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Conflict and the foundations of private property

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Conflict and the foundations of private property.

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Abstract

In this paper agents can create private property rights on a resource by making appropriative activities. We show that the value of the resource has a non monotonic effect on the emergence of private property. When the resource is sufficiently valuable, agents have an incentive to leave a sharing agreement and private property can appear. If the value of the resource increases beyond a given threshold, deviations from the sharing agreement lead to a very costly confrontation so in order to avoid that, agents stick to the agreement. In that case, private property is not sustainable. On the other hand, it is shown that populations size has an important effect on the size of the parameter set in which private property is sustainable.

JEL Classification: D74,D23,C72.

Keywords: Private property, conflict, coalition formation.

1 Introduction

Sanchez Pages (2006) shows that conflict leading to private property rights can be ex-ante Pareto superior to free access to a resource when the number of agents is large enough. In this paper, instead, we analyze the possibility of appearance of private property as result of a game between private agents. We focus on the relationship between value of the resource, conflict, and the appearance of private property, and analyze its sensibility to changes in the population size.

We analyze the agents' incentives for obtaining private property in Grossman's (2001)[1] model on the appearance of property rights. As in Sanchez Pages (2007), "free access" is an agreement (i.e. coalition) between all agents to share collectively a valuable resource. Then private property appears as a rational deviation[2] from this agreement.

In this framework we show that the value of the resource has a non monotonic effect on the emergence of private property. More specifically, when the resource is sufficiently valuable, agents have an incentive to leave the free access agreement. However, if the value of the resource increases enough, deviations from the free access agreement lead to a very costly conflict so in order to avoid it, agents stick to the agreement. Therefore we show that private property of the resource is only sustainable for intermediate values of the resource.

[1] Grossman (2001) presents a General Equilibrium model in which by making an effort people can obtain private property from a common pool of a valuable resource. The resource appropriated is used, together with labor, for production activities. Then he characterizes what would be the equilibrium allocation of private efforts to obtain property.

[2] Our concept of stability is based on Bloch (1996). See Bolgomolnaia and Jackson (2002), Chwe (1994) and Ray and Vohra (2015) for a discussion on stability.

Increases in the number of agents have also a non monotonic effect. We show that the set of parameter values for which "free access" agreement is unstable (i.e. private property appears as equilibrium) increases with the number of agents in a small enough economy, but it could get reduced when the number of agents is large enough.

Umbeck (1981) presents a theoretical investigation of how the initial distribution of property rights can arise starting from a situation of free access. Each agent can use labor time in violence to appropriate land or in getting gold. The marginal rate of substitution between land and labor in the production of gold is a measure of how much labor is willing to allocate to maintain the exclusivity of a marginal unit of land. The equilibrium allocation of land would be characterized to equal willingness to fight (and no conflict). In a symmetric model, the equal willingness to allocate labor to conflict implies an equal distribution of land.

This research is related with the literature of conflict with coalition formation summarized in Bloch (2009), and the corresponding sections in Garfinkel and Skaperdas (2007). On the other hand, our research is complementary to the literature on the appearance of property rights (see Alchian and Demsetz, 1973, Demsetz, 1967, and Grossman, 2001, among others).

The rest of the paper is organized as follows: Section 2 presents the basic model. Section 3 analyzes formation of property rights in a three agent economy. Section 4 analyzes coalition formation with exogenous responses. Section 5 presents some results when the process of coalition formation is endogenous. Section 6 analyzes the change in the equilibrium coalition structure when the number of agents changes. Finally section 7 presents some conclusions.

2 The Model

Assume that there is a valuable resource (i.e. a pool of "land") of size 1 in a n agent economy. Let $N = \{1, 2, \ldots, n\}$ denote the set of players. Players are identical. Each agent i has a stock of time of size 1 that can be used in production and appropriative activities. Agents can participate individually or collectively in these activities.

A coalition structure $\pi = [\{A_1, A_2, \ldots, A_k\}]$ is a partition of the set N. In other words, in a coalition structure each $A_m \subset N$, $A_m \cap \hat{A}_q = \emptyset$ and the union of all this coalitions $\cup_m^k A_m$ is equal to the set N.

