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preventing macroeconomic instability?

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# Are Consumption Taxes Preferable to Income Taxes in Preventing Macroeconomic Instability?

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## Abstract

This paper examines the local determinacy implications of using consumption taxes and income taxes to finance a balanced budget fiscal policy for a variety of popular monetary policy rules. It is shown using a New Keynesian framework that the severity of the indeterminacy problem that arises under each tax system depends not only on the specification of the interest-rate feedback rule, but also on the magnitude of the steady state tax rate, the steady state government debt-output ratio, and the degree of price stickiness. However, significant differences in the determinacy criteria across the two tax systems are found to exist. The robustness of the results are assessed by extending the baseline model to include capital accumulation and the taxation of bond interest income. From a policy perspective, our results suggest that future shifts towards indirect taxation could have non-trivial implications for the setting of monetary policy under balanced-budget rules, in particular the ability of the Taylor principle to achieve determinacy.

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**Keywords:** Equilibrium determinacy; Distortionary taxation; Income taxes; Consumption taxes; Taylor principle; Balanced-budget rules.

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# 1 Introduction

How should the government finance its spending under a balanced budget fiscal policy? Recent real business cycle studies suggest that the tax system used to finance a balanced-budget rule can have important implications for macroeconomic stability.<sup>1</sup> This literature has shown that income taxation may induce multiple equilibria, or equilibrium indeterminacy, which can destabilize the economy through the emergence of expectations-driven fluctuations. In contrast, Giannitsarou (2007) finds that determinacy, or local equilibrium uniqueness, is easily induced under a balanced budget policy if the government raises revenue using consumption taxes rather than income taxes, as the former do not exert a destabilizing influence on the economy.

An important omission in the above literature, however, is the notable absence of monetary policy from the determinacy analysis. As first highlighted by Leeper (1991), the determinacy properties of a rational expectations equilibrium depend crucially on the assumed interactions between fiscal policy and monetary policy. If monetary policy is implemented in terms of an interest-rate feedback rule and the government uses *lump-sum* taxation to continuously balance its budget, there is now a large literature that explores the suitability of the Taylor principle in preventing indeterminacy of equilibrium.<sup>2,3</sup>

This paper investigates the conditions for determinacy under a balanced budget fiscal policy, by augmenting a standard New Keynesian sticky-price model to include *distortionary* income and consumption taxation. Its main aim is to use the criteria for equilibrium determinacy to compare each tax system under alternative monetary policy specifications. It is assumed throughout that both tax systems are proportional, and that the government can only employ one of the two taxes to raise revenue to finance a fixed level of unproductive spending. Monetary policy is characterized by either a forward-looking or contemporaneous-looking interest-rate feedback rule that responds to inflation and output. It is shown that the necessary and sufficient conditions for determinacy are not equivalent across the two tax systems and this can have important implications on the effectiveness of the Taylor principle in preventing indeterminacy.<sup>4</sup>

<sup>1</sup>See, for example, Schmitt-Grohé and Uribe (1997), Guo and Harrison (2004), Giannitsarou (2007), Linnemann (2008), Ghilardi and Rossi (2014).

<sup>2</sup>The Taylor principle is a policy that raises the nominal interest rate by proportionally more than the increase in inflation.

<sup>3</sup>See, for example, Clarida et al. (2000), Carlstrom and Fuerst (2001, 2005), Bullard and Mitra (2002), Woodford (2003), Kurozumi (2006), Kurozumi and Van Zandweghe (2008), Surico (2008), Duffy and Xiao (2011), Huang et al. (2011), Zubairy (2014), Rannenberg (2015), McKnight and Mihailov (in press).

<sup>4</sup>Employing a New Keynesian framework, Motta and Rossi (2013) show that consumption and income taxes are not equivalent in terms of welfare.

To understand why differences in the determinacy criteria arise, consider how each tax system affects aggregate demand and aggregate supply in response to changes in the real interest rate. Under both consumption and income taxation, movements in the real interest rate induce changes in aggregate supply through a public finance channel of monetary policy: higher real interest rates imply larger government debt repayments and higher taxes to balance the budget, which exert upward pressure on real marginal cost and consequently inflation. However, under consumption taxation fiscal policy can additionally influence the economy via an aggregate demand channel: expected changes in the consumption tax rate that arise from the public finance channel, induce changes in the after-tax real interest rate, which directly affects aggregate demand via the consumption Euler equation.

Our main results are as follows. First, if the monetary authority sets the nominal interest rate in response solely to future inflation (i.e. a strict future inflation targeting policy), we find that under income taxation the Taylor principle easily renders the equilibrium indeterminate, and the problem of indeterminacy cannot be ameliorated by the incorporation of future output into the feedback rule (i.e. a flexible future inflation targeting policy). The severity of the indeterminacy problem is shown to be increasing in the steady state tax rate and the steady state government-debt ratio, and decreasing in the degree of price stickiness. In stark contrast, the Taylor principle always generates determinacy under consumption taxation.

Second, we find for a strict contemporaneous inflation targeting policy that the range of determinacy increases significantly under income taxation, and indeterminacy can easily be ameliorated if the feedback rule also responds to current output. While determinacy is also possible under consumption taxation, now the Taylor principle can additionally result in an explosive equilibrium, which is more likely to occur the greater is the steady state government debt-output ratio and the lower the degree of price stickiness. Moreover, a flexible contemporaneous inflation targeting policy exacerbates the problem of explosiveness: the larger the weight assigned to output in the interest-rate feedback rule, the greater the area of explosiveness associated with the Taylor principle. In this case, a passive monetary policy is shown to be appropriate for inducing determinacy.

The robustness of the above results are explored by modifying the baseline model to include either the taxation of bond interest income or capital and investment spending. When bond interest income is also subject to income taxation, the after-tax nominal interest rate enters the consumption Euler equation. This is shown to have two implications for determinacy. Similar to

Edge and Rudd (2007), the lower bound on the inflation response coefficient needs to be greater than what the Taylor principle prescribes, in order for increases in the after-tax nominal interest rate to result in a real interest rate increase. In addition, expected changes in the income tax rate that arise from the public finance channel, now also directly affect aggregate demand. However, unlike consumption taxation this aggregate demand channel is found to be weak. Consequently, the conclusions of the baseline model remain qualitatively unchanged. When capital accumulation is incorporated into the model, the severity of the indeterminacy problem under both tax systems is shown to be increasing in the degree of price stickiness and decreasing in the steady state government debt-output ratio. For a strict contemporaneous inflation targeting policy, the numerical exercise finds that the region of indeterminacy is always relatively larger under consumption taxation than income taxation. However, indeterminacy is easily preventable under both tax systems if the feedback rule also reacts to current output.

In many countries, there has been a shift away from direct taxation towards indirect taxation. For example, in the European Union the share of indirect taxation in total taxation has increased by 1.2 percentage points during the period 1995–2006 (Lipińska and von Thadden, 2009). Indeed, by 2006 indirect taxation had become the major source of tax revenue within the EU totaling 13.9% of GDP compared to 13.5% of GDP raised under direct taxation (Lipińska and von Thadden, 2009). Additional reforms have been advocated to further shift tax systems towards indirect taxation both at a European level and within individual European countries (OECD, 2006; European Commission, 2008). The findings from this paper suggest that such reforms could have non-trivial implications for the setting of monetary policy to prevent indeterminacy. As an example, a counterfactual exercise for the Euro area is performed, which considers the determinacy consequences of a revenue-neutral switch from income taxation to consumption taxation. For four variations in the underlying model environment, the tax reform can have both positive and negative repercussions for determinacy under the Taylor principle. Consequently, depending on the monetary policy reaction parameters chosen by the central bank, switching to consumption taxation could actually be harmful for the economy under balanced-budget rules.

The current paper is related to a small literature that has been exploring the determinacy implications of different monetary and fiscal policies under distortionary taxation (see Benhabib and Eusepi, 2005; Linnemann, 2006; Kurozumi, 2010).<sup>5</sup> Previous studies have focused exclusively on

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<sup>5</sup>In addition to balanced-budget fiscal rules, these studies also investigate the determinacy implications of debt-targeting fiscal rules that permit short-run fiscal deficits to arise. An earlier version of this paper additionally investigated the determinacy implications of income and consumption taxation under debt-targeting fiscal rules

the determinacy consequences of income taxation. Linnemann (2006) investigates the determinacy implications of income taxation under a strict future inflation targeting policy and shows that the Taylor principle cannot prevent indeterminacy under a balanced-budget rule. Benhabib and Eusepi (2005) find that indeterminacy can also arise under a current-looking interest-rate rule. Furthermore, they show that if the monetary policy rule additionally reacts to output, this can help in ameliorating the indeterminacy problem associated with a balanced budget fiscal policy. Kurozumi (2010) considers the determinacy implications when the monetary policy rule is designed to respond to forward-looking inflation and contemporaneous output. He finds that the conclusions of Linnemann (2006) can be overturned if seignorage revenues enter the government budget constraint.<sup>6</sup> The contribution of the paper to this literature is twofold. We make a first attempt at investigating the determinacy implications of consumption taxes with the aim of providing a direct comparison between direct and indirect taxation. Second, we additionally focus on a variety of popular interest-rate rules that respond to both inflation and output, which are implementable and empirically motivated, and hence should be of greatest interest to policymakers. This is important, since we find examples where reacting to output in the interest-rate feedback rule can both be beneficial and harmful for determinacy.

The remainder of the paper is organized as follows. Section 2 outlines the model. Section 3 compares the determinacy implications of budget-balancing using consumption taxes and income taxes under both forward-looking and contemporaneous-looking specifications of the interest-rate feedback rule. Section 4 extends the baseline model to allow for the taxation of bond interest income and capital accumulation, and discusses some of the policy implications of the results. Finally, Section 5 briefly concludes.

## 2 A Sticky Price Model with Distortionary Taxation

This section outlines the model. The economy is assumed to be cashless, where there exists a representative agent, a representative final good producer, a continuum of intermediate good producing firms that set prices according to Calvo (1983), and a fiscal and monetary authority. The fiscal authority follows a balanced-budget rule and can raise revenue by taxing *either* consumption

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where differences in the determinacy criteria across the two tax systems were also found to exist.

<sup>6</sup>As discussed by Kurozumi (2010), if seignorage revenues are rebated to the representative household, then the standard monetary economy with separable preferences is equivalent to a cashless economy model. For analytically tractability we follow Benhabib and Eusepi (2005) and Linnemann (2006) and assume a cashless economy.

or income to finance a constant level of (unproductive) spending. Monetary policy is specified as a Taylor-type feedback rule in which the nominal interest rate is a function of both inflation and output. Since we are concerned with issues of local determinacy, the following discussion is limited to a deterministic framework.

## 2.1 Representative Agent

The representative agent is infinitely lived, and chooses consumption  $C$  and labor  $L$  to maximize discounted lifetime utility:<sup>7</sup>

$$\sum_{t=0}^{\infty} \beta^t [u(C_t) - v(L_t)], \quad (1)$$

where the discount factor is  $0 < \beta < 1$ , subject to the period budget constraint

$$B_t + (1 + \tau_t^c)P_t C_t \leq R_{t-1}B_{t-1} + P_t(1 - \tau_t^l)(w_t L_t + \vartheta_t). \quad (2)$$

The agent carries  $B_{t-1}$  holdings of nominal government bonds into period  $t$ , which pay the gross nominal interest rate  $R_{t-1}$ . During period  $t$  the agent supplies labor to the intermediate good producing firms, receiving real income from wages  $w_t$  and real profits from the ownership of intermediate firms  $\vartheta_t$ . The government raises revenue, either by taxing consumption at a rate  $\tau_t^c$ , or by levying a proportional income tax  $\tau_t^l$  on the agent's total labor and profit income  $w_t L_t + \vartheta_t$ .<sup>8</sup> The agent's after-tax resources are then used to carry out bond trading  $B_t$  and for final good consumption  $C_t$ . The first-order conditions from the agent's maximization problem yields:

$$\frac{u_c(C_t)}{u_c(C_{t+1})} = \beta R_t \frac{P_t(1 + \tau_t^c)}{P_{t+1}(1 + \tau_{t+1}^c)}, \quad (3)$$

$$\frac{v_l(L_t)}{u_c(C_t)} = \frac{1 - \tau_t^l}{1 + \tau_t^c} w_t. \quad (4)$$

Equation (3) is the consumption Euler equation and equation (4) is the labor supply equation, where the trade-off between labor and consumption is the relevant after-tax wage rate. Optimizing behavior implies that the budget constraint (2) holds with equality in each period and the appropriate transversality condition is satisfied.

<sup>7</sup>As is standard in the literature, the utility function is assumed to be separable between consumption and leisure. Assuming a non-separable utility function could have important consequences for equilibrium determinacy, since tax-driven changes in labor supply would now also affect intertemporal consumption behavior.

<sup>8</sup>As is standard in the literature, we initially assume that interest income received from maturing bonds is not subject to taxation. The determinacy implications of relaxing this assumption is investigated in Section 4 below.

## 2.2 Firms

Following Yun (1996), the economy is comprised of a continuum of intermediate firms denoted  $y_t(i)$  each indexed by  $i \in [0, 1]$ . The final good  $Y_t$  is produced under perfect competition using intermediate goods as inputs according to the following CES aggregation technology index:

$$Y_t = \left[ \int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (5)$$

where  $\varepsilon > 1$  is the constant elasticity of substitution between intermediate goods. Letting  $p_t(i)$  denote the price of good  $i$ , cost minimization yields the demand schedule

$$y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \quad (6)$$

where the aggregate price index  $P_t$  is given by:

$$P_t = \left[ \int_0^1 p_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}. \quad (7)$$

Intermediate firms hire labor to produce output given a real wage rate  $w_t$ . It is assumed that a firm of type  $i$  has a linear production technology:

$$y_t(i) = L_t(i). \quad (8)$$

Thus, given competitive prices of labor, cost minimization requires that

$$mc_t = w_t, \quad (9)$$

where  $mc_t$  is real marginal cost.

