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VALUES AND PRICES IN JOINT PRODUCTION: DISCOVERING INNER-PRODUCTIVITIES

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Values and Prices in Joint Production:

Discovering Inner-Unproductivities.

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ABSTRACT

In this paper, we extend the concept of productivity developed in the context of simple production, to the case of joint production. We show, in the second section, that requiring the vector of sectorial net outputs to be semi-positive may still leave three types of "inner-unproductivities". If it is acknowledged that inner-unproductivities are socially undesirable, we wonder if society has signaling devices that could reveal their existence. In the third section, we center our attention on prices of production and labour values, and conclude that while the latter unequivocally detect two of the three types of inefficiencies that may occur, prices of production do not detect any. This explains our usage of the term inner-unproductivities: they characterize the technology independently of distributional aspects, and hence cannot be seen using magnitudes measured with prices. We conclude, contrary to Steedman (1977), that value calculations are more useful than price calculations.

RESUMEN

En este trabajo extendemos el concepto de productividad que ha sido definido, para el caso de producción simple, al caso de producción conjunta. En la segunda parte demostramos que el hecho de requerir que el vector de productos netos sectoriales sea semi-positivo, no elimina tres tipos de "improductividades propias". Si se acepta que las improductividades propias son socialmente indeseables, nos preguntamos si la sociedad tiene manera de detectar su presencia. En la tercera parte consideramos los casos de los precios de producción y de los valores trabajo, y concluimos que, si bien estos últimos detectan dos de los tres tipos de ineficiencias, los precios de producción no detectan ninguno. Esto explica nuestro uso de la expresión improductividades *propias*: no pueden ser percibidas en el ámbito de los precios porque estos últimos dependen tanto de la matriz tecnológica como de las variables de distribución. Concluimos, contrariamente a Steedman (1977), que el cómputo de valores trabajo es más util que el de los precios de producción.

I Introduction

In this paper, we extend the concept of productivity developed in the context of simple production, to the case of joint production. Traditionally, authors have required the vector of sectorial net outputs to be semi-positive. We show, in the second section, that this may still leave three types of "inner-unproductivities". To eliminate them, the appropriate restriction is that the inverse of the matrix of net outputs should be semi-positive.

If it is acknowledged that inner-unproductivities are socially undesirable, we wonder if society has signaling devices that could reveal their existence and their location. In the third section, we center our attention on prices of production and labour values, and conclude that while the latter unequivocally detect two of the three types of inefficiencies that may occur, prices of production will not detect any. This explains our usage of the term inner-unproductivities: they characterize the technology independently of distributional aspects, and hence cannot be seen using magnitudes measured with prices. We also generalize a result due to Stamatis (1983) about negative values being caused by the non-strict dominance of technologies. We conclude, contrary to Steedman (1977), that value calculations are more useful than price calculations.

To illustrate our conclusions, the fourth section presents three numerical examples. The first shows that, if labour values are negative, there are inner-unproductivities in production so that more output can be obtained with the same amount of labour. This anomaly is not detected by the prices of production or the profit rate since both are positive. The second illustrates the fact that, in simple production, negative labour values should not be interpreted as an indication of a flaw in the Labour Theory of Value (LTV) since they can be traced back to a mispecification of the technological matrix. We think that the same argument applies to joint production, so it is incorrect to interpret the negativity of labour values as an indictment of the LTV. The last shows that requiring the input coefficient matrix to be semi-positive is not a necessary condition to obtain a positive vector of prices for all values of the profit rate.

The fifth section presents a short summary of our conclusions.

II Defining Inner-Unproductivities

Consider a joint-production system with m processes and m goods, characterized by the following three coefficient matrices:

- A the $m \times m$ matrix of produced means of production (circulating capital only),
- Z[1] the $m \times 1$ vector of non-produced input,
- and B the $m \times m$ matrix of gross products.

We assume that A and B are defined in physical units, and Z in homogeneous units so that all three are semi-positive. The triad [A, Z, B] defines the Leontieff technological possibility set of this economy.

Call

	y	the $1 \times m$ vector of activity levels[2],
	N	the $m \times m$ matrix of net products = B − A,
	e	a $1 \times m$ vector of 1's
and	n	the $1 \times m$ vector of net products.