Since all players are assumed to be identical, payoffs for each player are dependent on the group size rather than on the specific players that are in the group (i.e. the game is symmetric in the sense of Bloch, 1996). Let $a_m = |A_m|$ be the cardinality of the coalition m, that also denotes the size of that alliance. Therefore we characterize a coalition structure by $\pi = [\{a_1, a_2, \ldots, a_k\}]$ which only depends on the number of members that are in each alliance[3]. From now on, i denotes the player and m the alliance a_m to which she belongs.

Agents that participate individually have a group size of $a_m = 1$, and members that participate collectively has a group size $a_m > 1$. Each agent i that belongs to the alliance a_m divides her available time in production (l_{im}) and appropriative activities (e_{im}). For each agent, assume that $1 = l_{im} + e_{im}$ is satisfied. Then in a coalition a_m production time is the sum of individual production time from each member ($L_m = \sum_{i \in a_m} l_{im}$). Analogously, coali-

[3]Note that with this notation there could be more than one partition of N that gives the same coalition structure. For example, suppose there are three players, $N = \{1, 2, 3\}$, two different partition of this set are $[\{12|3\}]$ and $[\{13|2\}]$, but both of them has the same coalition structure $\pi = [\{2, 1\}]$.

tion appropriative effort is the sum of individual effort within the coalition ($E_m = \sum_{i \in a_m} e_{im}$).

We assume the particular following functional form for appropriated land for a given coalition m,

$$r_m = \begin{cases} \sum_{i \in m} e_{im} / (\sum_{k_m = 1} \sum_{i \in m} e_{im}) & \text{if } \sum_{k_m = 1} \sum_{i \in m} e_{im} > 0 \\ \frac{1}{k} & \text{otherwise} \end{cases} \tag{1}$$

in which the amount appropriated depends on the relative coalition effort on appropriative activities. This functional form is a trivial extension of the Grossman's (2001) form to an economy with coalitions.[4]

We consider a sequential game of two stages. In the first stage, coalitions are formed. In the second stage, agents decide how much time to spent in appropriative e_{im} and productive l_{im} activities, given the coalition structure. The benefit that the alliance gets from productive activities is shared between the alliance members according to a proportional sharing rule $\frac{l_{im}}{L_m}$.[5] We assume that all agents in a coalition can freely use the common land and they get consumption in function of the labor they supply. The individual utility that agent $i \in m$ obtains is

$$U_{im} = \frac{l_{im}}{L_m} r_m^{\alpha} L_m^{1-\alpha} \tag{2}$$

Given a particular coalition structure π, players maximize their individual utility subject to the time constraint. Each agent solves

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[4] This is a simple form of what the literature knows as a Contest Success Function, and a particular simple form of the one analyzed in Skaperdas (1996).

[5] This is the typical assumption when agents exploit a common property resource. See, for instance, Miceli and Lueck (2001).

$$max_{e_{im}, l_{im}|\pi} \frac{l_{im}}{L_m} \left(\frac{E_m}{\sum_m^k E_m} \right)^{\alpha} L_m^{1-\alpha} \; s.t. \quad e_{im} + l_{im} = 1 \tag{3}$$

The marginal rate of substitution obtained from this maximization problem is[6]

$$\frac{L_q - \alpha l_{iq}}{\alpha l_{iq} L_q \frac{1-r_q}{r_q} \frac{1}{\sum_m^k E_m}} = 1 \tag{4}$$

From the solution of the maximization of problem (3) we obtain the following lemma.

Lemma 1: In equilibrium, players that belong to the same alliance make the same appropriative effort, e_{im} and offer the same productive labor supply l_{im}.

Proof: See the appendix.

From now on we omit the subindex i for $e_{im} = e_m$ and $l_{im} = l_m$ since it only depends on the alliance that they belong. Lemma 1 also implies that $L_m = a_m l_m$ and $E_m = a_m e_m$. Then if two different alliances p and q have the same group size the level of efforts will be the same.

We can also show that, given a coalition structure, a Nash equilibrium in efforts exists.

Lemma 2: Given the coalition structure π, for $0 < \alpha < 1$,there is a Nash equilibrium $e^* = (e_1^*, e_2^*, \ldots, e_k^*)$ and $l^* = (l_1^*, l_2^* \ldots, l_k*)$ corresponding to the second stage of the game.

Proof: See the appendix.

[6] See proof of lemma 1 in the appendix.

