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Investment and Forward-looking Monetary Policy: A Wicksellian Solution to the Problem of Indeterminacy

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Abstract

Recent research has shown that forward-looking Taylor rules are subject to indeterminacy in New Keynesian models with capital and investment spending. This paper shows that adopting a forward-looking Wicksellian rule that responds to the price level, rather than to inflation, is one potential remedy to the indeterminacy problem. This result is shown to be robust to variations in both the labor supply elasticity and the degree of price stickiness, the inclusion of capital adjustments costs, and if output also enters into the interest-rate feedback rule. Finally, it is shown that the superiority of Wicksellian rules over Taylor rules is not only confined to forward-looking policy, but also extends to both backward-looking and contemporaneous-looking specifications of the monetary policy rule.

JEL Classification Number: E22; E31; E52; E58

Keywords: Equilibrium determinacy; Interest-rate rules; Monetary policy; Investment; Taylor rule.

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1 Introduction

Due to the widely documented time lags in the transmission of monetary policy, the importance of conducting monetary policy in a forward-looking manner has long been recognized.[1] One important issue, relating to the design of monetary policy rules, is that they should avoid generating multiple equilibria, or indeterminacy, which can destablize the economy through the emergence of expectations-driven, welfare-reducing fluctuations. It has been well established in the literature that under the Taylor principle, that is a policy that raises the nominal interest rate by proportionally more than the increase in inflation, forward-looking Taylor rules can easily achieve local equilibrium uniqueness, or determinacy, in sticky-price New Keynesian models where labor is the only factor of production.[2] However, Carlstrom and Fuerst (2005) find that "determinacy is essentially impossible "under forward-looking Taylor rules once the economic environment allows for capital and investment spending.

To date, the literature has proposed few potential remedies for this indeterminacy problem. Kurozumi and Van Zandweghe (2008) advocate that the Taylor rule be amended to also include contemporaneous output. While this interest-rate policy is effective under an infinite labor supply elasticity, Huang et al. (2009) show that this amendment helps little in preventing indeterminacy with finite, empirically plausible values for the labor supply elasticity.

In this paper, we propose an alternative solution to overcome the indeterminacy problem of forward-looking monetary policy in the presence of investment spending. We show that equilibrium determinacy is easily induced if the interest-rate rule reacts to future fluctuations in the price level (a so-called Wicksellian rule) rather than future fluctuations in inflation, as prescribed under a Taylor rule. In order to obtain analytical results, the analysis initially assumes an infinite elasticity of labor supply. We then numerically show that the results are robust under a wide range of empirically plausible values for the labor supply elasticity and the degree of price stickiness. Further, we investigate the determinacy implications of including contemporaneous output in the Wicksellian rule and adding convex capital adjustment costs. It is shown that both these additional modifications help

[1] See Batini and Haldane (1999) for further discussion.

[2] See, for example, Clarida et al. (2000), Bullard and Mitra (2002), Woodford (2003).

to further enlarge the region of determinacy even under a finite labor supply elasticity calibration. Wicksellian rules offer an effective solution to the indeterminacy problem as they help reduce the sensitivity of the real interest rate to changes in monetary policy. Consequently, this weakens the cost channel of monetary policy, which as highlighted by Kurozumi and Van Zandweghe (2008), is responsible for causing the indeterminacy problem under inflation-targeting policy. Finally, we extend the baseline analysis to allow for feedback rules in which the interest rate either responds to the current or past price level. We find that even when the interest-rate rule is not forward-looking, Wicksellian rules remain superior to Taylor rules in achieving equilibrium determinacy.

This paper is related to Woodford (2003), Bauducco and Caputo (2013), and Giannoni (2014) who investigate the determinacy implications of Wicksellian rules in *labor-only* versions of the New Keynesian model.[3] These studies find that determinacy is easily achieved under Wicksellian rules as the Taylor principle is no longer a necessary condition for local stability.[4] In this paper, however, we show that Wicksellian rules also offer a possible solution to the indeterminacy problem in New Keyensian models that include capital and investment spending.

The remainder of the paper proceeds are follows. Section 2 outlines the model. Section 3 conducts the determinacy analysis under forward-looking interest-rate feedback rules and Section 4 investigates the robustness of the results when the feedback rule is specified to be backward-looking or current-looking. Finally, Section 5 briefly concludes.

2 The New Keynesian model with capital

We consider a (linearized) version of the popular New Keyensian model, where investment is endogenously introduced by assuming a competitive economy-wide rental market for capital. Details on the derivations can be found in Carlstrom and Fuerst (2005), Huang et al. (2009), and Duffy and Xiao (2011). In what follows, a variable with a hat $\widehat{X}_t$ denotes

[3]Woodford (2003) and Giannoni (2014) focus on interest-rate feedback rules that respond to the current price level, whereas Bauducco and Caputo (2013) also consider forward-looking and backward-looking Wicksellian rules. In addition, Giannoni (2014) shows that Wicksellian rules can perform better than optimal Taylor rules in terms of welfare.

[4]As discussed by Woodford (2003), the Taylor principle is automatically satisfied under a Wicksellian rule for any positive price response coefficient. Any deviation in the inflation rate from its target subsequently results in a change in the price level and the nominal interest rate responds by more than the excess inflation.

the percentage deviation of X_t with respect to its steady state value $\overline{X}$ $\left(\text{i.e., } \widehat{X}_t = \frac{X_t - \overline{X}}{\overline{X}}\right)$.

Model environment The economy is assumed to be cashless, where household preferences are separable between consumption C and leisure $1 - L_t$:

$$U_t = \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{L_t^{1+\phi}}{1+\phi} \right],$$

where $\beta \in (0,1)$ is the subjective discount factor, σ^{-1} is the intertemporal substitution elasticity of consumption, and ϕ^{-1} is the elasticity of labor supply. The period budget constraint is given by:

$$P_t C_t + P_t I_t + B_t = w_t L_t + rr_t K_t + R_{t-1} B_{t-1} + \Pi_t,$$

where K_t and B_{t-1} denote, respectively, the stock of capital and one-period nominal bonds (which pay the gross nominal interest rate R_{t-1}) at the beginning of period t, w_t and rr_t respectively denote the real wage and real rental return of capital, Π_t denotes nominal profits from firm ownership, and P_t and I_t denote the price level and investment. Capital accumulates according to:

$$\widehat{K}_{t+1} = (1-\delta)\widehat{K}_t + \delta \widehat{I}_t, \tag{1}$$

where $\delta \in (0,1)$ is the depreciation rate of capital. The first-order conditions for the household problem yield:

$$\widehat{R}_t - \widehat{\pi}_{t+1} = \sigma^{-1} \left[\widehat{C}_{t+1} - \widehat{C}_t \right], \tag{2}$$

$$\frac{\widehat{C}_{t+1} - \widehat{C}_t}{\sigma \left[1 - \beta(1-\delta)\right]} = \widehat{rr}_{t+1} = \left[\widehat{mc}_{t+1} + (1-\alpha)\left(\widehat{L}_{t+1} - \widehat{K}_{t+1} \right) \right], \tag{3}$$

$$\phi \widehat{L}_t + \sigma^{-1} \widehat{C}_t = \widehat{w}_t = \widehat{mc}_t + \alpha \widehat{K}_t - \alpha \widehat{L}_t. \tag{4}$$

Equation (2) is the consumption Euler equation, where $\widehat{\pi}_{t+1} \equiv \widehat{P}_{t+1} - \widehat{P}_t$ denotes the (next period) inflation rate, (3) is the Euler equation for capital, and (4) is the labor supply condition, where $\widehat{mc}_t$ denotes real marginal cost. In both equations (3) and (4), the second equality follows from the assumptions of competitive factor markets and Cobb-Douglas production technology:

$$\widehat{Y}_t = \alpha \widehat{K}_t + (1-\alpha)\widehat{L}_t, \tag{5}$$

where $\alpha \in (0,1)$ is the cost share of capital.

The supply side of the economy is characterized by a New Keynesian Phillips curve (NKPC), which under Calvo (1983) staggered price-setting is given by:

$$\widehat{\pi}_t = \kappa \widehat{mc}_t + \beta \widehat{\pi}_{t+1}, \tag{6}$$

where $\kappa \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi} > 0$ and $\psi \in (0,1)$ is the degree of price stickiness. Finally, goods market clearing requires:

$$\frac{\overline{Y}}{\overline{K}}\widehat{Y}_t = \frac{\overline{C}}{\overline{K}}\widehat{C}_t + \delta \widehat{I}_t, \tag{7}$$

where $\frac{\overline{C}}{\overline{K}} = \frac{1}{\alpha}\left[\frac{1}{\beta} - (1-\delta)\right]\frac{\lambda}{\lambda-1} - \delta$ is the steady state consumption-capital ratio, $\lambda > 0$ is the elasticity of substitution of differentiated goods, and $\frac{\overline{Y}}{\overline{K}} = \frac{\overline{C}}{\overline{K}} + \delta$ is the steady-state ratio of output to capital.