Intermediate firms set prices according to Calvo (1983), where in each period there is a constant probability  $1 - \psi$  that a firm will be randomly selected to adjust its price, which is drawn independently of past history. A firm  $i$ , faced with resetting its price at time  $t$ , chooses  $\tilde{p}_t(i)$  to maximize:

$$\max_{\tilde{p}_t(i)} \left\{ \sum_{s=0}^{\infty} (\psi\beta)^s X_{t,t+s} \left[ \left( \frac{\tilde{p}_t(i)}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} (\tilde{p}_t(i) - P_{t+s} mc_{t+s}) \right] \right\},$$

where  $\beta^s X_{t,t+s} = \beta^s [u_c(C_{t+s})/u_c(C_t)](P_t/P_{t+s})[(1 + \tau_t^c)/(1 + \tau_{t+s}^c)]$  is the discount factor. All



firms that are given the opportunity to reset their price in period  $t$ , all behave in an identical manner. The first-order condition for this maximization problem is given by:

$$\tilde{P}_t = \frac{\varepsilon}{\varepsilon - 1} \sum_{s=0}^{\infty} q_{t,t+s} m c_{t+s}. \quad (10)$$

The optimal price set is a mark-up  $\frac{\varepsilon}{\varepsilon-1}$  over a weighted average of future real marginal costs, where the weight  $q_{t,t+s}$  is given by:

$$q_{t,t+s} \equiv \frac{(\beta\psi)^s X_{t,t+s} P_{t+s}^{\varepsilon+1} Y_{t+s}}{\sum_{s=0}^{\infty} (\beta\psi)^s X_{t,t+s} P_{t+s}^{\varepsilon} Y_{t+s}}.$$

The aggregate price level evolves according to:

$$P_t^{1-\varepsilon} = \psi P_{t-1}^{1-\varepsilon} + (1 - \psi) \tilde{P}_t^{1-\varepsilon}. \quad (11)$$

### 2.3 Fiscal and Monetary Policy

The government purchases a fixed quantity  $G$  of the final good, which is financed by the issuing of new nominal debt  $B_t$  and revenues from levying taxes, either on consumption  $\tau_t^c C_t$  or on real income  $\tau_t^l Y_t$ , where  $Y_t = w_t L_t + \vartheta_t$ . Consequently, the government budget constraint is given by:

$$P_t G = P_t \tau_t^c C_t + P_t \tau_t^l Y_t + B_t - R_{t-1} B_{t-1}. \quad (12)$$

To close the model we need to specify a fiscal policy rule and an interest-rate feedback rule. Following Linnemann (2006) and Kurozumi (2010), we consider a balanced-budget rule, where the stock of real government debt is permanently fixed at its constant steady state level  $b$ :

$$b_t \equiv \frac{B_t}{P_t} = b. \quad (13)$$

Motivated by the studies of Clarida et al. (1999, 2000) and Orphanides (2001, 2004), the monetary authority is assumed to adjust the nominal interest rate in response to changes in both inflation  $\pi_t \equiv P_t/P_{t-1}$  and output according to the rule

$$R_t = R \left( \frac{\pi_{t+\kappa}}{\pi} \right)^{\mu_\pi} \left( \frac{Y_{t+\kappa}}{Y} \right)^{\mu_y}, \quad (14)$$

where  $\mu_\pi \geq 0$  is the inflation response coefficient,  $\mu_y \geq 0$  is the output response coefficient, and  $R = \pi/\beta > 1$ ,  $\pi$ , and  $Y$  respectively denote the steady state nominal interest rate, inflation, and output. The *Taylor principle* is represented by  $\mu_\pi > 1$ , implying that the nominal interest rate rises proportionally more than the increase in inflation. We consider two different specifications for the interest-rate feedback rule. A contemporaneous-looking feedback rule (i.e.  $\kappa = 0$ ), where the nominal interest rate reacts to both current inflation and output as first proposed by Taylor (1993), and a forward-looking feedback rule (i.e.  $\kappa = 1$ ), where the nominal interest rate reacts to expectations of future inflation and output.<sup>9</sup>

## 2.4 Market Clearing and Equilibrium

Market clearing in the factor and final goods market requires that  $L_t = \int_0^1 L_t(i)di$  and

$$Y_t = C_t + G. \tag{15}$$

Aggregating the production function (8) across intermediate firms yields

$$Y_t = \frac{L_t}{d_t}, \tag{16}$$

where  $d_t \equiv \int_0^1 (p_t(i)/P_t)^{-\varepsilon} di$  measures the price dispersion of intermediate goods.

**Equilibrium.** Given the constant  $G$  and the initial conditions  $B_{t_0-1}$  and  $d_{t_0-1}$ , a perfect foresight equilibrium consists of a sequence of prices  $\{w_t, mc_t, P_t, \tilde{P}_t, d_t\}$ , a sequence of allocations  $\{C_t, Y_t, L_t, B_t\}$ , a fiscal policy  $\{\tau_t^c\}$  or  $\{\tau_t^l\}$ , and a monetary policy  $\{R_t\}$  satisfying: (i) the optimality conditions of the representative agent (3)–(4) and the transversality condition holds; (ii) the optimality condition of intermediate firms (9), the price-setting rules (10) and (11), the aggregate production function (16), and a law of motion for price dispersion; (iii) the government budget constraint (12), the balanced-budget rule (13), and the monetary policy rule (14); (iv) the final goods market clears (15).

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<sup>9</sup>The determinacy implications of forward-looking interest-rate rules in the absence of distortionary taxation have been studied by Bullard and Mitra (2002), Evans and McGough (2005), and Duffy and Xiao (2011), amongst others.

## 2.5 Linearized Model

In order to analyze the equilibrium dynamics of the model, a first-order Taylor approximation is taken around the steady state. In what follows, a variable  $\widehat{X}_t$  denotes the percentage deviation of  $X_t$  with respect to its steady state value  $X$  (i.e.  $\widehat{X}_t \equiv \frac{X_t - X}{X}$ ). Noting from (15) that  $\widehat{Y} = s_c \widehat{C}$ , where  $0 < s_c < 1$  is the steady state consumption share in output, the linearized consumption Euler equation (3) is given by:

$$\widehat{Y}_t = \widehat{Y}_{t+1} - s_c \sigma \left( \widehat{R}_t - \widehat{\pi}_{t+1} \right) + \frac{s_c \sigma \tau^c}{1 + \tau^c} \left( \widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c \right), \quad (17)$$

where  $\sigma \equiv -C^{-1}(u_c/u_{cc}) > 0$  is the intertemporal elasticity of substitution in consumption. Combining the linearized versions of (4), (9), (10), (11), and (16) yields the AS equation:<sup>10</sup>

$$\widehat{\pi}_t = \beta \widehat{\pi}_{t+1} + \lambda \left( \omega + \frac{1}{s_c \sigma} \right) \widehat{Y}_t + \lambda \left( \frac{\tau^c}{1 + \tau^c} \right) \widehat{\tau}_t^c + \lambda \left( \frac{\tau^l}{1 - \tau^l} \right) \widehat{\tau}_t^l, \quad (18)$$

where  $\omega \equiv L(v_{ll}/v_l) > 0$  is the elasticity of labor disutility,  $\lambda \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi} > 0$  is the real marginal cost elasticity of inflation, and  $0 < \psi < 1$  is the degree of price rigidity. Linearizing the government budget constraint (12), the balanced-budget rule (13), and the monetary policy rule (14) yields:

$$\tau^c s_c \widehat{\tau}_t^c + \tau^l \widehat{\tau}_t^l + (\tau^c + \tau^l) \widehat{Y}_t + s_b \widehat{b}_t = \frac{s_b}{\beta} \left( \widehat{b}_{t-1} + \widehat{R}_{t-1} - \widehat{\pi}_t \right), \quad (19)$$

$$\widehat{b}_t = 0, \quad (20)$$

$$\widehat{R}_t = \mu_\pi \widehat{\pi}_{t+\kappa} + \mu_y \widehat{Y}_{t+\kappa}, \quad (21)$$

where  $s_b > 0$  is the steady state ratio of government debt to output. To summarize, for the income tax system we set  $\widehat{\tau}_{t+1}^c = \widehat{\tau}_t^c = \tau^c = 0$  in the linearized equations (17)–(21), whereas for the consumption tax system we set  $\widehat{\tau}_t^l = \tau^l = 0$ .

## 2.6 Parameterization

In order to illustrate the conditions for determinacy, the ensuing analysis uses the following baseline parameter values summarized in Table 2. Parameter  $\beta$  is standard in the literature and  $\omega$  is taken from Woodford (2003). We follow the related determinacy studies of Benhabib and Eusepi (2005),

<sup>10</sup>For the price dispersion variable  $d_t$ , its steady state value is  $d = 1$  and its first-order approximation is  $\widehat{d}_t = 0$ .

Table 1: Baseline parameter values

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$\beta$	Discount factor	0.99
$\omega$	Inverse of the elasticity of labor supply	0.47
$\sigma$	Intertemporal elasticity of substitution in consumption	1
$\psi$	Degree of price stickiness	0.75
$\lambda$	Real marginal cost elasticity of inflation	0.086
$\tau^l$	Steady state income tax rate	0.2, 0.3, or 0.4
$\tau^c$	Steady state consumption tax rate	0.05, 0.15, or 0.25
$s_b$	Steady state ratio of government debt to output	$0 < s_b \leq 3$
$\mu_\pi$	Inflation response coefficient	$0 \leq \mu_\pi \leq 4$
$\mu_y$	Output response coefficient	$0 \leq \mu_y \leq 3$

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Linnemann (2006), and Kurozumi (2010) and set  $\sigma = 1$  consistent with micro-level estimates (e.g., Vissing-Jorgensen, 2002). As noted by Benhabib and Eusepi (2005) and Huang et al. (2009), empirical estimates of  $\psi$  vary considerably. As is standard in the determinacy literature, we follow Taylor (1999) by setting  $\psi = 0.75$ , which constitutes an average price duration of one year and implies a real marginal cost elasticity of inflation  $\lambda \approx 0.086$ . However, the robustness of the numerical results are examined for variations in  $\psi$ . Estimated tax rates vary over time and across countries. Consequently, the numerical analysis considers three values for the income tax rate  $\tau^l = 0.2, 0.3, 0.4$  and three values for the consumption tax rate  $\tau^c = 0.05, 0.15, 0.25$  roughly in line with U.S. and European estimates (see, for example, Mendoza et al., 1994; Giannitsarou, 2007; Lipińska and von Thadden, 2009). Finally, given values for the tax rate and the ratio of government debt to output  $s_b$ , the consumption share in output  $s_c$ , is determined from the steady state version of the government budget constraint (12).

### 3 Consumption Taxation vs. Income Taxation

This section compares the determinacy implications of consumption taxes and income taxes under the baseline model. For both tax systems, the necessary and sufficient conditions for equilibrium determinacy are derived for two variants of the interest-rate feedback rule: a forward-looking specification, where the nominal interest rate is set contingent on future inflation and output, and a contemporaneous-looking specification.

### 3.1 Forward-Looking Interest-Rate Rules

We start by considering the consequences for determinacy if monetary policy is characterized by a forward-looking interest-rate rule.

**Proposition 1.** *If the monetary authority follows a forward-looking interest-rate rule, the necessary and sufficient conditions for local equilibrium determinacy are:*

#### A. Income Taxation

Case I:

$$1 - \beta + \frac{s_c \sigma \mu_y}{1 - s_c \sigma \mu_y} > \frac{\lambda s_b (\mu_\pi - 1)}{\beta (1 - \tau^l) (1 - s_c \sigma \mu_y)}, \quad (22)$$

$$\frac{\lambda \Lambda_1^l (\mu_\pi - 1)}{1 - s_c \sigma \mu_y} + \frac{\mu_y (1 - \beta)}{1 - s_c \sigma \mu_y} > 0, \quad (23)$$

$$(1 + \beta) \left[ 2 + \frac{s_c \sigma \mu_y}{1 - s_c \sigma \mu_y} \right] > \frac{\lambda (\mu_\pi - 1)}{1 - s_c \sigma \mu_y} \left[ s_c \sigma \Lambda_1^l + \frac{2s_b}{\beta (1 - \tau^l)} \right]. \quad (24)$$

Case II: The two strict inequalities opposite to (23) and (24) hold.

#### B. Consumption Taxation

Case I:

$$1 - \beta + \frac{\beta (1 + \tau^c) s_c \sigma \mu_y}{\Lambda_2^c} + \frac{\lambda s_b \sigma \omega (\mu_\pi - 1)}{\Lambda_2^c} > 0, \quad (25)$$

$$\frac{\lambda \Lambda_1^c (\mu_\pi - 1)}{\Lambda_2^c} + \frac{\mu_y (1 - \beta)}{\Lambda_2^c} > 0, \quad (26)$$

$$(1 + \beta) \left[ 2 + \frac{\beta (1 + \tau^c) s_c \sigma \mu_y}{\Lambda_2^c} \right] > \frac{\lambda \sigma (\mu_\pi - 1)}{\Lambda_2^c} [\beta (1 + \tau^c) s_c \Lambda_1^c - 2\omega s_b]. \quad (27)$$

Case II: The two strict inequalities opposite to (26) and (27) hold.

where  $\Lambda_1^l \equiv \omega + \frac{1}{s_c \sigma} - \frac{\tau^l}{1 - \tau^l}$ ,  $\Lambda_1^c \equiv \omega + \frac{1}{s_c \sigma} - \frac{\tau^c}{(1 + \tau^c) s_c}$ , and  $\Lambda_2^c \equiv \beta [1 + \tau^c (1 - \sigma)] + \sigma \mu_y [s_b - \beta s_c (1 + \tau^c)]$ .