3 A Three Agent Economy.

We fully work the process of coalition formation in a three agent economy in order to illustrate one of the main insights of the model

3.1 Efforts and Utilities for Each Coalition Structure

The possible coalition structures are $[\{1,1,1\}], [\{1,2\}]$, and $[\{3\}]$. The first (degenerate coalition structure) occurs when the tree agents make individual appropriation efforts. The second coalition structure occurs when an agent makes individual appropriation efforts and the other two make collective appropriation efforts. The third coalition is the grand coalition implies a free access agreement between the three agents. In the first and second cases private property arises and the valuable resource is divided in parts from which agents can exclude non coalition members.

We start computing utilities corresponding to the grand coalition $[\{3\}]$. In this case, appropriation efforts are zero for each agent and the resource is shared and exploited among the three agents. Then, given the parameter values, each agent i has a payoff of

$$U_{\mathrm{i}} = U_{\mathrm{G}} = (1/3)^{\alpha}. \tag{5}$$

For the coalition structure $[\{1,1,1\}]$, from Grossman's (2001) all players receives the same payoff

$$U_{\mathrm{i}} = U_{\mathrm{d}} = (\frac{1}{3})^{\alpha}(\frac{3(1-\alpha)}{3-\alpha})^{1-\alpha} \tag{6}$$

Consider now the coalition structure $[\{1,2\}]$. The agent remaining a singleton, s, decides the appropriating effort by maximizing consumption. Therefore its reaction function solves equation.

$$\frac{dU_s}{de_s} = \alpha(\frac{l_s}{r_s})^{1-\alpha}(\frac{\sum_{j=1}^{2} e_j}{(\sum_{j=1}^{3} e_j)^2}) - (1-\alpha)(\frac{r_s}{l_s})^{\alpha} = 0 \quad (7)$$

For an agent remaining in a coalition exploiting and defending collectively the land against the deviant (i.e. for $i \in \{2\}$), the first order condition is:

$$\frac{dU_2}{de_2} = \frac{l_2}{L_2}[\alpha(\frac{L_2}{r_2})^{1-\alpha}\frac{e_s}{(\sum_{j=1}^{3} e_j)^2} - (1-\alpha)(\frac{L_2}{r_2})^{-\alpha}] - \frac{L_2 - l_2}{L_c^2} r_2^{\alpha} L_2^{1-\alpha} = 0 \quad (8)$$

Table 1 presents solutions of the equation system (7) and (8) for explicit values of α. Finally, table 2 presents the explicit values of α (first column) and corresponding utility levels for the two different coalitions in the $[\{2,1\}]$ coalition structure (second and third columns), the grand coalition (fourth column), and the degenerated coalition structure (fifth column)[7]. We have the following remarks about table 2.

Remark 1: A particularly interesting value $\alpha = 0.348$ is obtained as a solution of the system of three equations (7),(8) and $U_G = U_s$ in α. It is easy to show (see table 2) that for numerical values $\alpha < 0.348$, U_G is larger than U_s. However for $\alpha > 0.348$ this inequality is reversed. The immediate consequence is that for low values of α it does not pay for agent s to deviate from the grand coalition.

Remark 2: Another interesting value is $\alpha = 0.549$ which is obtained as the solution of the system of three equations (7), (8) and $U_2 = U_d$ in α. It is easy to show (see table 2) that for α less than 0.549, $U_2 > U_d$, and that for α greater than 0.549, $U_2 < U_d$. Therefore, for large enough α, doing

[7] The solution from the system of equations for general values of α can be obtained by using Lemma 3 presented in section 6.

Table 1: Appropriative Effort Values

α	e_2	e_s
0.1	0.025	0.052
0.3	0.082	0.173
0.348	0.0978	0.2066
0.4	0.114	0.243
0.5	0.148	0.323
0.549	0.1659	0.3669
0.6	0.184	0.414
0.7	0.224	0.519
0.9	0.314	0.798

private appropriation efforts is better than doing efforts in a coalition, when an agent deviates from the free access agreement.

Remark 3: A trivial observation is that the grand coalition structure is always better for each agent than the degenerate coalition structure (i.e. $U_G > U_d$) for every value of α.

3.2 The Sequential Equilibrium

We find equilibrium coalition structures as the result of a game of sequential coalition formation[8]. Following Bloch (1996), in a symmetric game, a perfect equilibrium coalition structure can be reached as the outcome of a finite game of choice of coalition sizes. In the Bloch's game, an exogenous protocol sets an order in which agents propose coalition sizes. The initiator, proposes

[8]Following a bargaining protocol as proposed by Bloch (1996) and Ray and Vohra (1999).