Labor-only model For comparison purposes, we also present results for a labor-only version of the model under a linear production technology $\widehat{Y}_t = \widehat{L}_t$. Now, goods market clearing requires $\widehat{Y}_t = \widehat{C}_t$ and the linearized model simplifies to:

$$\widehat{R}_t - \widehat{\pi}_{t+1} = \sigma^{-1}\left[\widehat{Y}_{t+1} - \widehat{Y}_t\right], \tag{8}$$

$$\widehat{\pi}_t = \kappa\left[\phi + \sigma^{-1}\right]\widehat{Y}_t + \beta\widehat{\pi}_{t+1}. \tag{9}$$

Monetary policy Monetary policy is initially specified as an interest-rate feedback rule in which the nominal interest rate is a function of either future inflation (a forward-looking Taylor rule):

$$\widehat{R}_t = \mu^T \widehat{\pi}_{t+1}, \tag{10}$$

or the future price level (a forward-looking Wicksellian rule):

$$\widehat{R}_t = \mu^w \widehat{P}_{t+1}, \tag{11}$$

Table 1: Parameter values

β	Discount factor	0.99
α	Cost share of capital	0.33
δ	Depreciation rate of capital	0.02
σ^{-1}	Intertemporal elasticity of substitution in consumption	2
λ	Elasticity of substitution of differentiated goods	11
ψ	Degree of price stickiness	$\psi \in [0.33, 0.75]$
ϕ	Inverse of the labor supply elasticity	$\phi \in [0, 10]$

where $\mu^T, \mu^w \geq 0$.

Determinacy analysis The determinacy analysis proceeds as follows. When the labor supply elasticity is infinite (i.e. $\phi = 0$), in some cases we can analytically derive the determinacy properties of the model. For finite labor supply elasticities, analytical results are not possible and a numerical investigation is carried out employing the parameterization of Huang et al. (2009). The parameter values given in Table 1 for β, α, δ, σ^{-1}, and λ are standard in the literature. As justified in Huang et al. (2009), we consider values for the degree of price stickiness $0.33 \leq \psi \leq 0.75$ and the inverse of the labor supply elasticity $0 \leq \phi \leq 10$, to adequately cover the range of empirical estimates.

3 Results

This section considers the issue of local determinacy for forward-looking interest-rate rules.

3.1 The indeterminacy problem under forward-looking Taylor rules

Let $\widehat{x}_t = \widehat{L}_t - \widehat{K}_t$ denote the labor-capital ratio. Under a forward-looking Taylor rule (10), the linearized model (1)–(7) can be reduced to the following four-dimensional system after setting $\phi = 0$:

$$\mathbf{z}_{t+1}^T = \mathbf{A}^T \mathbf{z}_t^T, \quad \mathbf{z}_t^T = \left[\widehat{mc}_t \ \widehat{x}_t \ \widehat{\pi}_t \ \widehat{K}_t\right]',$$

$$\mathbf{A}^T \equiv \begin{bmatrix} 1-\alpha-\frac{\kappa(\mu^T-1)}{\beta}J_1 & -\alpha(1-\alpha) & \left(\frac{\mu^T-1}{\beta}\right)J_1 & 0 \\ -1-\frac{\kappa(\mu^T-1)}{\beta}J_2 & \alpha & \left(\frac{\mu^T-1}{\beta}\right)J_2 & 0 \\ -\frac{\kappa}{\beta} & 0 & \frac{1}{\beta} & 0 \\ -\frac{\overline{C}}{\overline{K}}\sigma & (1-\alpha)\frac{\overline{Y}}{\overline{K}}+\frac{\overline{C}}{\overline{K}}\sigma\alpha & 0 & 1+\frac{\overline{C}}{\overline{K}} \end{bmatrix},$$

where $J_1 \equiv \left[1+\frac{\alpha(1-\Lambda)}{\Lambda}\right]$ and $J_2 \equiv \frac{(1-\Lambda)}{\Lambda}$.

Proposition 1 *If monetary policy is characterized by a forward-looking Taylor rule, then the necessary and sufficient conditions for determinacy with an infinite labor supply elasticity are:*

$$1 < \mu^T < \min\left\{1+\frac{\Lambda(1-\beta)}{\alpha\kappa} \equiv \Gamma_1^T, 1+\frac{\Lambda 2(1+\beta)}{\kappa\left[\Lambda(1-\alpha)+2\alpha\right]} \equiv \Gamma_2^T\right\} \tag{12}$$

where $\Lambda \equiv 1-\beta(1-\delta)$.

Proof. One eigenvalue of $\mathbf{A}^T$ is given by $1+\frac{\overline{C}}{\overline{K}} > 1$, which is outside the unit circle, and another is zero. The remaining eigenvalues of $\mathbf{A}^T$ are solutions to the quadratic equation $r^2+a_1r+a_0=0$, where $a_1 = -1-\frac{1}{\beta}+\frac{\kappa(\mu^T-1)}{\beta}J_1$ and $a_0 = \frac{1}{\beta}-\frac{\alpha\kappa(\mu^T-1)}{\beta}(1+J_2)$. With one predetermined variable $\widehat{K}_t$, the remaining two eigenvalues must lie outside the unit circle for determinacy, which requires that one of the following two cases is satisfied (see, e.g., Proposition C.1 of Woodford, 2003). Case I: (i) $a_0 > 1$, (ii) $1+a_0+a_1 > 0$, (iii) $1+a_0-a_1 > 0$; Case II: (iv) $1+a_0+a_1 < 0$, (v) $1+a_0-a_1 < 0$. For Case I, inequality (ii) requires $\mu^T > 1$ and the remaining inequalities yield the upper bounds Γ_1^T and Γ_2^T given in (12). For Case II, inequality (iv) requires $\mu^T < 1$ which implies $1+a_0-a_1 > 0$, contradicting inequality (v). ■

Note that in the labor-only model (8)–(9), the upper bound Γ_1^T no longer applies and determinacy requires:[5]

$$1 < \mu^T < 1+\frac{2(1+\beta)}{\kappa} \equiv \Gamma_L^T. \tag{13}$$

It is clear from (12) and (13) that in both the labor-only and capital versions of the model the Taylor principle (i.e. $\mu^T > 1$) is a necessary condition for determinacy. Under a passive

[5] The proof of this case is provided in McKnight and Mihailov (2015).

policy response (i.e. $0 < \mu^T < 1$), indeterminacy arises from the aggregate demand channel of monetary policy: an increase in inflationary expectations leads to a decrease in the real interest rate, which increases real marginal cost (4), and via the NKPC (6), results in a self-fulfilling increase in inflation. For the capital model, the numerical analysis suggests that $\Gamma_1^T < \Gamma_2^T$, so that Γ_1^T given in (12) is the empirically relevant upper bound on the inflation response coefficient. By inspection, this upper bound is independent of σ, and it is straightforward to verify that Γ_1^T is increasing in the degree of price stickiness $\partial\Gamma_1^T/\partial\psi > 0$ and decreasing with respect to the cost share of capital $\partial\Gamma_1^T/\partial\alpha < 0$. Moreover, the numerical analysis finds that this upper bound is very close to 1 rendering the equilibrium indeterminate for nearly all possible combinations of μ^T and ψ. For example, using the parameter values of Table 1 the interval of inflation response coefficients that induce determinacy are $1 < \mu^T < 1.0007$ if $\psi = 0.33$, and $1 < \mu^T < 1.0105$ if $\psi = 0.75$.[6] This is in stark contrast to the labor-only model, where Γ_1^T no longer applies, $\Gamma_2^T < \Gamma_L^T$, and the upper bound Γ_L^T binds for high values of μ^T. For example, the interval of inflation response coefficients that now induce determinacy is $1 < \mu^T < 3.9115$ for $\psi = 0.33$ and $1 < \mu^T < 47.3689$ for $\psi = 0.75$. As discussed by Kurozumi and Van Zandweghe (2008), the indeterminacy problem is more severe with the presence of capital and investment due to the cost channel of monetary policy. Under the Taylor principle, an increase in inflationary expectations results in an increase in the real interest rate. This puts upward pressure on the future rental price of capital and future marginal cost (see equations (2) and (3) above), which via the next-period NKPC (6), results in a self-fulfilling increase in inflation.