**Proof.** See Appendix A. □

**Income taxes.** Proposition 1.A suggests that in the presence of government debt (i.e.  $s_b > 0$ ) indeterminacy is a serious problem when a balanced budget policy is financed using income taxation. To see this, first consider the case when the monetary authority adopts a strict future inflation targeting policy, whereby the interest-rate feedback rule (21) reacts only to future inflation (i.e.,  $\mu_y = 0$ ).<sup>11</sup> For all parameter values employed in the numerical analysis  $\Lambda_1^l > 0$ , and Case II

<sup>11</sup>This was the interest-rate feedback rule originally studied by Linnemann (2006).

of Proposition 1.A never applies. Hence, for determinacy, condition (23) requires that the Taylor principle is satisfied (i.e.  $\mu_\pi > 1$ ), and conditions (22) and (24) simplify to:

$$\mu_\pi < 1 + \frac{\beta(1-\beta)(1-\tau^l)}{\lambda s_b} \equiv \Gamma_1, \quad (28)$$

$$\mu_\pi < 1 + \frac{2(1+\beta)}{\lambda \left[ s_c \sigma \Lambda_1^l + \frac{2s_b}{\beta(1-\tau^l)} \right]} \equiv \Gamma_2. \quad (29)$$

The numerical analysis suggests that for  $s_b > 0$ ,  $\Gamma_1 < \Gamma_2$  so that  $\Gamma_1$  given in (28) above is the empirically relevant upper bound on the inflation response coefficient  $\mu_\pi$ . By inspection, this upper bound is independent of  $\sigma$ , and it is straightforward to verify that  $\Gamma_1$  is increasing in the degree of price stickiness  $\partial\Gamma_1/\partial\psi > 0$  and decreasing in both the debt level  $\partial\Gamma_1/\partial s_b < 0$  and the steady state tax rate  $\partial\Gamma_1/\partial\tau^l < 0$ . The numerical analysis finds that even with only a small degree of  $s_b$ , the upper bound  $\Gamma_1$  is of a magnitude to be likely to bind. For example, using the baseline parameter values and setting  $s_b = 0.1$ , the interval of inflation response coefficients that induce determinacy are:  $1 < \mu_\pi < 1.923$  for  $\tau^l = 0.2$ ,  $1 < \mu_\pi < 1.807$  for  $\tau^l = 0.3$ , and  $1 < \mu_\pi < 1.692$  for  $\tau^l = 0.4$ . This is in stark contrast to when government debt is absent from the model (i.e.  $s_b = 0$ ), where the upper bound  $\Gamma_1$  no longer applies, and the upper bound  $\Gamma_2$  given in (29) binds only for unrealistically high values of  $\mu_\pi$ . For instance, under the baseline parameter values the interval of inflation response coefficients that now induce determinacy is  $1 < \mu_\pi < 40.43$  for  $\tau^l = 0.2$ ,  $1 < \mu_\pi < 46.06$  for  $\tau^l = 0.3$ , and  $1 < \mu_\pi < 53.57$  for  $\tau^l = 0.4$ .

Figure 1 gives a graphical representation of these results using two alternative values of the steady state income tax rate  $\tau^l = 0.2, 0.4$ . The top half of Fig. 1 graphs the (in)determinacy regions for combinations of  $\mu_\pi$  and  $s_b$  setting  $\psi = 0.75$ . The bottom half of Fig. 1 graphs the (in)determinacy regions for combinations of  $\mu_\pi$  and  $\psi$  setting  $s_b = 2.0$ .<sup>12</sup> For alternative combinations of  $\tau^l$ ,  $\psi$ , and  $s_b$ , the upper bound  $\Gamma_1$  on  $\mu_\pi$  changes only modestly, such that the determinacy region always remains extremely narrow.

Figure 2 illustrates the determinacy implications if the interest-rate feedback rule (21), in addition to reacting to future inflation, also reacts to future output (i.e.  $\mu_y > 0$ ). Figure 2 depicts the areas of (in)determinacy for combinations of  $\mu_\pi$  and  $\mu_y$  setting  $\tau^l = 0.3$  for three alternative debt levels  $s_b = 1.0, 2.0, 3.0$  and four alternative values of the degree of price stickiness  $\psi = 0.3, 0.5, 0.75, 0.85$ . By inspection, the effectiveness of a flexible future inflation targeting policy

<sup>12</sup>Setting  $s_b = 2.0$  implies a yearly government debt-output ratio of 50%.

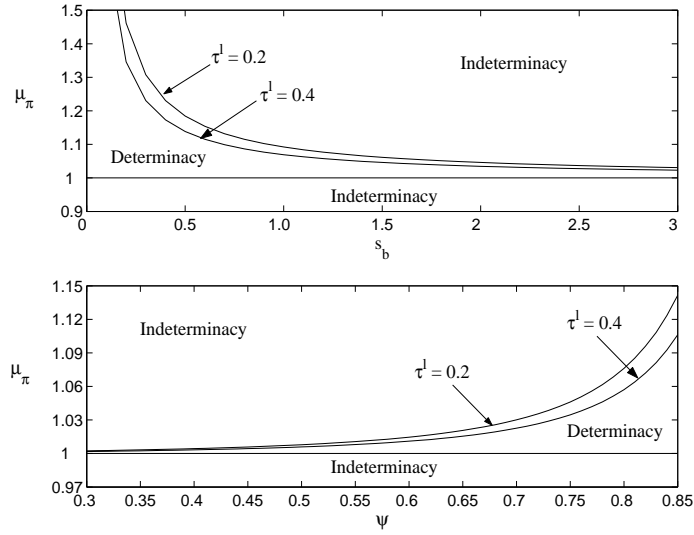


Figure 1: Income Taxes – Regions of (in)determinacy under a strict future inflation targeting policy

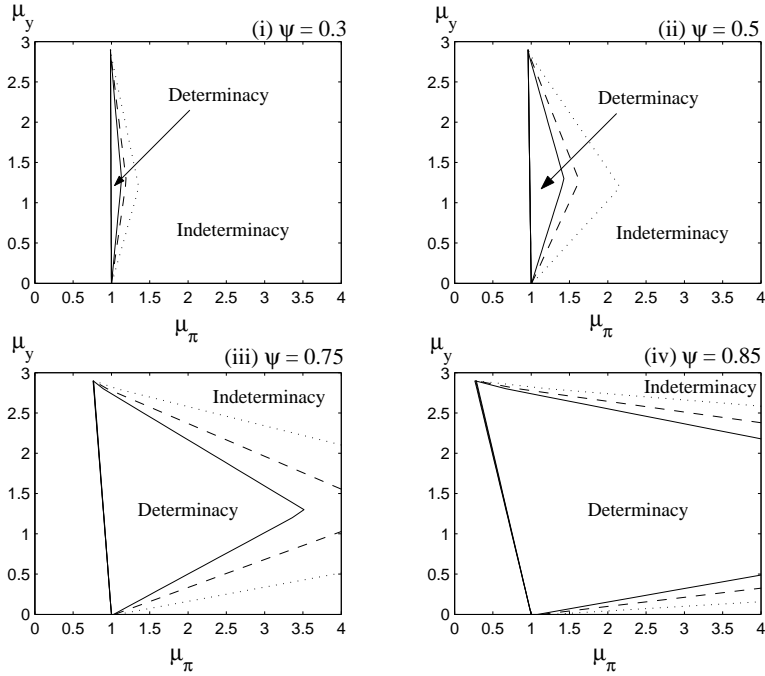


Figure 2: Income Taxes – Regions of (in)determinacy under a flexible future inflation targeting policy ( $\tau^l = 0.3$ ):  $s_b = 1.0$  ( $\cdots$ ),  $2.0$  ( $- - -$ ),  $3.0$  ( $-$ )

in inducing determinacy crucially depends on the magnitude of  $\psi$ : the lower is  $\psi$ , the less effective is such a monetary policy in preventing indeterminacy.<sup>13</sup>

<sup>13</sup>The sensitivity analysis shows that this result is robust to variations in  $\tau^l$ .

**Consumption taxes.** For the consumption tax system, Proposition 1.B outlines the conditions for determinacy under a forward-looking interest-rate rule. First note that  $\Lambda_1^c > 0$  for all parameter values used in the numerical analysis. As before, let us first consider the case when the interest-rate feedback rule reacts only to future inflation (i.e.  $\mu_y = 0$ ). Then it is straightforward to verify that  $\Lambda_2^c > 0, \forall \tau^c$  under the baseline parameterization. Consequently, conditions (25) and (26) are always satisfied under the Taylor principle, and condition (27) simplifies to:<sup>14</sup>

$$\mu_\pi < 1 + \frac{2\beta(1+\beta)}{\lambda[\beta(1+\omega) - \omega s_b(1+\beta)]} \equiv \Gamma_3, \quad (30)$$

which by inspection is independent of the steady state tax rate  $\tau^c$ . The numerical analysis suggests that the upper bound  $\Gamma_3$  given in (30) has little practical significance. For example, under the baseline parameterization the interval of inflation response coefficients that induce determinacy is  $1 < \mu_\pi < 34.71$  with  $s_b = 0.1$ . Since  $\Gamma_3$  is increasing in  $s_b$ , determinacy is therefore easily attainable for any debt level.

Figure 3 illustrates the determinacy implications if the interest-rate feedback rule also reacts to future output (i.e.  $\mu_y > 0$ ).<sup>15</sup> Fig. 3 illustrates the areas of (in)determinacy for combinations of  $\mu_\pi$  and  $\mu_y$  for four alternative values of the degree of price stickiness  $\psi = 0.3, 0.5, 0.75, 0.85$ , setting  $s_b = 2.0$ .<sup>16</sup> By inspection, regardless of the value of  $\psi$  and  $\mu_y$  the Taylor principle easily ensures equilibrium determinacy under consumption taxation.

To get some intuition behind these results first suppose that the government raises revenue using income taxation. With  $\mu_\pi > 1$  and  $\mu_y = 0$ , then an increase in inflationary expectations  $\uparrow \hat{\pi}_{t+1}$  can be validated through the public finance channel of monetary policy. Under a balanced budget fiscal policy, an increase in the real interest rate raises future government debt repayments and future taxation from (19). Since taxes are distortionary, higher future income taxes  $\uparrow \hat{\tau}_{t+1}^l$  increase future marginal cost, which via the next-period AS equation (18), results in a self-fulfilling increase in  $\uparrow \hat{\pi}_{t+1}$ . The higher is the steady state tax rate  $\tau^l$ , the higher the government debt-output ratio  $s_b$ , and the lower the degree of price stickiness  $\psi$ , the more severe the indeterminacy problem becomes. The key difference under consumption taxation is that the public finance channel now also directly affects aggregate demand. By inspection of (17), higher future consumption taxes  $\uparrow \hat{\tau}_{t+1}^c$  shift consumption towards the present, thereby reducing future output. Consequently, the aggregate

<sup>14</sup>Clearly, if  $\beta(1+\omega) - \omega s_b(1+\beta) < 0$  then condition (27) is also always satisfied under the Taylor principle.

<sup>15</sup>The numerical analysis suggests that under the baseline parameterization the determinacy conditions are once again independent of the steady state tax rate  $\tau^c$ .

<sup>16</sup>The sensitivity analysis shows that this result is robust to variations in  $s_b$ .



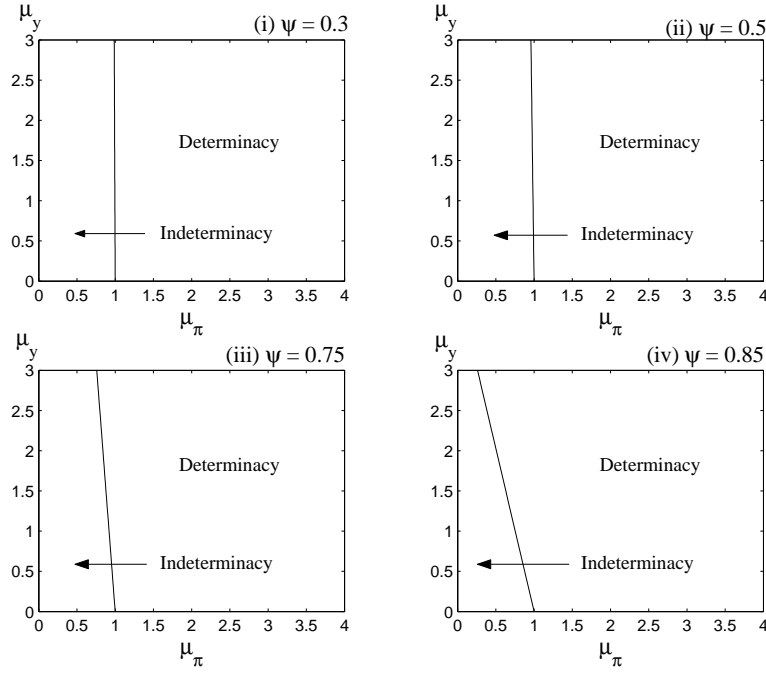


Figure 3: Consumption Taxes – Regions of (in)determinacy under a flexible future inflation targeting policy ( $s_b = 2.0$ )

supply effects of higher future inflation can now be offset by the reduction in future inflation generated via lower future aggregate demand. Therefore, under forward-looking interest-rate rules consumption taxation helps to prevent the emergence of self-fulfilling inflationary expectations.

### 3.2 Contemporaneous-Looking Interest-Rate Rules

How sensitive are the previous results in relation to the specification of the monetary policy rule? Here we consider the determinacy implications of the two tax systems when the interest-rate feedback rule reacts to both current inflation and output. The Appendix proves the following.