Table 2: Comparison of Utilities for Different Coalition Structures

α	U_2	U_s	U_G	U_d
0.1	0.849	0.889	0.895	0.84
0.3	0.616	0.715	0.719	0.603
0.348	0.572	0.682	0.682	0.559
0.4	0.526	0.649	0.644	0.516
0.5	0.451	0.594	0.577	0.447
0.549	0.418	0.571	0.547	0.418
0.6	0.387	0.550	0.517	0.392
0.7	0.332	0.519	0.463	0.349
0.9	0.246	0.505	0.372	0.306

a coalition size. All the prospective members of the coalition respond in turn to the offer. If all the agents accept the offer, the cooperative agreement takes effect and they leave the game. If one of the agents rejects the offer, the proposed coalition is not formed and the agent that rejected the offer becomes the initiator in the next round.

Remark 4 Private property is sustainable as a perfect equilibrium coalition structure for intermediate values of α (i.e $0.348 < \alpha < 0.549$). For the rest of the values of α the grand coalition is the only perfect equilibrium coalition structure.

Proof of the remark: For α small (i.e. $\alpha < 0.348$) no individual agent has incentives to deviate from the grand coalition as $U_2 < U_d < U_s < U_G$, so the grand coalition is the only stable coalition structure.

Consider now intermediate values of α (i.e $0.348 < \alpha < 0.549$). In the 3 agent economy the Bloch's protocol could choose randomly any player.

Without loss of generality assume that it is player s. As $U_s > U_G$, and $U_2 > U_d$ (so, upon its deviation, the other two players would stick together in a complementary coalition), player s would rationally offer a coalition of size one that is accepted and the coalition is formed. In the second stage, one of the remaining players (1 or 2) offers a coalition of size two that is accepted by the other agent (as $U_2 > U_d$ for the corresponding values of α) and the coalition is formed. Then the coalition structure [{2,1}] arises.

Assume now that we are in the region $\alpha > 0.549$. In the first stage player s will not offer a coalition size of one as it knows that $U_2 < U_d$ and upon its deviation from the grand coalition, the rest of players will become singletons (and $U_G > U_d$). Therefore the grand coalition is an equilibrium structure. QED.

As α is an index of the value of the resource (the share of the resource in production), the conclusion of this section is that private property is only sustainable as a perfect equilibrium coalition structure for intermediate values of the resource. Unless the resource is sufficiently valuable, agents do not have incentives to deviate from the free access agreement. If the value of the resource increases enough, deviations from the free access agreement are too costly in terms of conflict.

4 Coalition Formation with Exogenous Responses.

We say that a coalition structure is stable if no member can unilaterally o collectively deviate. Testing stability is difficult as we would need to specify the responses of the other members of a coalition once a member o group of members deviated.

Hart and Kurz (1983) present two models of stability[9] that assume two types of responses of members of a coalition once a member deviates. Each model corresponding to a coalitional game, and in each one stability is based on the strong equilibria concept. The first one called the γ game and corresponds to the case in which each agent chooses the coalition to which she wants to belong, and a coalition forms if all its members have chosen to form it. The players not belonging to these unanimous consent coalitions become singletons. This means that if a player leaves a given coalition, the rest of the players become singletons (the coalition breaks). As Hart and Kurz claim, this game is supported by the view of coalitions as the result of an unanimous agreement among all its members to act together. Then, if one of the players leave, the agreement breaks down.

The second is called the δ game and corresponds to the case in which each player chooses the largest set of players he is willing to be associated with in the same coalition. Coalitions are formed among all the players that choose to be in the same coalition. In Hart and Kurz's words "a coalition corresponds to an equivalence class, with respect to equality of strategies". This means that if a player leaves a given coalition, the rest of the members form one new coalition. As Hart and Kurz claim, this model is justified specially in large games in which the fact that a player leaves a coalition has no influence in the others agreement to act together.

We characterize stable coalition structure for each of the games proposed by Hart and Kurz.