3.2 The desirability of price-level rules

Under a forward-looking Wicksellian interest-rate rule (11), equations (1)–(7) can be reduced to the following five-dimensional system after setting $\phi = 0$:

$$\mathbf{z}_{t+1}^w = \mathbf{A}^w \mathbf{z}_t^w, \quad \mathbf{z}_t^w = \left[\widehat{mc}_t \ \widehat{x}_t \ \widehat{\pi}_t \ \widehat{P}_{t-1} \ \widehat{K}_t\right]',$$

[6] The numerical analysis indicates that the range of determinacy is even smaller under finite labor supply elasticities.

$$\mathbf{A}^w \equiv \begin{bmatrix} 1-\alpha-\frac{\kappa(\mu^w-1)}{\beta}J_1 & -\alpha(1-\alpha) & \left(\frac{\mu^w-1}{\beta}+\mu^w\right)J_1 & \mu^w J_1 & 0 \\ -1-\frac{\kappa(\mu^w-1)}{\beta}J_2 & \alpha & \left(\frac{\mu^w-1}{\beta}+\mu^w\right)J_2 & \mu^w J_2 & 0 \\ -\frac{\kappa}{\beta} & 0 & \frac{1}{\beta} & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ -\frac{\overline{C}}{\overline{K}}\sigma & (1-\alpha)\frac{\overline{Y}}{\overline{K}}+\frac{\overline{C}}{\overline{K}}\sigma\alpha & 0 & 0 & 1+\frac{\overline{C}}{\overline{K}} \end{bmatrix},$$

where J_1 and J_2 are defined as before.

Proposition 2 *If monetary policy is characterized by a forward-looking Wicksellian rule, then the necessary and sufficient condition for determinacy with an infinite labor supply elasticity is:*

$$0 < \mu^w < 2 + \frac{\Lambda 4(1+\beta)}{\kappa\left[\Lambda(1-\alpha)+2\alpha\right]} \equiv \Gamma^w, \tag{14}$$

where $\Lambda \equiv 1-\beta(1-\delta)$.

Proof. For the coefficient matrix $\mathbf{A}^w$, one eigenvalue is given by $1+\frac{\overline{C}}{\overline{K}} > 1$, which is outside the unit circle, and another eigenvalue is zero. With two predetermined variables, $\widehat{P}_{t-1}$ and $\widehat{K}_t$, determinacy requires that two of the three remaining eigenvalues lie outside the unit circle. The remaining eigenvalues of $\mathbf{A}^w$ are solutions to the cubic equation $r^3+a_2r^2+a_1r+a_0=0$, where $a_2=-2-\frac{1}{\beta}+\frac{\kappa(\mu^w-1)}{\beta}J_1$, $a_1=1+\frac{2}{\beta}-\frac{\kappa(\mu^w-1)}{\beta}\left(1-\alpha+\frac{2\alpha}{\Lambda}\right)+\frac{\kappa\mu^w}{\beta}J_1$, and $a_0=-\frac{1}{\beta}-\frac{\alpha\kappa}{\beta\Lambda}$. Since $1+a_2+a_1+a_0>0$, the equilibrium is determinate if and only if (see, e.g., Proposition C.2 of Woodford, 2003): (i) $-1+a_2-a_1+a_0<0$, and (ii) either $|a_2|>3$ or $a_0^2-a_0a_2+a_1-1>0$. The first inequality yields (14) and the final two inequalities can be expressed respectively as:

$$3 < \left|\frac{\kappa(\mu^w-1)}{\beta}\left(1-\alpha+\frac{\alpha}{\Lambda}\right)-\left[\frac{1}{\beta}+2\right]\right|, \tag{15}$$

$$\mu^w > \frac{(1-\alpha)\left[\frac{1-\beta}{\beta}+\frac{\alpha\kappa}{\Lambda\beta}\right]}{\left(1-\alpha+\frac{\alpha}{\Lambda}\right)\left[\frac{1}{\beta}+\frac{\alpha\kappa}{\Lambda\beta}\right]-\frac{\alpha}{\Lambda}} < 1. \tag{16}$$

By inspection, (15) is always satisfied with $\mu^w<1$, whereas (16) is always satisfied with $\mu^w>1$. This completes the proof. ■

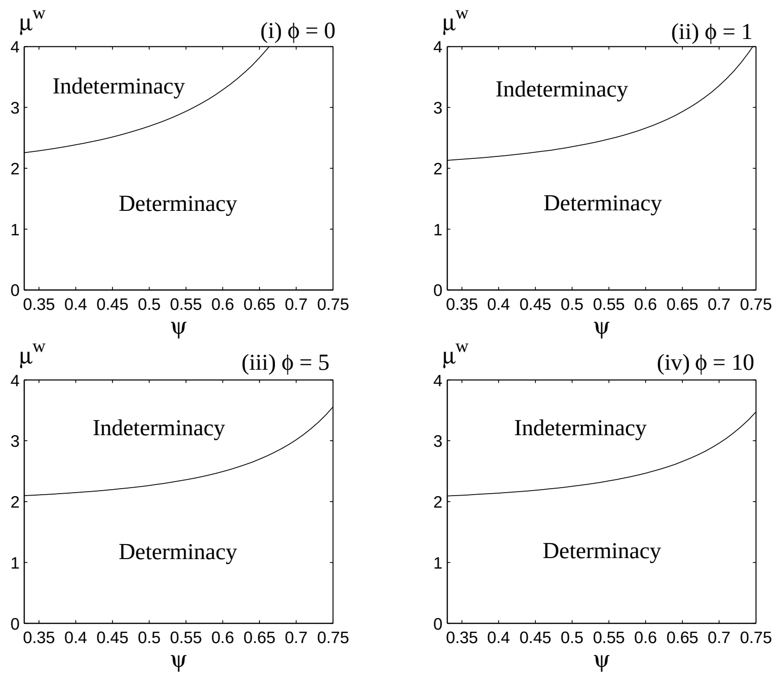

Figure 1: Region of (in)determinacy under a forward-looking Wicksellian rule

One immediate implication of (14) is that under a Wicksellian rule the Taylor principle is not a necessary condition for determinacy. Indeed, provided μ^w does not exceed 2, determinacy of equilibrium is always ensured. By inspection, the upper bound Γ^w is independent of σ, and it is straightforward to verify that Γ^w is increasing in the degree of price stickiness $\partial\Gamma^w/\partial\psi > 0$ and decreasing with respect to the cost share of capital $\partial\Gamma^w/\partial\alpha < 0$. Figure 1 depicts the region of (in)determinacy for combinations of the price response coefficient μ^w and the degree of price stickiness ψ, using the parameter values given in Table 1. The top left-hand panel of Figure 1 illustrates the (in)determinacy region when $\phi = 0$. The remaining panels illustrate the impact on determinacy for finite values of the labor supply elasticity $\phi = 1, 5, 10$. By inspection, while the indeterminacy region expands as the value of ϕ increases, the range of determinacy remains sizeable for all values of ψ.

The above results complement the findings of Bauducco and Caputo (2013), who showed the irrelevance of the Taylor principle under forward-looking Wicksellian rules in a labor-only economy. In the labor-only model (8)–(9), determinacy requires (with $\phi = 0$):

$$0 < \mu^w < 2 + \frac{4(1+\beta)}{\kappa} \equiv \Gamma_L^w. \tag{17}$$

Comparing conditions (13) and (17) reveals that determinacy is also greater under a Wicksellian rule in the labor-only model, since the upper bound is relatively lower ($\Gamma_L^T < \Gamma_L^w$) and the lower bound implied by the Taylor principle is absent. While the key difference between the labor-only and capital versions of the model is the presence in the latter of the cost channel of monetary policy ($\Gamma^w < \Gamma_L^w$), the above analysis has shown that the ability of this cost channel to induce indeterminacy is significantly reduced under a Wicksellian rule. In response to higher inflationary expectations, policymakers under the Wicksellian rule (11) can raise the nominal interest rate by less than what is possible under a Taylor rule: $\widehat{R}_t = \mu^w \widehat{P}_{t+1} = \mu^w[\widehat{\pi}_{t+1} - \widehat{P}_t]$. Intuitively, by reducing the sensitivity of the real interest rate to policy changes, this weakens the cost-channel of monetary policy and the likelihood of indeterminacy subsequently decreases under a Wicksellian rule.