**Proposition 2.** *If the monetary authority follows a contemporaneous-looking interest-rate rule, the necessary and sufficient conditions for local equilibrium determinacy are:*

#### A. Income Taxation

Case I:

$$\mu_y(1 - \beta) + \lambda\Lambda_1^l(\mu_\pi - 1) > 0, \quad (31)$$

$$(1 + \beta) \left[ 2 + s_c\sigma\mu_y \right] + \lambda(\mu_\pi - 1) \left[ s_c\sigma\Lambda_1^l + \frac{2s_b}{\beta(1 - \tau^l)} \right] > 0, \quad (32)$$

$$|a_2^l| > 3 \quad \text{or} \quad a_0^l a_0^l - a_0^l a_2^l + a_1^l - 1 > 0. \quad (33)$$

Case II: The two strict inequalities opposite to (31) and (32) hold.

### B. Consumption Taxation

Case I:

$$\frac{\mu_y(1-\beta)}{\Lambda_3^c} + \frac{\lambda\Lambda_1^c(\mu_\pi - 1)}{\Lambda_3^c} > 0, \quad (34)$$

$$(1+\beta) \left[ 2 - \frac{\sigma\mu_y}{\Lambda_3^c} \left( \frac{2s_b}{\beta(1+\tau^c)} - s_c \right) \right] > \frac{\lambda\sigma(\mu_\pi + 1)}{\Lambda_3^c} \left[ \frac{2s_b\omega}{\beta(1+\tau^c)} - s_c\Lambda_1^c \right], \quad (35)$$

$$|a_2^c| > 3 \quad \text{or} \quad a_0^c a_0^c - a_0^c a_2^c + a_1^c - 1 > 0. \quad (36)$$

Case II: The two strict inequalities opposite to (34) and (35) hold.

where  $\Lambda_1^l = \omega + \frac{1}{s_c\sigma} - \frac{\tau^l}{1-\tau^l}$ ,  $\Lambda_1^c = \omega + \frac{1}{s_c\sigma} - \frac{\tau^c}{(1+\tau^c)s_c}$ , and  $\Lambda_3^c = 1 - \frac{\sigma\tau^c}{1+\tau^c}$  and  $a_j^i$ ,  $i = l, c$ ;  $j = 0, 1, 2$ , are given in Appendix B.

**Proof.** See Appendix B. □

**Income taxes.** In contrast to Section 3.1, Proposition 2.A suggests that under a contemporaneous-looking feedback rule, indeterminacy is no longer a serious problem when the balanced-budget rule is financed using income taxation. For example, under a strict contemporaneous inflation targeting policy, conditions (31) and (32) are satisfied for any  $\mu_\pi > 1$ .<sup>17</sup> Thus, the Taylor principle is consistent with determinacy provided one of the conditions given in (33) hold. The numerical analysis finds that there are many values of  $s_b > 0$  that induce determinacy. Employing the baseline parameterization, Figure 4 illustrates the (in)determinacy regions using two alternative values for the steady state income tax rate  $\tau^l = 0.2, 0.4$ . The top half of Fig. 4 graphs the (in)determinacy regions for combinations of  $\mu_\pi$  and  $s_b$  setting  $\psi = 0.75$ , where by inspection, indeterminacy only arises under  $\tau^l = 0.4$  and for particular combinations of  $\mu_\pi$  and  $s_b$ . The bottom half of Fig. 4 graphs the (in)determinacy regions for combinations of  $\mu_\pi$  and  $\psi$  setting  $s_b = 2.0$ . By inspection, a sufficient degree of price stickiness is required to induce indeterminacy, which decreases as the steady state tax rate is increased. For example, if  $\tau^l = 0.4$  then indeterminacy is eliminated under the Taylor principle with  $\psi \leq 0.737$ .

<sup>17</sup>Recall that for all parameter values used in the numerical analysis  $\Lambda_1^l, \Lambda_1^c > 0$ .

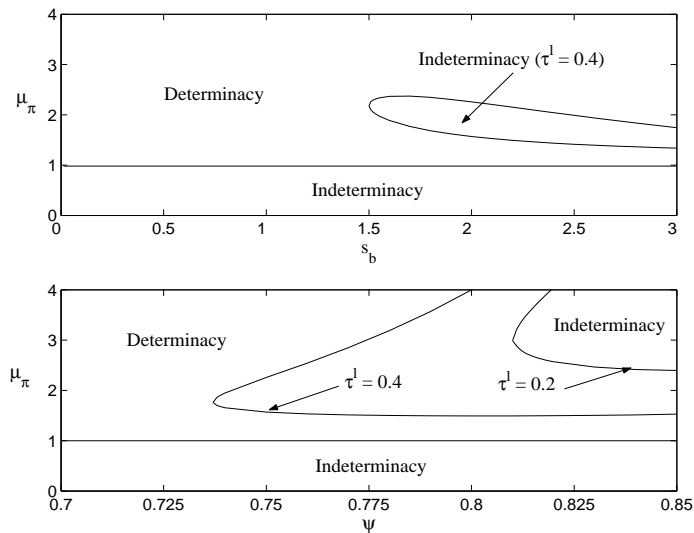


Figure 4: Income Taxes – Regions of (in)determinacy under a strict contemporaneous inflation targeting policy

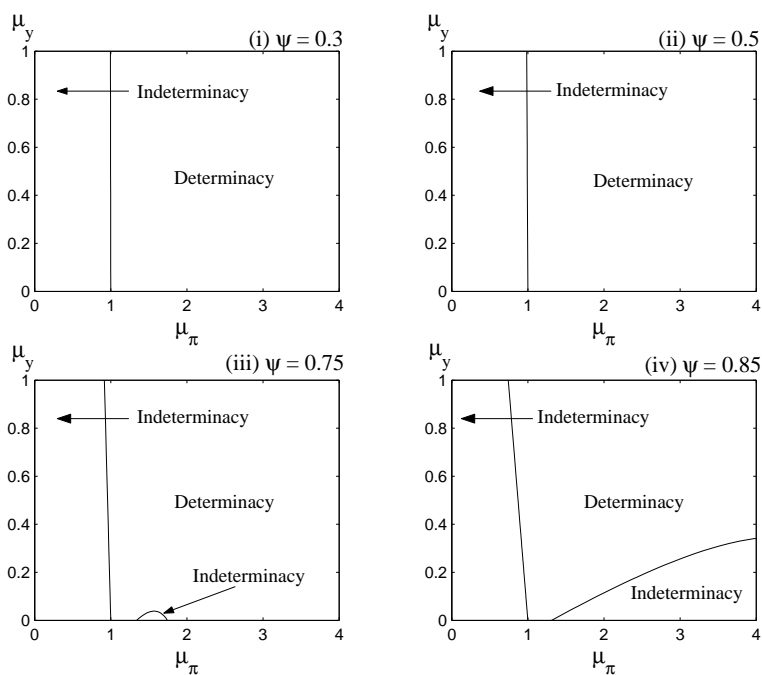


Figure 5: Income Taxes – Regions of (in)determinacy under a flexible contemporaneous inflation targeting policy ( $\tau^l = 0.4$ ;  $s_b = 3.0$ )

Figure 5 illustrates the determinacy implications if the interest-rate feedback rule also responds to current output. For alternative combinations of  $\mu_\pi$  and  $\mu_y$ , Fig. 5 depicts the (in)determinacy regions using four alternative values for the degree of price stickiness  $\psi = 0.3, 0.5, 0.75, 0.85$ , setting

$\tau^l = 0.4$  and  $s_b = 3.0$ . By inspection, the indeterminacy problem that arises under the Taylor principle with relatively high degrees of price stickiness can easily be ameliorated if the interest-rate feedback rule reacts sufficiently strongly to current output.<sup>18</sup>

**Consumption taxes.** For the consumption tax system, we first illustrate Proposition 2.B using the baseline parameter values under a strict contemporaneous inflation targeting policy. Figure 6 shows the regions of determinacy, indeterminacy, and explosiveness under this monetary policy. If the equilibrium is explosive no perfect foresight equilibrium exists (locally). The numerical analysis suggests that under the baseline parameterization the determinacy conditions are independent of the steady state tax rate  $\tau^c$ . The top half of Fig. 6 graphs the (in)determinacy regions for combinations of  $\mu_\pi$  and  $s_b$  setting  $\psi = 0.75$ , where by inspection, the Taylor principle guarantees determinacy for all values of  $s_b$ . The bottom half of Fig. 6 graphs the (in)determinacy regions for combinations of  $\mu_\pi$  and  $\psi$  setting  $s_b = 3.0$ . By inspection, a sufficient degree of price stickiness is required to induce determinacy. Otherwise, when prices are sufficiently flexible  $\psi < 0.32$ , condition (35) cannot be satisfied, thereby making Case II of Proposition 2.B relevant for determinacy. In this case, the Taylor principle results in a locally explosive equilibrium and determinacy is only possible under a passive monetary policy (i.e.  $\mu_\pi < 1$ ).

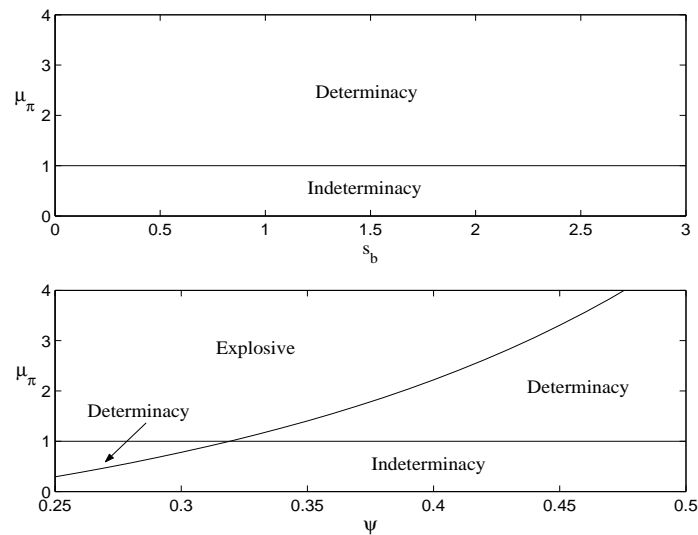


Figure 6: Consumption Taxes – Regions of (in)determinacy under a strict contemporaneous inflation targeting policy

<sup>18</sup>This conclusion is consistent with the numerical findings of Benhabib and Eusepi (2005).

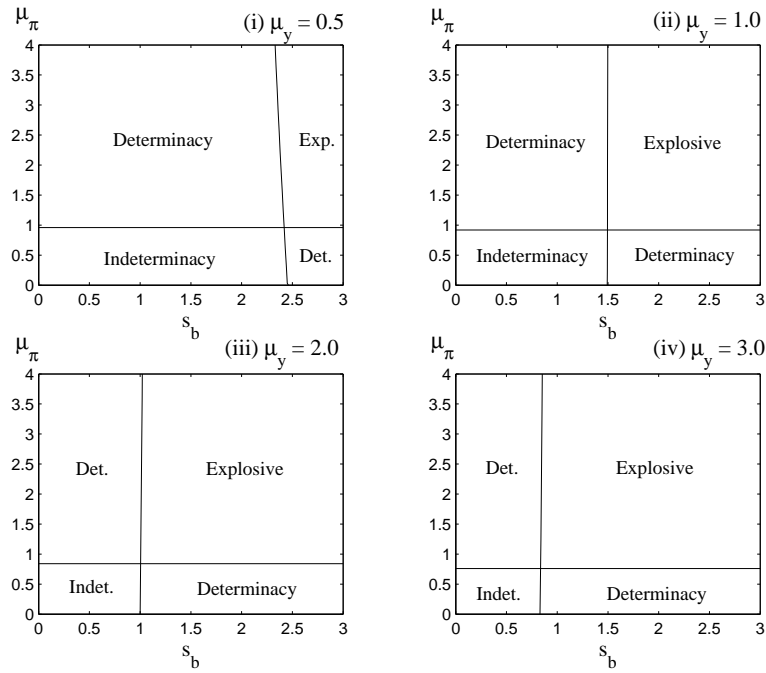


Figure 7: Consumption Taxes – Determinacy regions under a flexible contemporaneous inflation targeting policy ( $\psi = 0.75$ )

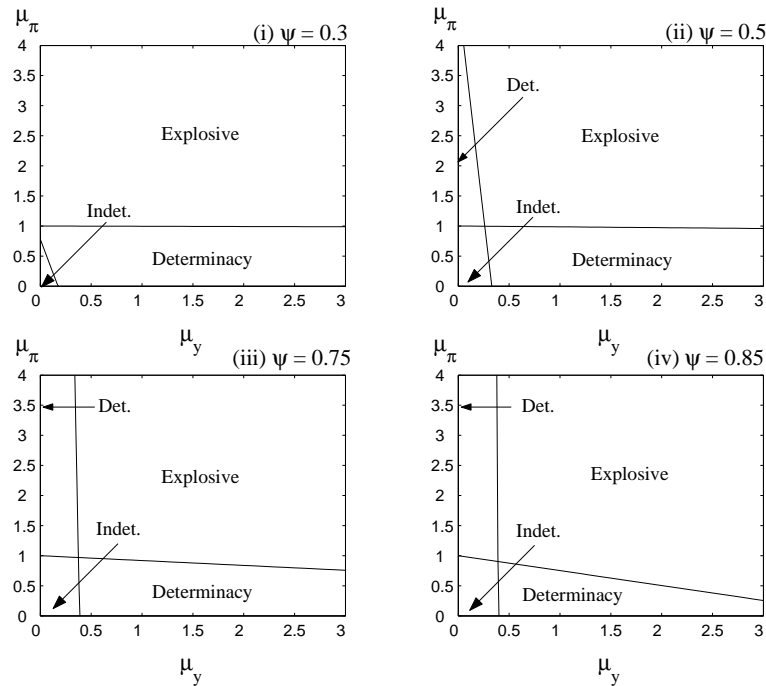


Figure 8: Consumption Taxes – Determinacy regions under a flexible contemporaneous inflation targeting policy ( $s_b = 3.0$ )

By inspection of Proposition 2.B it is possible to see why assigning a positive weight to output in the interest-rate feedback rule cannot help the Taylor principle to induce determinacy. Noting that  $\Lambda_3^c > 0$  under the baseline parameterization, condition (35) is less likely to be satisfied with  $\mu_y > 0$ . When (35) cannot be satisfied, then Case II of Proposition 2.B becomes the relevant case for determinacy. Rearranging the Case II equivalent to (34) yields the following upper bound on  $\mu_\pi$ :

$$\mu_\pi < 1 - \frac{\mu_y(1-\beta)}{\lambda\Lambda_1^c} \equiv \Gamma_4 < 1, \quad (37)$$

where  $\Gamma_4$  is decreasing in  $\mu_y$ . Consequently, regardless of the magnitude of the steady state consumption tax rate, determinacy can only occur under a passive monetary policy in this case.