Obviously, when deciding her appropriation effort the agent has to consider that the rest of the agents would also make appropriating efforts to

[9] Also analyzed in Bloch, 2012, for the particular case of contests by coalitions

keep some of the land. Otherwise the single agent would keep all the land. To analyze the appropriating efforts in the economy we have to compute the reaction functions of the deviating agent and also of the agents remaining in the coalition.

Proposition 1: Assume that we are in the δ model (i.e. upon a deviation by one agent from the grand coalition the rest of agents remain in a complementary coalition). There is a finite $\bar{n}$ such that for any $n \geq \bar{n}$ the grand coalition is not stable for any $0 < \alpha < 1$.

Proof: See appendix.

This proposition establish that if we are in the δ model defined by Hart, S. and Kurz, M. (1983) private property would be sustainable for a large enough number of agents.

Proposition 2: Assume that we are in the γ model (i.e. upon a deviation by one agent from the grand coalition the rest of agents become singletons). The only stable coalition is the grand coalition.

This results from the simple comparation between the individual utility in case all agents form the grand coalition $U_i = (\frac{1}{n})^{\alpha}$ and the individual utility in the case all agents make individual appropriative efforts $U_i = (\frac{1}{n})^{\alpha} \left(\frac{n(1-\alpha)}{n-\alpha}\right)^{1-\alpha}$.

In the case we are in the γ model, private property would not be sustainable in Grossman's model.

5 Endogenous Coalition Formation

In the general case with an arbitrary number of agents, closed form solutions for the strategies and utilities associated to each coalition structure are impossible to obtain. This is only possible for particular coalition structures.

One of them is a coalition structure that divides the set of agents into two. The other is for symmetric coalition structures.

A closed form solution can be obtained when the alliance structure is $\pi = [\{c, s\}]$ where s is an integer, $1 \leq s \leq \frac{n}{2}$, and $c + s = n$.

Lemma 3: The optimal effort level for the problem of s players in the coalition structure $\pi = [\{c, s\}]$ is given by

$$e_s = \frac{2}{3}\sqrt{f(6+f)}\cos\left(\frac{\theta}{3} + \frac{4\pi}{3}\right) - \frac{f}{3} \tag{8}$$

where $f(n, s, \alpha) \equiv \frac{c\alpha}{(n-\alpha)(s-\alpha)}$ and $\theta(n, s, \alpha) \equiv \cos^{-1}\left(-\frac{1}{2}\frac{f(18f+27\alpha+2f^2)}{\sqrt{(f(6+f))^3}}\right)$.

Proof: See the appendix.

A symmetric coalition structure is $\pi = \{a, a, \ldots, a\}$ where a is repeated k times and $a = \frac{n}{k}$ with $\frac{n}{k}$ an integer.

Lemma 4: In a symmetric coalition structure every player obtains a payoff given by

$$U_{i \in a_m} = \frac{1}{a}\left(\frac{1}{k}\right)^{\alpha}\left(\frac{ak(a-\alpha)}{ak-\alpha}\right)^{1-\alpha}. \tag{9}$$

Proof: See the appendix.

A symmetric coalition structure cannot be an equilibrium of the game of sequential coalition formation.

Proposition 3 A symmetric coalition structure is strictly dominated by the grand coalition

This result comes from the observation that $ak = n$

$$U_{i \in a_m} = \frac{1}{a}\left(\frac{1}{k}\right)^{\alpha}\left(\frac{ak(a-\alpha)}{ak-\alpha}\right)^{1-\alpha}$$

can be written as

$$\left(\frac{1}{n}\right)^{\alpha}\left(\frac{n(a-\alpha)}{a(n-\alpha)}\right)^{1-\alpha}$$

and $\left(\frac{n(a-\alpha)}{a(n-\alpha)}\right)^{1} < 1$. Therefore $U_{i \in a_m} < (\frac{1}{n})^{\alpha}$.

6 The Role of Changing the Number of Players

The closed form computation of coalitional equilibria with arbitrary number of agents is impossible as the number of coalitions to be considered is also arbitrary. We analyze the role of changing the number of agents by computing equilibrium coalitions for economies with different number of agents. We detail the computation of equilibria when the number of agents are four and five. Three important conclusions are obtained. The first is that the non-monotonic effect of changing the α holds. The second is the appearance of new equilibrium coalition structures that would imply differente private property regimes. The third is a non-monotonic effect of changing the number of agents on the size of the set of α for which the grand coalition is stable.