As should be obvious, the total amount of net output produced depends on the level at which each process operates:

$$n = y \, . \, N \qquad (1)$$

Call

$$a = y \, . \, A$$

and

$$z = y \, . \, Z$$

the amounts of means of production and non-produced input used to produce the amount

$$b = y \, . \, B$$

of gross output. The triad [a, z, b] defines the state of the economy.

The usual assumption about 'productiveness'[3] is that positive amounts of net output are produced at unit levels:

$$e \, . \, N > 0 \qquad (2)$$

We assume this restriction to be true and prove it does not guarantee that the economy will, in fact, be productive. To do so, take n to be the exogenous (target) variable and inquire about the activity levels which are necessary to obtain that level of n. We assume, throughout the paper, that the system is regular in the sense that N^{-1} exists[4]. Equation (1) can then be solved for y,

$$y = n \cdot N^{-1} \qquad (3)$$

We want to know by how much will the activity levels and resource usage have to vary to sustain a different amount of output. To change the amount of net output produced by dn[5], we need to change activity levels by:

$$dy = dn \cdot N^{-1} \qquad (4)$$

Notice that, while dy may contain negative elements, they are bounded below for each y, since $y \pm dy \geq 0$. Equation (4) can be rewritten as:

$$dy_k = \sum_{j=1}^{m} N^{-1}_{jk} \, dn_j \qquad (5)$$

and hence,

$$N^{-1}_{jk} = \frac{\partial y_k}{\partial n_j} \qquad (6)$$

Equation (6) allows us to interpret N^{-1}_{jk} as the increase in the level of activity of process k per unit increase in the net amount produced of good j (when that is the only change). We can similarly calculate the changes in intermediate inputs and total non-produced input necessary for the change in the target level of net output. Since

$$a = y \cdot A = n \cdot N^{-1} \cdot A$$

and

$$z = y \cdot Z = n \cdot N^{-1} \cdot Z,$$

we get:

$$da = dn \cdot A^{*} \qquad (7)$$

and

$$dz = dn \cdot Z^{*} \tag{8}$$

with A^{*} and Z^{*} 6 defined by:

$$A^{*} = N^{-1} \cdot A \tag{9}$$

and

$$Z^{*} = N^{-1} \cdot Z \tag{10}$$

Notice that equations (7) and (8) are valid for any value of y and that they can be rewriten as:

$$da_k = \sum_{j=1}^{m} A^{*}_{jk} \cdot dn_j \tag{11}$$

and

$$dz = \sum_{j=1}^{m} Z^{*}_j \cdot dn_j \tag{12}$$

Hence,

$$A^{*}_{jk} = \frac{\partial a_k}{\partial n_j} \tag{13}$$

and

$$Z^{*}_j = \frac{\partial z}{\partial n_j} \tag{14}$$

Equations (13) and (14) allow us to interpret A^{*}_{jk} and Z^{*}_{j} as the increases in input k and non-produced input necessary to produce a unit increase in the net amount of good j (again, when that is the only change considered). The elements of N^{-1}, A^{*} and Z^{*} can be seen as the variational production coefficients since they show us the required changes in input usage required to bring about a change in net output. Indeed we have gone from the state [a, z, b] to a new state [a', z', b']; da = a' − a, dl = z' − z, and db = b' − b summarize the changes that were needed for dy to occur.

We will define an input to be productive if and only if no less of it is required to produce more net output. Similarly, a process will be productive iff its activity level has to not decrease when an attempt is made to increase net output.Therefore, if all inputs and all proceses are productive A^{*}, Z^{*} and N^{-1} will all three be semi-positive. On the other hand, we will say that "inner-unproductivities" exist if A^{*}, Z^{*} or N^{-1}have negative elemenents. Hence, there may be three sources of inner-unproductivities in:

-the usage of the non-produced input ($Z^{*}_{j} < 0$ for some j).

-the usage of an intermediate good ($A^{*}_{jk} < 0$ for some j or some k).

-or the activity levels of a process of production ($N^{-1}_{jk} < 0$ for some j or some k).

We now come to the four results of this section.

Proposition I

The economy can produce positive amounts of net outputs and still have inner-unproductivities.

Obviously, N^{-1}, Z^{*}, or A^{*} could have negative elements and equation (2) still be fulfilled.□

Proposition II

If N^{-1} has negative elements, some levels of net output cannot be reached.