Figure 7 illustrates the regions of determinacy, indeterminacy, and explosiveness for alternative combinations of  $\mu_\pi$  and  $s_b$  using four different values of the output response coefficient  $\mu_y = 0.5, 1.0, 2.0, 3.0$  setting  $\psi = 0.75$ .<sup>19</sup> By inspection, for empirically plausible values of  $s_b$  the equilibrium is rendered explosive under a flexible contemporaneous inflation targeting policy: the more aggressive the monetary authority is in its setting of  $\mu_y$ , the larger is the interval of  $s_b$  that generates explosiveness under the Taylor principle. Figure 8 illustrates the regions of determinacy, indeterminacy, and explosiveness for alternative combinations of  $\mu_\pi$  and  $\mu_y$  using four different values of the degree of price stickiness  $\psi = 0.3, 0.5, 0.75, 0.85$  setting  $s_b = 3.0$ . By inspection of Fig. 8, the interval of output response coefficients that induce determinacy under the Taylor principle is small for all values of  $\psi$ : as prices become more flexible, the smaller is the interval of  $\mu_y$  that supports determinacy with  $\mu_\pi > 1$ . In summary, Figs. 7 and 8 highlight a key danger of blindly following the Taylor principle under consumption taxation when the interest-rate feedback rule is contemporaneous looking. With consumption taxation, it is critical for determinacy that the monetary authority does not respond to output to avoid rendering the equilibrium explosive.

What is the intuition behind these results? Recall that for forward-looking interest-rate rules indeterminacy arose under income taxation via the public finance channel of monetary policy. However, under current-looking interest-rate rules indeterminacy now depends on the effect of the public finance channel, relative to the aggregate demand channel of monetary policy. Under the Taylor principle, an increase in inflationary expectations  $\uparrow \hat{\pi}_{t+1}$  increases current inflation  $\uparrow \hat{\pi}_t$  and the real interest rate. Under a balanced budget policy, from (19) the increase in the real

<sup>19</sup>If  $\sigma = 1$ , it can be shown that the determinacy conditions (34) and (35) are independent of the steady state tax rate  $\tau^c$ . For the baseline parameter values, one of the conditions given in (36) always holds. Consequently, Figure 7 is appropriate for  $\tau^c = 0.05, 0.15, 0.25$ .

interest rate raises the debt repayments of the government, resulting in an increase in income taxation, marginal cost, and from the AS equation (18), upward pressure on  $\uparrow \hat{\pi}_t$ . However, the increase in the real interest rate also reduces aggregate demand via (17), which reduces marginal cost, exerting downward pressure on  $\downarrow \hat{\pi}_t$ . Therefore, the initial inflationary expectations are self-fulfilling only if the public finance channel, brought about by the need for higher tax revenues, outweighs the aggregate demand channel of monetary policy. Consequently, indeterminacy is less likely to arise under a strict contemporaneous inflation targeting policy. Indeed, indeterminacy can easily be ameliorated under income taxation if the interest-rate feedback rule also responds to current output, since this magnifies the aggregate demand response to interest rate changes.

A key difference between the two tax systems is that locally explosive equilibrium can emerge under consumption taxation. Recall that with consumption taxes the public finance channel also directly affects aggregate demand from the AD equation (17). Consequently, higher real interest rates not only exert downward pressure on inflation via the aggregate demand channel of monetary policy, but such decreases in inflation can be reinforced under a balanced budget policy with the need for higher consumption taxes, further reducing output, and thus inflation can diverge away from the steady state. Responding to current output exacerbates this problem under the Taylor principle, in which case, a passive monetary policy is required to induce determinacy.

## 4 Extensions

This section investigates the robustness of the results presented in Section 3 in two important directions. Section 4.1 first considers the determinacy implications of income taxation when the taxation of bond interest income is also permitted, whereas Section 4.2 introduces capital and investment spending into the analysis. In addition, Section 4.3 discusses some of the policy implications of the results.

### 4.1 Taxing Bond Interest Income

So far we have ignored bond interest income as a source of tax revenue for the government. However, as originally shown by Edge and Rudd (2007) this can have important implications for determinacy. We now assume that the interest income received from maturing bonds is taxed at the same rate  $\tau^l$  as the agent's total labor and profit income  $w_t L_t + \vartheta_t$ . Hence, the individual and

government period budget constraints are now given by:

$$B_t + (1 + \tau_t^c)P_t C_t \leq [1 + (1 - \tau_t^l)(R_{t-1} - 1)] B_{t-1} + P_t(1 - \tau_t^l)(w_t L_t + \vartheta_t),$$

$$P_t G = P_t \tau_t^c C_t + P_t \tau_t^l Y_t + \tau_t^l (R_{t-1} - 1) B_{t-1} + B_t - R_{t-1} B_{t-1}. \quad (38)$$

Consequently, the future labor income tax rate  $\tau_{t+1}^l$  now enters into the consumption Euler equation:

$$\frac{u_c(C_t)}{u_c(C_{t+1})} = \beta [R_t - \tau_{t+1}^l (R_t - 1)] \frac{P_t(1 + \tau_t^c)}{P_{t+1}(1 + \tau_{t+1}^c)}. \quad (39)$$

The other features of the model remain unchanged from the baseline model of Section 2. The Appendix proves the following.

**Proposition 3.** *If bond interest income is also subject to taxation, the necessary and sufficient conditions for local equilibrium determinacy under a forward-looking interest-rate rule are:*

*Case I:*

$$1 - \beta + \left[ \frac{\mu_y(1 - \beta\tau^l) + \frac{(1-\beta)\tau^l}{1-\tau^l}}{\Lambda_3^l} \right] \left[ \frac{s_c\sigma\beta(1 - \tau^l)}{\beta(1 - \tau^l) + s_b(1 - \beta)} \right] > \frac{\lambda s_b}{\Lambda_3^l} \left[ \frac{(1 - \beta\tau^l)\mu_\pi - 1}{\beta(1 - \tau^l) + s_b(1 - \beta)} \right], \quad (40)$$

$$\frac{\lambda [(1 - \beta\tau^l)\mu_\pi - 1]}{\Lambda_3^l} \left[ \Lambda_2^l - \frac{\tau^l}{1 - \tau^l} \left( \frac{s_b(1 - \beta)}{\beta(1 - \tau^l) + s_b(1 - \beta)} \right) \right] + \frac{(1 - \beta)}{\Lambda_3^l} \left[ \mu_y(1 - \beta\tau^l) + \frac{(1 - \beta)\tau^l}{1 - \tau^l} \right] > 0, \quad (41)$$

$$(1 + \beta) \left[ 2 \left( 1 + \frac{s_b(1 - \beta)}{\beta(1 - \tau^l)} \right) + \frac{s_c\sigma}{\Lambda_3^l} \left( \mu_y(1 - \beta\tau^l) + \frac{(1 - \beta)\tau^l}{1 - \tau^l} \right) \right] >$$

$$\frac{\lambda [(1 - \beta\tau^l)\mu_\pi - 1]}{\Lambda_3^l} \left[ s_c\sigma\Lambda_2^l + \frac{2s_b}{\beta(1 - \tau^l)} - \frac{s_c s_b \sigma(1 - \beta)}{\beta(1 - \tau^l) + s_b(1 - \beta)} \left( \frac{\tau^l}{1 - \tau^l} \right) \right]. \quad (42)$$

*Case II: The two strict inequalities opposite to (41) and (42) hold.*

where  $\Lambda_2^l \equiv \omega + \frac{1}{s_c\sigma} - \frac{\beta\tau^l}{\beta(1-\tau^l)+s_b(1-\beta)}$  and  $\Lambda_3^l \equiv 1 - \frac{s_c\sigma\beta(1-\tau^l)}{\beta(1-\tau^l)+s_b(1-\beta)} \left[ \mu_y(1 - \beta\tau^l) + \frac{(1-\beta)\tau^l}{1-\tau^l} \right]$ .

**Proof.** See Appendix C. □

To see the determinacy implications of taxing bond interest income, first consider the case of a strict future inflation targeting policy. For all parameter values employed in the numerical analysis  $\Lambda_3^l > 0$ , and Case II of Proposition 3 never applies. Hence, for determinacy, conditions (40)–(42) simplify to:

$$\max \{0, \Gamma_3^{BI}\} < \mu_\pi < \min \{\Gamma_1^{BI}, \Gamma_2^{BI}\} \quad (43)$$



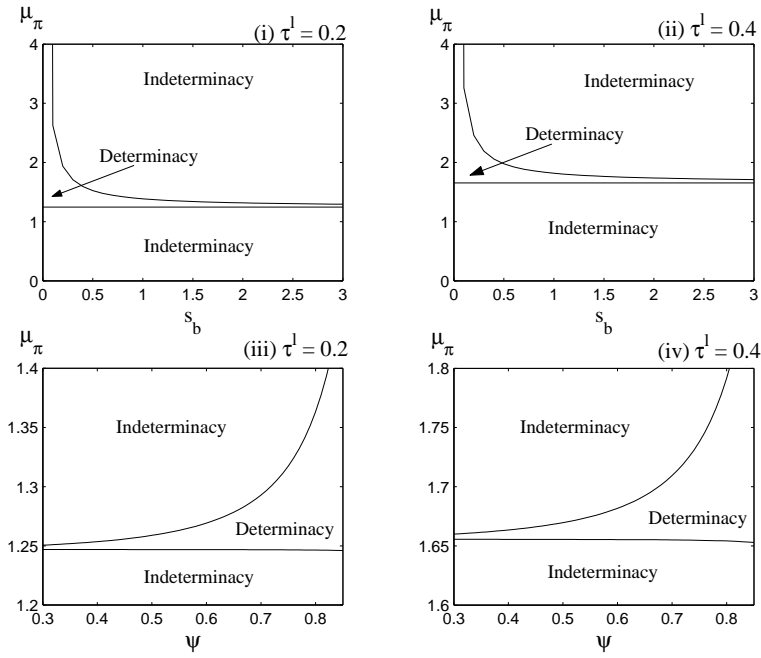


Figure 9: Bond Interest Income Taxes – Regions of (in)determinacy under a strict future inflation targeting policy

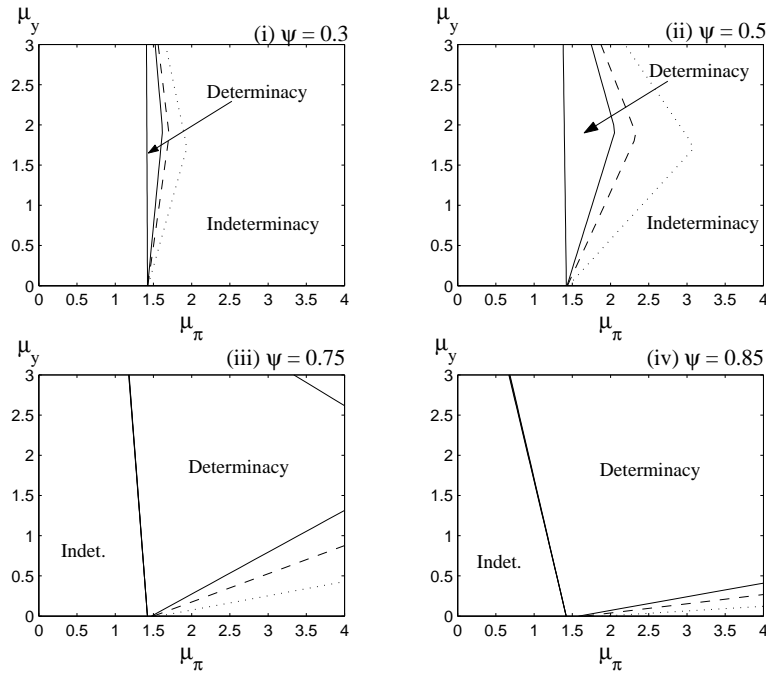


Figure 10: Bond Interest Income Taxes – Regions of (in)determinacy under a flexible future inflation targeting policy ( $\tau^l = 0.3$ ):  $s_b = 1.0$  ( $\cdots$ ),  $2.0$  ( $- - -$ ),  $3.0$  ( $-$ )

where

$$\Gamma_1^{BI} \equiv \frac{1}{1 - \beta\tau^l} + \frac{(1 - \beta)\beta\sigma s_c\tau^l}{\lambda s_b(1 - \beta\tau^l)} + \frac{\Lambda_3^l(1 - \beta)[\beta(1 - \tau^l) + s_b(1 - \beta)]}{\lambda s_b(1 - \beta\tau^l)},$$

$$\Gamma_2^{BI} \equiv \frac{1}{1 - \beta\tau^l} + \frac{(1 + \beta)(1 - \beta)s_c\sigma\tau^l}{\lambda(1 - \tau^l)(1 - \beta\tau^l) \left[ s_c\sigma X^l + \frac{2s_b}{\beta(1 - \tau^l)} \right]} + \frac{2(1 + \beta)\Lambda_3^l}{\lambda(1 - \beta\tau^l) \left[ s_c\sigma X^l + \frac{2s_b}{\beta(1 - \tau^l)} \right]} \left[ 1 + \frac{s_b(1 - \beta)}{\beta(1 - \tau^l)} \right],$$

$$\Gamma_3^{BI} \equiv \frac{1}{1 - \beta\tau^l} - \frac{(1 - \beta)^2\tau^l}{\lambda(1 - \tau^l)(1 - \beta\tau^l)X^l}.$$

The numerical analysis suggests that  $X^l \equiv \Lambda_2^l - \left( \frac{\tau^l}{1 - \tau^l} \right) \frac{s_b(1 - \beta)}{\beta(1 - \tau^l) + s_b(1 - \beta)} > 0$ , and for  $s_b > 0$ ,  $\Gamma_1^{BI} < \Gamma_2^{BI}$ . Comparing the empirical relevant upper bound  $\Gamma_1^{BI}$  with  $\Gamma_1$  of the baseline model given in (28), the numerical analysis suggests that  $\Gamma_1 < \Gamma_1^{BI}$ . However, while taxing bond interest income increases the upper bound on  $\mu_\pi$ , the numerical analysis suggests that the lower bound  $\Gamma_3^{BI} > 1$ . For example, Figure 9 illustrates the (in)determinacy regions for two alternative values of the steady state income tax rate  $\tau^l = 0.2, 0.4$ . Panels (i) and (ii) of Fig. 9 depict the (in)determinacy regions for combinations of  $\mu_\pi$  and  $s_b$  setting  $\psi = 0.75$ , whereas panels (iii) and (iv) depict the regions for combinations of  $\mu_\pi$  and  $\psi$  setting  $s_b = 2.0$ . By inspection of Fig. 9, taxing bond interest income increases both the lower and upper bound on the inflation response coefficient, the net effect of which, is an expansion of the determinacy region relative to the baseline results illustrated in Fig. 1. However, indeterminacy continues to be a serious problem under income taxation as the determinacy region still remains narrow.

