poll subsidy. These sets are compact. So there must be some subsequence which converges to a limit point which will be denoted by \( [(x^*_i, y^*_i, s^*_k)_{k \in K}, p^*] \). Given that all the \( \hat{x}^*_k \), \( \hat{y}^*_k \) are integrable, we can apply Fatou’s Lemma in many dimensions to show that there are subsequences \( h(m) \) \((m = 1, 2, \ldots)\) of \( h = 1, 2, \ldots \) with some \( p \in \bar{B} \) and \( s_k \) in the set defined above, and measurable functions \( x^{*i} \) such that: (i) \( \int_I x^{*i} d\nu \leq x^* \); and, as \( m \to \infty \), so: (ii) \( p_{h(m)} \to p^* \); (iii) \( s_{kh(m)} \to s^*_k \); (iv) \( x_{kh(m)} \to x^*_k \); (v) \( \hat{x}_{h(m)} \to x^{*i} \) a.e. in \( I \).

By the properties of the fixed point and given that compensated demands and supply correspondences have closed graphs, \( x^{*i} \in \xi^i(s^*_k) \) and \( x^* - \bar{x} = \hat{y} \in \eta^i(p^*) \). From the discussion above \( \bar{x} = x^* - \hat{y} \). So it defines a compensated equilibrium. \( \square \)

If \( s^*_k = 0 \), then the definition of the national poll subsidy implies that

\[ 0 = \bar{m}_k - \bar{q}_k \bar{x}_k = p^* \hat{y}_k + p^* \bar{q}_k - p^* \bar{x}_k \]

Given that pre-reform demand was single valued and that the pre-reform allocation was balanced, this contradicts our assumption of an improvement in productive efficiency. This implies that the poll subsidy is positive for every country forming the union. In a convex economy this would be enough to have a cheaper point for each consumer and so to avoid Arrow’s exceptional case. Here, as discussed in section 3, a non-null set of consumers could migrate from one of the countries to some other, and even after receiving a poll subsidy still be on the lower boundary of the conditional consumption set. This could imply the existence of some points of discontinuity in mean Walrasian demand, preventing the existence of a competitive equilibrium. To avoid such discontinuities in mean demand, the obvious assumption in our continuum economy is to require that individuals are sufficiently dispersed. For each \( i \in I_k \), define the set

\[ S^i = \{ s_k \mid \exists \hat{\xi}^i \in \xi^i(s_k) \cap X^i_k, k' \} \]

\(^3\)Remember that mean demand was assumed to be single-valued at \( s_k = 0 \).
and,

\[
\forall \hat{x}^i \in X_{kk}^i : \hat{q}_k \hat{x}^i \geq \tilde{m}^i + s_k \}
\]

and,

\[
I(s) = \{ i \in I \mid i \in I_k \Rightarrow s_k \in S^i \}\]

Our assumption is:

**DISPERSED COMPENSATION 2:** At the compensated equilibrium, \( \nu(I(s)) = 0 \).

As in section 2, what is required is that the distribution of the budget sets conditioned to the consumption sets is non-atomic in the points of discontinuity. In this case, given that the poll subsidy is the same for all individuals of the same country, what is needed is enough dispersion of the points of possible discontinuity of individual demand. This requires enough dispersion in the consumption sets and pre-reform income. As in section 3, this is enough to have a cheaper point, so

**LEMMA:** Assuming that the transfers satisfy "Dispersed compensation 2", \( x^i \succ_i \hat{x}^i \implies px^i > p\hat{x}^i \nu-a.e. \) at the compensated equilibrium.

**PROOF:** Trivial extension of the proof of the lemma in section 3. \( \square \)

And we also conclude that

**Proposition 2** Under the assumptions of section 2 and assuming "dispersed compensation 2", there exists a competitive equilibrium \( \nu-a.e. \) In this equilibrium, every consumer is better off than at the pre-reform allocation.

To conclude this section we remark that the compensatory mechanism is incentive compatible. In this case it was also required every consumer to receive a subsidy dependent of where were they currently living.