3.3 Reacting to output

We now examine the determinacy implications if contemporaneous output also enters into the interest-rate feedback rule. Specifically, we consider the following Taylor- and Wicksellian-type feedback rules:

$$\widehat{R}_t = \mu^T \widehat{\pi}_{t+1} + \mu_y \widehat{Y}_t, \tag{18}$$

$$\widehat{R}_t = \mu^w \widehat{P}_{t+1} + \mu_y \widehat{Y}_t, \tag{19}$$

where $\mu_y \geq 0$ is the output response coefficient. A Taylor-type rule of the form (18) has been advocated by Kurozumi and Van Zandweghe (2008) as a possible remedy for the indeterminacy problem of forward-looking monetary policy. Using the parameter values of Table 1 and a value of $\phi = 10$ for the inverse of the labor supply elasticity, Figures 2 and 3 illustrate the region of determinacy for alternative values of the degree of price stickiness $\psi = 0.75, 0.67, 0.5, 0.33$ and the output response coefficient $0 \leq \mu_y \leq 1$. By inspection of Figure 2, including contemporaneous output into the Taylor rule can only help increase the determinacy region, provided the central bank responds sufficiently to output, when the degree of price stickiness is high. As the degree of price stickiness is reduced, the upper bound on μ^T becomes less sensitive to increases in μ_y. Consequently, as illustrated by the bottom panels of Figure 2, for lower degrees of price stickiness the indeterminacy problem

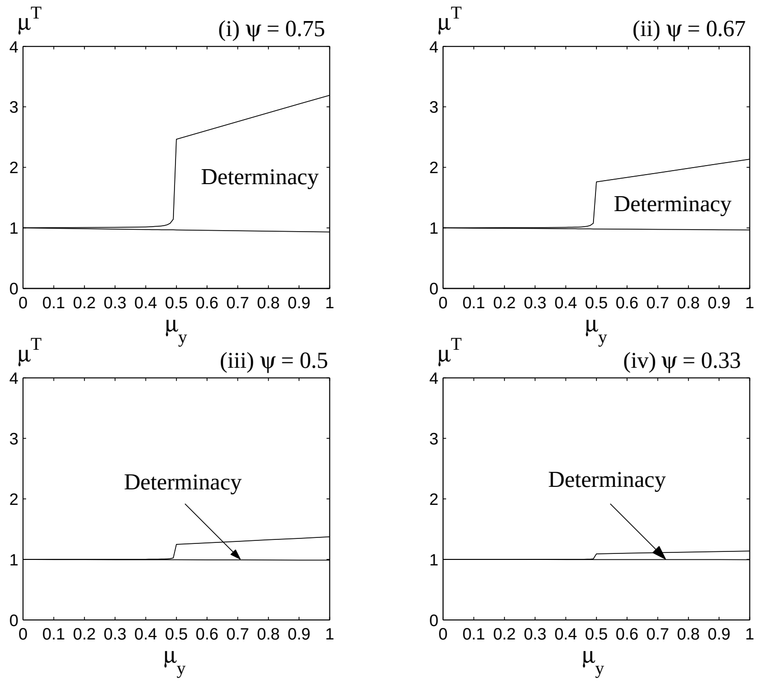

Figure 2: Determinacy region under a Taylor rule with contemporaneous output ($\phi = 10$)

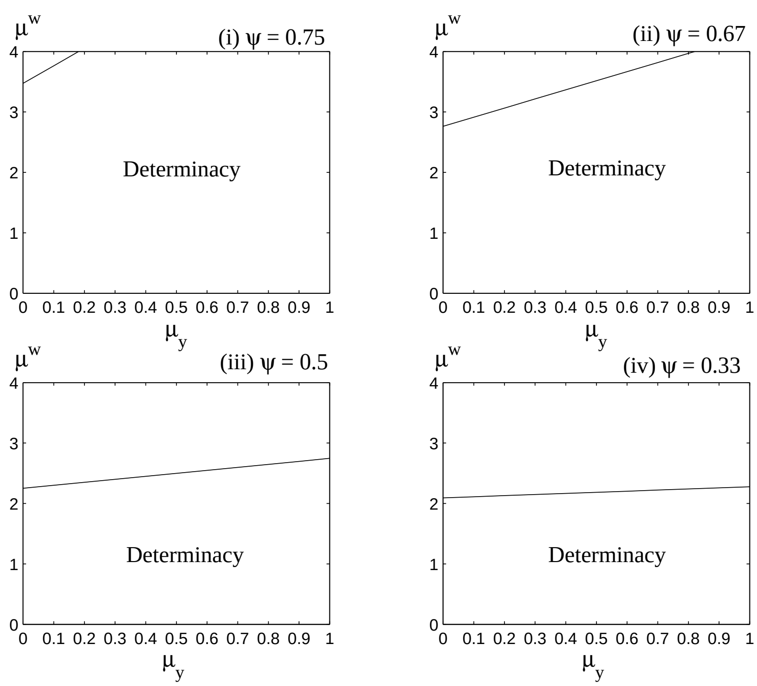

Figure 3: Determinacy region under a Wicksellian rule with contemporaneous output ($\phi = 10$)

remains.[7] By inspection of Figure 3, the superiority of the Wicksellian rule is clear: even with medium to low levels of price stickiness and low values of μ_y, determinacy is easily achieved. For example, setting $\mu^T = \mu^w = 1.5$ and $\mu_y = 0.125$ in accordance with Taylor (1993), the Taylor rule always results in indeterminacy for all variations in ψ, whereas the equilibrium is always rendered determinate under the Wicksellian rule.

3.4 Capital adjustment costs

We now consider the robustness of the above results by incorporating capital adjustment costs into the analysis. Following the existing literature, we assume that the capital accumulation condition is given by:[8]

$$I_t = I\left(\frac{K_{t+1}}{K_t}\right) K_t,$$

where in the steady state $I(\cdot)$ satisfies: $I(1) = \delta$, $I'(1) = 1$, and $I'' = \varepsilon_q > 0$. The parameter ε_q measures the (steady-state) elasticity of the investment to capital ratio with respect to Tobin's q. Consequently, the Euler equation for capital (3) now becomes:

$$\begin{aligned}\sigma^{-1}\left(\widehat{C}_{t+1} - \widehat{C}_t\right) + \varepsilon_q\left[\widehat{K}_{t+1} - \widehat{K}_t - \beta\left(\widehat{K}_{t+2} - \widehat{K}_{t+1}\right)\right] \\ = [1-\beta(1-\delta)]\left[\widehat{mc}_{t+1} + (1-\alpha)\left(\widehat{L}_{t+1} - \widehat{K}_{t+1}\right)\right].\end{aligned} \tag{20}$$

The other features of the model remain unchanged.

We first consider the determinacy implications of capital adjustment costs for the Wicksellian rule (11). Using the parameter values given in Table 1, Figures 4 and 5 show the (in)determinacy region for combinations of the price response coefficient μ^w and the degree of price stickiness ψ for alternative values of the inverse of the labor supply elasticity $\phi = 0, 1, 5, 10$. Figure 4 follows Huang et al. (2009) and Duffy and Xiao (2011) in setting the capital adjustment cost parameter $\varepsilon_q = 3$, whereas Figure 5 sets $\varepsilon_q = 10$. First, by comparing Figure 4 with Figure 1 (where capital adjustment costs are absent), it is clear that the inclusion of capital adjustment costs results in a sizeable expansion of the determinacy region for all values of ϕ and ψ. Moreover, the numerical analysis suggests that the upper

[7] Huang et al. (2009) were the first to show that hybrid Taylor rules of the type given by (18) fail to prevent indeterminacy under a finite labor supply elasticity.

[8] See, for example, Sveen and Weinke (2005), Huang et al. (2009), Duffy and Xiao (2011).

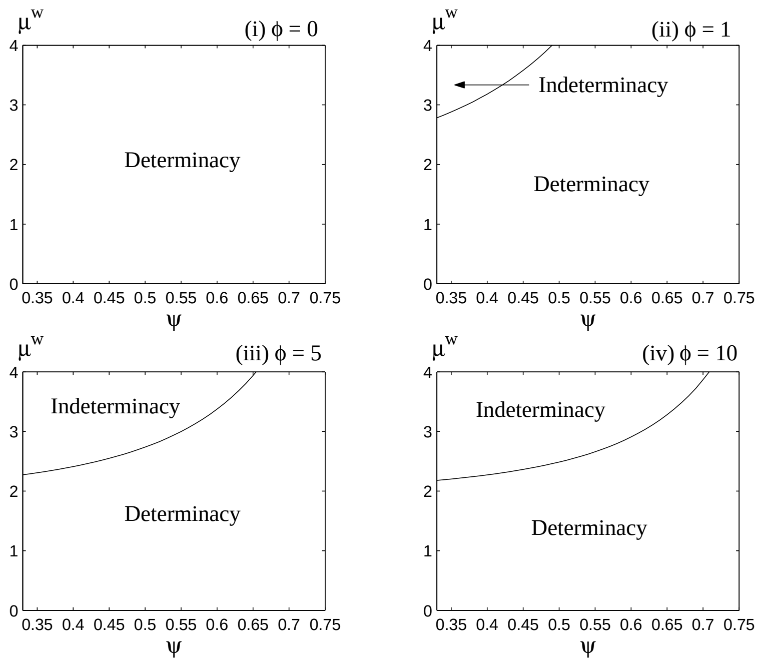

Figure 4: Region of (in)determinacy under a forward-looking Wicksellian rule with capital adjustment costs ($\varepsilon_q = 3$)