We now briefly consider the determinacy implications if the interest-rate feedback rule also reacts to future output. Figure 10 illustrates the areas of (in)determinacy for combinations of  $\mu_\pi$  and  $\mu_y$  setting  $\tau^l = 0.3$  for alternative debt levels  $s_b = 1.0, 2.0, 3.0$  and degrees of price stickiness  $\psi = 0.3, 0.5, 0.75, 0.85$ . Comparing Fig. 10 against the baseline results given in Fig. 2, the lower and upper bound on  $\mu_\pi$  are relatively larger for each value of  $\mu_y$  when bond interest income is taxed. However, despite these differences the qualitative conclusions remain unchanged: the lower is  $\psi$ , the less effective is such a monetary policy in alleviating the indeterminacy problem under income taxation.

To understand these results, consider the linearized version of the Euler equation (39) after using  $\widehat{Y}_t = s_c\widehat{C}_t$ :

$$\widehat{Y}_t = \widehat{Y}_{t+1} - s_c\sigma \left[ (1 - \beta\tau^l)\widehat{R}_t - \widehat{\pi}_{t+1} \right] + \frac{s_c\sigma\tau^c}{1 + \tau^c} (\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c) + \frac{s_c\sigma\tau^l(1 - \beta)}{1 - \tau^l} \widehat{\tau}_{t+1}^l. \quad (44)$$

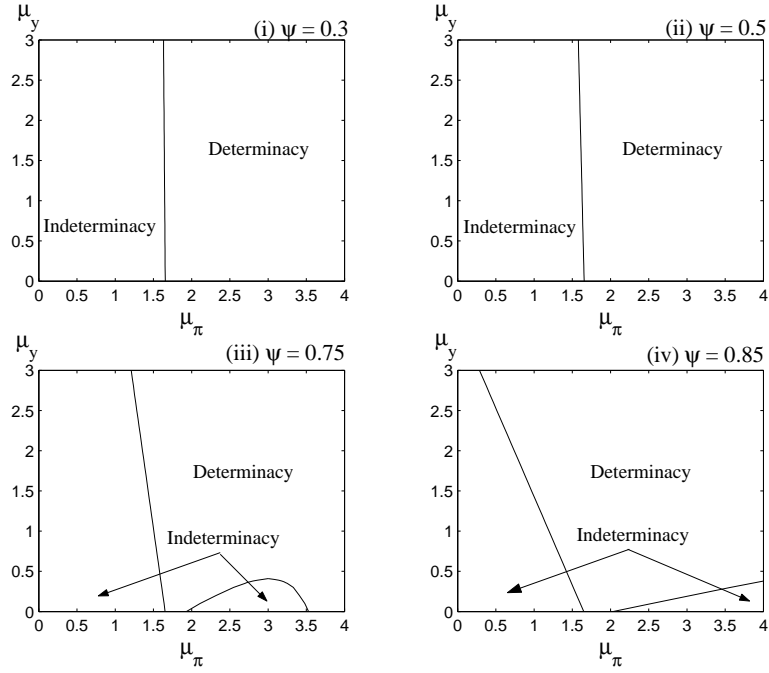


Figure 11: Bond Interest Income Taxes – Regions of (in)determinacy under a flexible contemporaneous inflation targeting policy ( $\tau^l = 0.4$ ;  $s_b = 3.0$ )

There are two differences between (44) and its baseline version (17). First, the future income tax rate  $\hat{\tau}_{t+1}^l$  enters into the Euler equation (44). Consequently, adjustments in the income tax rate now have direct implications for aggregate demand. Recall that indeterminacy arises under distortionary taxation via the public finance channel of monetary policy. In order for the government to balance its budget, increases in the real interest rate result in higher taxes, which exert upward pressure on real marginal cost and inflation. By taxing bond interest income, this increases the upper bound on  $\mu_\pi$ , since higher income taxes now reduce aggregate demand helping partially offset the increase in inflation. However, as highlighted by the numerical analysis and in stark contrast to consumption taxation, this aggregate demand effect is found to be small under income taxation. Second, in the baseline version of the income tax model it is the nominal interest rate adjusted for inflation that influences aggregate demand (17), whereas by also taxing bond interest income (44) it is the inflation adjusted *after-tax* nominal interest rate:  $(1 - \beta\tau^l)\hat{R}_t - \hat{\pi}_{t+1}$ . Therefore, the lower bound on the inflation response coefficient needs to be greater than what the Taylor principle prescribes, in order for increases in the after-tax nominal interest rate to result in increases in the real interest rate.

For completeness, Figure 11 illustrates the areas of (in)determinacy under a flexible *contempo-*

*aneous* inflation targeting policy for alternative degrees of price stickiness  $\psi = 0.3, 0.5, 0.75, 0.85$  setting  $\tau^l = 0.4$  and  $s_b = 3.0$ . Comparing Fig. 11 against the baseline results given in Fig. 5, shows a reduction in the determinacy region in the presence of bond interest income taxation, since as discussed above, in order to prevent indeterminacy the lower bound on  $\mu_\pi$  is required to be larger relative to the baseline model. Furthermore, the aggregate demand effect now present with bond interest income taxation increases the additional indeterminacy region that arises under  $\psi = 0.75, 0.85$ . However, unlike consumption taxation locally explosive equilibrium do not emerge under income taxation.

In summary, the above analysis suggests that while taxing bond interest income has interesting implications for the determinacy conditions under income taxation, the general conclusions of Section 3 remain unaffected.

## 4.2 Introducing Capital and Investment Spending

We now introduce capital into the baseline model by assuming an economy-wide rental market for the capital stock. The changes are briefly outlined below.

**Firms** To produce output intermediate firms hire labor  $L$  and rent capital  $K$  from the representative household, given the real wage rate  $w_t$  and the rental cost of capital  $rr_t$ . A firm of type  $i$  now has the following production technology:

$$y_t(i) = K_t(i)^\alpha L_t(i)^{1-\alpha}, \quad (45)$$

where the input share is  $0 < \alpha < 1$ . Given competitive prices of labor and capital, cost-minimization yields:

$$rr_t = mc_t(i) \alpha \left( \frac{L_t(i)}{K_t(i)} \right)^{1-\alpha}, \quad (46)$$

$$w_t = mc_t(i) (1 - \alpha) \left( \frac{K_t(i)}{L_t(i)} \right)^\alpha. \quad (47)$$

The price-setting problem of intermediate firms remains unchanged.

**Households** The representative household owns the capital stock  $K$  and makes all investment decisions  $I$  according to the following law of motion:

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (48)$$

where  $0 < \delta < 1$  is the depreciation rate of capital. The household period budget constraint (2) is now given by:

$$B_t + (1 + \tau_t^c)P_t C_t + P_t I_t \leq R_{t-1}B_{t-1} + P_t r r_t K_t + P_t(1 - \tau_t^l)(w_t L_t + \vartheta_t).$$

Consequently, there is an additional first-order condition for optimal household investment:

$$\frac{U_c(C_t)}{U_c(C_{t+1})} = \frac{(1 + \tau_t^c)}{(1 + \tau_{t+1}^c)} \beta [r r_{t+1} + (1 - \delta)]. \quad (49)$$

Noting that with capital real income is  $Y_t = r r_t K_t + w_t L_t + \vartheta_t$ , the government period budget constraint (12) can be expressed as:

$$P_t G = P_t \tau_t^c C_t + P_t \tau_t^l (Y_t - r r_t K_t) + B_t - R_{t-1} B_{t-1}. \quad (50)$$

Finally, the market clearing condition (15) now becomes:

$$Y_t = C_t + I_t + G. \quad (51)$$

The complete linearized model is given by the following equations:

$$\widehat{C}_{t+1} - \sigma (\widehat{R}_t - \widehat{\pi}_{t+1}) + \frac{\sigma \tau^c}{1 + \tau^c} (\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c) = \widehat{C}_t, \quad (52)$$

$$\widehat{C}_{t+1} + \frac{\sigma \tau^c}{1 + \tau^c} (\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c) = \widehat{C}_t + \sigma [1 - \beta(1 - \delta)] \left[ \widehat{m}c_{t+1} + (1 - \alpha) (\widehat{L}_{t+1} - \widehat{K}_{t+1}) \right], \quad (53)$$

$$\widehat{\pi}_t = \beta \widehat{\pi}_{t+1} + \lambda \widehat{m}c_t, \quad (54)$$

$$\widehat{m}c_t = (\omega + \alpha) \widehat{L}_t + \sigma^{-1} \widehat{C}_t - \alpha \widehat{K}_t + \left( \frac{\tau^c}{1 + \tau^c} \right) \widehat{\tau}_t^c + \left( \frac{\tau^l}{1 - \tau^l} \right) \widehat{\tau}_t^l, \quad (55)$$

$$\widehat{K}_{t+1} = (1 - \delta) \widehat{K}_t + \delta \widehat{I}_t, \quad (56)$$

$$\widehat{Y}_t = \alpha \widehat{K}_t + (1 - \alpha) \widehat{L}_t = s_c \widehat{C}_t + s_I \widehat{I}_t, \quad (57)$$

$$\tau^c s_c (\widehat{\tau}_t^c + \widehat{C}_t) + \tau^l (\widehat{\tau}_t^l + \widehat{Y}_t) - \frac{\tau^l s_I}{\beta \delta} [1 - \beta(1 - \delta)] (\widehat{\tau}_t^l + \widehat{m}c_t + \widehat{Y}_t) = \frac{s_b}{\beta} (\widehat{R}_{t-1} - \widehat{\pi}_t), \quad (58)$$

$$\widehat{R}_t = \mu_\pi \widehat{\pi}_t + \mu_y \widehat{Y}_t, \quad (59)$$

where  $0 < s_I < 1$  is the steady state output share of investment. Equation (59) only considers a contemporaneous specification for the monetary policy rule, since it is well established that determinacy is almost impossible under forward-looking interest-rate rules.<sup>20</sup>

The linearized model (52)–(59) can be reduced to a five-dimensional system:  $\mathbf{Z}_{t+1} = \mathbf{A}\mathbf{Z}_t$ , where  $\mathbf{Z}_t = [\widehat{m}c_t, \widehat{L}_t, \widehat{K}_t, \widehat{R}_t, \widehat{R}_{t-1}]'$ . Since there are two predetermined variables,  $\widehat{K}_t$  and  $\widehat{R}_{t-1}$ , determinacy requires that three eigenvalues lie outside the unit circle and two eigenvalues must lie inside the unit circle. As analytical results are not possible, a numerical investigation is carried out. Following Kurozumi and Van Zandweghe (2008), we set the cost share of capital  $\alpha = 0.33$ , the depreciation rate of capital  $\delta = 0.025$ , and the steady state output share of investment  $s_I = 0.3$ . For the remaining parameters, we use the parameterization given in Table 2.

To see the determinacy implications of allowing for capital and investment spending, first consider the case of a strict contemporaneous inflation targeting policy. Using the above parameterization, the numerical analysis suggests that the determinacy conditions are independent of the steady state tax rate  $\tau^c$ . Figures 12 and 13 illustrate the areas of (in)determinacy for combinations of the inflation response coefficient  $\mu_\pi$  and the degree of price stickiness  $\psi$  using two alternative values of the steady state income tax rate  $\tau^l = 0.2, 0.4$ . Fig. 12 graphs the (in)determinacy regions setting  $s_b = 2.0$ , whereas Fig. 13 graphs the regions setting  $s_b = 3.0$ . By inspection of Figs. 12 and 13, indeterminacy can only arise with capital and investment spending for high degrees of price stickiness. For both tax systems, indeterminacy is greater the lower is  $s_b$ , and for income taxation indeterminacy is greater the lower is the steady state tax rate. But, the numerical analysis suggests that the region of indeterminacy is always relatively larger under consumption taxation than under income taxation.