### 6 Gains from Trade without Survival

On the relationship between international trade and famines there is not a practical definitive conclusion. As explained very clearly by Drèze and Sen (1989) in their important contribution to the analysis of famines, on one hand we have the idea that when the scarcity of food shows up, the increase of needs would be partly balanced by more international trade which would also reduce food prices serving as a means of entitlement protection. But
it is also observed in the history of famines that there have often been exports of food from famine affected areas. The argument is that if people do not have enough income to buy food at current prices, then the response of private traders would be to sell their product in the foreign market. In the latter, more international trade would obviously worsen the situation in the area unless people are given enough cash to buy food. Sen (1981) considers a situation of starvation as one in which, in the exchange entitlements set (in competitive economies, the budget set), there is not enough food. This would define the starvation set as those commodity vectors such that the consumer cannot meet his food requirements through exchange. He suggests introducing the survival question into general equilibrium models by bringing in social security transfers preventing starvation or minimum entitlements transfers. Dasgupta and Ray (1987) characterize an equilibrium with malnourishment and involuntary unemployment in a model with workers' characteristics related with food intake. The work of Coles and Hammond (1991) shows by standard general equilibrium techniques that all the usual theorems of General Equilibrium theory remain valid when the assumption of full survival of the population gets removed. In particular, the very important conclusion of their paper is that famines and starvation can occur as a consequence of the inequalities derived from the market mechanism without any reference to market failures. This would justify public intervention favouring a better wealth distribution.

In this section, we consider the model of section 2 with the Coles and Hammond (1991) consumption set. We will analyse conditions for Pareto gains to result from market integration in customs unions when the survival assumption is removed and the kind of compensation used in sections 4 and 5 is used.

6.1 Lump-Sum Compensation

As Drèze and Sen (1989) point out, a possible way to fight famines that is compatible with the market mechanism is to give cash to people affected. In the case analysed now, assuming that the government is able to use lump-sum compensation, the price system is left undistorted and people are given

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4Two cases reported in Drèze and Sen (1989) concern the famine of 1880 in India and the Irish famine of 1845-8.
enough income to buy the allocation they would have purchased before the trade reform. By using this system, the entitlements to food that people had before the reform would be guaranteed by giving them cash.

Now we require the lump-sum system to satisfy $m^i(p) > px^i$ so each one has more income than that needed to purchase their pre-union allocation. This is feasible in aggregate given our assumption of improvement in productive efficiency. Following the same argument as in section 4, we could conclude that there is a compensated equilibrium in which each consumer would gain. This means that all survivors and non-survivors gain, no survivor is made a non-survivor and some non-survivors could be made survivors. In order to show that Pareto gains result from such a reform, we have to show existence of competitive equilibrium. The problem here, as noticed by Coles and Hammond (1991), is that none of our assumptions rules out the existence of a non-null set of individuals who, after getting the transfers, remain at the limit of survival. This fact would impede the upper hemi-continuity of mean demand, preventing the existence of competitive equilibrium. To rule out this case we make an additional assumption, based on the assumption of dispersed needs of Coles and Hammond (1991). To express this assumption formally, define first the sets

$$P^i = \{p \mid \exists \bar{x}^i \in \xi^i(p, m^i(p)) \cap X^i_S \text{ and } \forall \bar{x}^i \in X^i_S : px^i \geq m^i(p)\}$$

and,

$$I(p) = \{i \in I \mid p \in P^i\}$$

Now assume:

**Dispersed Compensation 3**: The lump-sum transfers satisfy $\nu(I(p)) = 0$ $\forall p \neq 0$.

In this case what we require is that the distribution of transfers conditioned on the consumption sets is dispersed. The main difference from Coles and Hammond's assumption is that here we could allow people to have very similar needs, as long as the transfers are dispersed enough. This assumption ensures that the set of people on the margin of survival has null measure. Then the discontinuities in individual demand will be dispersed enough that
they will not cause mean demand to be discontinuous. With this assumption and following the reasoning of section 4, the existence of competitive equilibrium and so a Pareto gain would be ensured.