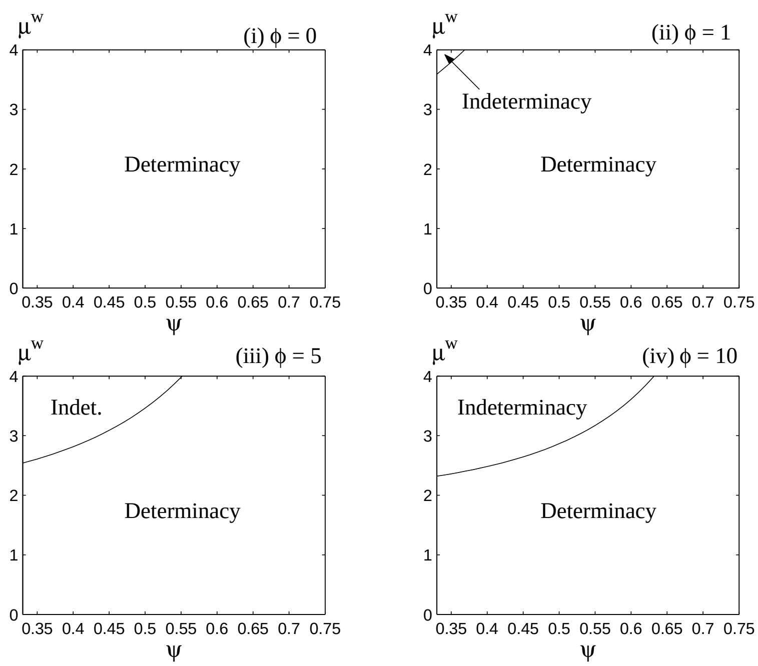

Figure 5: Region of (in)determinacy under a forward-looking Wicksellian rule with capital adjustment costs ($\varepsilon_q = 10$)

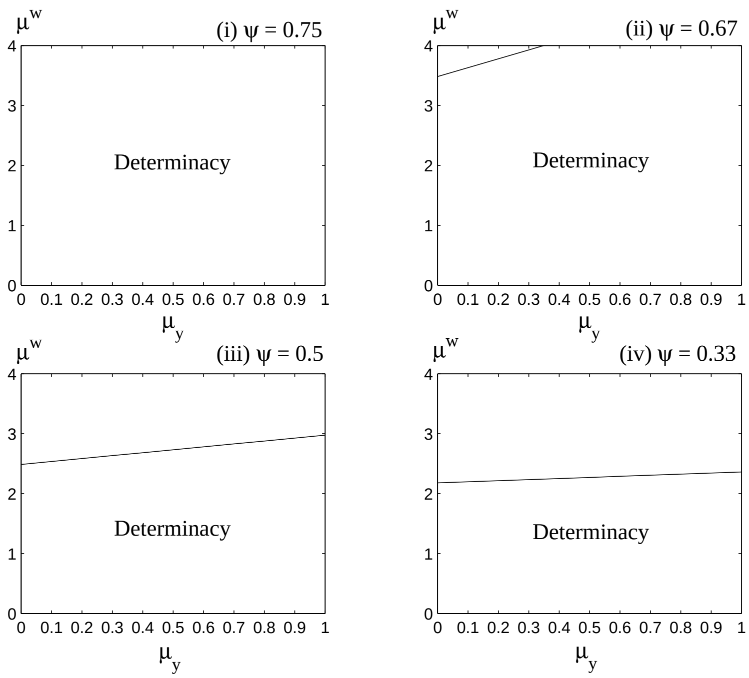

Figure 6: Capital adjustment costs: Determinacy region under a Wicksellian rule with contemporaneous output ($\phi = 10$, $\varepsilon_q = 3$)

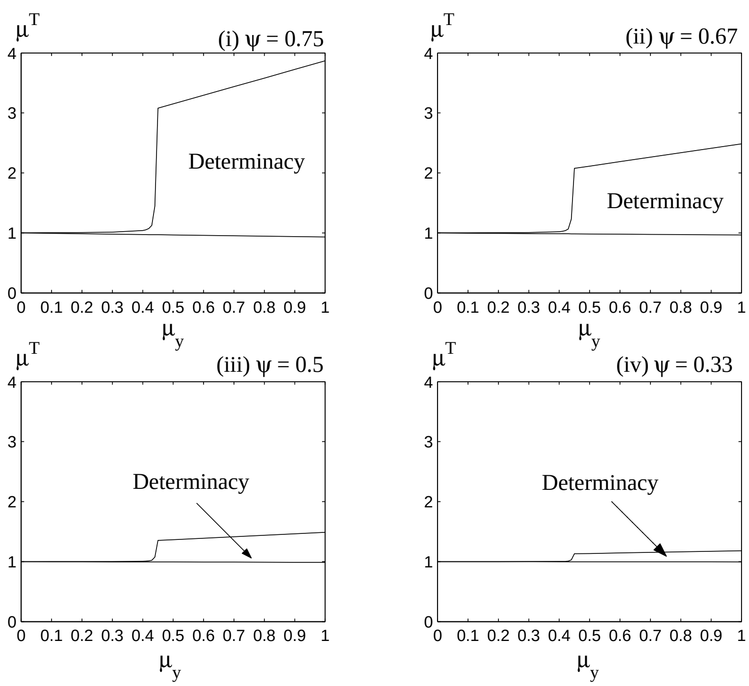

Figure 7: Capital adjustment costs: Determinacy region under a Taylor rule with contemporaneous output ($\phi = 10$, $\varepsilon_q = 3$)

bound on μ^w is increasing in ε_q. By comparing Figure 4 with Figure 5, it is evident that the determinacy region gets larger as ε_q increases. Intuitively, increases in ε_q help to reduce the likelihood of indeterminacy by making capital accumulation more costly, which weakens the cost channel of monetary policy. Figure 6 illustrates the region of determinacy when contemporaneous output also enters into the Wicksellian rule (19) for alternative values of the degree of price stickiness $\psi = 0.75, 0.67, 0.5, 0.33$. The determinacy region in Figure 6 is depicted for combinations of μ^w and the output response coefficient μ_y setting $\phi = 10$ and $\varepsilon_q = 3$. By comparing Figure 6 with Figure 3, capital adjustment costs also increase the upper bound on μ_y, further increasing the determinacy region.

The above results are in stark contrast to a Taylor rule, where in the absence of a policy response to output $\mu_y = 0$, capital adjustment costs of the size $\varepsilon_q \in (0, 10]$ have a negligible effect on the upper bound Γ_1^T given in (12).[9] Setting $\phi = 10$ and $\varepsilon_q = 3$, Figure 7 illustrates the region of determinacy for combinations of μ^T and μ_y under the Taylor rule (18) for alternative values of the degree of price stickiness $\psi = 0.75, 0.67, 0.5, 0.33$. By inspection, capital adjustment costs have little effect on the determinacy region for zero/low values of μ_y. By comparing Figures 2 and 7, capital adjustment costs can increase the determinacy region when prices are sufficiently sticky, $\psi = 0.75, 0.67$, provided μ_y is sufficiently large. However, the determinacy region remains very narrow for lower degrees of price stickiness, $\psi = 0.5, 0.33$.

4 Backward- and current-looking feedback rules

In this section we examine the robustness of the previous results to alternative specifications of the interest-rate feedback rule. Specifically, we consider the following Taylor- and Wicksellian-type feedback rules:

$$\widehat{R}_t = \mu^T \widehat{\pi}_{t-i} + \mu_y \widehat{Y}_{t-i}, \tag{21}$$

$$\widehat{R}_t = \mu^w \widehat{P}_{t-i} + \mu_y \widehat{Y}_{t-i}, \tag{22}$$

[9] See Duffy and Xiao (2011) for an excellent discussion of the determinacy implications of Taylor rules under capital adjustment costs.

where $\mu^T, \mu^w, \mu_y \geq 0$ and $i = 0, 1$. If $i = 0$, the interest-rate feedback rules are contemporaneous-looking, whereas $i = 1$ corresponds to backward-looking rules.

4.1 Contemporaneous-looking rules

Under a strict contemporaneous-looking Taylor rule, whereby the interest-rate feedback rule only reacts to current inflation (i.e. $\mu_y = 0$ in (21)), the linearized model (1)-(7) can be reduced to the following four-dimensional system after setting $\phi = 0$:

$$\mathbf{z}_{t+1}^T = \mathbf{A}_c^T \mathbf{z}_\mathbf{t}^\mathbf{T}, \quad \mathbf{z}_t^T = \left[\widehat{mc}_t \; \widehat{x}_t \; \widehat{\pi}_t \; \widehat{K}_t\right]',$$

$$\mathbf{A}_c^T \equiv \begin{bmatrix} 1-\alpha+\frac{\kappa}{\beta}J_1 & -\alpha(1-\alpha) & \left(\mu^T - \frac{1}{\beta}\right)J_1 & 0 \\ -1+\frac{\kappa}{\beta}J_2 & \alpha & \left(\mu^T - \frac{1}{\beta}\right)J_2 & 0 \\ -\frac{\kappa}{\beta} & 0 & \frac{1}{\beta} & 0 \\ -\frac{\overline{C}}{\overline{K}}\sigma & (1-\alpha)\frac{\overline{Y}}{\overline{K}} + \frac{\overline{C}}{\overline{K}}\sigma\alpha & 0 & 1+\frac{\overline{C}}{\overline{K}} \end{bmatrix},$$

where J_1 and J_2 are defined as in Section 3.