Similar to the baseline model, indeterminacy arises under the Taylor principle in the capital version of the model when the cost-push effects exerting upward pressure on inflation outweigh the downward pressure on inflation due to reductions in aggregate demand caused by higher real interest rates. However, while the aggregate demand effect is stronger under consumption taxation,

<sup>20</sup>See, for example, Carlstrom and Fuerst (2005), Kurozumi and Van Zandweghe (2008), Huang et al. (2009), Duffy and Xiao (2011).

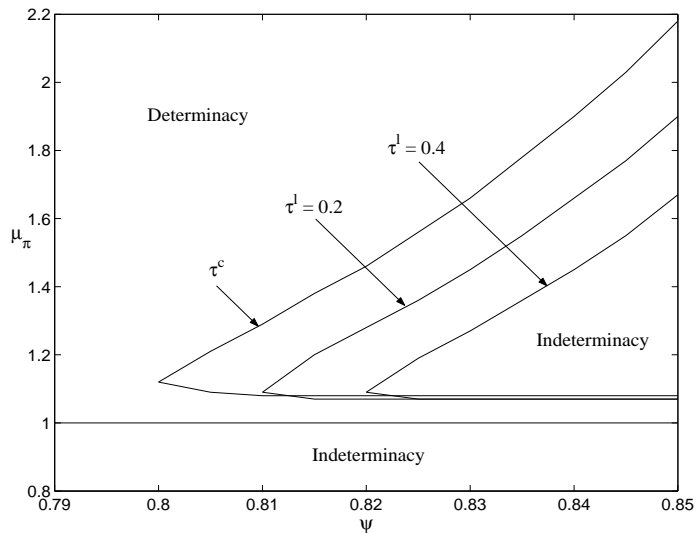


Figure 12: Income Taxes vs. Consumption Taxes – Regions of (in)determinacy under a strict contemporaneous inflation targeting policy with capital ( $s_b = 2$ )

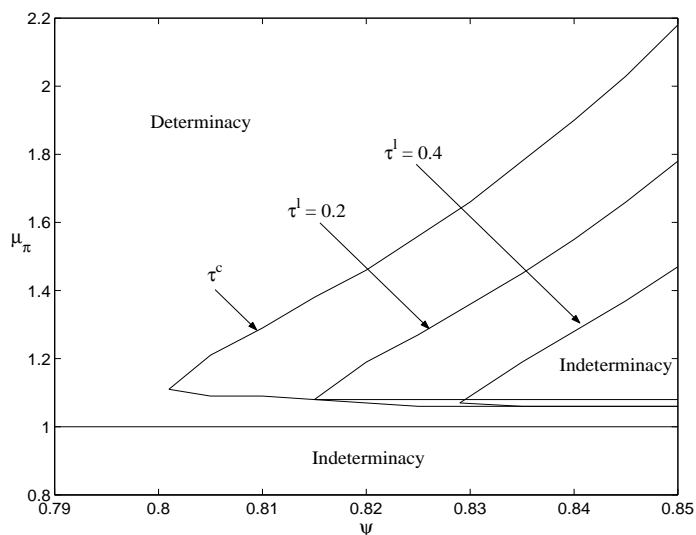


Figure 13: Income Taxes vs. Consumption Taxes – Regions of (in)determinacy under a strict contemporaneous inflation targeting policy with capital ( $s_b = 3$ )

since the public finance channel also directly affects aggregate demand via (52), the numerical analysis suggests that the indeterminacy problem is less severe under income taxation. Comparing the linearized government budget constraint under investment (58) with its baseline version (19), reveals an additional negative term under income taxation:  $-\frac{\tau^l s_l}{\beta \delta} [1 - \beta(1 - \delta)] (\hat{\tau}_t^l + \hat{m}c_t + \hat{Y}_t) < 0$ . From equations (52) and (53), an increase in the real interest rate results in a rise in the rental cost of capital. This decreases the capital stock and output, which via lower consumption and leisure,

increases the supply of labor. Consequently, with a larger tax base, income taxes do not need to rise as much as consumption taxes in order to balance the budget, suggesting a relatively weaker public finance channel under income taxation.

Finally, the numerical analysis finds that indeterminacy can easily be eliminated under both tax systems if the interest-rate feedback rule also reacts to output, as this magnifies the aggregate demand response to changes in the interest rate. For example, setting  $\psi = 0.85$  and  $s_b = 2.0$ , then indeterminacy is eliminated if  $\mu_y \approx 0.032$  for  $\tau^l = 0.2$  and  $\mu_y \approx 0.03$  for  $\tau^c > 0$ . Since the aggregate demand channel is stronger under consumption taxation, the monetary authority can target output slightly less aggressively to prevent indeterminacy.

### 4.3 Policy Implications

To highlight some of the policy implications of the above results, a counterfactual exercise is now performed for the Euro area, which as discussed in the introduction is currently contemplating further tax reform in the direction of indirect taxation. Specifically, we consider the determinacy consequences of a revenue-neutral switch from income taxes to consumption taxes for four variations in the underlying model environment: the baseline model under both a forward-looking interest-rate rule (FLR) and a current-looking interest-rate rule (CLR), the inclusion of bond interest income taxation under a CLR, and the modified model with capital under a CLR.

The parameter values are chosen to broadly match several features of Euro area data. Following Lipińska and von Thadden (2009), we set the steady state income tax rate  $\tau^l = 0.3$  and the steady state government debt-output ratio  $s_b = 2.64$ , consistent with the Euro area average for the period 1996–2006.<sup>21</sup> For the baseline model, it then follows that  $G/Y = 0.273$ .<sup>22</sup> For the modified model with endogenous capital and investment, we follow Smets and Wouters (2003) and set the steady state output share of investment  $s_I = 0.22$ . It then follows that  $G/Y = 0.18067$  and the steady state output share of consumption  $s_c = 0.6$ . These values are consistent with the average share of investment and consumption in total Euro area output for the period 1970–2000.<sup>23</sup> Following Lipińska and von Thadden (2009), we set the degree of price stickiness  $\psi = 0.85$ , which implies an average price duration of 6.67 quarters. As discussed by Blattner and Margaritov (2010), estimates of the inflation response coefficient  $\mu_\pi$  and the output response coefficient  $\mu_y$  for the Euro area

<sup>21</sup>Setting  $s_b = 2.64$  implies a yearly steady state government debt-output ratio of 66% which is slightly higher than the 60% threshold expressed in the Maastricht Treaty.

<sup>22</sup>In the presence of bond interest income taxation we set  $\tau^l = 0.2922$  to keep  $G/Y$  unchanged.

<sup>23</sup>See, for example, Smets and Wouters (2003).



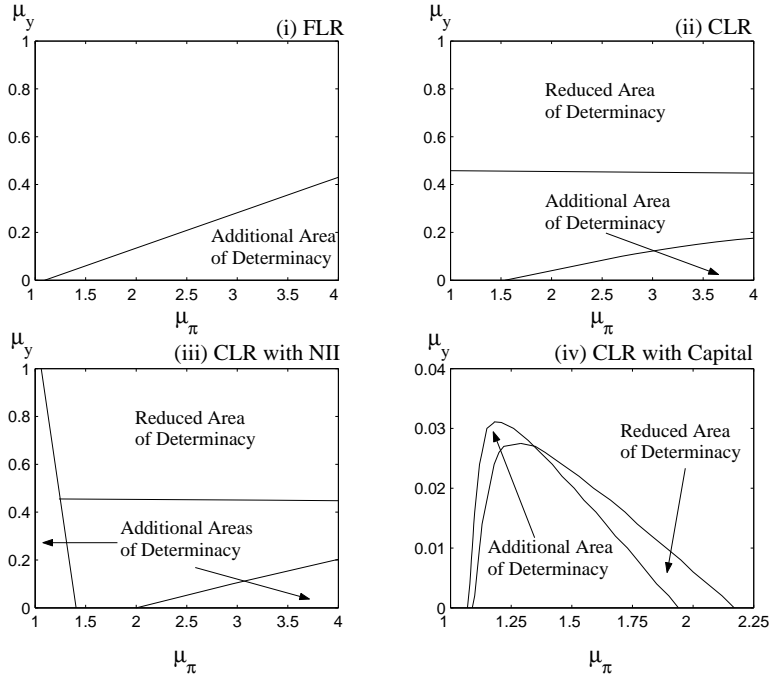


Figure 14: Revenue-Neutral Switch from Income to Consumption Taxation – Determinacy Implications under the Taylor Principle

vary considerably. Consequently, we vary the monetary policy reaction parameters  $1 \leq \mu_\pi \leq 4$  and  $0 \leq \mu_y \leq 1$ , to cover a number of estimates for the Euro area. The remaining parameter values are unchanged from Table 2.

Figure 14 summarizes the results from this experiment. For the baseline model under a FLR, the top, left-hand panel of Fig. 14 shows that for particular combinations of  $\mu_\pi$  and  $\mu_y$  there are potential determinacy gains from switching to consumption taxation. While similar gains also arise under a CLR, as shown by the top, right-hand panel of Fig. 14, there are now dangers associated with the tax reform, since a  $\mu_y \geq 0.46$  results in explosiveness under consumption taxation. If bond interest income is also taxed, then there is an additional determinacy gain with consumption taxes, since the inflation response coefficient must be larger than one under income taxation. However, as shown by the bottom, left-hand panel of Fig. 14, the reduced area of determinacy under consumption taxation remain sizable. Finally, for the capital version of the model, the bottom, right-hand panel of Fig. 14 indicates that the tax reform would increase the area of indeterminacy under a strict inflation targeting policy. In summary, the above exercise for the Euro area suggests that at least in terms of macroeconomic stability, switching from income to consumption taxation could have potentially harmful repercussions.

## 5 Conclusions

This paper has examined how financing a balanced budget fiscal policy using different tax systems can alter the conditions for determinacy under a variety of popular interest-rate feedback rules. The analysis has shown that indeterminacy can arise under both consumption and income taxation, the severity of which can depend on the magnitude of the steady state tax rate, the steady state government-debt output ratio, and the degree of price rigidity. However, importantly our analysis reveals that the determinacy criteria are not equivalent across the two tax systems. From a policy perspective, the findings from this paper suggest that future shifts away from income taxation towards consumption taxation could have non-trivial implications for how monetary policy should best be conducted under a balanced-budget fiscal rule in order to prevent macroeconomic instability.

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## Appendix A. Proof of Proposition 1

If monetary policy is characterized by a forward-looking interest-rate rule, then the set of linearized equations (17)–(21) can be reduced to a two-dimensional system:  $\mathbf{Z}_{t+1} = \mathbf{A}^j \mathbf{Z}_t$ ; where  $\mathbf{Z}$  is the column vector of non-predetermined endogenous variables  $[\widehat{Y}, \widehat{\pi}]'$  and  $\mathbf{A}^j, j = l, c$ , is the respective coefficient matrix under income taxation or consumption taxation:

$$\mathbf{A}^l \equiv \begin{bmatrix} \frac{1}{1-s_c\sigma\mu_y} - \frac{\lambda s_c\sigma(\mu_\pi-1)}{\beta(1-s_c\sigma\mu_y)} \left[ \Lambda_1^l + \frac{s_b\mu_y}{\beta(1-\tau^l)} \right] & \frac{s_c\sigma(\mu_\pi-1)}{\beta(1-s_c\sigma\mu_y)} \left[ 1 - \frac{\lambda s_b(\mu_\pi-1)}{\beta(1-\tau^l)} \right] \\ -\frac{\lambda}{\beta} \left[ \Lambda_1^l + \frac{s_b\mu_y}{\beta(1-\tau^l)} \right] & \frac{1}{\beta} \left[ 1 - \frac{\lambda s_b(\mu_\pi-1)}{\beta(1-\tau^l)} \right] \end{bmatrix},$$

$$\mathbf{A}^c \equiv \begin{bmatrix} 1 + \frac{s_c\sigma\mu_y\beta(1+\tau^c)}{\Lambda_2^c} + \frac{\lambda\sigma J_1^c(\mu_\pi-1)}{\beta\Lambda_2^c} \left[ \Lambda_1^c + \frac{s_b\mu_y}{\beta s_c(1+\tau^c)} \right] & \frac{\sigma(\mu_\pi-1)}{\Lambda_2^c} \left[ s_b - \frac{J_1^c J_2^c}{\beta} \right] \\ -\frac{\lambda}{\beta} \left[ \Lambda_1^c + \frac{\mu_y s_b}{\beta s_c(1+\tau^c)} \right] & \frac{J_2^c}{\beta} \end{bmatrix},$$

where  $\Lambda_1^l \equiv \omega + \frac{1}{s_c\sigma} - \frac{\tau^l}{1-\tau^l}$ ,  $\Lambda_1^c \equiv \omega + \frac{1}{s_c\sigma} - \frac{\tau^c}{(1+\tau^c)s_c}$ ,  $\Lambda_2^c \equiv \beta[1+\tau^c(1-\sigma)] + \sigma\mu_y[s_b - \beta s_c(1+\tau^c)]$ ,  $J_1^c \equiv s_b - \beta s_c(1+\tau^c)$ , and  $J_2^c \equiv 1 - \frac{\lambda s_b(\mu_\pi-1)}{\beta(1+\tau^c)s_c}$ . Equilibrium determinacy requires that both eigenvalues of  $\mathbf{A}^j$  are outside the unit circle. By Proposition C.1. of Woodford (2003) this is the case if and only if either of the following two cases are satisfied. Case I:  $\det \mathbf{A}^j > 1$ ,  $1 + \det \mathbf{A}^j - \text{tr} \mathbf{A}^j > 0$ ,  $1 + \det \mathbf{A}^j + \text{tr} \mathbf{A}^j > 0$ . Case II:  $1 + \det \mathbf{A}^j - \text{tr} \mathbf{A}^j < 0$ ,  $1 + \det \mathbf{A}^j + \text{tr} \mathbf{A}^j < 0$ ; where

$$\det \mathbf{A}^l = \frac{1}{\beta(1-s_c\sigma\mu_y)} \left[ 1 - \frac{\lambda s_b(\mu_\pi-1)}{\beta(1-\tau^l)} \right],$$

$$\text{tr} \mathbf{A}^l = \frac{1}{1-s_c\sigma\mu_y} - \frac{\lambda s_c\sigma(\mu_\pi-1)}{\beta(1-s_c\sigma\mu_y)} \left[ \Lambda_1^l + \frac{s_b\mu_y}{\beta(1-\tau^l)} \right] + \frac{1}{\beta} \left[ 1 - \frac{\lambda s_b(\mu_\pi-1)}{\beta(1-\tau^l)} \right],$$

$$\det \mathbf{A}^c = \frac{1}{\beta} + \frac{s_c\sigma\mu_y(1+\tau^c)}{\Lambda_2^c} + \frac{\lambda\sigma\omega s_b(\mu_\pi-1)}{\beta\Lambda_2^c},$$

$$\text{tr} \mathbf{A}^c = 1 + \frac{1}{\beta} + \frac{s_c\sigma\mu_y\beta(1+\tau^c)}{\Lambda_2^c} - \frac{\lambda s_b(\mu_\pi-1)}{\beta^2 s_c(1+\tau^c)} + \frac{\lambda\sigma J_1^c(\mu_\pi-1)}{\beta\Lambda_2^c} \left[ \Lambda_1^c + \frac{s_b\mu_y}{\beta s_c(1+\tau^c)} \right].$$

For the income tax system, the three inequalities in Case I can be reduced to equations (22)–(24), whereas under the consumption tax system the inequalities of Case I are given by equations (25)–(27).