6.1 Four and Five Agent Example

In the four agent example the possible coalition structures are $[\{a, b, c, d\}]$, $[\{a, b, cd\}]$, $[\{ab, cd\}]$, $[\{a, bcd\}]$ and $[\{abcde\}]$. We omit the coalition structures that have associated the same payoff. In the first stage players compare five possible outcomes. Table 3 shows the utility levels for each coalition structure at different levels of α, U_a denotes the utility for player 1, U_b denotes the utility for player 2, and so on.

Remark 5: Private property is sustainable as a perfect equilibrium solution for low values of α, (i.e $\alpha \leq 0.493$). If $\alpha > 0.493$], then the perfect equilibrium coalition structure is the grand coalition, $\pi = [\{abcd\}]$.

Proof of the remark:

First notice that the grand coalition dominates two and three players coalition strategies. Hence player a never proposes a two or three member coalition. In the following analysis we do not consider these strategies.

If $\alpha \leq 0.073$, then player a proposes to form the grand coalition and all players receive the same payoff. If she deviates and chooses to form a singleton, then player b forms a singleton also. Player c offers a two-member coalition to player d which is accepted because the payoffs for the two-member in $[\{a, b, cd\}]$ are bigger than those corresponding to the degenerated coalition structure. In the coalition structure in $[\{a, b, cd\}]$ player a receives $U_a^{[\{a,b,cd\}]}$ which is lower than the payoff corresponding to the grand coalition.

If $0.74 \geq \alpha \leq 0.492$, player a chooses to form a singleton. Player b proposes a three-member coalition to players c and d, which is accepted. Notice that Player b never proposes a two-member coalition because the payoffs are lower than in the three-member coalition. If player b deviates to a one-member coalition, then player c and d become singletons, and all players would receive the degenerated coalition structure payoff which is lower than $U_b^{[\{a,bcd\}]}$. Therefore player b would not deviate (and symmetrically, neither c nor d) and, as player a receives her best payoffs, she does not deviate.

If $0.493 \geq \alpha$, then player a chooses the grand coalition again which is accepted by all players. Notice that for these values of α the degenerated structure dominates coalitions with three and two members if a singleton is formed. Therefore, if player a deviates and chooses to play alone, then players b to d do the same and all of them receive the degenerated payoff. Hence player a does not deviates. QED.

The five agent case is analyzed similarly. Table 5 shows the perfect equilibrium coalitions structures for given values of α. There are only four equilibrium coalition structures.

Remark 6: Private property is sustainable as a perfect equilibrium solution for intermediate values of α, (i.e $0.081 \leq \alpha \leq 0.469$). If $\alpha \in (0.469, 0.566]$, then the perfect equilibrium coalition structure is $\pi = [\{ab, cde\}]$. Finally for the rest of values of α the grand coalition is the perfect equilibrium coalition structure.

Proof of the remark: If $\alpha \leq 0.081$, then player a does not have incentives to deviate from the grand coalition. Player a never deviates and chooses a coalition of size two, three, or four; because the grand coalition gives a higher payoff. If player a deviates and chooses a coalition of size one, then player b and d offer a size two coalition which is accepted by player c and e, respectively, because $U_{\mathrm{b}}^{[\{\mathrm{a,bc,de}\}]} > U_{\mathrm{b}}^{[\{\mathrm{a,b,c,de}\}]}$ and $U_{\mathrm{b}}^{[\{\mathrm{a,bc,de}\}]} > U_{\mathrm{b}}^{[\{\mathrm{a,b,c,d,e}\}]}$. Hence player a obtains a payoff of $U_{\mathrm{a}}^{[\{\mathrm{a,bc,de}\}]}$ which is worse than $U_{\mathrm{a}}^{[\{\mathrm{abcde}\}]}$. Player b does not choose a coalition of size one because he knows that the next three players would choose to form a singleton in that case. He never offers a three or four member coalition because this gives lower payoffs than the grand coalition.