Even if $dn > 0$, $dy = dn \,.\, N^{-1}$ may have negative elements. But since negative elements of dy are bounded below for each y, not all values of dn nor of $n+dn$ can be reached.□

Proposition III

Iff there are inner-unproductivities in the system, it is possible to obtain more net output with smaller amounts of some inputs (but possibly more of others) with no change in the technological input mix defined by $[A, Z, B]$.

Under which conditions can we simultaneously have $dn \geq 0$ on the one hand, and $da_k \leq 0$, $dz \leq 0$ or $dy_k \leq 0$ on the other? Given equations (5), (11) and (12), these restrictions are pairwise mutually compatible only when N^{-1}, A^{*}, or Z^{*} have negative elements.□

This result is not obtainable if $N^{-1} \geq 0$. Since the vectors of inputs are not directly comparable, this theorem would not be very interesting if it were not for the following corollary. Suppose that labour is the only non-produced input, call L the vector of direct labour coefficients and define $L^{*} = N^{-1}.\, L$

Corollary

If L^{*} has some negative components, more net output can be obtained with less total employment by redistributing the work force amongst processes, with no change in the technological input mix.

This is a direct application of the previous theorem to the case of labour which is a non-produced input of production. □

Definition

We will say that an economy is totally productive if it is not saddled by any of the previously mentioned unproductivites[7].

Proposition IV

An economy is "totally productive" iff $N^{-1} \geq 0$.

The proof of propositon IV is trivial once we remember definitions (9) and (10), and the fact that both A and Z are semi-positive.□

Proposition V

If $A^* \geq 0$ and $L^* \geq 0$, the vector of prices of production p, the profit rate r and the wage rate w which satisfy the equation of prices of production:

$$B \cdot p = (1 + r)(A \cdot p + w \cdot L) \qquad (15)$$

are all semi-positive within the economically meaningful interval $0 \leq r \leq R$, where $1/R$ is the dominant eigenvalue of A^*.

Multiplying (15) to the left by N^{-1} and solving for p/w, we get:

$$p/w = (1 + r).(I - r \cdot A^*)^{-1} \cdot L^*$$

If A^* is semi-positive, there is a well known theorem, a consequence of that of Perron Frobenius, which asserts that the matrix $(I - r \cdot A^*)^{-1}$ exists and is semi-positive within the interval $0 \leq r \leq R$[8]. Since $L^* \geq 0$, it is clear that p/w must also be semi-positive within the interval considered. Moreover, since for $w = 0$, the price equation (15) becomes $p = r \cdot A^*$, with $r = R$, we see that in this case p is the (semi-positive) dominant eigenvector of A^*. □

Corollary

If there are no inner-unproductivities in production, there exists a set of prices (including wages and rates of profit) which assures the reproduction of the system.

This is a direct extension of Proposition V.□

III Detecting Inner-Unproductivities.

As we showed in the previous section, inner-unproductivities in technology provoke an allocation of resources which is not efficient for all inputs. We inquire, in this section, into which (if any) societal mechanisms can correct or at least reveal the presence of those inner-unproductivities.

In a descentralized economy, individual productive agents try to maximize their profits using the information contained in prices. As proved by Manara (1977), there are five conditions for the vector of prices p to be strictly positive[9]. None of these conditions implies, however, that N^{-1} will be semi-positive. Since prices can be positive even if there are inner-unproductivities in the system, they can hardly function as an indicator of their presence.

L^{*} -the vector of marxist values as defined by Morishima (1973)- is another candidate. If the value of all commodities is positive ($L^{*} \geq 0$) there will be no inner-unproductivities in the usage of labour. But even in that case, nothing can be said about the usage of raw materials and the operation of processes, since A^{*} and N^{-1} could still have negative elements.

This sheds a different light on Steedman's(1977) discussion. Negative values are not an anomaly that weakens the Labour Theory of Value (LTV); on the contrary, they reveal the presence of inner-unproductivities in the usage of labour. Production prices, the profit rate and the wage rate could be positive because they are defined at a different level of abstraction where the information about technical characteristics may have been distorted. Indeed as can be infered from the previous discusion, the two measures rely on different information sets. For value computations we only need the technological characteristics of the system: [A, Z, B]. To calculate prices of production, however, we also need distributional variables which could obscure the previous information set.