Proposition 3 *If monetary policy is characterized by a Taylor rule that reacts only to current inflation, then the necessary and sufficient conditions for determinacy with an infinite labor supply elasticity are* $\mu^T > 1$ *and either:*

$$(2\beta - 1)\Lambda < \kappa\left[1 - \beta(1-\delta)(1-\alpha)\right] \quad \textit{or} \tag{23}$$

$$1 - \beta + \frac{\kappa\alpha}{\Lambda} + \kappa\mu^T(1-\alpha) + \frac{\kappa\alpha\mu^T}{\beta\Lambda}\left[\frac{\kappa\alpha(\mu^T - 1)}{\Lambda} - 1 - \kappa(1-\alpha)\right] > 0, \tag{24}$$

where $\Lambda = 1 - \beta(1-\delta)$.

Proof. One eigenvalue of $\mathbf{A}_c^T$ is given by $1 + \frac{\overline{C}}{\overline{K}} > 1$, which is outside the unit circle. With one predetermined variable $\widehat{K}_t$, determinacy requires that two of the three remaining eigenvalues lie outside the unit circle. The remaining eigenvalues of $\mathbf{A}_c^T$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -1 - \frac{1}{\beta} - \frac{\kappa J_1}{\beta}$, $a_1 = \frac{1}{\beta} + \frac{\kappa\mu^T J_1}{\beta} + \frac{\kappa\alpha}{\beta\Lambda}$, and $a_0 = -\frac{\kappa\alpha\mu^T}{\beta\Lambda}$. Since $-1 + a_2 - a_1 + a_0 < 0$, the equilibrium is determinate if and only if

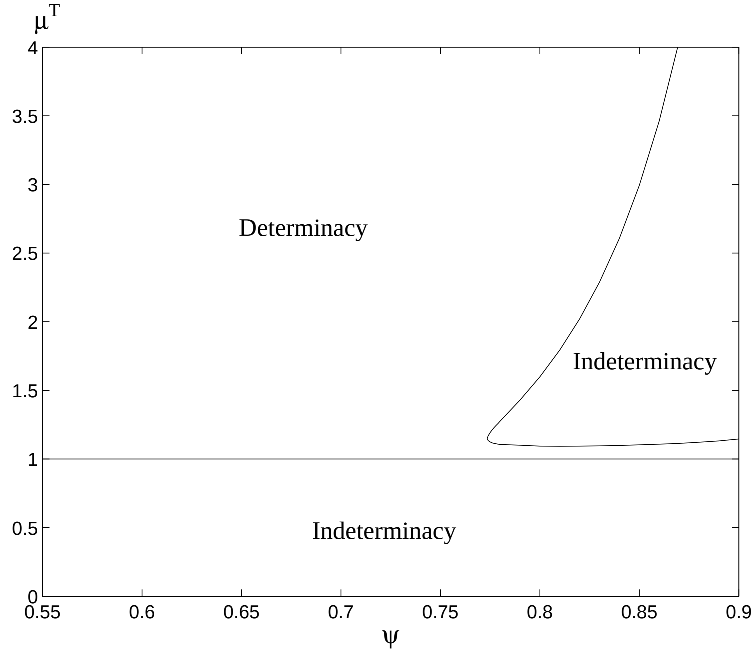

Figure 8: Regions of (in)determinacy under a strict current-looking Taylor rule ($\phi = 0$)

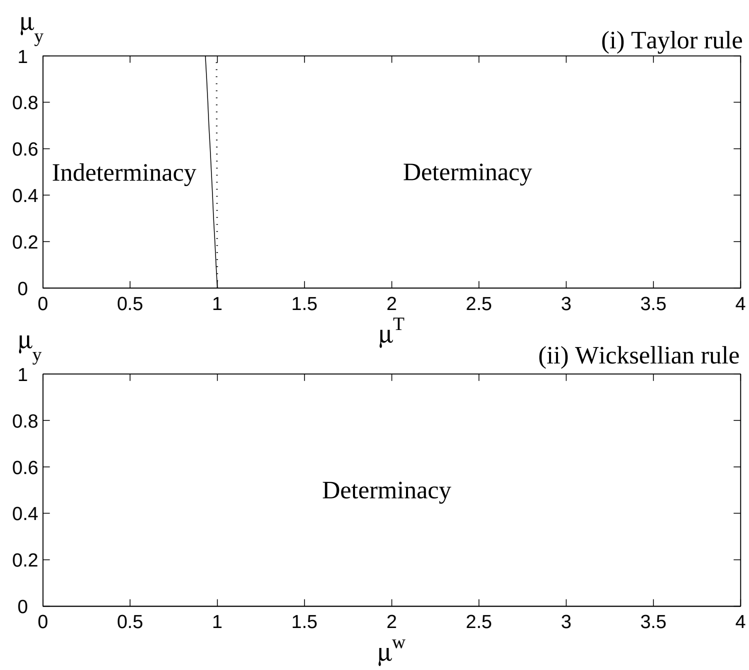

Figure 9: Region of (in)determinacy under current-looking rules with contemporaneous output ($\phi = 10$): $\psi = 0.33$ ($\cdots$) vs. $\psi = 0.75$ (—)

(see, e.g., Proposition C.2 of Woodford, 2003) $1 + a_2 + a_1 + a_0 > 0$, and either $|a_2| > 3$ or $a_0^2 - a_0 a_2 + a_1 - 1 > 0$. The first inequality requires $\mu^T > 1$ and the final two inequalities yield (23) and (24) respectively. ■

Figure 8 illustrates the areas of (in)determinacy for combinations of the inflation response coefficient μ^T and the degree of price stickiness ψ using the parameter values of Table 1.[10] By inspection, while indeterminacy can arise under the Taylor principle, condition (23) can only be violated if prices are very sticky ($\psi > 0.77$). The finding that indeterminacy is significantly less likely to occur under a current-looking Taylor rule is well known from the studies of Carlstrom and Fuerst (2005), Sveen and Weinke (2005), and Duffy and Xiao (2011). As previously discussed, indeterminacy arises under forward-looking interest-rate rules from the cost channel of monetary policy. However, under current-looking Taylor rules the upward pressure on inflation arising from the cost channel is now offset by the downward pressure on inflation generated from a real interest rate-induced fall in aggregate demand. Only if prices are sufficiently sticky can the cost channel outweigh the aggregate demand channel of monetary policy resulting in indeterminacy.

Under a strict contemporaneous-looking Wicksellian rule, whereby the interest-rate feedback rule only reacts to the current price level (i.e. $\mu_y = 0$ in (22)), equations (1)-(7) can be reduced to the following five-dimensional system after setting $\phi = 0$:

$$\mathbf{z}_{t+1}^w = \mathbf{A}_c^w \mathbf{z}_t^w, \quad \mathbf{z}_t^w = \left[\widehat{mc}_t \ \widehat{x}_t \ \widehat{\pi}_t \ \widehat{P}_{t-1} \ \widehat{K}_t\right]',$$

$$\mathbf{A}_c^w \equiv \begin{bmatrix} 1-\alpha+\frac{\kappa}{\beta}J_1 & -\alpha(1-\alpha) & \left(\mu^w - \frac{1}{\beta}\right)J_1 & \mu^w J_1 & 0 \\ -1+\frac{\kappa}{\beta}J_2 & \alpha & \left(\mu^w - \frac{1}{\beta}\right)J_2 & \mu^w J_2 & 0 \\ -\frac{\kappa}{\beta} & 0 & \frac{1}{\beta} & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ -\frac{\overline{C}}{\overline{K}}\sigma & (1-\alpha)\frac{\overline{Y}}{\overline{K}} + \frac{\overline{C}}{\overline{K}}\sigma\alpha & 0 & 0 & 1+\frac{\overline{C}}{\overline{K}} \end{bmatrix}.$$

where J_1 and J_2 are defined as in Section 3.

[10] The sensitivity analysis suggests that the indeterminacy region that arises under the Taylor principle gets smaller with a finite labor supply elasticity or by including capital adjustment costs.