If $0.081 < \alpha \leq 0.469$, then player a prefers to be a singleton, and player b proposes a four member coalition which is accepted. If player b deviates and proposes a one member coalition, then the other three player choose a singleton, and all players obtain the degenerated payoff which is worse than $U_{\mathrm{b}}^{[\{\mathrm{a,bcde}\}]}$. If he proposes a two member coalition to player c, then the proposal is rejected because players d and e become a singleton as $U_{\mathrm{d}}^{[\{\mathrm{a,bc,d,e}\}]} > U_{\mathrm{d}}^{[\{\mathrm{a,bc,de}\}]}$, and player b and c receives a lower payoff than $U_{\mathrm{b}}^{[\{\mathrm{a,bcde}\}]}$. He never proposes a three member coalition because his payoff would be worse than the degenerated coalition structure payoffs. Player a does not have incentives to deviate as $U_{\mathrm{a}}^{[\{\mathrm{a,bcde}\}]}$ is the best payoff for her.

If $0.469 < \alpha \leq 0.566$, player a proposes a two member alliance to player

b who accepts, and player c proposes a three member coalition which is accepted by players d and e. If player c deviates and chooses a singleton then players d and e prefer to be singletons, and they obtain $U_c^{[\{ab,c,d,e\}]} < U_c^{[\{ab,cde\}]}$. This player never proposes a two member coalition because it gives a lower payoff as $U_c^{[\{ab,cd,e\}]} < U_c^{[\{ab,cde\}]}$. Player a never chooses a coalition of three, four or five members, because it gives lower payoffs in any structure that it is formed. If she deviates and chooses a singleton, then the remaining players become singletons too, and the degenerated structure is formed which gives worse payoffs than $\pi = [\{ab, cde\}]$. In the case that player a chooses to form a singleton, player b never chooses a two, three or four members coalition because if it is accepted, the remaining players become singletons and then they obtain lower payoffs. The same argument applies for players c and d.

If $0.566 < \alpha \leq 1$ then the grand coalition is formed again. Player a never deviates and chooses a coalition of three or four members because it gives lower payoffs than the grand coalition. If she deviates and chooses to form a singleton, then all the players has incentives to become singletons too, and they obtain the payoff corresponding to the degenerate coalition structure which is lower than the grand coalition payoff. If she deviates and chooses a two member coalition then the offer is rejected, because if it is accepted then the remaining players prefer to be singletons since $U_c^{[\{ab,c,d,e\}]} > U_c^{[\{ab,cde\}]} > U_c^{[\{ab,cd,e\}]}$. Players' strategies are symmetric so the same arguments apply for the rest of the players. QED.

The two remarks show that private property is sustainable as a perfect equilibrium for intermediate values of α. Another conclusion is that if the number of agents increases there are new equilibria that neither imply strictly private property nor common land. In this example, the grand coalition is a perfect equilibrium structure for a greater set of values of α, than in the tree agents example.

Table 5, shows the α values for which the grand coalition is a perfect equilibrium structure in the sequential coalition formation game, as the number of agents increases. We calculate the perfect equilibrium, from $n = 3$ to $n = 8$, as described previously. In the table, we can see that the values of α for which the grand coalition is a perfect equilibrium is non monotonic.

In this section, we conclude that as the number of players increases, private property is sustainable for intermediate values of α and there are new forms of property that are neither strictly private nor common property in the equilibrium[10].

[10] Sánchez-Pagés (2007) concludes that in in the sequential coalition formation game there are a bigger set of equilibrium coalition structures. Our conclusion is similar.

7 Final Remarks

The paper contributes to the literature on the foundations of private property rights by setting a model in which private property rights can emerge as an equilibrium allocation. The paper analyzes conditions such that private property arises as equilibrium in the model by Grossman (2001). For private property to arise as coalitional equilibrium, the resource has to be valuable enough to incentive some agents to do private appropriation efforts on the resource. However, if the resource is too valuable then too many agents will be doing appropriation efforts and too much effort in conflict is wasted in equilibrium. This can make not worthwhile to attain private property rights on the resource for any agent.

Our paper would imply that,as in Demsetz (1967), increases in the value of land lead to the appearance of private property. However, if land value increases too much then the appearance of private property is through too much conflict. The lost of resources can be large enough and could make the appearance of private property not desirable for any of the individuals. The implication is that the appearance of private property, apart from private gains, may also require the existence of institutions that reduce the amount of conflict. One of such institutions can be a superior authority. Others can be family links between the agents that reduce conflict.

8 References

Alchian, A. and Demsetz, H. (1973), "The property rights paradigm," Journal of Economic History 33, 16-27.

Bloch, F., 1996. "Sequential formation of coalitions in games with exter-