Labour values cannot detect all three types of inner-unproductivities, but they can help a social planer pinpoint their existence in both the usage of labour, and in the levels of activity. It is doubtful that the market could provide corrective measures when some labour values are negative, because, in general, prices and profit rates will be positive[10] so there would be no incentive for capitalists to change their behavior. The market would be in a trap and its atomization would prevent it from eliminating inefficiencies.

The restriction that N^{-1} be semi-positive is weaker than the one usually invoqued with the help of the Perron-Frobenius Theorems to guarantee that the technology is productive. Indeed in the case of simple production -when B is a diagonal matrix- A could have negative elements and A^{*} still be semi-positive if N^{-1} is. In that case, as was shown in section II, values (and prices as will be illustrated in section IV with a numerical example), will be positive since there are no inner-unproductivities. Notice, furthermore, that condition (2) is neither necessary nor sufficient since there is no relationship between $e . N > 0$ and N^{-1} being semi-positive does not imply that

To offer a more intuitive explanation for the meaning of inner-unproductivities, let us define a composite process as a linear combination of the "original"[11] ones, and characterize it by its vector of activity levels y. Obviously, the amounts of net output and employment it entails will be given by $n = y . N$ and $l = y . L$. Let p and q be two such composite processes. Without loss of generality, we can assume that total labour usage is the same in both processes (adjusting their scale of operation if necessary) and set $l_p = l_q$.

Definition

We shall say that a process p dominates (in labour productivity) another process q iff:

$$n_p \geq n_q \text{[12]} \quad \text{for } l_p = l_q \qquad (16)$$

Proposition VI

The labour value of a commodity is non-positive iff it is possible to find, within the economy, at least one process (original or composite) which dominates another (original or composite).

i) If L^* has a non-positive element, then according to (8), it is possible to find a $dn \geq 0$ such that $dl = 0$. We can choose processes p and q to be $y_p = y_q + dy$ with any $y_q \geq 0$, for which $y_q \geq -dy$; thus $y_p \geq 0$. Then, since from (4) we have $dy = dn \cdot N^{-1}$:

$$\begin{aligned} n_p &= y_p \cdot N \\ &= (y_q + dy) \cdot N \\ &= n_q + dn \\ &\geq n_q \end{aligned}$$ □

ii) If (15) is fulfilled, then

$$dl = l_p - l_q = 0$$

and $$dn = n_p - n_q \geq 0.$$

Therefore, according to (8), L^* must have non-positive elements. □

We consider this to be a generalization of Stamatis's (1983) result since if a process dominates another, their "partial productivities of labour" will not be equal in the positive orthon.

IV A Numerical Illustration

In this section, we illustrate some of the conclusions of the previous two sections with three numerical exercises.

First we use one of Steedman's (1977) numerical examples to show that if a technology is productive in the usual sense and labour values are negative, this is not be interpreted as an abnormality of the labour value calculation, but as an indication that there are inner-unproductivities in the system, and that hence more output can be produced with less labour.

As presented in his Chapter 11 (p.151), we consider a technology with the following characteristics:

$$A = \begin{vmatrix} 5 & 0 \\ 0 & 10 \end{vmatrix} \quad \mathbb{L} = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \quad B = \begin{vmatrix} 6 & 1 \\ 3 & 12 \end{vmatrix}$$

so that,

$$N = \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$$

Obviously this system satisfies condition (2):

$$e . N > 0$$

because

$$[\,4 \;\; 3\,] > [\,0 \;\; 0\,]$$

The system is viable fo a real wage bundle (w) equal to:

$$\hat{w} = [\,1/2 \quad 5/6\,]$$

since we can solve for the price vector (p) and the profit rate (r):

$$p^T = [\,1/3 \quad 1\,] > [\,0 \quad 0\,]$$

$$r = 20\% > 0$$

Values on the other hand will equal:

$$\mathbb{L}^{*T} = [\,-1 \quad 2\,]$$

so " there appears to be no good reason for not abandoning all reference to such magnitude" [13].