The critique of the compensation mechanism relies first on incentive incompatibility. Its feasibility depends on the knowledge of preferences and income of individuals by the government. This is hardly true in real economies. A different sort of critique is based on the fact that, even though survival of people who would survive in the absence of a reform is assured, this is compatible with some proportion of the population remaining in a situation of starvation. It would probably be better to use the gains in productive efficiency derived from trade in order to increase the proportion of survivors in the population, even if those who are far from the margin of survival are made worse off.

6.2 Incentive Compatible Compensation

In this case, we use the mechanism used in section 5 to distribute the gains from trade. Now the government freezes consumer prices and uses a uniform subsidy to distribute the gains. Relating this mechanism to those proposed by Drèze and Sen to fight famines, we would protect the food entitlements of citizens by controlling food prices and we would also increase those entitlements by providing cash.

To prove the existence of Pareto gains is equivalent to proving existence of competitive equilibrium when consumer prices are frozen, producer prices adjust to clear markets, and people are given a poll subsidy. With the same argument as in section 5 we can show existence of a compensated equilibrium in which each survivor is made better off, no survivor becomes a non-survivor and some survivors could be made survivors. Again, none of our assumptions rules out the existence of a non-null set of consumers who after the reform find themselves stuck at the lower boundary of the survival set. To avoid this possibility we use an assumption similar to the one used in section 6.1 based in turn on the dispersed needs devised in Coles and Hammond (1991). Define first the sets, for \( i \in I_k \) and \( k \in K \)

\[
S^i = \{ s_k \mid \exists \tilde{z}^i \in \xi_c^i(s_k) \cap X^i_S \ \text{and} \ \forall \tilde{z}^i \in X^i_S : \tilde{q}_k \tilde{z}^i \geq \tilde{m}^i + s_k \} \]
and,

$$I(\tilde{s}) = \{i \in I \mid \tilde{s}_k \in P^i, k \in K\}$$

The new assumption is:

**Dispersed Compensation 4:** At the compensated equilibrium,\( \nu(I(\tilde{s})) = 0 \).

In this case what is required is enough dispersion of the lower boundary of the consumption sets and pre-reform income. It would make the discontinuities of individual demand dispersed enough so that mean demand preserves continuity properties. With this assumption, the same argument as in section 5 proves the existence of Pareto gains.

By using this mechanism, we would assure the survival of all those who would have survived in absence of the trade reform. It is also true that some of the people who would not have survived without the reform could be made survivors. But, as remarked in section 6.1, some people may not survive even after the reform. So it is probably a socially better policy to increase the number of survivors, using the gains from productive efficiency, without looking for a Pareto gain.

7 Final Remarks

In this paper we have given conditions to get Pareto gains from market integration in customs unions when individual non-convexities in consumer sets and preferences are allowed. The main assumption, different from that used in standard general equilibrium analysis, was that the type of compensation had to be dispersed. This let us consider two particular examples. The first happened when people were allowed to migrate from one country to other among those forming the union. In the second we removed the assumption of full survival of the population. In both cases, given that the formation of the union could have made some people better off, but that it also could have harmed some others, it was necessary to find a redistribution mechanism to convert the gains in productive efficiency into effective Pareto gains.

In the case of migration, we need very important informational and institutional requirements to be able to compensate the losers (to know the evolution of prices, to be able to freeze prices and dividends, etc...). We also
had to do the compensation of people depending on their location in absence of the reform.

When we removed the assumption of full survival, we associated our compensation mechanisms with different ways of protecting people's food entitlements. In the two cases analysed, we could find a Pareto gain in which all survivors were made better off and some of the non-survivors could be made survivors. Apart from the problems with the practical implementation of the redistribution mechanisms, we find that it would be socially desirable to use the gains in productive efficiency to maximize the number of survivors, instead of looking for a Pareto gain.

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