## Appendix B. Proof of Proposition 2

If monetary policy is characterized by a contemporaneous-looking interest-rate rule, then the set of linearized equations (17)–(21) can be reduced to a three-dimensional system:  $\mathbf{Z}_{t+1} = \mathbf{A}^j \mathbf{Z}_t$ ; where  $\mathbf{Z}_t = [\widehat{Y}_t, \widehat{R}_t, \widehat{R}_{t-1}]'$  and  $\mathbf{A}^j, j = l, c$ , is the respective coefficient matrix under income taxation or

consumption taxation:

$$\mathbf{A}^l \equiv \begin{bmatrix} 1 + \frac{s_c \sigma}{\beta} \left[ \lambda \Lambda_1^l + \frac{\mu_y J_1^l}{\mu_\pi} \right] & s_c \sigma \left[ 1 - \frac{J_1^l}{\beta \mu_\pi} \right] & \frac{\lambda s_c \sigma s_b}{\beta^2 (1 - \tau^l)} \\ \mu_y - \frac{[\mu_\pi - s_c \sigma \mu_y]}{\beta} \left[ \lambda \Lambda_1^l + \frac{\mu_y J_1^l}{\mu_\pi} \right] & s_c \sigma \mu_y + \frac{\Lambda_4^l (\mu_\pi - s_c \sigma \mu_y)}{\beta \mu_\pi} & - \frac{\lambda s_b (\mu_\pi - s_c \sigma \mu_y)}{\beta^2 (1 - \tau^l)} \\ 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{A}^c \equiv \begin{bmatrix} 1 + \frac{\sigma}{\beta \Lambda_3^c \mu_\pi} \left[ \frac{\mu_y s_b}{1 + \tau^c} + s_c J_3^c J_4^c \right] & \frac{s_c \sigma J_3^c J_5^c}{\Lambda_3^c} - \frac{\sigma s_b}{\beta (1 + \tau^c) \Lambda_3^c \mu_\pi} & \frac{\sigma s_b}{\beta (1 + \tau^c) \Lambda_3^c} \left[ 1 + \frac{\lambda J_3^c}{\beta} \right] \\ \mu_y + \frac{\mu_y^2 \sigma s_b}{\beta (1 + \tau^c) \Lambda_3^c \mu_\pi} - \frac{J_4^c J_6^c}{\beta} & J_6^c J_7^c + \frac{\sigma \mu_y J_8^c}{\Lambda_3^c} & \frac{\sigma \mu_y s_b}{\beta (1 + \tau^c) \Lambda_3^c} - \frac{\lambda s_b \mu_\pi J_6^c}{\beta^2 (1 + \tau^c) s_c} \\ 0 & 1 & 0 \end{bmatrix},$$

where  $\Lambda_1^l \equiv \omega + \frac{1}{s_c \sigma} - \frac{\tau^l}{1 - \tau^l}$ ,  $J_1^l \equiv 1 + \frac{\lambda s_b}{\beta (1 - \tau^l)}$ ,  $\Lambda_1^c \equiv \omega + \frac{1}{s_c \sigma} - \frac{\tau^c}{(1 + \tau^c) s_c}$ ,  $\Lambda_3^c \equiv 1 - \frac{\sigma \tau^c}{1 + \tau^c}$ , and  $J_3^c \equiv 1 - \frac{s_b}{\beta (1 + \tau^c) s_c}$ ,  $J_4^c \equiv \lambda \Lambda_1^c \mu_\pi + \mu_y \left[ 1 + \frac{\lambda s_b}{\beta (1 + \tau^c) s_c} \right]$ ,  $J_5^c \equiv 1 - \frac{1}{\beta \mu_\pi} - \frac{\lambda s_b}{\beta^2 (1 + \tau^c) s_c \mu_\pi}$ ,  $J_6^c \equiv 1 - \frac{s_c \sigma \mu_y J_3^c}{\Lambda_3^c \mu_\pi}$ ,  $J_7^c \equiv \frac{1}{\beta} + \frac{\lambda s_b}{\beta^2 (1 + \tau^c) s_c}$ , and  $J_8^c \equiv s_c J_3^c - \frac{s_b}{\beta (1 + \tau^c) \mu_\pi}$ . The three eigenvalues of  $\mathbf{A}^j$  are solutions to the cubic equation  $r^3 + a_2^j r^2 + a_1^j r + a_0^j = 0$ , where under income taxation:

$$a_2^l = -1 - s_c \sigma \mu_y - \frac{J_1^l}{\beta} - \frac{\lambda \Lambda_1^l s_c \sigma}{\beta},$$

$$a_1^l = \frac{J_1^l}{\beta} + \frac{\lambda \Lambda_1^l s_c \sigma \mu_\pi}{\beta} + \frac{J_1^l s_c \sigma \mu_y}{\beta} + \frac{\lambda s_b \mu_\pi}{\beta^2 (1 - \tau^l)} - \frac{\lambda s_b s_c \sigma \mu_y}{\beta^2 (1 - \tau^l)},$$

$$a_0^l = -\frac{\lambda s_b \mu_\pi}{\beta^2 (1 - \tau^l)},$$

and under consumption taxation:

$$a_2^c = -1 - \frac{1}{\beta} - \frac{\lambda s_b}{\beta^2 (1 + \tau^c) s_c} - \frac{s_c \sigma J_3^c}{\Lambda_3^c} \left[ \mu_y + \frac{\lambda \Lambda_1^c}{\beta} \right],$$

$$a_1^c = \frac{1}{\beta} + \frac{\lambda s_b}{\beta^2 (1 + \tau^c) s_c} + \frac{s_c \sigma J_3^c J_4^c}{\beta \Lambda_3^c} + \frac{\lambda s_b}{\beta^2 (1 + \tau^c)} \left[ \frac{\mu_\pi}{s_c} - \frac{\sigma \mu_y J_3^c}{\Lambda_3^c} - \frac{\sigma \Lambda_1^c}{\Lambda_3^c} \right] - \frac{\sigma \mu_y s_b}{\beta (1 + \tau^c) \Lambda_3^c},$$

$$a_0^c = \frac{\sigma s_b J_4^c}{\beta^2 (1 + \tau^c) \Lambda_3^c} - \frac{\lambda s_b \mu_\pi}{\beta^2 (1 + \tau^c) s_c} - \frac{\lambda \sigma \mu_y s_b^2}{\beta^3 (1 + \tau^c)^2 \Lambda_3^c s_c}.$$

With one predetermined variable  $\widehat{R}_{t-1}$ , equilibrium determinacy requires that two eigenvalues are outside the unit circle and one eigenvalue is inside the unit circle. By Proposition C.2 of Woodford (2003) this is the case if and only if either of the following two cases are satisfied. Case 1:  $1 + a_2^j + a_1^j + a_0^j > 0$ ,  $-1 + a_2^j - a_1^j + a_0^j < 0$ , &  $|a_2^j| > 3$  or  $a_0^j a_1^j - a_0^j a_2^j + a_1^j - 1 > 0$ . Case 2:  $1 + a_2^j + a_1^j + a_0^j < 0$ ,  $-1 + a_2^j - a_1^j + a_0^j > 0$ . For the income tax system, the first two inequalities in Case I can be reduced to equations (31)–(32), whereas under the consumption tax system, these inequalities are given by equations (34)–(35).

## Appendix C. Proof of Proposition 3

If the government taxes bond interest income, the linearized version of equations (38) and (39) can be expressed as:

$$\tau^c s_c \widehat{\tau}_t^c + \tau^l \left[ 1 + \frac{s_b(1-\beta)}{\beta(1-\tau^l)} \right] \widehat{\tau}_t^l + (\tau^c + \tau^l) \widehat{Y}_t + s_b \widehat{b}_t = \frac{s_b}{\beta} (\widehat{b}_{t-1} - \widehat{\pi}_t) + \frac{s_b(1-\beta\tau^l)}{\beta} \widehat{R}_{t-1}, \quad (60)$$

$$\widehat{Y}_t = \widehat{Y}_{t+1} - s_c \sigma \left[ (1-\beta\tau^l) \widehat{R}_t - \widehat{\pi}_{t+1} \right] + \frac{s_c \sigma \tau^c}{1+\tau^c} (\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c) + \frac{s_c \sigma \tau^l (1-\beta)}{1-\tau^l} \widehat{\tau}_{t+1}^l. \quad (61)$$

If monetary policy is characterized by a forward-looking interest-rate rule, then the set of linearized equations (18), (20), (21), (60), and (61) can be reduced to a two-dimensional system:  $\mathbf{Z}_{t+1} = \mathbf{A}^l \mathbf{Z}_t$ ; where  $\mathbf{Z}$  is the column vector of non-predetermined endogenous variables  $[\widehat{Y}, \widehat{\pi}]'$  and  $\mathbf{A}^l \equiv$

$$\begin{bmatrix} \frac{1}{\Lambda_3^l} - \frac{\lambda s_c \sigma (1-\tau^l) [(1-\beta\tau^l)\mu_\pi - 1]}{\Lambda_3^l [\beta(1-\tau^l) + s_b(1-\beta)]} \left[ \Lambda_2^l + \frac{s_b(1-\beta\tau^l)\mu_y}{\beta(1-\tau^l) + s_b(1-\beta)} \right] & \frac{s_c \sigma (1-\tau^l) [(1-\beta\tau^l)\mu_\pi - 1]}{\Lambda_3^l [\beta(1-\tau^l) + s_b(1-\beta)]} \left[ 1 - \frac{\lambda s_b [(1-\beta\tau^l)\mu_\pi - 1]}{\beta(1-\tau^l) + s_b(1-\beta)} \right] \\ -\frac{\lambda}{\beta} \left[ \Lambda_2^l + \frac{s_b(1-\beta\tau^l)\mu_y}{\beta(1-\tau^l) + s_b(1-\beta)} \right] & \frac{1}{\beta} \left[ 1 - \frac{\lambda s_b [(1-\beta\tau^l)\mu_\pi - 1]}{\beta(1-\tau^l) + s_b(1-\beta)} \right] \end{bmatrix},$$

where  $\Lambda_2^l \equiv \omega + \frac{1}{s_c \sigma} - \frac{\beta\tau^l}{\beta(1-\tau^l) + s_b(1-\beta)}$  and  $\Lambda_3^l \equiv 1 - \frac{s_c \sigma \beta (1-\tau^l)}{\beta(1-\tau^l) + s_b(1-\beta)} \left[ \mu_y (1-\beta\tau^l) + \frac{(1-\beta)\tau^l}{1-\tau^l} \right]$ . Equilibrium determinacy requires that both eigenvalues of  $\mathbf{A}^l$  are outside the unit circle. By Proposition C.1. of Woodford (2003) this is the case if and only if either of the following two cases are satisfied. Case I:  $\det \mathbf{A}^l > 1$ ,  $1 + \det \mathbf{A}^l - \text{tr} \mathbf{A}^l > 0$ ,  $1 + \det \mathbf{A}^l + \text{tr} \mathbf{A}^l > 0$ . Case II:  $1 + \det \mathbf{A}^l - \text{tr} \mathbf{A}^l < 0$ ,  $1 + \det \mathbf{A}^l + \text{tr} \mathbf{A}^l < 0$ ; where

$$\det \mathbf{A}^l = \frac{1}{\beta \Lambda_3^l} \left[ 1 - \frac{\lambda s_b [(1-\beta\tau^l)\mu_\pi - 1]}{\beta(1-\tau^l) + s_b(1-\beta)} \right],$$

$$\begin{aligned} \text{tr} \mathbf{A}^l = \frac{1}{\Lambda_3^l} - \frac{1}{\Lambda_3^l} \left[ \frac{\lambda s_c \sigma (1-\tau^l) [(1-\beta\tau^l)\mu_\pi - 1]}{\beta(1-\tau^l) + s_b(1-\beta)} \left( \Lambda_2^l + \frac{s_b(1-\beta\tau^l)\mu_y}{\beta(1-\tau^l) + s_b(1-\beta)} \right) \right] \\ + \frac{1}{\beta} \left[ 1 - \frac{\lambda s_b [(1-\beta\tau^l)\mu_\pi - 1]}{\beta(1-\tau^l) + s_b(1-\beta)} \right]. \end{aligned}$$

The three inequalities in Case I can be reduced to equations (40)–(42).