$L_1 < 0$ indicates that there are inner-unproductivities in the system undetected by the variables in the price sphere. This diagnosis is confirmed by inspection of N^{-1}:

$$N^{-1} = \begin{vmatrix} -2 & 1 \\ 3 & -1 \end{vmatrix}$$

To show that this production scheme is not efficiently using labour, "suppose that, in a certain period, 6 units of labour are employed, 5 operating in the first process and 1 operating the second"[14]. For

$$y_0 = [5 \quad 1]$$

$$\mathbf{n}_0 = \mathbf{y} . N = [8 \quad 7]$$

If, on the contrary, 5 units of labour are used in the second process and 1 in the first, we will have

$$y_1 = [1 \quad 5]$$

$$\mathbf{n}_1 = [16 \quad 11] > \mathbf{n}_0$$

N^{-1}, A^* and $\mathbf{L}^*$ still have negative elements so we may go to:

$$y_2 = [0 \quad 6]$$

$$\mathbf{n}_2 = [18 \quad 12] > \mathbf{n}_1 > \mathbf{n}_0$$

Why would an economic system use its ressources to produce net output $\mathbf{n}_0$ when it could reorganize them and obtain $\mathbf{n}_2 > \mathbf{n}_0$? When the system is in the state described by y_0, there are no market forces at work that will move it towards y_2. The price system does not offer such a mechanism since $p > 0$, $\omega > 0$ and $r > 0$, so that no individual capitalist has an incentive to change its behavior.

The social planner who would try to induce the state described by y_2

may face a problem because the original stocks of raw materials are inadequate to sustain those production levels. Indeed, y_0 requires 25 units of good 1 and 10 units of good 2 to get started, while y_1 requires 5 units of 1 and 50 units of 2, and y_2 requires 0 units of 1 and 60 units of 2. The problem of "proportionality", as it is known in the literature[15] is outside of the purview of this paper. We assume that initial stocks are sufficient and exclude a discussion of the trajectory from one scheme to another. Notice, however, that if an economy possesses the required initial stocks, it is self-sustaining.

II

In the second and third examples, we use a diagonal B matrix (simple production) because, in that case, the characteristics of the input matrix are easily understandable. With the second we try to show that if labour values are negative it is <u>because</u> the tecnological matrix violates some obvious productivity conditions. It has been shown in the literature that a sufficient condition for a system to be "viable" (totally productive and with positive prices and profit rate) is that the A matrix be semi-positive and that condition (2) be fulfilled. We show in the third example that, contrary to received wisdom, this is not a necessary condition but that N^{-1} being semi-positive is.

Consider a technology described by:

$$A = \begin{vmatrix} 60 & -10 \\ -5 & 20 \end{vmatrix}, \quad L = \begin{vmatrix} 1 & 1 \\ & 1 \end{vmatrix}, \quad B = \begin{vmatrix} 69 & 0 \\ 0 & 23 \end{vmatrix}$$

As can easily be checked:

$$e.N = [\ 14 \ \ 13\ ] > [\ 0 \ \ 0\ ]$$

$$N^{-1} = \frac{1}{23} \begin{vmatrix} -3 & 10 \\ 5 & -9 \end{vmatrix}$$

$$A^{*} = \begin{vmatrix} -10 & 10 \\ 15 & -10 \end{vmatrix}$$

$$L^* = \begin{vmatrix} -1 \\ 2 \end{vmatrix}$$

If the nominal wage rate is

$$w.p^T = .1010$$

$$p^T = [.3031 \quad .4041]$$

and

$$r = 40\%$$

In this case, prices are positive and so is the rate of profit, but labour values are negative because the N^{-1} matrix has negative elements. This "pathology" (negative values and $p_1 \leq 0$, if $0 < r \leq .1016$) should be attributed to a non-productive technology; it can hardly be argued that it shows that labour values should be abandonned in favor of production prices. This example illustrates the fact that the negative elements of A^*, L^* and N^{-1} can be explained by an ill-specified technology not by a flaw in the Labour Theory of Value (LTV). The same should be done in the case of joint production by calculating those same matrices and checking that they do not contain negative elements before indicting the LTV.