Proposition 4 *If monetary policy is characterized by a Wicksellian rule that reacts only to the current price level, then the necessary and sufficient condition for determinacy with an infinite labor supply elasticity is* $\mu^w > 0$.

Proof. One eigenvalue of $\mathbf{A}^w$ is given by $1 + \frac{\overline{C}}{\overline{K}} > 1$, which is outside the unit circle, and another eigenvalue is zero. The remaining eigenvalues of $\mathbf{A}^w$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -2 - \frac{1}{\beta} - \frac{\kappa}{\beta} J_1$, $a_1 = 1 + \frac{2}{\beta} + \frac{\kappa}{\beta}(1 + \mu^w) J_1 + \frac{\kappa\alpha}{\beta\Lambda}$, and $a_0 = -\frac{1}{\beta} - \frac{\kappa\alpha}{\beta\Lambda}(1 + \mu^w)$. With two predetermined variables, $\widehat{P}_{t-1}$ and $\widehat{K}_t$, determinacy requires that two of the three remaining eigenvalues lie outside the unit circle. Using Proposition C.2 of Woodford (2003), this requires (i) $1 + a_2 + a_1 + a_0 > 0$, (ii) $-1 + a_2 - a_1 + a_0 < 0$, and (iii) $|a_2| > 3$. The first inequality is satisfied provided $\mu^w > 0$ and the remaining two inequalities are always satisfied. This completes the proof. ■

Proposition 4 reveals that indeterminacy is not possible under current-looking Wicksellian rules regardless of the degree of price-stickiness.[11] A similar result was found by Woodford (2003), Bauducco and Caputo (2013), and Giannoni (2014) for labor-only economies. Under a current-looking Wicksellian rule, for no parameter constellations can the cost channel of monetary policy outweigh the aggregate demand channel and induce indeterminacy. Figure 9 gives a graphical representation of the results when the interest-rate feedback rules (21) and (22) also react to current output (i.e. $\mu_y > 0$) using two alternative values of the degree of price stickiness $\psi = 0.33, 0.75$ and setting $\phi = 10$. The top half of Figure 9 illustrates the (in)determinacy region for combinations of μ^T and μ_y under the Taylor rule, whereas the bottom half depicts the determinacy region for combinations of μ^w and μ_y under the Wicksellian rule.[12] By inspection, even in the case when the indeterminacy problem caused by the cost channel of monetary policy is ameliorated, Wicksellian rules are still superior to Taylor rules, since in the former the Taylor principle is not a necessary condition for determinacy.

[11] The numerical analysis suggests that this result is robust under both finite values for the labor supply elasticity and the inclusion of capital adjustment costs.

[12] The sensitivity analysis suggests that the results illustrated in Figure 9 are robust to the inclusion of capital adjustment costs.

4.2 Backward-looking rules

Finally, we present results for backward-looking interest-rate rules. Figures 10 and 11 depict the regions of determinacy, indeterminacy, and explosiveness for alternative values of the inverse of the labor supply elasticity $\phi = 0, 1, 5, 10$ setting $\mu_y = 0$. If the equilibrium is explosive no equilibrium exists (locally). Figure 10 shows the combinations of the inflation response coefficient μ^T and degree of price stickiness ψ that induce determinacy under a strict backward-looking Taylor rule. By inspection, the determinacy region is narrow under the Taylor principle for all values of ϕ, and the upper bound on μ^T tightens as prices become more flexible. Similar to the labor-only economy findings of Bullard and Mitra (2002), the violation of the upper bound on μ^T now results in a locally explosive equilibrium. Figure 11 illustrates the combinations of μ^w and ψ that generate determinacy under a strict backward-looking Wicksellian rule. By inspection, in stark contrast to the Taylor rule indeterminacy is not possible under the Wicksellian rule, and even though the explosive region expands as ϕ increases, the range of determinacy remains sizeable for all values of ψ.

Figures 12–15 examine the robustness of these conclusions in the presence of capital adjustment costs and if lagged output also enters into the interest-rate feedback rule. Setting $\phi = 10$, Figures 12 and 13 illustrate the regions of determinacy, indeterminacy, and explosiveness for combinations of μ^T and μ_y under a backward-looking Taylor rule for alternative values of the degree of price stickiness $\psi = 0.75, 0.67, 0.5, 0.33$. In Figure 12 capital adjustment costs are absent ($\varepsilon_q = 0$), whereas Figure 13 sets $\varepsilon_q = 3$. By inspection, assigning a positive weight to lagged output in the Taylor rule has little positive effect on inducing determinacy. For example, the top left-hand corner of Figure 12 shows the determinacy properties of the model with $\psi = 0.75$. The more aggressive the monetary authority is in its setting of μ_y, the smaller is the interval of μ^T that generates determinacy under the Taylor principle. Indeed, if μ_y is sufficiently large the Taylor principle results in a locally explosive equilibrium and determinacy is only possible under a passive monetary policy (i.e. $0 < \mu^T < 1$). As depicted by the other three panels of Figure 12, as prices become more flexible, these determinacy regions become extremely narrow. As shown by Figure 13, the inclusion of capital adjustment costs has a positive effect on the ability of the Taylor principle to induce determinacy. But the determinacy region remains very small as ψ is