III

Lest some inatentive reader think that requiring A to be semi-positive is a necessary condition for the technology to be viable, we offer the following example. Let

$$A = \begin{vmatrix} 22 & -5 \\ -5 & 21 \end{vmatrix}, \quad L = \begin{vmatrix} 12 \\ -1 \end{vmatrix}, \quad B = \begin{vmatrix} 19 & 0 \\ 0 & 19 \end{vmatrix}$$

This example is highly suspect since both L and A have negative elements. It can easily be checked, however, that N^{-1}, A^* and L^* are all positive, so that the price vector is positive for all values of the profit rate. Indeed,

$$N^{-1} = \frac{1}{19}\begin{vmatrix} 2 & 5 \\ 5 & 3 \end{vmatrix}$$

$$A^{*} = \begin{vmatrix} 1 & 5 \\ 5 & 2 \end{vmatrix}$$

$$L^{*} = \begin{vmatrix} 1 \\ 3 \end{vmatrix}$$

$$0 \leq r \leq .15326$$

$$1 \leq p_1 \leq 1.9428$$

$$3 \leq p_2 \leq 2.1468$$

V Conclusions

In this paper, we have offered a measure of productivity for the case of joint production. Authors have traditionally required the vector of sectorial net outputs to be semi-positive. We have required that restriction to apply to the inverse of the matrix of net outputs instead; that condition is sufficient to guarantee that the technology is productive.

We have shown that prices of production cannot function as a signaling device for the existence of such inner-unproductivities because they mix technological and distributional parameters. Labour values, on the other hand, detect two of the three types of inefficiencies. We concluded, contrary to Steedman (1977), that value calculations are much more useful than price calculations since they can detect a mispecifications of the technological matrix which the latter cannot. Hence, we argued it is incorrect to interpret the negativity of labour values as an indictment of the Labour Theory of Value when it is one of its strengths.

Footnotes.

(1) We use bold characters to denote vectors, upper case letters for matrices and lower case letters for scalars. Indexed letters will represent the elements of that array.

(2) Obviously $y \geq 0$.

(3) We use quotation marks around productiveness because we will define that notion carefully latter on.

(4) If N^{-1} did not exist, some levels of net output would not be obtainable. For those that could be obtained, there would be an infinite number of y's that would produce them.

(5) Negative elements of dn indicate that less of that net output is desired.

(6) $dN^{-1} = dA = dA^* = dZ^* = 0$ because we assume the technological possibility set to be given and invariant.

(7) Hence we have the following relationships:

If there are negative elements in:	There will be inner-unproductivities in Means of Production	Non-Produced Input	Processes
A^*	Yes	?	Yes
Z^*	?	Yes	Yes
A^* and Z^*	Yes	Yes	Yes
N^{-1}	?	?	Yes

and

Semi-Positive Array	INNER-UNPRODUCTIVITIES IN Means of Production	Non-Produced Input	Processes
A^{*}	No	?	?
Z^{*}	?	No	?
A^{*} and Z^{*}	No	No	?
N^{-1}	No	No	No

(8) See Pasinetti (1977), Theorem 5A p.276.

(9) See Manara in Pasinetti (1980). Let us cite them briefly:

- $-e\,.\,N > 0$ — equation 1.13
- –There exists a positive $p^{\dagger}$ such that $N\,.\,p^{\dagger} > 0$ — equation 1.14
- $-\det[\,N\,] \neq 0$ — equation 1.25
- –if r is in $\mathfrak{J}$, L is in $V'(r)$ — equation 1.38
- –There exists a $r^{\dagger}>0$ such that $\det[\,B-A(1+r^{\dagger})\,]=0$ — equation 1.51

for r is the system's profit rate, and $\mathfrak{J}$ the interval where:

$-r \geq 0$

$-\det[\,B - A(1+r)] \neq 0$

–neither $\mathfrak{U}(r)$, nor $\mathfrak{V}(r)$ are empty

$\mathfrak{U}(r)$ is the set of non-negative column vectors belonging to X ($X=\{x|x \geq 0\}$), and such that, for every vector x of $\mathfrak{U}(r)$ the following relationship holds:

$$[\,B - A(1+r)\,]\,x \geq 0$$

Similarly, $\mathfrak{V}(r)$ is the set of non-negative vectors belonging to P ($P=\{y|y \geq 0\}$), and such that, for every vector y of $\mathfrak{V}(r)$ the following relationship holds:

$$y\,[\,B - A\,(1+r)\,] \geq 0$$

(10) No process of reproduction can occur if one of the prices of

production is not positive.

(11) The original ones are characterized by the triad [A, **L**, B].

(12) Which is to say:

$n_{pk} > n_{qk}$ for all k, and the unequality holds strictly for, at least one k.

(13) See Steedman (1977), p.162.

(14) See Steedman (1977), p.152.

(15) See Bhaduri and Robinson (1980).

(16) This proposition can easily be shown to be true but its proof is excluded for the sake of brevity.

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