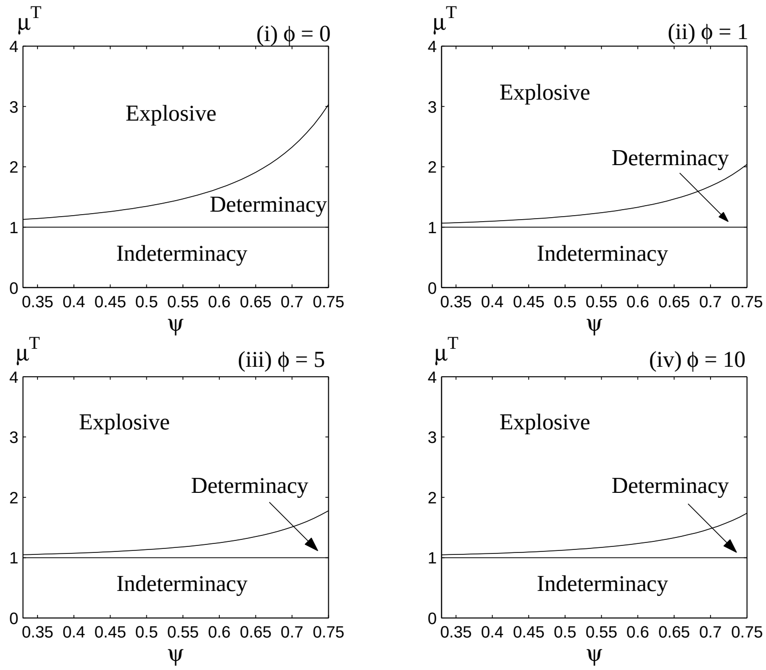

Figure 10: Determinacy region under a strict backward-looking Taylor rule

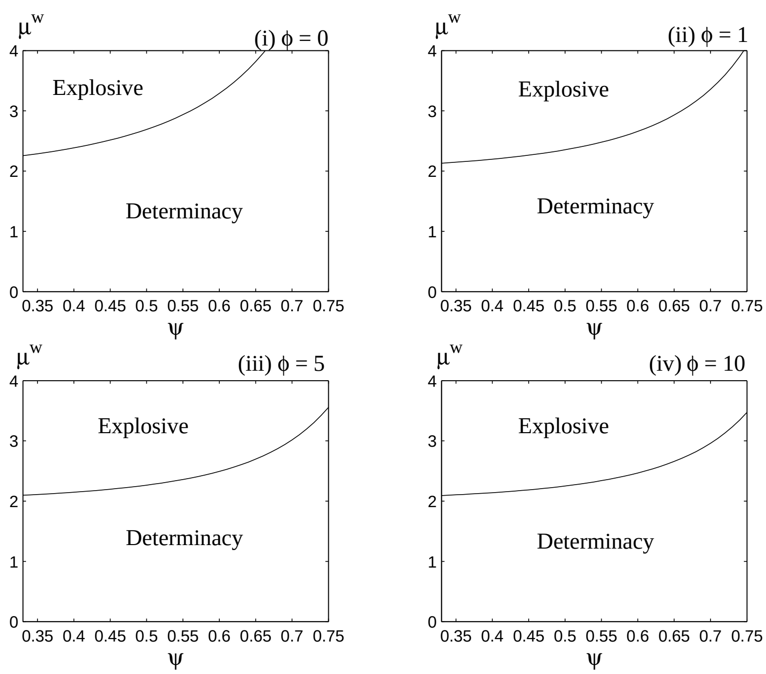

Figure 11: Determinacy region under a strict backward-looking Wicksellian rule

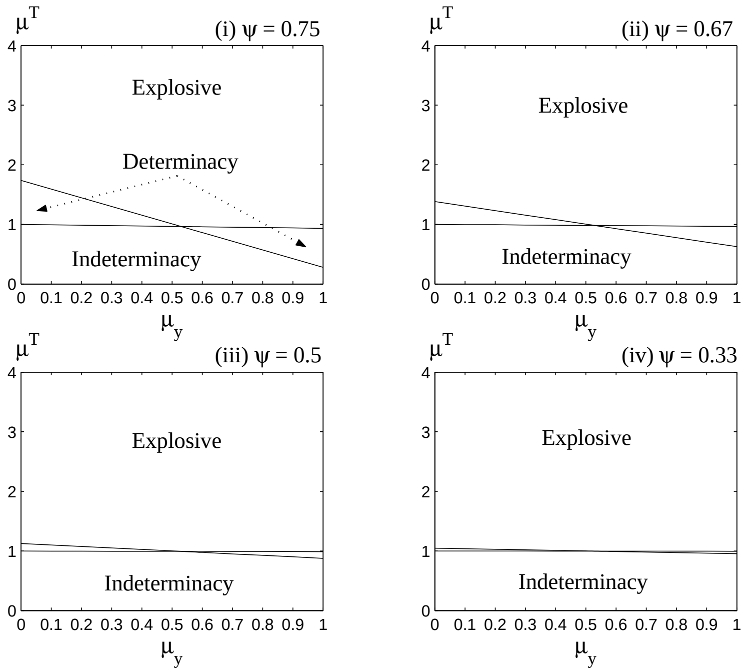

Figure 12: Determinacy regions under a backward-looking Taylor rule when reacting to output ($\phi = 10$, $\varepsilon_q = 0$)

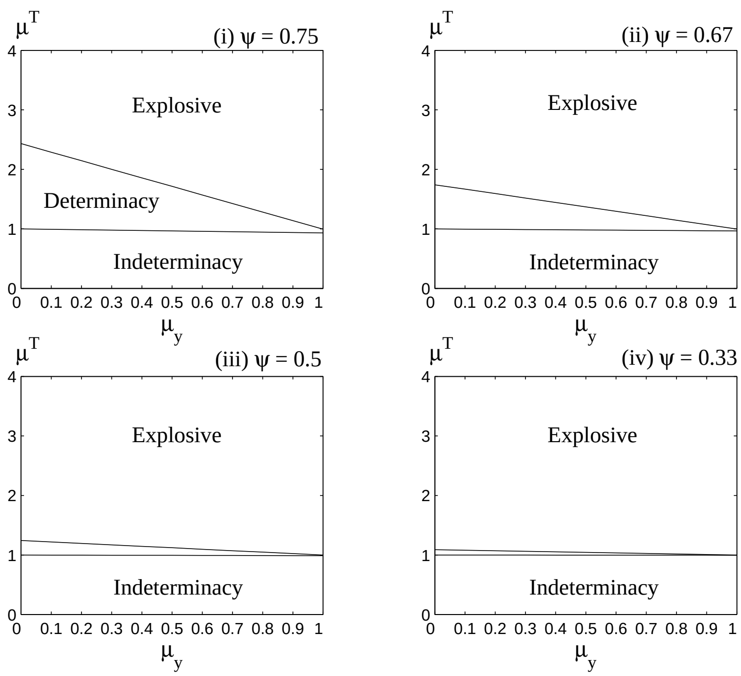

Figure 13: Determinacy regions under a backward-looking Taylor rule when reacting to output ($\phi = 10$, $\varepsilon_q = 3$)

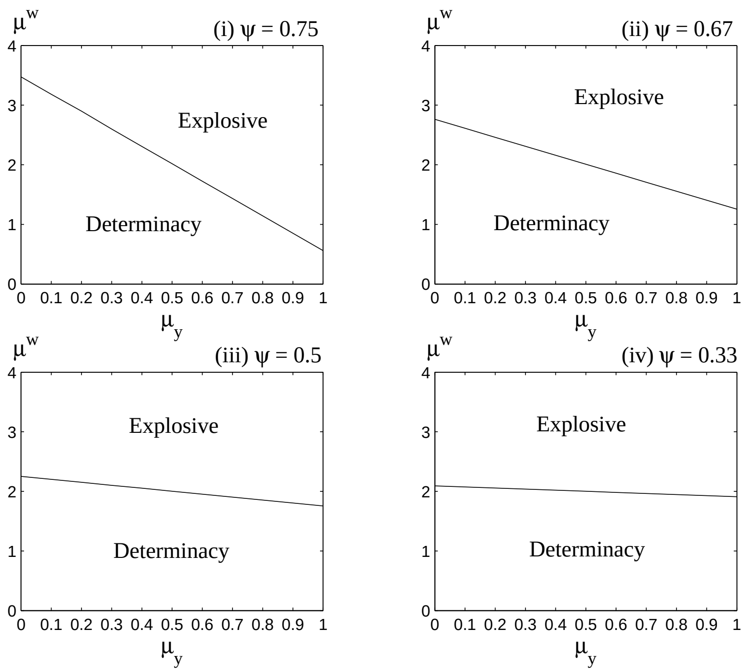

Figure 14: Determinacy regions under a backward-looking Wicksellian rule when reacting to output ($\phi = 10$, $\varepsilon_q = 0$)

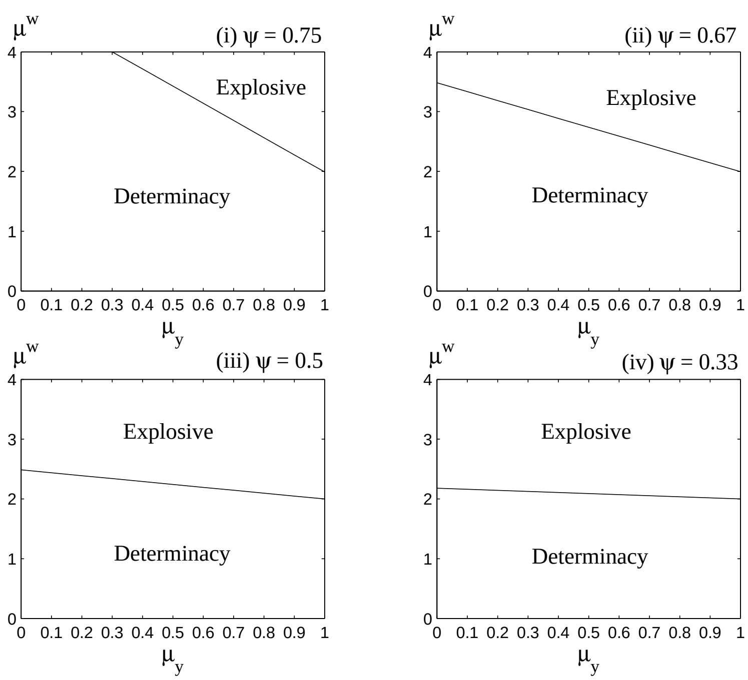

Figure 15: Determinacy regions under a backward-looking Wicksellian rule when reacting to output ($\phi = 10$, $\varepsilon_q = 3$)

reduced and/or μ_y is increased.

Figures 14 and 15 repeat the previous exercise but under a backward-looking Wicksellian rule. Similar to the Taylor rule, the determinacy region is decreasing in μ_y and increasing in ψ. However, the superiority of the Wicksellian rule is clear. Indeed, with small, empirically plausible capital adjustment costs ($\varepsilon_q = 3$), determinacy is achieved for any $0 < \mu^w \leq 2$ for all values of $\mu_y \in [0, 1]$ and $\psi \in [0.33, 0.75]$.

5 Conclusion

Using a popular New Keynesian model with capital and investment spending, it has been shown that the indeterminacy problem that arises under forward-looking Taylor rules can be alleviated under a Wicksellian interest-rate rule. This finding has been shown to be robust to variations in the labor supply elasticity and the degree of price stickiness, the inclusion of capital adjustment costs, and if contemporaneous output enters the feedback rule. Finally, we have shown that Wicksellian rules continue to be superior to Taylor rules in achieving determinacy under both current-looking and backward-looking specifications of the interest-rate rule.

In the conduct of monetary policy, traditional arguments against the adoption of price-level targeting rules suggest that inflation and output variability would be higher than under an inflation targeting regime.[13] For example, under price-level targeting all inflationary shocks are required to be offset, since temporarily higher inflation today must be reversed by lower inflation in the future in order to return to target. This is in stark contrast to inflation targeting which allows for price-level drift, since the price level remains permanently higher if inflation exceeds its target. Our analysis suggests that any potential costs associated with abandoning the inflation targeting regime should be weighed against the clear advantages of adopting price-level targeting in terms of equilibrium determinacy.

[13] For further discussion on the arguments for and against price-level targeting, see Mishkin and Schmidt-Hebbel (2001), Ambler (2009), and Reis (2013), among others.

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